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Mathematical Modeling of Sampling, Quantization, and Coding in Sigma Delta Converter using Matlab

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ABSTRACT

The received analog signal must be digitized before the digital signal processing can demodulate it. Sampling, quantization, and coding are the separate stages for the analog-to-digital adaptation procedure. The procedure of adapting an unceasing time-domain signal into a separate time-domain signal is called sampling. While, the procedure of adapting a separate-time, continuous-valued signal into a discrete-time, discrete-valued signal is known as quantization. Thus, quantization error is the mismatch between the unquantized sample and the quantized sample. The method of demonstrating the quantized samples in binary form is known as coding. This investigation utilized Matlab® program to recommend a proper scheme for a wireless-call button network of input signal, normalized frequency, and over-sampling ratio against signal-to-quantization noise ratio. Two vital characteristics of this wireless network design are cost-effective and low-power utilization. This investigation, through reducing the in-band quantization error, also studied how oversampling can enhance the accomplishment of an analog-to-digital adapter.

Index Terms: Analog-to-digital Adapter, Coding, Matlab, Quantization Error, Wireless Network

1. INTRODUCTION

It is not easy to decide precisely when and how the first data converter was established. The most primitive documented binary analog-to-digital adaptor recognized is not electronic at all but hydraulic. To the best of our knowledge, the optimum historical review regarding the analog-to-digital adapters, in general, can be found in the study of Kester *et al.* [1].

The analog domain is unceasing with both time and signal magnitude, while the digital domain is independent on both time and magnitude. A single binary value signifies a variety of analog values in the quantization band nearby its code center point. Analog values that are not precisely at the code center point have an allied amount of quantization error [2].

It can be stated that sigma-delta [3] analog-to-digital adapter is a most common approach of over-sampling analog-to-digital adapter. The map processor of a sigma-delta analog-to-digital adapter is displayed in Fig. 1 [4].

The sigma-delta analog-to-digital adapter can be divided into two lumps, the quantizing and the decimating parts. Essentially, decimation is the act of decreasing the data rate down from the over-sampling rate without losing information. The quantizing part contains the analog integrator, the 1-bit analog-to-digital adapter, and the 1-bit digital-to-analog adapter [5]. The task of the quantizing part is to adapt the data in the analog input into digital shape. The input-output relationship of the sigma-delta quantizer is

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non-linear; nevertheless, the capacity of frequency depression for the analog input, explicitly $x(t)$, might be retrieved from the quantizer yield, namely, $y[n]$ as shown in Fig. 1. $y[n]$ is a restricted order with sample values equivalent to −1 or +1. $y[n]$ may just stay restricted if the output collector, $w[n]$, is bounded similarly. As a result, the typical value of $y[n]$ is needed to be equivalent to the mean value of the input $x(t)$. Accordingly, the authors have been capable of solving $w[n]$ to obtain the constant of $x(t)$ [6].

This investigation suggests a 1-bit analog-to-digital converter which can be utilized as an alternative of a more costly multi-bit analog-to-digital converter. This can be done through studying two divergent procedures that permit a 1-bit analog-to-digital converter to attain the enactment of a multi-bit analog-to-digital converter. The authors will also investigate the superiority and drawbacks of both these procedures. Relying on this exploration, one of the two procedures is selected for our data radios.

2. THEORY

$x_a(t)$ is an analog signal ant it behaves similar to the input to an analog-to-digital adapter. Equations 1 and 2 describe $x_a(t)$ and the average power in $x_a(t)$, correspondingly [7].

$$x_a(t) = \frac{A}{2}\cos\omega t \tag{1}$$

$$\sigma_x = \frac{1}{T}\int_0^T [x_a(t)]^2\, dt = \frac{A^2}{8} \tag{2}$$

To prototype this analog signal, assume a b-bit analog-to-digital adapter. Conditionally, if the analog signal possesses

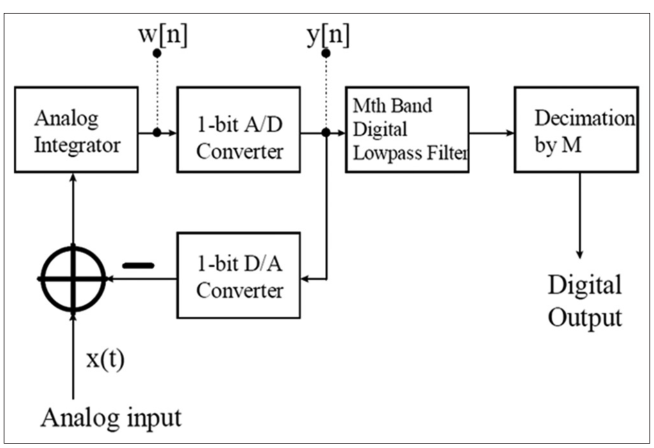

Fig. 1. Sigma-delta analog-to-digital converter

peak-to-peak amplitude of A, and subsequently, the minimum potential step, ΔV, by means of b bits is given by Equation 3:

$$\Delta V = \frac{A}{(2^b - 1)} \cong \frac{A}{(2^b)} \tag{3}$$

Quantization noise, or quantization error, is a unique restricting parameter for the effective range of an analog-to-digital adaptor [8]. This error is essentially the "round-off" error that happens when an analog signal is quantized.

A quantized signal may be different as of the analog-signal by just about $\pm(\Delta V/2)$. Supposing a quantization error is equivalently distributed ranging from $-\Delta V/2$ to $\Delta V/2$, then the root mean square of the quantization noise power, σ_e, is identified by Equation 4 [9].

$$\sigma_e^{\,2} = \frac{(\Delta V)^2}{12} = \frac{A^2}{\left(2^{2b}\right)12} \tag{4}$$

Using Equations 2 and 4, the signal-to-quantization noise ratio (SQNR) for our b-bit analog-to-digital converter might be assessed [10] as follows:

$$SQNR = \frac{\sigma_x}{\sigma_e} = \frac{3}{2}2^{2b} \tag{5}$$

The SQNR in decibels is assumed through Equation 6 [11].

$$SQNR(\mathrm{dB}) = 10\ log_{10} SQNR = 1.76 + 6.02b \tag{6}$$

Equation 6 is an illustration of the SQNR of an analog-to-digital adaptor which rises through almost 6 dB per every single added bit.

One can realize that the assessed signal through the analog-to-digital adaptor is at baseband. Thus, a uniform spectrum in the frequency ranges from 0 to $F_s/2$ is the characteristic of the root mean square quantization noise power, σ_e. The noise power for each unit of bandwidth can be assumed using Equation 7.

$$N_o = \frac{\sigma_e}{(F_s/2)} = \frac{A^2}{\left(2^{2b}\right)6F_s} \tag{7}$$

The Nyquist frequency, which termed after Harry Nyquist, is basically twice the input signal bandwidth F_m. Recalling that F_m for a baseband signal expands from 0 to F_m [12]. The entire quantization noise power in the concerned band or the in-band noise is specified through Equation 8.

$$N_o(F_m) = \frac{A^2}{(2^{2b})6F_s}(F_m) = \frac{A^2}{(2^{2b})6}\left(\frac{F_m}{F_s}\right) \quad (8)$$

Equation 8 might be examined to classify which limits disturb the in-band quantization error in analog-to-digital adaptor. Where A is the amplitude of the signal and F_m is half Nyquist frequency, both of them are relying on signal, while the analog-to-digital adaptor has no dominance above these. Nevertheless, the available analog-to-digital bits number, b, and the specimen frequency, F_s, are organized through the analog-to-digital adaptor scheme [13].

The act of sampling the input signal at frequencies considerably higher than the Nyquist frequency is called oversampling.

By means of Equation 8, a correlation might be resulting for the over-sampling segment, $M = F_s/2F_m$, like that the two analog-to-digital adaptors offer an identical in-band error power [14]. After allowing $F_s = 2F_m$, Equation 8 becomes:

$$\frac{A^2}{(2^{2b})6}\left(\frac{F_m}{F_s}\right) = \frac{A^2}{(2^{2\beta})12} \quad (9)$$

By means of Equation 9, one can acquire Equation 10.

$$(\beta - b) = +\frac{1}{2}\log_2(M) \quad (10)$$

($\beta - b$) means the additional determination bits which are in consequence gained out of a b-bit adaptor by means of oversampling. The above equation, also, indicates that each duplication of the over-sampling proportion rises the actual bits at the Nyquist frequency by 0.5 [15].

$$H(z) = \frac{1 - z^{-M}}{1 - z^{-1}} \quad (11)$$

Mitra [16] relates the enactment of a sigma-delta analog-to-digital adaptor through that of a linear over-sampling analog-to-digital adaptor. The enhancement in enactment achieved by means of a sigma-delta analog-to-digital adaptor is illustrated in Equation 10 [17].

$$Enhancement(M) = -\ 5.1718 + 20\log_{10}(M) \quad (12)$$

The power spectral density, $S_x(f)$, is the strength of the variations as a function of frequency. For an arbitrary time signal $x_a(t)$, the power spectral density can be given by Equation 13.

$$S_x(f) = \lim_{T\to\infty} E\left\{\frac{1}{2T}\left|\int_{-T}^{T} x(t)e^{-i\omega t}dt\right|^2\right\} \quad (13)$$

Power spectral density computation can be made straightforwardly through the fast Fourier transform method.

3. RESULTS AND DISCUSSIONS

A. Power Spectral Density

A first-order integrator which might be demonstrated as a collector has been utilized by the straightforward sigma-delta quantizer [18]. The Matlab® program imitates the occupied first-order sigma-delta adaptor. The power spectral density, Equation 13, of the stimulus signal can be illustrated in Fig. 2. In addition, the stimulus signal has been over sampled at a level of 50 times Nyquist. It can be notice in Fig. 2 indicated that the normalized frequency is schemed against power spectral density possessing a range from 0 to 1 knowing that 1 indicating 50 Nyquist frequency [19].

Analogous tendency has been obtained by Belitski *et al.* [20] which indicates that the proposed model is appropriate for sampling, quantization, and coding in sigma-delta converter.

$y[n]$ is the digital signal characterized by means of 1-bit. The power spectral density of $y[n]$ is plotted in Fig. 3 against the normalized frequency.

Fig. 3 shows the noise forming aptitudes of the sigma-delta analog-to-digital converter. As stated previously, a straightforward over-sampling analog-to-digital converter is capable to diffuse the overall quantization error power

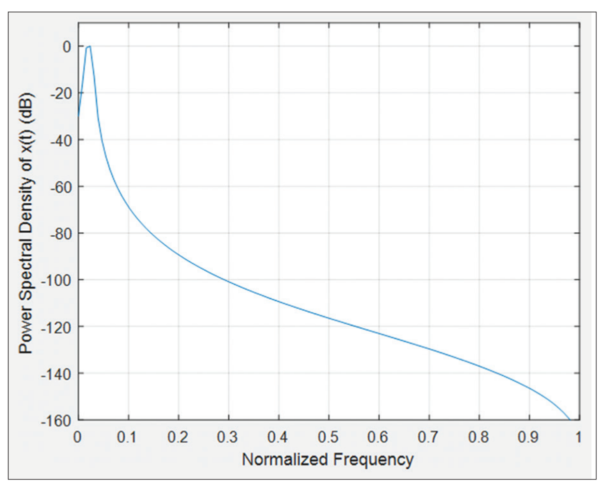

Fig. 2. Power spectral density of input signal after oversampling

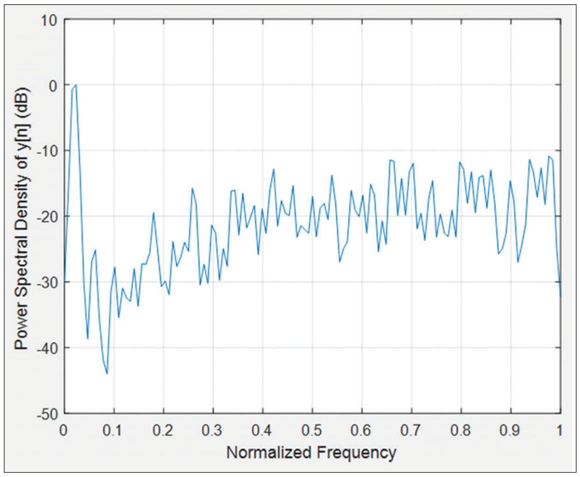

Fig. 3. Power spectral density of $y[n]$ versus normalized frequency

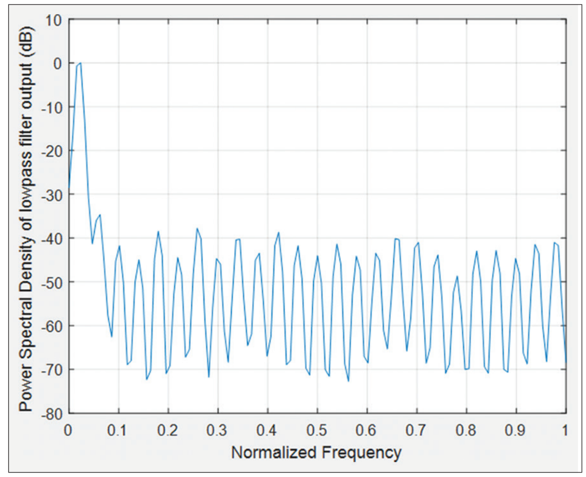

Fig. 4. Power spectral density versus normalized frequency of low-pass filter output

above a longer band, thus reducing the in-band error power. Alternatively, Overney *et al.* [21] utilize Josephson voltage standard to achieve logical description of higher level resolution analog-to-digital adaptor. Their method might be used in many metrological applications for different analog-to-digital adaptors with frequencies up to a few kHz. In addition, Posselt *et al.* [22] utilized a reconfigurable analog-to-digital converter which was suggested with aptitudes of digitalizing completely related wireless facilities for vehicular usage with frequency ranging from of 600 MHz to 6 GHz.

In addition, sigma-delta adaptors are normally capable to achieve error modeling just like that the error power is centralized in upper frequencies [23]. Fig. 3 demonstrated that the bottom error is significantly sophisticated at upper frequencies and rather beneath the concern band. Furthermore, the signal $y[n]$ is the quantizer yield and is low pass clean by means of a M^{th} band low-pass filter, where $M = 50$ is the over-sampling ratio. The transmission function, H(z), of the M^{th} band low-pass filter is known from Equation 10. The power spectral density of the clarified yield is shown in Fig. 4.

Fig. 5 displays the power spectral density of the real analog-to-digital converter production. It can be obvious that, at this point, the signal has been downsampled to the Nyquist frequency once again.

Taking into consideration that for the sigma-delta adaptor, exhibited in the Matlab® program, the utilized over-sampling ratio, M, was 50. It can be realized that, through Equation 12, a sigma-delta adaptor might offer an enhancement of about 29 dB once likened through a straight over-sampling adaptor that similarly works at 50 times Nyquist frequency.

B. SQNR

The SQNR generated using a 1-bit sigma-delta adaptor can be linked through Fig. 6 with the SQNR of a straight over-sampling analog-to-digital converter at numerous over-sampling ratios.

Moreover, it can be noticed that from Fig. 6 when the over-sampling ratio is 15, then the SQNR is just round 20 dB. Accordingly, the Nyquist frequency for an analog signal modulator utilizing a binary phase shift keying and conveying 80 Kbps of data will be 160 KHz. On the other hand, oversampling in 15 would necessitate sampling at 2.4 MHz. Otherwise stated, considering the present digital signal processing, which can treat samples at an order of nearly 2.4 MHz, one might be capable to carry out a regulate over-sampling analog-to-digital adaptor. In similar work, Brooks *et al.* [24] stated that their analog-to-digital converter works at a 20 MHz and it attains a signal-to-noise ratio of about 90 dB exceeding a 1.25 MHz signal bandwidth. Fig. 7 displays a scheme of the extra accuracy bits achieved against the over-sampling percentage.

Fig. 7 indicates that for a 10 dB corresponded input, once the signal-to-quantization error ratio of the analog-to-digital adaptor is 20 dB, and the digitized output possesses a signal-to-noise ratio of approximately 9.5 dB. This, perhaps, shows that using a SQNR of 20 dB, the digitizing process solely increases the bottom error by 0.5 dB. The bottom error is increased by even below 0.5 dB if the equivalent contribution's signal-to-noise ratio is less than 10 dB. Accordingly, the first and the last goal behind this study was directing over-sampling analog-to-digital adaptor to possess

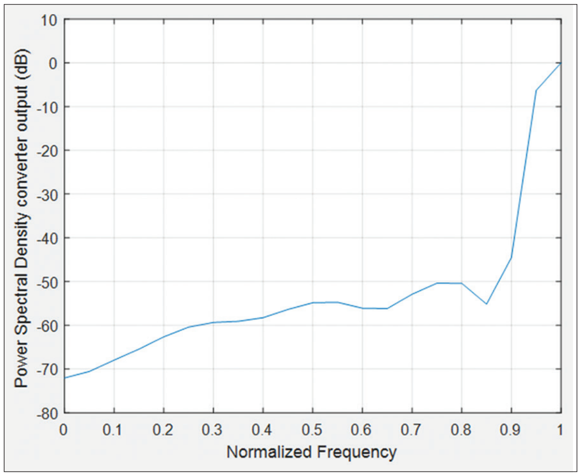

Fig. 5. Power spectral density of analog-to-digital converter output signal

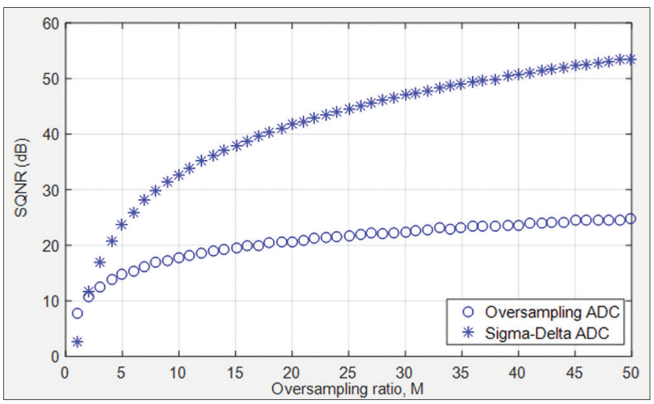

Fig. 6. Over-sampling ratio versus signal-to-quantization noise ratio

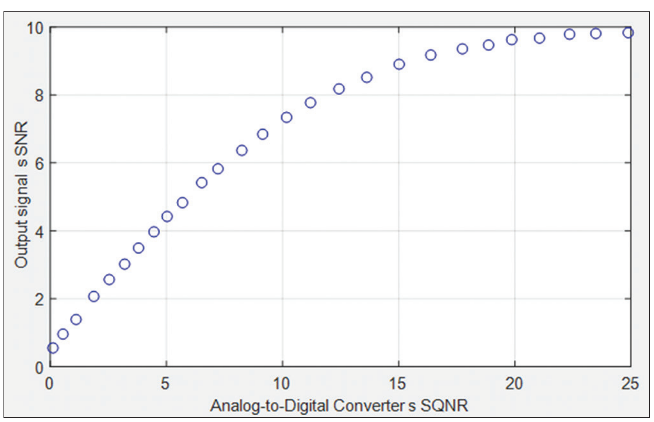

Fig. 7. Analog-to-digital adaptor yield's signal-to-noise ratio (SNR) (with contribution's SNR = 10 dB)

a SQNR of 20 dB or less. Fujimori *et al.* [25], alternatively, stated that in their study, no signal-to-noise ratio decay caused by numerical swapping error has been inspected, showing the strength of the error combination avert methods in combination with the low master clock of 20 MHz.

4. CONCUSSION

Running the analog-to-digital adaptor beyond the input signal's Nyquist frequency enhances the improvement of a low accuracy analog-to-digital adaptor. This is the evidence behindhand the operating of continuous over-sampling analog-to-digital converters. The quantization noise addition to the analog-to-digital adaptation procedure supplementary enhances enactment. Sigma-delta analog-to-digital adaptors apply both noise affecting and oversampling. Sigma-delta analog-to-digital converters propose significantly superior enactments than uninterrupted over-sampling adaptors. Nevertheless, it has been recommended that a straightforward sampling adaptor be utilized due to the difficulty of a sigma-delta analog-to-digital adaptor. Similarly, it has been observed that a straight over-sampling analog-to-digital converter can be utilized without any kind of signal humiliation.

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