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Stochastic Allocation of Transmit Power for Realistic Wireless Channel Models

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Abstract: Control of transmitted power is crucial for the successful operation of multi-user wireless channels communications. There are practical situations in which the transmitted power cannot be adjusted by feedback information; hence, only forward transmit power allocation can be applied, especially in situations where a feedback channel is not available in a wireless network or when wireless nodes are only transmit types. Conventionally, transmitted power can be fixed. Higher gain may be observed if the sensors' transmitted power is randomized. In this work, random power allocation for a Nakagami-m distributed wireless channel model was investigated, and a number of random distributions were evaluated theoretically and tested by simulations. The outage probability was evaluated theoretically and validated by Monte Carlo simulations.

Keywords: Random power control, Nakagami channel, Weibull distribution.

الضبط العشوائي لقدرة الإشارة المرسلة في نماذج قنوات الاتصال اللاسلكية الواقعية

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الملخص: التحكم في قدرة الإشارة المرسلة يعد أمرا في غاية الأهمية لإنجاح عملية الاتصالات اللاسلكية متعددة الأطراف. هناك حالات واقعية لا يمكن فيها ضبط مستوى قدرة الإشارة باستخدام التغذية الراجعة للبيانات، حينها يجب ضبط المستوى المناسب مسبقا. على سبيل المثال قد لا تكون هناك قناة اتصال للتغذية الراجعة، أو أن يكون مسار الاتصال في اتجاه واحد فقط (إرسال دون استقبال). من المتعارف عليه أن مستوى القدرة يمكن ضبطه مسبقا ليكون ثابتاً. ومن الملحوظ أن عشوائية مستوى القدرة يؤدي عادة إلى رفع فعالية الاتصال. تبحث هذه الدراسة أثر الضبط العشوائي لمستوى قدرة الإشارة باستخدام نموذج نكاجامي- م للتوزيع العشوائي للإشارات المرسلة عبر قنوات الاتصال اللاسلكية، كما تختبر الدراسة عددا من التوزيعات العشوائية من الناحية النظرية وتقوم بتقييمها عن طريق المحاكاة البرمجية. تم تقييم احتمالية انقطاع الإشارة نظريا والتصديق عليها من قبل محاكاة مونت كارلو.

الكلمات المفتاحية: التحكم في قدرة الإشارة عشوائياً، قناة نكاجامي، توزيع ويبُل.

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1. Introduction

Wireless sensor networks are ideally suited for applications ranging from scientific data gathering, and environmental hazard and battlefield monitoring because of their rapid and inexpensive deployment. Many sensor network applications have been investigated in the literature. For example, one can find applications to target tracking, environmental monitoring, and the detection and monitoring of car theft (Brooks, Ramanathan, Sayeed 2003; Bonnet, Gehrke, Seshadri 2000; Pottie and Kaiser 2000). However, sensor networks have varying characteristics and limitations, and sensors are usually deployed in the region of interest without intensive preplanning.

One positive characteristic is that the required quality of service (QoS) of sensors is usually much more flexible than mobile networks. The sensors send their data to access points in single or multi-hop wireless channels. The required data rate depends on the application, but generally it is less than the cellular mobile requirements. Further, for intensive sensor deployment, sensed data can be highly redundant which can be exploited in post processing. The maximum allowable delay depends also on the application. For example, if the sensors are part of a closed loop control system, then the maximum delay is critical for the system stability and robustness.

Sensors are energy-constrained devices as it is not practical to periodically recharge or replace the sensors' batteries, especially for large sensor networks where hundreds or even thousands of sensors may be deployed in an area of interest. When in receive mode, practical sensors can consume power levels comparable to the consumption rate experienced during transmit mode. Therefore, deaf sensors can prolong network life significantly. Moreover, the sensor nodes should be efficient in terms of transmitted power and data coding in order to reduce any unnecessary extra power transmission and improve the overall system performance.

Because of the characteristics of sensor networks, sensors should be designed to be cheap, simple, and highly energy efficient. To keep the total cost and complexity of sensor networks at reasonable levels, it is preferable to design the sensors as broadcasters. In any multiuser wireless network with non-perfectly orthogonal multiple access (such as code division multiple access [CDMA], time division multiple access [TDMA], or orthogonal frequency-division multiple access [OFDMA] with frequency reuse, etc.), power control is essential to limit interference, improve network capacity, enhance performance, improve energy efficiency, and prolong the battery life of devices. Prolonging the lifetime of batteries is a critical objective in wireless sensors that have batteries that cannot be recharged or replaced, which is the case of many applications with outdoor sensor network deployment. The optimum transmit power value should be the minimum power needed to achieve a transmit-receive link objective (such as signal-to-noise ratio [SNR], throughput, bit error rate [BER], etc.). Hence, the transmitter should have enough information about the channel.

This information about the channel is called channel state information (CSI), which is measured, or estimated, at the receiver side with the assistance, for example, of a pilot signal. Nevertheless, the receiver should send full or partial information to the transmitter using a feedback channel. If the coherence time of the channel is longer than the feedback delay, then this CSI could be used to optimize the transmit power or other resources such as the modulation level. This procedure is known as closed-loop transmitter power control. These topics have been widely covered in research on wireless networks for more than a decade (Elmusrati *et al*. 2007; Fan *et al*. 2015; Zander 1992). However, there are some extreme cases where optimizing the transmit power with closed loop power control is not feasible. Two examples follow: First, when the feedback delay is greater than the coherence time of the channel, the CSI becomes outdated and even uncorrelated with the channel at the time of transmission. Therefore, using CSI is useless in this case as the feedback delay can be larger than the coherence time of the channel. When the channel is changing with high dynamics such as in very fast mobiles or when the duty cycle of the feedback channel is very large, such as in some energy-limited sensor devices, using CSI is not tenable.

The second case is observed when there is a practical obstacle for the feedback channel such as when receivers do not have transmission capabilities, the transmitters do not have reception capabilities, or there is no bandwidth available for the feedback channel. In such situations, algorithms for optimizing the transmit power according to the channel variations cannot be used. This extreme situation of lack of reception capability can be found, for example, in some primitive sensor terminals where the sensors sense some physical quantity and transmit its value without any knowledge about the CSI at the receiver side.

In such extreme situations, the conventional and practical power allocation is to use fixed (can be different) power transmission for all sensors. However, there are several problems associated with this fixed power allocation such as high unnecessarily energy consumption and high interference profile. To handle such problems, a randomized power allocation strategy was suggested by Kim and Kim (2005) whereas uniformly randomly distributed transmitted power levels were used to mitigate the near-far effect in congested systems without any channel feedback. It was based on the so-called "snapshot analysis approach", neglecting the effects of channel fading. An evaluation of the performance of random power for Rayleigh fading channel was discussed by Elmusrati *et al*. (2009, 2012). Recently, the randomization of power and modulation resources for device-to-device (D2D) scenarios were investigated by Zhang and Haenggi (2012); Lee, Lin *et al*. (2015); and Erturk *et al*. (2013).

In this paper, the performance of random power allocation strategies for wireless sensor networks were investigated taking into account realistic channel fading. The assumed channel model is the Nakagami-m model, which includes Rayleigh and Rician fading channels, and the unfaded channels as special cases. In section II, the system model is described. In section III, the analysis of the fixed transmit power allocation is given. In section IV, random transmit power allocation is considered. Finally, conclusions are drawn in section V.

2. System Model

In the considered scenario, the sensors are scattered randomly and uniformly over the area of interest and send their packets over a wireless link to an access point. As shown in Fig. 1, the measured quantities (*eg*. temperature, pressure, speed, etc.) are detected by sensors and then multiplexed.

Error correction coding is added to improve performance, and then the signal is power amplified to be injected to the antenna for transmission. The amplification factor of the power amplifier stage is controlled by a power allocation scheme which can choose to fix or randomize the power according to a selected probability distribution. Every packet contains coded transmitted information, pilot bits, and time and sequence stamps. The transmitted packets arrive from the N sensors to the access point in a multi-path manner with or without a dominant path. In this work, a Nakagami-m fading channel with mean channel gain is assumed and is presumed to be inversely proportional to the sensor distance as $G_{ij} = 1/d_{ij}^4$. The lost packets cannot be retransmitted because there is no feedback channel. However, the likelihood of packet loss can be compensated by increasing the level of redundancy. Assuming the envelope of the received signal follows the Nakagami distribution, then the link gain G_{ij} connecting transmitter j and receiver i will be gamma distributed as,

$$f_{G_{ij}}(g) = \left(\frac{m_{ij}}{g_{ij}}\right)^{m_{ij}} \frac{g^{m_{ij}-1}}{\Gamma(m_{ij})} e^{-\frac{m_{ij}g}{g_{ij}}}, \quad g \geq 0 \tag{1}$$

where g_{ij} denotes the mean channel gain. If m_{ij}=1, the distribution reduces to exponential distribution corresponding to Rayleigh fading. As m_{ij} increases, the channel behaves like Rician fading. As $m_{ij} \to \infty$ the channel becomes additive white Gaussian noise (AWGN). The characteristic function of the gamma-distributed link gain is well known and given by Papoulis (2002) as

$$\Psi_{G_{ij}}(s) = \int_0^\infty e^{sg} f_{G_{ij}}(g) dg = \left(1 - \frac{g_{ij}}{m_{ij}} s\right)^{-m_{ij}} \tag{2}$$

for $m_{ij} > 1/2$. If m_{ij} is a positive integer, then the cumulative distribution function for the link gain is (Papoulis 2002).

$$F_{G_{ij}}(g) = 1 - e^{-m\frac{g}{g_{ij}}} \sum_{k=0}^{m-1} \left(\frac{mg}{g_{ij}}\right)^k \frac{1}{\Gamma(k+1)} \tag{3}$$

For simplicity, consider a case in which all links have the same channel statistics of $m_{ij} = m$ for all (*i, j*). However, it is straightforward to generalize the analysis to the case where m_{ij} also depends on the link (*i, j*). Therefore, a normalized probability density function (PDF) $f_G(g)$, cumulative density function (CDF) $F_G(g)$ and characteristic function $\Psi_G(s)$ that corresponds to a channel with a unit mean that can be defined. The SINR Γ_i for each sensor node can be written as

$$\Gamma_i = \frac{G_{ii}P_i}{\sum_{j \neq i} G_{ij}P_j + \sigma^2} \tag{4}$$

where P_i is the i-th sensor transmit power and σ^2 denotes the noise power.

3. Fixed Transmit Power Allocation

When there is no feedback channel available and no channel information, the conventional way is to

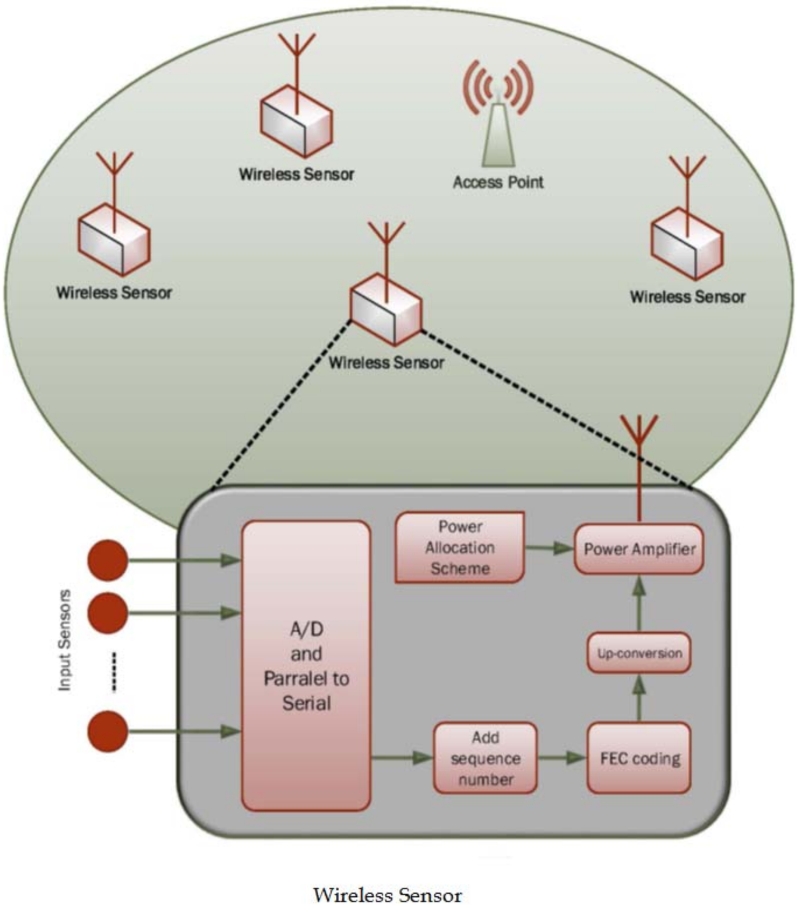

Figure 1. Sensor Network System Layout.

set the transmit power of all sensors to a fixed value. In case of single transmitter and fixed transmission power $P_i = p_i$, the outage probability p_{out} can be written as,

$$p_{out} = \Pr\{G_{ii}P_i < \gamma_i\sigma^2\} = F_G\left(\frac{\gamma_i\sigma^2}{g_{ii}p_i}\right) \quad (5)$$

Where γ_i is the minimum detectable signal-to-interference-plus-noise ratio (SINR). If m is a positive integer, the outage in case of multiuser interference can be derived as $\Pi\rho\{G_{ii}p_i < \gamma_i(I_i + \sigma^2)\}$ where $I_i = \sum_{j\neq i} G_{ij}p_j$ denotes the interference power at receiver i. Let a random variable be defined as $\beta_i = \frac{m\gamma_i(I_i+\sigma^2)}{g_{ii}p_i}$. The characteristic function of β_i for fixed transmission power can be directly derived from the characteristic equation of the channel; hence,

$$\Psi_{\beta_i}(s) = e^{s\frac{m\gamma_i\sigma_i^2}{g_{ii}p_i}} \prod_{j\neq i} \Psi_G\left(\frac{m\gamma_i g_{ij}p_j}{g_{ii}p_i}s\right) \quad (6)$$

The outage probability $O_i = \Pr\{G_{ii}p_i < \gamma_i(I_i + \sigma^2)\}$ can be evaluated as,

$$O_i = \int_0^\infty F_G\left(\frac{\beta}{m}\right) dF_{\beta_i}(\beta) =$$

$$1 - \sum_{k=0}^{m-1} \frac{1}{\Gamma(k+1)} \int_0^\infty e^{-\beta}(\beta)^k dF_{\beta_i}(\beta) \quad (7)$$

where $F_{\beta_i}(\beta)$ denotes the CDF of the random variable β_i. Note that by definition,

$$\Psi_{\beta_i}(s) = \int_0^\infty e^{s\beta} dF_{\beta_i}(\beta) \quad (8)$$

and

$$\Psi_{\beta_i}^{(k)}(s) = \frac{d^k}{ds^k}\Psi_{\beta_i}(s) = \int_0^\infty \beta^k e^{s\beta} dF_{\beta_i}(\beta) \quad (9)$$

Thus, the outage can be written in the form

$$O_i = 1 - \sum_{k=0}^{m-1} \frac{1}{\Gamma(k+1)} \Psi_{\beta_i}^{(k)}(-1) \quad (10)$$

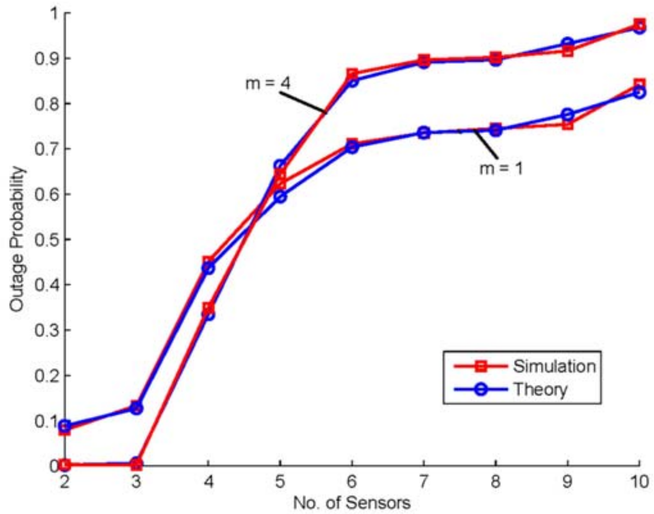

Figure 2. Outage versus network size.

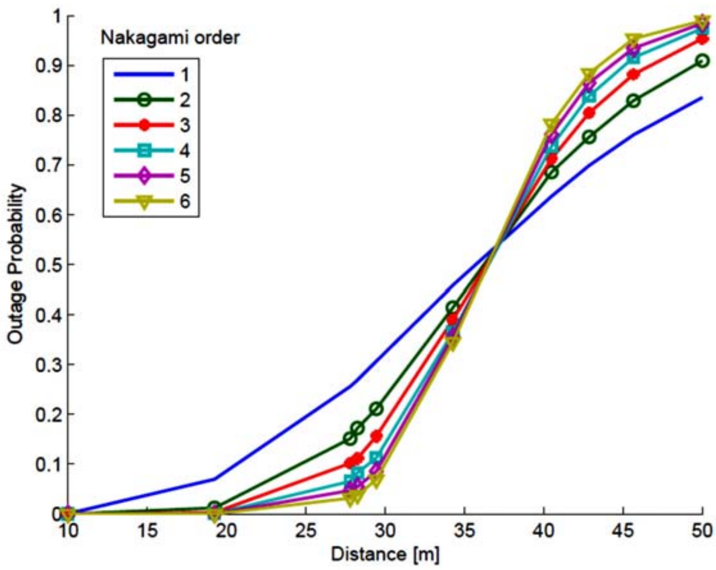

Figure 3. Outage versus sensor location.

The result in Eqn. 10 is validated numerically as follows. Assume 10 sensors are distributed randomly such that $10 \leq d_i \leq 50$, where d_i is the distance of sensor i to the access point. Let the sensors be arranged in increasing distance from the access node with d_1 spaced at 10 meters and d_N at 50 meters. The first simulation is carried out by assuming that the sensors at d_1 and d_2 are active and the others are set to OFF. Then the outage of the sensor at d_2 is evaluated theoretically and numerically. Next the sensor at d_3 is switched ON while keeping the first ones set to ON, and its outage is calculated, and so on. This means that the outage of the farthest sensor is evaluated for an increased network size. The transmitted power of all sensors is fixed at 0 dBm, the SINR at 5 dB and the noise at -70 dBm. Figure 2 shows that the theoretically evaluated outage is consistent with the numerically evaluated outage for m = 1 and 4. Figure 3 shows the outage of each sensor of a network with 10 sensors for different values of the Nakagami-m parameter m. It can be clearly seen that closer sensors will benefit from stronger paths. On the other hand, the far sensors outage will increase due to the increased interference from closer sensors.

4. Random Transit Power Allocation

Let $Q_{ij} = G_{ij}P_j$ denote the power received at receiver *i* from transmitter *j*. In the current study, it was assumed that the power values were drawn from a distribution having finite moments. Therefore, the characteristic function of the received power can be expressed as:

$$\Psi_{Q_{ij}}(s) = \int_0^\infty \left(1 - \frac{g_{ij}p}{m}s\right)^{-m} dF_P(p)dp \quad (11)$$

Let us define the normalized characteristic function $\Psi_Q(s)$ corresponding to the case $g_{ij} = 1$. It follows that the characteristic function for β_i becomes

$$\Psi_{\beta_i}(s) = e^{s\frac{m\gamma_i\sigma^2}{g_{ii}P_i}} \prod_{j\neq i} \Psi_Q\left(\frac{m\gamma_i g_{ij}}{g_{ii}P_i}s\right) \quad (12)$$

Using the same argument as was used in the last section, the outage probability can be written as for a general random power distribution as

$$O_i = 1 - \sum_{k=0}^{m-1} \frac{1}{\Gamma(k+1)} \int_0^\infty \Psi_{\beta_i}^{(k)}\left(\frac{1}{p}\right) dF_P(p) \quad (13)$$

Evaluating the outage for a specific Tx power distribution is tedious. It involves the evaluation of the integral m - 1 derivatives of the characteristic function of β_i which is composed of the product of *N* - 1 characteristic functions of the received signal power. Therefore, a numerical evaluation of the outage for some selected Tx power distributions was employed and the average outage for the fixed, uniform and Weibull distributed transmit power allocations were compared numerically.

The Weibull PDF was selected for the following reasons. First, it has large shaping flexibility as shown in the next section, meaning that wide statistical behavior of the transmit power can be obtained. Second, it has a tractable mathematical form. It should be stressed at this point that optimizing the power allocation distribution is beyond the scope of this work, and other distributions may produce similar or better behavior compared to the Weibull allocations.

5. Weibull Distributed Transmit Power

The PDF of the Weibull distribution of the random variable X is given by

$$f_X(x) = \begin{cases} ab^{\;b-1}e^{-ax^b}, & x > 0 \\ 0, & otherwise \end{cases} \quad (14)$$

where, $a > 0$ is the scale parameter, and $b > 0$ is the shape parameter. When applied as the PDF of the transmit power, a truncated Weibull distribution is used. Also for the sake of easier control of the transmit power distribution, the scale parameter was fixed to $a = 1$ and let the shape parameter b be a free control variable. Then the truncated Weibull PDF is given by

$$f_P(p) = \begin{cases} \frac{1}{1-e^{-P_{max}^b}} bp^{b-1}e^{-p^b}, & 0 < p < P_{max} \\ 0, & otherwise \end{cases} \quad (15)$$

Figure 4 shows this PDF for several values of the shape parameter with P_{max} set to 1 dBm. As can be seen by changing the shape parameter, different shapes of the truncated Weibull power distribution can be obtained. For b <1 the transmitted power will be more biased toward zero. On the other hand, for b >1, the transmitted power will be closer to P_{max}. Because P_{max} <1, b = 1 results in an approximately uniformly distributed power. Therefore, if b is tuned separately for the different sensors according to their location, it is expected that the outage of the sensors will be equalized such that the far and close sensors to the access node will perform comparably in terms of outage probability. The average power is given by means of a distribution that can be evaluated by

$$\bar{P} = \frac{1}{1-e^{-P_{max}^b}} \gamma\left(1 + \frac{1}{b}, P_{max}^b\right) \quad (16)$$

where $\gamma(a,b) = \int_0^b x^{a-1}\exp(-x)dx$ is the incomplete gamma function.

6. Simulation Results

In this section, the investigated technique of random power allocation was tested via several simulation runs. In the simulation setup, the assumption was a network with 10 sensor nodes distributed randomly but uniformly over an area such that the closest sensor was at a distance of 10 meters and the farthest sensor was at 50 meters. The target SINR of the nodes was set to 5 dB, the noise power was set to -70 dBm, and the maximum transmitted power was 0 dBm. The channel was assumed to be a Nakagami-m with m = 1 (Rayleigh fading channel). The channel realizations were generated by first producing average channel gains following the fourth order path loss factor, such that $g_{ij} = 1/d_{ij}^4$ in Eqn. 1, where d_{ij} is the distance to the *i*-th sensor, and G_{ij} is generated using the Nakagami-m PDF.

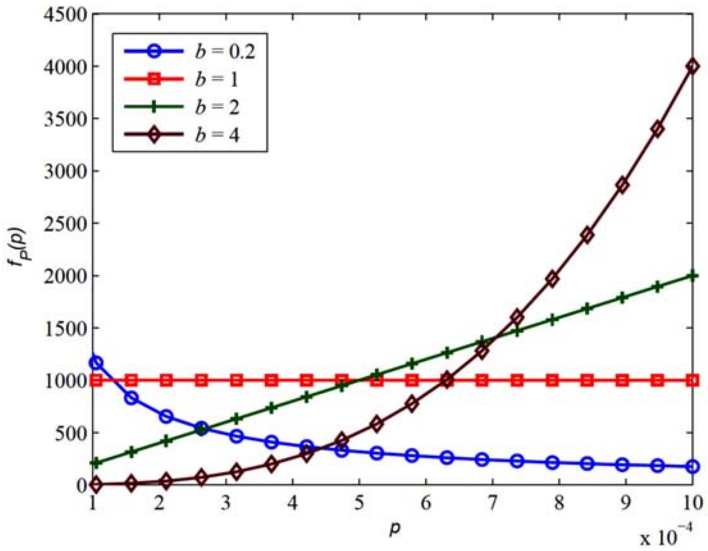

Figure 4. Truncated Weibull PDF for several values of the shape parameter b.

Next, the transmitted power by each sensor was generated according to the selected distribution and the received signal power was evaluated. Hence, the probability of outage for each sensor was evaluated by checking if Γ_i in Eqn. 4 was less than the target γ_i. The simulation was conducted such that 20 network deployments were performed, and for each deployment 10^5 channels and power realizations were generated. The final outputs were found by averaging the results of the 20 network deployments. Five scenarios were simulated to test the following transmit power allocations:

- Scenario I: All sensors transmit at their maximum fixed power of P_{max}
- Scenario II: All sensors use a randomly and uniformly distributed power (~0 to P_{max})
- Scenario III: All sensors transmit a truncated Weibull random power with fixed b = 0.2
- Scenario IV: All sensors transmit a truncated Weibull random power with fixed b = 4
- Scenario V: Sensors transmit following a truncated Weibull random power with b assigned linearly to sensors such that the closest sensor has b = 0.2 and the farthest sensor has b = 4

In all Weibull random power scenarios, the scale parameter was fixed at a to one. As depicted in Fig. 5, scenario III showed the worst performance due to low b which tends to produce a low transmit power that is insufficient to combat noise. For most of the nodes, scenarios I, II and IV resulted in comparable performance with Scenario I, showing slightly better results. However, scenario I consumed the highest average power of P_{max} per node. The total average power of scenarios I–IV was 10 dBm, 7 dBm, 1.8 dBm, and 9 dBm, respectively. In scenario V, it was assumed that the shape parameter b can be controlled before the sensor is deployed to its location such that its value is tuned to lower values for close sensors and higher values for distant sensors.

For simplicity and without any effort to optimize its value, b was set to depend linearly on the distance by setting the close sensor to b = 0:2 and the farthest sensor to b = 4. The b for sensors located in between were interpolated linearly from these end points. In this case, the power of close sensor would be biased towards zero and therefore have higher outage compared to fixed and uniform power allocations. On the other hand, far sensors would benefit from regulated power allocation and enhanced performance can be clearly observed in Fig. 5. The network size of 10 sensors was used for the sake of demonstration. It is expected that as the network size increases, the performance will deteriorate. The factors affecting the performance are the target SINR, the mean channel gain, the transmit power, and the noise.

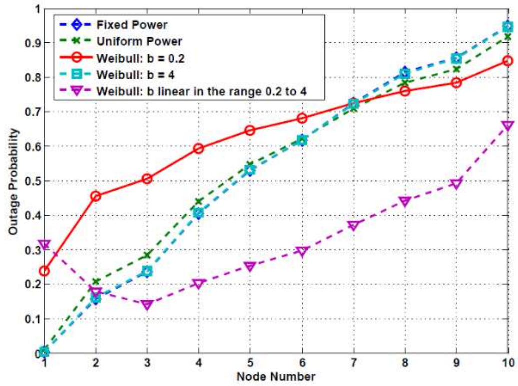

Figure 5. Outage versus transmit power distribution.

7. Conclusion

In this paper, the performance of a random power allocation scheme in a Nakagami-m wireless channel was analyzed. The Nakagami-m model can emulate several scenarios, including the line of sight, the non-line of sight, and the AWGN channel as special cases. Fixed power allocation was compared to uniform random power and truncated Weibull random power allocation schemes. It was found that the fixed power allocation outperforms the other methods with the disadvantage of increased average transmitted power. On the other hand, the Weibull distribution outperformed the other methods if the control parameter of the distribution was tuned such that far sensors had higher values and closer sensors had lower values.

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