A Role for Bose-Einstein Condensation in Astrophysics

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Abstract

We revive a 60-year-old idea that might explain a remarkable new observation of a periodic low-frequency radio emission from a source at galactic distances (GLEAM-X J162759.5-

523504.3). It derives from the observation that a high-density high-temperature charged boson plasma is a superconducting superfluid with a Meissner effect.

Keywords: Bose-Einstein condensate, Charged Bose gas, Astrophysical Chemistry

Introduction

Sporadic forays over the years have explored the possibility that the physics of Bose-Einstein condensation ought to play some role in Astrophysics, e.g. [1,2]. Many of the particles involved in stellar evolution are bosons, i.e. have zero or integer spin. Bose-Einstein condensation is a fundamental macroscopic manifestation of quantum physics. It would seem remiss of the Creator not to have employed the phenomenon somewhere in building the Universe. Especially is this so since Fermi-Dirac and Classical Statistical Mechanics do figure largely. A suggestion of a role for Bose-Einstein condensation was made 60 years ago when quasars were first observed [1], and forgotten. Later attempts failed because they considered superconductivity and Bose condensation as classical low-temperature phenomena like that which occurs for electrons in metals. But the phenomena are not limited and exist for very high-temperature high-density charged particles [1].

We here revive that 60-year-old idea and suggest it might explain a recent extraordinary observation.

The Phenomenon

Hurley-Walker et al. [3] recently reported an unusually slow periodic low-frequency radio emission from a source at galactic distances (GLEAM-X J162759.5-523504.3) with a pulse

period of 18.18 minutes. One clue to its origin is that high linear polarization has been shown to be characteristic of a source with strongly ordered magnetic fields [4-6]. The observations are unlike emissions characteristic of stars, white dwarfs, white binaries, or exoplanets. Furthermore, the 0.5-light-second upper limit on the object's size and estimated brightness temperature of 10^{16} K led Hurley-Walker et al. [3] to propose that a radiation source is a compact object with a rotational origin.

The Idea

With that in mind, we give reasons to consider if Bose-Einstein condensation [1] might have something to do the phenomenon:

(i) Stable nuclei in stellar interiors have zero or integer spin. Nuclei of higher and higher atomic numbers built up during the evolution of stars. They are charged bosons.

(ii) A dense charged high-temperature boson plasma becomes nearly perfect as density increases (i.e., the Coulomb collective interactions become so weak that they can be ignored, and we can work with the perfect gas approximation).

(iii) It can undergo Bose-Einstein condensation to a superfluid state.

(iv) A conducting superfluid is a superconductor. A rotating superconducting superfluid has a Meissner effect. That is, it expels the magnetic field generated by rotation.

(v) Such a magnetic field would be trapped in the lower-density surface region. This process continues as the star collapses and its rotation speeds up.

(vi) Massive synchrotron radiation follows that dissipates this increasing build-up of energy.

The assumptions i-vi were originally made 60 years ago to explain the newly discovered quasars. Schafroth [8], Blatt [9], and Butler [10] had shown earlier that an ideal charged Bose gas below the critical point for superfluidity is a superconductor (see also Refs. [7,8-17]). These theories [7-17] call on electron pairing to generate charged bosons that then lead to Bose condensation and superconductivity *at very low temperatures.* Our situation is quite different. The stellar objects involve real boson nuclei of even spin. The high-density, high-temperature plasmas are close to ideal.

We recall the process of nucleosynthesis in stellar interiors [18-21]. The theory explains how nuclear reactions convert lighter elements into heavier ones through the fusion of atomic nuclei. As the star evolves, the fuel elements involve successive steps, with H, He, C, O, Ne, Si, Fe, and U providing increasingly heavier energy sources that drive the stellar evolution to completion [18]. We need to estimate the critical temperatures and core densities for Bose condensation for stars with different fuel elements to check that they can have a Boson core region.

Calculations

Consider an assembly of ions of even spin, mass *M and charge Ze*, in a background electron gas. Under extreme high-density and high-temperature conditions, the system is expected to behave like a mixture of ideal gases. That can be achieved by ensuring that the average energy of Coulomb interactions between two ions is small compared to their kinetic energy. At densities approaching the critical value for Bose-Einstein condensation of ions, i.e., when their

chemical potential approaches zero, the mean energy, per particle of the ideal Bose gas is approximately equal to kT. The average distance between the particles is then [1]

$$\mathrm{r} \approx 2\sqrt[3]{3\mathrm{M}/[4\pi\rho]} \approx \lambda = h/(Mv) = h/\sqrt{2\ \mathrm{M}\ \mathrm{kT}}, \qquad (1)$$

where λ is the de Broglie wave length of an ion of mass M and kinetic energy $\sim kT$ (taken in Eq (1) equal to $Mv^2/2$). The condition that the actual gas be nearly ideal is [1]

$$\frac{2Z^2 e^2}{r} \ll kT. \qquad (2)$$

Hence, taking the requirement Eq. (2) with Eq. (1), the critical expressions for temperatures, density, and particle separation can be estimated to be of the order

$$T_C \sim \frac{8MZ^4 e^4}{h^2 k} \qquad (3)$$

$$\rho_C \sim \frac{384}{\pi}\frac{M^4 Z^6 e^6}{h^6} \sim \frac{6}{8\pi}\frac{Mk^3}{Z^6 e^6}T_C^3, \qquad (4)$$

$$\mathrm{r}_C \sim \frac{h^2}{4MZ^2 e^2} \sim \frac{2Z^2 e^2}{kT_C}. \qquad (5)$$

The numerical values are summarized in Table 1. We expect that highly charged nuclei will be "dressed" by an inhomogeneous adsorbed relativistic electron cloud (mesons in another guise): just as for charged micelles or highly charged ions in electrolyte solutions. In that case, "bound" counterions are typically 80-90% of the bare charge. The effective charge is 10-20% of the actual charge. Without recognising such screening, estimated critical parameters for the separation of heavy ions become unphysical and ridiculous. In Table 1, we take two

extreme estimates to bound these uncertainties: the unscreened Z and Z=1. To illustrate our point, we present also the critical temperatures with 10% and 20% effective charges in Table 2. For Uranium, from Table 2, the estimated critical temperatures are 10^{14} K<T_c<10^{15} K. These bounds are similar in magnitude to the observed brightness temperature of 10^{16} K discussed by Hurley-Walker et al. [3].

Fuel element [A,Z]	T_C (Z=1) [K]	ρ_C (Z=1) [g/cm³]	T_C (Z) [K]	ρ_C (Z) [g/cm³]
He [4,2]	4.7×10^8	3.5×10^{10}	8×10^9	2×10^{12}
C [12,6]	1.4×10^9	2.9×10^{12}	2×10^{12}	1×10^{17}
O [16,8]	1.8×10^9	9.0×10^{12}	8×10^{12}	2×10^{18}
Ne [20,10]	2.3×10^9	2.2×10^{13}	2×10^{13}	2×10^{19}
Si [28,14]	3.2×10^9	8.5×10^{13}	1×10^{14}	6×10^{20}
Fe [56,26]	6.6×10^9	1.3×10^{15}	3×10^{15}	4×10^{23}
U [238,92]	2.8×10^{10}	4.4×10^{17}	2×10^{18}	3×10^{29}

Table 1. The critical temperatures and mass densities for different fuel elements in the core of a Boson star are derived from Equations (3)-(5). Since we did not include screening we present two different estimates (a) based on setting Z=1 (columns 2 and 3) and (b) using Z

from column 1 (columns 4 and 5). Here $h = 6.626 \times 10^{-27}$erg s, $k = 1.381 \times 10^{-16}$erg/K,

$e = 4.803 \times 10^{-10}$esu, and we estimate the mass as $M = A \times 1.66 \times 10^{-24}$g (Atomic

number x proton mass).

Fuel element [A,Z]	T_C $(Z \rightarrow Z \times 0.1)$ [K]	T_C $(Z \rightarrow Z \times 0.2)$ [K]
He [4,2]	8×10^5	1×10^7
C [12,6]	2×10^8	3×10^9
O [16,8]	8×10^8	1×10^{10}
Ne [20,10]	2×10^9	4×10^{10}
Si [28,14]	1×10^{10}	2×10^{11}
Fe [56,26]	3×10^{11}	5×10^{12}
U [238,92]	2×10^{14}	3×10^{15}

Table 2. The critical temperatures for different fuel elements in the core of a Boson star are

derived from Equations (3)-(5). Estimates (a) based on setting $Z \rightarrow Z \times 0.1$ (column 2) and $Z \rightarrow$

$Z \times 0.2$ (column 3). Constants used are same as in Table 1.

With even the lowest densities *N(He)*, the accompanying electron gas is highly relativistic and degenerate. We intend to go beyond these simple estimates using a screened intervening plasma including both magnetic and dielectric susceptibilities. Notably, the core temperatures for the seven ages of a 20M$_o$ star discussed by Trimble are T=1.9×10^8K for a star with Helium as fuel element and T=3.7×10^9K for a star with Silicon as fuel element [18]. These numbers are in surprisingly good agreement with our estimates based on using Z=1 in Table 1.
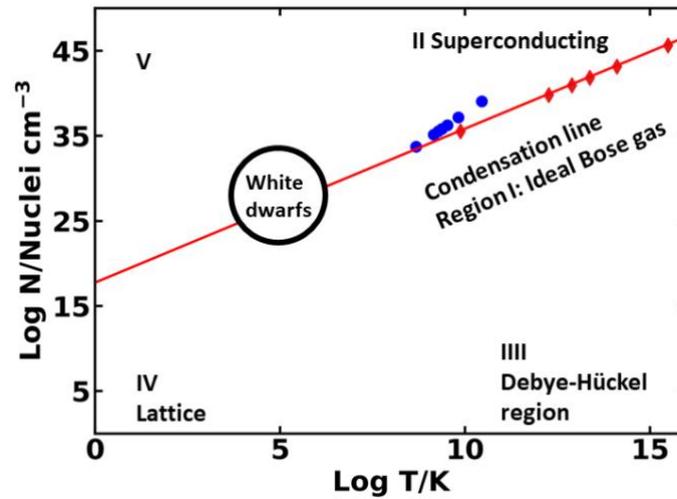
Figure 1. Number density (N) of stable nuclei vs. temperature phase diagram for the charged Bose gas. The condensation line shown use Eq. (4) and the data from Table 1. Adapted from Figure 1 in Ref. [1]. The data points from the unscreened model (Z from Table 1) are shown as red symbols while the results corresponding to Z=1 model are shown as blue symbols. We indicate schematically a region relevant for white dwarfs (large black circle) (from Figure 1 in Ref. [1]).

In region I, near the condensation line, $\rho \sim \frac{6}{8\pi} \frac{Mk^3}{Z^6 e^6} T^3$, the system is nearly an ideal Bose gas. Above the condensation line in region II (high density, high temperature), a rotating gas should become superconducting, Region III corresponds to the classical Debye-Hückel (high temperature, low density) region, and in region IV conditions are favourable for the formation

of a lattice (low temperature, low densities). Region V (low temperature, high density), exhibits an energy gap (c.f. eq. (7)), where the quasi-particle elementary excitation energy has the form [1,12,13]

$$\varepsilon(p) = \sqrt{\left(\frac{p^2}{2m}\right)^2 + \left(\hbar\omega_p\right)^2},$$ (7)

where ω_p is the classical plasma frequency.

A Diversion. With Bose Einstein condensation in stellar systems, further unanticipated complications may occur. As they contract under the influence of gravitational forces, their core densities and temperatures will increase, leading to charged Boson core regions with increasingly heavy fuel elements. Available nuclear mass models show that not all numbers of proton-neutron combinations would be stable. For each state there is a minimum and a maximum number of neutrons and protons that are stable: a phenomenon known as the neutron and proton drip line [22]. The nuclear mass models demonstrated that certain ensembles of protons and neutrons are inherently unstable. That is, the ground-state configurations of such species are energetically unstable to the emission of a constituent nucleon [22]. The matter is still open.

Even the established sign of nucleon-nucleon interactions which can be used for plausible models of Bose condensation with neutron pairing [2] has been questioned. The data dating back to Seaborg's work from which that conclusion was reached has never been questioned. See references in [23,24]. Be that as it may, the possibility that Bose condensation and consequent magnetic events of even quarks and other exotic objects in the cores of neutron

stars has seriously been broached [25]. Returning to our main theme, Hurley-Walker et al. [3] speculated that their observations might be due to a compact rotating magnetar. However magnetars are made of neutrons, that is fermions. Our considerations apply to bosons. Our propositions might then be appropriate rather to the early stages of the collapse of one of a pair of rotating massive binary stars. This would be consistent with the very low frequency of the pulses.

Conclusions

Quantum coherent Bose-Einstein condensation is a general property of matter with particles of even spin. Therefore, it might reasonably be expected to play a role in astrophysics one way or another, just as Fermi-Dirac statistics does in the theory of white dwarf and neutron stars. Attempts to invoke Bose condensation and superconductivity are few [1-2]. The present proposal is unlike the original low temperature theories of superconductivity [10-11, 14-17]. It is concerned with a phenomenon that occurs with charge bosons at high temperature and high density [1]. The charged ideal Bose gas is superfluid below its critical point [1,12]**.** And a rotating superfluid has a Meissner effect [8-10] with consequent, expulsion and buildup of large magnetic effects. The observed linearly polarized ultra-long period low-frequency radio emission is known to be related to magnetic fields [5-7]. The Bose-Einstein condensation could be the origin of the magnetic fields and the phenomenon, not previously observed, might be of wider occurrence, as anticipated in [25] A. Mann, "The strange hearts of neutron stars", *Nature* **579**, 20-22 (2020).

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