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A knowledge-Induced Operator Model

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نموذج الدالة منتجة المعرفة

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خلاصة: نظام التعليم في مقدمة التحقيقات التحليلية في العلوم. وفي العلوم الإجتماعية تقوم نظم التعليم بدراسة تعقيدات نظم الكلمات. وعندما يرتبط بعبارات المنطق يطلق عليه اسم "مكمل الأخر" أو علم الرموز. في علوم الرياضيات تعتمد طرق النظام التعليمي على المنطق الرسمي أو تحليل الأنظمة. في هذه الورقة سوف يتم التوصل إلى أن النظام التعليمي يتخطى زمان ومكان التحقيقات العلمية ليصل إلى أفاق المعرفة. وأفاق المعرفة هذه ستفسر بتفاعل واسع يؤدي إلى تكامل يتبعه تطور مستمر يعتبر عملية مكاملة ما بين الوحدات والمجموع في نظم الكلمات. في هذه الورقة أيضاً سيتم ايجاد توصيف رياضي لخواص العمليات المنتجة للمعرفة في نظم الكلمات في ضوء علم المعرفة. والنتيجة هي تشكيل تعميم رياضي لظاهرة التعليم، وهذا التعميم يعتمد على احدى الدوال. وهذه الدالة التي تم استعمالها تلخص خواص التفاعل والتكامل والتطور في نظام التعليم. كما تمت بعد ذلك دراسة الحالة المعالجة أي التجهيل.

ABSTRACT: Learning systems are in the forefront of analytical investigation in the sciences. In the social sciences they occupy the study of complexity and strongly interactive world-systems. Sometimes they are diversely referred to as symbiotics and semiotics when studied in conjunction with logical expressions. In the mathematical sciences the methodology underlying learning systems with complex behavior is based on formal logic or systems analysis. In this paper relationally learning systems are shown to transcend the space-time domain of scientific investigation into the knowledge dimension. Such a knowledge domain is explained by pervasive interaction leading to integration and followed by continuous evolution as complementary processes existing between entities and systemic domains in world-systems, thus the abbreviation *IIE*-processes. This paper establishes a mathematical characterization of the properties of knowledge-induced process-based world-systems in the light of the epistemology of unity of knowledge signified in this paper by extensive complementarities caused by the epistemic and ontological foundation of the text of unity of knowledge, the prime example of which is the realm of the divine laws. The result is formalism in mathematical generalization of the learning phenomenon by means of an operator. This operator summarizes the properties of interaction, integration and evolution (*IIE)* in the continuum domain of knowledge formation signified by universal complementarities across entities, systems and sub-systems in unifying world-systems. The opposite case of 'de-knowledge' and its operator is also briefly formalized.

KEYWORDS: Learning systems, epistemology, topology, evolutionary set, functional, dynamic continuity, knowledge-induced operators and variables, knowledge flow.

1. Introduction

The objective of this paper is to formulate an analytical methodology and model of knowledge induction premised on systemic unity of knowledge. Such a model premised on the epistemology of unity of knowledge is found to apply to theoretical conception and positive perspectives relating to a vast domain of epi-phenomena. Furthermore, if we can formulate the model by means of a mathematical 'operator' based on the underlying episteme of unity of knowledge, such an operator can be used to develop well-defined mappings for generating the objective criteria for all socio-scientific problems. We undertake this task in this paper. Along the passage of developing our detailed model and operator of the relationally learning topological system we introduce important concepts and explain them in detail. These important concepts are of the Social Wellbeing Function and the **I**nteractive, **I**ntegrative and **E**volutionary (*IIE*) learning processes across continuity and continuums.

1.1 Definition of the operator

The operator developed in this paper means a summarized way of depicting and bringing together the function and application of systemic unity of knowledge to bear upon sets of learning relations by the use of the operator. The operator is thereby a concept imbued in a mathematical form to explain, quantify and measure the implications of unity of knowledge relating to the problem under investigation. Our operator is thus a conceptual-quantitative functional form that generalizes the functions of unity of knowledge as relational epistemology underlying all learning entities intra- and inter- systems.

The singular axiom of our methodology is unity of knowledge both in its epistemological and ontological sense of existence and quantification of a unified worldview of entities intra- and inter- systems. This choice of the axiom is premised on the historical permanence of the search for unity in the sciences and world-systems in reference to the most reduced totality of laws that can overarch all systemic explanations. The prime example of such an axiom is the episteme of unity of knowledge in the divine laws.

Another example of the search for unity of knowledge from the sciences is the classical contribution on The Unity of Science by Neurath *et al.* (1970). In this revolutionary work the role of ethics as relational learning is explained well by Edel [p. 285 in Neurath *et al* (1970)]: "Causal-explanatory approaches look for the conditions and contexts of the occurrence of moral phenomena, experiences and processes. In whatever terms these phenomena be described, there is always the question of relating them, as they occur, to other phases of life and internal economy of the individual and to the other phases of operation of the society at a given point of development of its history and culture."

1.2 Rationale for the topological method used

The topological method used in the study of the epistemology, ontological and applicative nature of the operator as conceptual-quantitative functional form imputed on a general body of socio-scientific problems is found to be the most suitable one for studying relationally learning entities of 'every' world-system. The word 'every' and 'everything' are used to convey a generalized theory of unity of relationally learning entities within and across entities and their sub-systems in the domain of learning. Such a totality of knowledge flows and knowledge-induced variables characterizing entities intra- and inter- systems is defined by the generalized understanding of topological mappings that universalize continuously relational learning among all such entities.

When certain entities do not learn due to the absence of relational continuity attained by the topological mappings, such entities are enclosed within their own universe as a segmented domain of study. Now once again, the persistence of differentiated entities caused by their systemic independence and individualism means, continuity of such entities within their own domain of differentiation and methodological individualism.

Hence there is continuity of the relational worldview for both cases treated in this paper. First, there is continuity of the relationally unifying entities of their world-systems. Secondly, there is continuity of the differentiated entities in their own and disjoint world-systems. The topological method applied to both kinds of world-systems in this context generalizes the methodology of unity of knowledge to explain such ‘everything’ (Barrow, 1990).

The conceptual, quantitative and applied perspectives of the unified worldview explained through the operator have serious implications on a scientific way of exclaiming the powerful methodology of unity of knowledge. Some limited applications of the methodology of unity of knowledge to constructed and estimated models are brought out. But for reason of space, a full development of the applied content cannot be divulged in the paper and the technical appendix. The reader may refer elsewhere for this interest [Choudhury (2004, 2006a), Choudhury & Hossain (2006) forthcoming, Johannessen (1998), Xuemoue and Dinghe (1999)].

2. Principal definitions

At the very outset we define some of the foundational terms and concepts used in this paper to establish the methodological background of the knowledge-induced operator. Some definitions appear by reference to the literature; others are original contributions by the authors in the field of relationally learning systems under unity of knowledge.

2.1 Epistemology

What is the epistemology of unity of knowledge used here? Epistemology means theory of knowledge and its study explains the foundations and structure by which knowledge is derived, classified and disseminated. In the case of unity of knowledge epistemology becomes the field of studying relational unification between multidimensional and multivariate entities that learn by interaction leading to integration between them and thereafter evolving to subsequent phases of knowledge flows arising from the level of experiences encountered in the previous phases of learning. Thus knowledge flows get forward lagged.

The scientific research program of unity of knowledge has a singularly strict axiom. That is the axiom of epistemology of unity of knowledge that springs from the premise of the unified laws of ‘everything’. The epistemology of the unified laws leads to a worldview of relational unity of knowledge in the systemic sense. The epistemological premise is formally used to establish the ontological model of unity of knowledge. Finally, generically formulated ontological model of unity of knowledge explaining relational unity of knowledge between variables and their systems leads to its application through data simulation. Empirical consequences, application and inferences follow from such a methodological treatment of the relationally learning *IIE*-processes emanating from the epistemological origins of oneness in the ontological model. The search thereby is for the unique and universal foundations of the epistemological and ontological text of laws. These are firstly established in the absolutely unified nature of such laws (e.g. the divine laws). Secondly, the unity of relationally unified learning systems is reflected in ‘everything’ of diverse world-systems. Thirdly, the totality of such unified laws and their causation generated by relational unification in world-systems reproduces unity of being in the problem at hand between and within entities and their sub-systems.

The socio-scientific and historical search for such a law leads straight to the foundations of oneness of God, which is the established premise of the divine laws. Hence the ensuing concepts and quantitative systems are premised on this epistemology and ontology of relational unity of knowledge as the axiom to be found in the fundamental epistemology. Such a precept of unity of the divine laws is referred to as *Tawhid* in the *Qur’an*. In every other epistemic reasoning, for example in the Ghaia notion of a relational worldview, the epistemological premise is human reason (Primavesi, 2000). In Aquinas (1946) the notion of oneness of God is a speculative metaphysical concept.

2.2 Uniqueness and universality

In the introduction we have pointed out the meaning of 'everything' in the context of the methodological generalization of the epistemology of unity of knowledge and the role of the operator as a conceptual-quantitative generalized functional form to explain this context of systemic unity. We now go on to extend this idea to the concept of uniqueness and universality in spaces that are induced by relational learning, and hence must be explained by probability limits (*p*lim) taken over domains of interaction, integration and evolutionary (*IIE*) learning between the entities and their functions as topological relations.

2.2.1 Universality

A formal definition of universality and uniqueness that endows it with a generalized methodology is given below (Wiener, 1961):

Let P_n be a proposition. P_n is extended to all numbered propositions 'n' if the following properties of P_n hold:

P_n is proved true for $n = 1$

If P_n is true then P_{n+1} is true

Therefore P_n is true for all positive integer n. Furthermore, if 'n' is a real number then too the above conditions abiding establish the extended inductiveness of the proposition P_n.

2.2.2 Uniqueness

The relationally learning methodology of unity of knowledge is unique. The definition of uniqueness follows by virtue of the relationally learning methodology that remains permanent in explaining unity of knowledge in 'every' world-system (issues and problems) in the sense of universality. The methodology of unity of knowledge thus explains both learning by relational unity as well as 'de-knowledge' (differentiated relations). Thus issues and problems under investigation may be diverse but their socio-scientific treatment remains unique in the methodology of unity of knowledge. Choudhury (1998, pp. 20) writes, "The unification of knowledge under a unique set of universal laws that apply equally within and between systems, given the perceived differences in the specifications of the problems underlying the systems, is the essence of the worldview."

We note then that in connection with the concepts of paradigms and scientific revolutions, the concept of the worldview means reduction of all paradigms and scientific revolutions to a unique praxis of theory. This core of all theories that enables the interdisciplinary systems to get linked up is premised on unity of knowledge. When the premise of unity of knowledge is identified as having a vaster explanatory and applicative power than do specific theories on an ever-expanding class of issues and problems under investigation, we then prove the universality and uniqueness of such a super paradigm. We have then discovered a socio-scientific revolution.

3. Formalizing epistemological concepts by means of probabilistic topology

When translated into probability language, the concept of universality and uniqueness are formalized as follows:

Let $\{\theta_{ij}\} \in (T, s)$ denote a space of learning parameters. $\{\theta_{ij}\}$ are treated as knowledge flows, being premised on the unity of relational systems as defined under the epistemology of (*T, s*). Here *i* denotes numbered interaction intra- and inter- entities and sub-systems; *j* denotes numbered entities or their sub-systems. Such knowledge flows are derived from the epistemology of unity of knowledge as the reference law (*T*). They are mapped into learning possibility by the ontological mapping (*s*). Hence (*T*, s) denotes the fundamental epistemology (foundation of knowledge).

With the above definition we now reframe the definition of universality given above, as follows:

$$\theta = p\lim_{ij} \cup_i \cap_j \{\theta_{ij}\}$$

The postulate, issue or the problem, $P_n(\theta)$ is proved true for a certain number $n = 1$.
If $P_{n+1}(\theta)$ is proved true on the assumption that $P_n(\theta)$ is true, then it is true for all n. Therefore $P_n(\theta)$ is true for every knowledge-induced configuration

$$K_{ij}(\theta_{ij}) \subseteq K(\theta\,;P_n(\theta))$$

existing in continuum.

Thus due to the positive monotonic continuous relationship between θ and $K(\theta\,;P_n(\theta))$, and between $\{\theta_{ij}\}$ and $\{K_{ij}(\theta_{ij}\,;P_n(\theta_{ij}))\}$, probability functions and limits apply to all the class of functions defined over these domains, if and only if $\theta \in (T, \mathrm{s})$ in the sense of probability limit taken over relationally learning processes between entities intra- and inter- systems.

Within the above formalization, uniqueness is characterized by the application of the universal topology (T, s) to the learning domain $\{\theta\,, K(\theta\,;P_n(\theta))\}$ and all its specific sub-domains of issues and problems, $\{\theta_{ij}\,, K_{ij}(\theta_{ij}\,;P_n(\theta_{ij}))\}$.

$$\{K_{ij}(\theta_{ij}\,;P_n(\theta_{ij}))\} \subseteq K(\theta\,;P_n(\theta))$$

is therefore true in the general case of θ being knowledge flows denoting relational unity of knowledge.
What is true of $K(\theta\,;P_n(\theta))$ with respect to the given $P_n(\theta)$ is equally true of $K_{ij}(\theta_{ij})$ with respect to the application of $P_n(\theta_{ij})$ i, j defined as above. This result is readily proved by noting that monotonically positive and continuous mappings over sub-domains of a topology (T, s) form topological sets (Maddox, 1970). Hence such a function defined on $\{\theta\}$ applies to its components $\{\theta_{ij}\}$ and vice versa. It is likewise the case for $K(\theta\,;P_n(\theta))$ and $K_{ij}(\theta_{ij}\,;P_n(\theta_{ij}))$. More along these lines can be read in the recent contributions by Choudhury (2005), Choudhury and Zaman (2006) and Shakun (1988).

An example of normed positive monotonic transformation on the domain $\{\theta_{ij}\,, K_{ij}(\theta_{ij}\,;P_n(\theta_{ij}))\}$ and the domain $\{\theta\,, K(\theta\,;P_n(\theta))\}$ is a probability measure (Halmos, 1974). Let such a probability measure be denoted by μ (.) operating on the domains shown here that can be enclosed in (.). Thus,

$$p\lim\{\theta_{ij}\} = \theta \;\Rightarrow\; p\lim\; \mu(\theta_{ij}) = \mu(\theta)$$

is a normed density function of θ. A similar normed probability density function ν maintains,

$$p\lim\{K_{ij}(\theta_{ij})\} = K(\theta) \;\Rightarrow\; p\lim\; \nu\,(K_{ij}(\theta_{ij})) = \nu\,(K(\theta)).$$

All probability limits are taken over interaction and entities. Such measures can be extended by matrix transformations over interconnecting systems of entities. Over still more complex interacting, integrating and evolutionary domains of learning, tensors can be used (Gel′fand, 1961; Choudhury, 2006a,b). Any further normed and positively monotonic and continuous transformations of these functions will behave in the same way (Henderson and Quandt, 1971 p. 20).

4. Formulation of the knowledge-centered methodology

To address our stated objective we now formulate the methodology of unity of knowledge. (*T*, *s*) was defined above as the epistemological core of unity of knowledge. It comprises the irreducible concept, hence the universality and uniqueness of the divine laws. (*T*, *s*) is the topology of the complete, perfect and absolute stock of knowledge from which emanate flows of knowledge, $\{\theta\ ,\theta_{ij}\}$. The stock by its very concept of completeness and absoluteness is never changed, as it recycles back the flows into itself continuously across processes of learning (*IIE*-processes). Thus (*T*, *s*) remains the exogenous source of knowledge in every relationally learning problem. On the other hand, the derived $\{\theta\ , K(\theta\ ; P_n(\theta))\}$ and its sub-spaces are permanently endogenous in nature in the relationally learning systems of unity of knowledge. $\{\theta\} \in$ (*T*, *s*) denotes an open set of knowledge flows emanating from the epistemology of the unchanging knowledge stock *T*. The very property of topology makes *T* an open set containing its members, which too are open. Thus even at this early stage of the paper the process-oriented nature of learning, and thereby the impossibility of optimization as opposed to simulation to occur among the relations generated by knowledge flows, is established (Choudhury and Korvin, 2001). Complexity signified by non-linearity caused by the topological mappings becomes the general rule of relationally learning systems as opposed to linearity of relations in differentiated systems that mutate or reflect the preferences of methodological individualism. We deal with this issue of 'de-knowledge' briefly later on in this paper.

s denotes a mapping from *T* 'into' the domain of knowledge flows. This mapping was identified earlier by the example of the ontology of the Prophetic guidance in Islam. We denote the direction of the epistemic mappings as,

$$T \to s(T) \to \{\theta\}$$

Following up on the properties of topology (Maddox, 1970; Robertson and Robertson, 1966), $\{\theta\}$ forms an open subset contained within a subset of *T*, denoted by $s(T)$. Thus for a particular knowledge value pertaining to a range of interaction *i*, say θ_i, we write,

$$\theta_i \in \{\theta\} \in T$$

The following case holds as well:

$$\cap_i\ \theta_i \neq \phi\ \ \in T$$

Besides, $\cap_i\ \theta_i$ (as also the previous intersection property) is under the impact of dynamic θ_i- values. Thus, $\cap_i\ \theta_i$ forms an open subset of *T*.

In the more general sense with say *i* number of interaction and *j* entities in their systems (variables) we have,

$$\cup_i\ \cap_i \{\theta_{ij}\} \in T \tag{1}$$

Also the complement of $\{\theta\}$, say $\{\theta\}^c \in$ *T*. Since *T* is complete as the stock of knowledge, therefore $\{\theta\}^c$ also forms an open subset of *T*. We can therefore write,

$$\{\theta_i\} \cap \{\theta_i\}^c = \phi \in T \tag{2}$$

In reference to our earlier characterization of the universality and uniqueness properties of (*T, s*), this specific topology cannot be probabilistic in nature because of its irreducible and complete nature that enables it to contain 'every' sub-systemic sets. That is (*T, s*) determines all the functions and processes ensuing as knowledge-induced systems of entities. But such sub-systemic members of the topology, as in the case of $\{\theta , \theta_{ij}\}$ and $\{\theta^c , \theta^c_{ij}\}$ and likewise, $\{K(\theta) , K_{ij}(\theta_{ij})\}$ and $\{K^c(\theta^c) , K^c_{ij}(\theta^c_{ij})\}$ are all probabilistic in nature because of their openness and incompleteness in learning ('de-learning') spaces within the topologies defined by (*T, s*). Expression (2) now assumes $p\lim_{ij}$ transformation, as was noted earlier. The symbolization of *p*lim (.) over interaction and systems of entities is subsumed in this notation.

In expression (2), *i* denotes the number of interaction taken in the set of interactions *I* across entities contained in the set of entities *A* in their sets of systems denoted by *S*. As was explained earlier relational learning is identical with interaction that lead to integration and are followed by further evolutionary learning (*IIE*-process). Thus implicitly, each knowledge flow $\{\theta_i\}$ spans (*I*, *A*, *S*). That is knowledge-flows occur over interaction, integration and evolution. ($i \in I$) of entities (*A*) in their sub-systems (*S*). Both intra-systemic and inter-systemic *IIE*-processes are implied. These *IIE*-processes lead to the formation of $\{\theta_i\}$, *i* = 1, 2,… The idea of the *IIE*-process is further brought out in Figure 1.

Interactive, integrative and evolutionary (*IIE*) learning processes:

$T \to s(T) \to \{\theta\} \to \{\mathbf{x}(\theta)\}$ New $\{\theta^*_{N_i}\}$ values etc. in continuum

Social Wellbeing Function /Generalized topological mapping

Simulation $_{(\theta N_i)}\ W \circ s = f(\mathbf{x}^*(\theta^*_{N_i}) , \theta^*_{N_i}) = h(T)$

Subject to circular causation in the multiple $\{\mathbf{x}^*(\theta^*_{N_i}), \theta^*_{N_i}\}$ - variables.

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Continuity of the relationally learning *IIE* processes :

$\{\theta^*_{N_i}\}$

$T \to s(T) \to \{\theta\}$ → continuity across continuums

$\mathbf{x}^*(\theta_{N_i})$

↓

Simulation $_{(\theta N_i)}\ W \circ s = f(\mathbf{x}^*(\theta^*_{N_i}) , \theta^*_{N_i}) = h(T)$

Subject to circular causation in the multiple $\{\mathbf{x}^*(\theta^*_{N_i}), \theta^*_{N_i}\}$ - variables

Figure 1. The string relation of unity of knowledge and it's implications.

Any temporary phase of learning is characterized by a given level of interaction converging to integration followed by evolution, whereby a subsequent phase of similar learning emerges from recalling of (*T, s*) and a fresh knowledge flow is derived from (*T, s*). Such process-oriented phases carry onwards fresh knowledge flows that are learned out of the experience of previous probability limiting domains of $\{K(\theta)\,,\,K_{ij}(\theta_{ij})\}$ and $\{K^c(\theta^c)\,,\,K^c_{ij}(\theta^c_{ij})\}$ over (*I*, *A*, *S*).

We write in the probability limiting sense of relational learning with the simplified case of *i* denoting interaction only over *N*-rounds:

$$\underset{i \to N_i}{p\lim}\{\theta_i\} = \theta^*_{N_i}\ , \quad N_i \in \{N_1, N_2, \ldots\} \tag{3}$$

This probability limiting property can be further extended across entities and systems. The result will then be,

$$\underset{\substack{i \to N_i \\ A \to A_i \\ S \to S_i}}{p\lim}\{\theta_{i,\,A,\,S}\} = \theta^*_{N_i,\,A_i,\,S_i}\ , \quad N_i \in \{N_1, N_2, \ldots\} \tag{4}$$

Here, $N_i \in \{N_1, N_2, \ldots\}$, $A_i \in \{A_1, A_2, \ldots\}$, $S_i \in \{S_1, S_2, \ldots\}$.

The probabilistic nature of relational learning across evolutionary phases (processes) signifies that evolutionary dynamics is intrinsic in knowledge-induced systems of entities.

The properties of the knowledge stock (*T*) and flow (θ) shown in expressions (1) and (2), are also true for the extended system of relations shown in expression (4). Thus both (*T*, *s*) and its derived knowledge flows comprise topologies (Maddox, 1970).

The property of the mapping *s* maintains the topological nature of learning in unity of knowledge derived from the core of *T* and of the knowledge induction of systemic variables, $\{\mathbf{x}(\theta_i)\}$ and their topological relations.

Let $\mathbf{x}(\theta_i)$ denote the knowledge-induced vector variables. Because of the properties in expressions (3) and (4) we can write,

$$\underset{i \to N_i}{p\lim}\{\mathbf{x}(\theta_i)\} = \mathbf{x}^*(\theta^*_{N_i})\ , \quad N_i \in \{N_1, N_2, \ldots\} \tag{5}$$

$$\underset{\substack{i \to N_i \\ A \to A_i \\ S \to S_i}}{p\lim}\{\mathbf{x}(\theta_{i,\,A,\,S})\} = \mathbf{x}^*(\theta^*_{N_i,\,A_i,\,S_i}) \tag{6}$$

for, $N_i \in \{N_1, N_2, \ldots\}$, $A_i \in \{A_1, A_2, \ldots\}$, $S_i \in \{S_1, S_2, \ldots\}$.

I denotes the totality of the interaction domain, $I = \{N_1, N_2, \ldots, N_i, \ldots\}$; *A* denotes the totality of the agents domain, $A = \{A_1, A_2, \ldots, A_k \ldots\}$; *S* denotes systems $S = \{S_1, S_2, \ldots, S_l, \ldots\}$.

5. The social wellbeing expression (*W*(.))

We have now defined a mapping from *T* to (θ_i) by *s* and from (θ_i) to $\{\mathbf{x}(\theta_i)\}$, say by *W*. Because of the irreducible nature of *T*, this being the complete topology and because all other sets are proper subsets of (*T*, *s*) as topologies in their own, we refer to the self-referencing nature of these topologies as a property of unity of knowledge derived from the fundamental root of (*T*, *s*).

We define a functional map W characterized by all the properties of topological mapping defined on the ($\mathbf{x}^*(\theta^*_{N_i})$, $\theta^*_{N_i}$) - domain and which carry the probabilistic transformation as,

$$W = W(\mathbf{x}^*(\theta^*_{N_i}),\ \theta^*_{N_i}) \tag{7}$$

with

$$s(T) = \theta^*_{N_i} \tag{8}$$

$s(T)$ is taken as a proper and open subset of T. Furthermore,

$$\mathbf{x}^*(\theta^*_{N_i}) = f_1(\theta^*_{N_i}), \tag{9}$$

because of the Jacobian inversion of $\mathbf{x}^*(\theta^*_{N_i})$ in terms of $\theta^*_{N_i}$ denoted by the non-vanishing function f_1. Therefore, W^{-1} exists as a result of Jacobian inversion by the non-vanishing of f_1. Hence by composite mapping '$\circ$' we write,

$$W \circ s = f(\mathbf{x}^*(\theta^*_{N_i}), \theta^*_{N_i}) = h(T) \tag{10}$$

$W \circ s$ is a continuously differentiable function of T mapped into a proper open subset of $\{\mathbf{x}^*(\theta^*_{N_i}), \theta^*_{N_i}\}$. The sets of $\{\mathbf{x}^*(\theta^*_{N_i}), \theta^*_{N_i}\}$ in terms of the open topological set $\{\theta^*_{N_i}\}$ and their functional relations comprise topologies as proper open subsets of the complete topology T.

Yet it is not necessarily true that $W^{-1}(\mathbf{x}(\theta^*_{N_i})) = \theta^*_{N_i}$, because there can be multiple probability limiting values in the mapping between $\theta^*_{N_i}$ and $\mathbf{x}(\theta^*_{N_i})$, causing perturbation sets to appear in the domains of these variables. Recursive and discursive learning also causes such consequences in social systems.

Such learning processes are current themes of sociology (.), philosophy (Whitehead, 1978) and the process worldview of economics (Georgescu-Roegen, 1961; Boulding, 1971). They have also entered the study of science as process in recent times (Prigogine, 1980; Hull, 1988). The essential point underlying all these is the relational viewpoint of learning systems. Yet it is true that socio-scientific disciplines despite their search for relational processes (Thayer-Bacon, 2003) have not been able to develop a cogent methodology (Sztompka, 1991) in the field of unity of knowledge. The problem lies in the fundamental epistemology of received socio-scientific reasoning.

Epistemologies are over-determined in mainstream sciences, even in those with social meaning (Resnick and Wolff, 1987). Contrarily, our methodology of unity of knowledge in the study of processual world-system issues in this paper is premised on a unique and universal epistemology that cannot thereby accept over-determination of competing epistemologies.

The meaning of expression (10) is now particularized from the generalized form of this topological mapping over its domain into a Social Wellbeing Function (W(.)). Such a function is central for evaluating the degree to which the domain of the function has been relationally unified as a participatory social system. Social Wellbeing Function defined on the knowledge space of the compound mapping, $T \to s(T) \to \{\theta\} \to \{\mathbf{x}(\theta)\}$ has an intermediary function that causes further mappings of this kind to emerge from each completed learning *IIE*-process. This generates θ -values in continuum. The function W post-evaluates a process of relational learning experience subject to the circular causation between the variables of the domain.

6. The formal nature of interactive, integrative and evolutionary learning: circular causation

The following characterization of the relational learning process gives an explicit epistemological background of learning consequences mentioned above in the probabilistic sense and its ontological form (methodological formulation). The meaning of ontology used is of the engineering category as opposed to being of a metaphysical nature (Gruber, 1993).

Figure 1 as a whole points out the simulation nature of probabilistic learning defined by the epistemology and ontology of (*T*, *s*) on the (θ, $\mathbf{x}(\theta)$) - domain. In Figure 1, $\{\theta^*_{N_i}\}$ denotes new $\{\theta^*_{N_i}\}$ - values in continuum of relational learning. The probabilistic nature is carried across in such a simulative and topological space of evolutionary learning over the domain (θ , $\mathbf{x}(\theta)$). Despite this open nature of the learning domain there always comes about temporary evolutionary equilibriums in every phase of the simulative relational learning over IIE-processes (Grandmont, 1989). Consequently, there is temporary boundedness in the probability limiting values of the domain variables. Such evolutionary equilibriums are only expectational in nature, in the sense that the core of equilibrium cannot be attained, as learning continuously shifts the expectational equilibrium along the path of evolutionary equilibriums (Choudhury, 2006c; Thurow, 1996).

Probability limits of expectational equilibriums that do not attain the core equilibrium but are deflected away by relational learning across processes in continuum can be explained by the measure theoretic understanding of probability measure. We have explained the measure-theoretic implication of probability limit equilibriums earlier in this paper.

IIE-processes continue across continuums post-evaluation of unity of knowledge by the simulation of Social Wellbeing Function taken over *IIE* processual domains of relational learning.

7. Dimensionality of the social wellbeing function in learning spaces

Finally, the matter of dimensionality of *W* needs to be addressed in this section. *W* by definition is the positive monotonic measure of unity of knowledge attained by relational learning along the *IIE*-processes and circular causation between the variables of the domain of the *W*-function. *W* can assume various forms. As a complex non-linear function of the domain variables in any one system (e.g. economy) *W* represents one objective function in a vector of variables of this system. When *W* is compounded with multiple social wellbeing indices (W_s say, s = 1, 2,….*N*) of similar relationally learning types then,

$$W = \prod_{s,\, s'=1}^{N} [W_s \circ W_{s'}] \, , \; s \neq s' = 1, 2, \ldots N \, .$$

Each W_s has similar character of relational unity of knowledge both between their individual domains and across domains, as diverse but relationally learning systems in unity of knowledge interact, integrate and evolve. Finally, if the objective is to explicate inter-systemic *IIE*-processes, as for example in the case of specifically interdependent securities within a portfolio of financial securities, then *W* assumes matrix characterization denoted by [.] here (Jean, 1970). Now, $W = [W_{ss'}]$, s , $s' = 1, 2, \ldots$, which will include the identity case of $s = s'$, with W_{ss} = W_s. Besides, the interdependent variables across circular causation processes of relational learning will also be expressed interactively.

According to Figure 1, we now have a simulation problem of the Social Wellbeing objective criterion over θ -variables. Once again, θ -variables are understood in the sense of $p\lim_{ij}\{\theta_{ij}\}$, as mentioned earlier in this paper. The dimensional issue of *W*-function is now established as $dimW = N$ in the case of *N* variables within a system or compounded system of sub-systems of variables and $dimW = N \times N$ in the case of matrix system.

The dimensionality of the circular causation variables is as follows:

dim(variables ; $dimW = N$) = $N \times N$ variables and equations, respectively.

dim(variables ; dim$W$ = N^2) = $N^2 \times N^2$ number of variables and equations.

Matters of actually estimating such systems with policy and instrumental revisions to the estimated dynamic coefficients of such a simulation system, as implicated in Figure 1, is beyond the scope of this paper. The interested reader may refer to Choudhury and Hossain (2006 forthcoming). A simple empirical version of the simulation problem is given in Technical Appendix 2.

8. Implications of the methodology of relational learning in unity of knowledge in the social sciences

It can be noted from Figure 1 and the implications of the positive monotonic and continuously differentiable property of the probability measure-theoretic transformation over the domains of the learning sets (i.e. over phases of *IIE*-processes between entities and their sub-systems) that these entities and systems learn recursively between processes in a circular causal way. That is along a history of such evolutionary learning processes that recursive share experiences forward and backward both over time and continuums.

In the social sciences as an example, the recursive learning across *IIE*-processes in the mode of circular causation conveys how $s(T)$ generates $\{\theta\}$ values and the resulting measure-theoretic functional forms denoted in general by say $f(.)$. Now the Social Wellbeing Function W appears as one such evaluative criterion of the positive monotonic, continuously differentiable function over its domain that evaluates the degree of unification attained by circular causation interrelations between the included domain variables (vectors) and systems (matrices). This kind of plethora of circular causations between interactions, entities and their systems signify relational unity of knowledge between them. They are extended further over learning *IIE*-processes and convey the moral and ethical nature of the entire methodology. The knowledge flows are representations of the moral and ethical values in the socio-scientific studies. They are further endogenously extended by relevant state and policy variables. Such extended domains of the variables learn perpetually over time and space along the *IIE*-processes. All such variables are influenced by the epistemology emanating from $T \rightarrow s(T)$.

9. The knowledge-induced operator

We now set up the following expression in view of expression (1) to mean interaction ($i \in I$) among entities with integration ($\cap_A$) across systems (systemic integration = $\cap_S$).
Thus we write these properties together as,

$$(\cup_I \cap_A \cap_S)[\mathrm{W}(\mathbf{x}^* (\theta^*_{N_i}), \theta^*_{N_i})]_A{}^S{}_{, \mathrm{I}}$$

Here any function or variable within the operator would be carried over interactions $i \in I$, I being the totality of interaction domain, and across the totality of systems S and the totality of agents A. ${}_A{}^S{}_{, \mathrm{I}}$ means the number of agents (A) in S number of systems that go through numbered interaction in the interaction set I.

The dynamic continuity of knowledge-flows $\{\theta^*_{N_i}\}$, and their monotonically positive effect on $[W(\mathbf{x}^* (\theta^*_{N_i}), \theta^*_{N_i})]_A{}^S{}_{, I}$ is expressed as,

$$\frac{d}{d\theta^*_{N_i}} \{\cup_I (\cap_A \cap_S)[W(\mathbf{x}^* (\theta^*_{N_i}), \theta^*_{N_i})]_A{}^S{}_{, I}\} > 0. \tag{11}$$

Note that we have changed f to W in order not to add yet another new symbol. Note also that $W(\mathbf{x}^*(\theta^*_{N_i}), \theta^*_{N_i})_A{}^S{}_{,\ I}$ being continuously differentiable over $\{\mathbf{x}^*(\theta^*_{N_i}), \theta^*_{N_i}\}\ {}_A{}^S{}_{,\ I}$, means $(\cup_I \cap_A \cap_S)[W(\mathbf{x}^*(\theta^*_{N_i}), \theta^*_{N_i})]\ {}_A{}^S{}_{,\ I}$ is also continuously differentiable in the same open sets of variables defined in their epistemology denoted by $T \to s(T)$.

Because of the above properties of the continuously differentiable functions the differentiation as shown in expression (11) exists over values of $\{\mathbf{x}^*(\theta^*_{N_i}), \theta^*_{N_i}\}_A{}^S{}_{,\ I}$. $[W(\mathbf{x}^*\ (\theta^*_{N_i}), \theta^*_{N_i})]\ {}_A{}^S{}_{,\ I}$ is thus differentiable around the limiting knowledge-induced variables $\{\mathbf{x}^*\ (\theta^*_{N_i}), \theta^*_{N_i}\}_A{}^S{}_{,\ I}$ as a series by Taylor's theorem.

Now by dropping the specificity of the limiting values, $\{\mathbf{x}^*(\theta^*_{N_i}), \theta^*_{N_i}\}$, we define the knowledge-induced operator as the interactive (I), integrative (I) and evolutionary (E) – thus the *IIE*-process in the form given by,

$$\frac{d}{d\,\theta_i}\{(\cup_I \cap_{A^S})[.]\ {}_A{}^S{}_{,\ I}\} > 0 \tag{12}$$

This operator summarizes the entire idea of unity of knowledge in terms of its application to relational unity of knowledge that is derived from the epistemic core, (T, s). This is signified by the fact that any included function and its variables within [.] is monotonically positive transformation in the *IIE* relational learning processes. This is the principle of pervasive and continuous complementarities in the relationally learning system of unity of knowledge as derived from the epistemology of (T, s). In the case of the differentiated system of 'de-knowledge' θ^c (mathematical complementation as opposite of θ) de-learns, but [.] is still positive monotonic and continuously differentiable in terms of θ^c.

Contrarily, this is not the case in non-(T, s) epistemic cores. In the case of neoclassical economic theory the variables of the objective function enclosed by [.] remain marginal substitutes of each other and complementarities between them are problematic (Holton, 1992). Now θ^c is derived from the realm of individualism, the ego, which is a matter of the ontological origins of knowledge as perceived in the liberal order (Heidegger, 1988; Minogue, 1963). Consequently, by the concept of 'de-knowledge' in neoclassical economic theory the sign of expression (12) will be negative.

In the special case of the Social Wellbeing Function $W(\mathbf{x}(\theta_i), \theta_i)$ in [.] we get the following result:

$$\frac{d}{d\,\theta_i}\{\,(\cup_I \cap_{A^S})[W(\mathbf{x}(\theta_i), \theta_i)]\ {}_A{}^S{}_{,\ I}\} > 0, \tag{13}$$

where , $\cap_A{}^S = \cap_A \cap_S$; $A = \{A_1, A_2,\ldots, A_i \ldots\}$; $S = \{S_1, S_2,\ldots, S_i ,\ldots\}$;
$I = \{N_1, N_2 , \ldots, N_i, \ldots\}$ and $i = 1,2,\ldots\ldots$

The properties of W-function in terms of the variables and θ-values were given earlier. All the notations were defined.

In the language of social theory expression (13) signifies that integration resulting from inter- and intra-system interaction are continuously evolved under the impact of new sequences of knowledge values $\{\theta_i\}$ derived from the epistemology $T \to s(T)$.

The operator conveys many knowledge-induced properties all at once. That is, there are extensive interactions (I) in generating knowledge flows. These interactions lead to integration across agents and systems in terms of the knowledge flows (I). The interactively formed integration then evolves continuously by the passage of knowledge flows into more of the same kind of processes circularly in continuously differentiable sequences (E). The higher is the positive value of the operator $(d/d\theta_i)[.]_{A}{}^{S}{}_{,\ I} > 0$ in reference to $W(.)$, the more integrated becomes the system and the more capable it is of evolving to higher levels of complementarities that are formed by interrelations between the entities with systemic diversity (Choudhury, 2000).

The above kind of densification of knowledge and the *IIE*-process define the continuously learning character of the system (13) as defined by $(d/d\theta_i)[.]_{A}{}^{S}{}_{,\ I} > 0$. In other words, existence of complementarities across diversity is the sure sign of realizing unity within diversity through the *IIE*-process (see Technical Appendix 1). The expression within [.] can be of specific types, though this operator is capable of examining the three attributes of interaction, integration and evolution (*IIE*-process) across all possible complementary networks of diverse systems, agents, variables and their intra- and inter- systemic interrelations.

Let A denote the set of entities as agents and variables together with their relations ; S denote systems of agents and variables together with their relations and I denote the total number of interaction within which $\{\mathbf{x}(\theta_i),\theta_i\}$ are generated and then acquire their limiting values as shown earlier. In this way, expressions (1) - (13) have already been built into the operator.

A note on the continuously positive value of $(d/d\theta_i)[.]_{A}{}^{S}{}_{,\ I} > 0$ needs to be explained. Three particular kinds of learning can constrain the continuously evolutionary nature of this operator in the *IIE*-processes. Firstly, when the learning has lags between the *IIE*-processes particular knowledge values θ_i in $\{\theta_i\}$ across ${}_{A}{}^{S}{}_{,\ I}$ are temporarily halted from evolution**.** Then we do not have evolutionary equilibriums with punctuated limiting θ_i (Grandmont, 1989). Secondly, when θ_i = constant, then learning ceases. The result conveys neoclassical marginal substitution between alternatives at the optimal and steady-state point of equilibrium resource allocation. Now all novelty of learning is lost (Shackle, 1971). The variables cannot participate and therefore cannot complement in any social interrelationship. Thirdly, the ideal case is of continuously learning, which is a phenomenon of symbiotic socio-scientific world-systems (Yolles, 1998).

10. An application

$W = W(\mathbf{x}^* (\theta^*_{N_i}), \theta^*_{N_i})$ as the social well-being function describes complementarities between the vectors of knowledge flows and their induced variables. Functional relations of such paired complements signify relational learning in systemic unity of knowledge. This is the meaning of interaction (i.e. participation). The diversities of interrelations lead to integration (complementarities). Next the creative evolution of $W(.)$ is caused by the evolutionary epistemology of $\{\theta_i\}$, and hence of $\{\mathbf{x}(\theta_i),\theta_i\}$and $\{W(\mathbf{x}(\theta_i),\theta_i)\}$ over *IIE*-processes. The Social Wellbeing Function as a criterion signifying degrees of complementarities attained establishes the spirit of unity of knowledge. Such a social criterion contrasts sharply with the neoclassical economic meaning of social welfare. The latter is based on marginal substitution between variables and agents, and hence between systems. The neoclassical principle of marginal substitution imparts the causal relationship between the axioms of methodological individualism, competition and independence in an optimal state of resource allocation with scarcity of resources.

The UNDP has presented a specific form of a wellbeing index. *World Development Report* writes as follows with regards to the human development while addressing the topic of poverty alleviation in developing countries (World Bank, 2000, p.7):

"There is no hierarchy of importance. The elements are deeply complementary. Each part of the strategy affects underlying causes of poverty addressed by the other two."

The Report then carries on (p.19): "One approach to addressing comparability is to define a multidimensional welfare function or a composite index. An alternative is to define a poor as anybody who is poor in any of the dimensions without attempting to estimate tradeoffs among the dimensions."

The stand of the World Bank and UNDP on the complementary social approach to poverty alleviation is the newest well-thought out methodology of socio-economic development. It requires a framework of theorizing and application to poverty alleviation, entitlement and empowerment formation and institution-market interaction that can be addressed by the principle of pervasively complementary relations across diversity of issues and problems. So far the World Bank and the UNDP have not come up with another model of the type we are prescribing in the framework of unity of knowledge with the social wellbeing function as a quantitative measure of complementarities between the knowledge-induced variables.

11. Implications on complexity

Systemic interaction, integration and evolution under a continuum of knowledge-flows and their induced socio-scientific variables lead to networking. Networking may be either over orderly domains of knowledge or over chaotic domains of 'de-knowledge'. In the case of equilibrium dynamics of the punctuated type conveyed by the knowledge-induced operator of expression (12) yields the following result (we have the specificity of $i \in I$ in expression (12)):

$$\frac{d}{d\theta}\{(\cup_I \cap_{A^S})[W(\mathbf{x}(\theta),\theta)]_{A^S, I}\} > 0, \text{ for } \left|\theta - \theta^*\right| < \varepsilon(\theta^*) \qquad (14)$$

$\varepsilon(\theta^*)$ is an arbitrarily small quantity based on θ^*- induction**.** The knowledge-induced form has a similar property:

$$|\mathbf{x}(\theta) - \mathbf{x}^*(\theta^*)| < \eta(\theta^*)$$

where $\eta(\theta^*)$ is an arbitrarily small quantity defined around $\mathbf{x}^*(\theta^*)$ based on θ^*.

The limiting values $\{(\mathbf{x}(\theta),\theta)\}_{A^S, I}$, and thereby $\{[W(\mathbf{x}(\theta),\theta)]_{A^S,I}\}$, are probabilistic in nature. This signifies the incompleteness of learning from the perfect state of knowledge and knowledge-induced forms at any given *IIE*-process relating to the problem under investigation in the dynamically (*IIE*) learning socio-scientific space. Equivalently the expressions,

$$\left|\theta - \theta^*\right| < \varepsilon(\theta^*)$$

and

$$|\mathbf{x}(\theta) - \mathbf{x}^*(\theta^*)| < \eta(\theta^*)$$

establish the continuity of the variables and function relating to,

$$\{(\cup_I \cap_{A^S})[W(\mathbf{x}(\theta),\theta)]_{A^S, I}\}$$

Note that because of the topological nature of the mappings that occur between the knowledge-flows and their induced forms, diverse forms of the distance functions can be defined, depending upon the incomplete nature of learning with evolutionary equilibriums. In the simplest case with linearity in the learning process such distances are metric in nature. In the complex case they can be extended to higher dimensions, such as in Banach space (Choudhury, 1993).

12. A brief note on 'de-knowledge' relations in the light of the (*T*, *s*)-methodology

The formalism for the case of 'de-knowledge' experience of differentiated world-system of methodological individualism is also addressed by the same universal and unique episteme of (*T*, *s*), but as a consequence that is disjoint of relational learning in unity of knowledge. The 'de-knowledge' character is conveyed by the continuously evolutionary processes that cause differentiation instead of integration out of interaction in mutations. Such mutative social processes reflect social Darwinism (Dawkins, 1976). The circular causation processes now define temporary bundles of interactive social organisms that share in their over-determined epistemologies, but subsequently cause plethora of mutations by competition, conflict and independence between the present generations and their parents. Methodological individualism following subsequently from the early temporary interactive processes becomes the permanent character of the mutated world-system (Buchanan, 1971, 1999).

13. 'De-knowledge' operator

In the above-mentioned case when learning ceases with θ_i = constant, we have the case of a 'de-knowledge' operator. We now replace all the variables by the ones induced by 'de-knowledge'. The 'de-knowledge' connotation of the variables-domain is depicted by $(\mathbf{x}^c(\theta^c), \theta^c)$. Previously, θ^c was defined as the opposite (mathematical complementation of θ). Now two cases arise: Firstly, when the problem objective is one of optimization as opposed to simulation of continuous relational learning then,

$$\frac{d}{d\theta'}\{(\cup_I \cap_{A^S})\,[W(\mathbf{x}^c(\theta^c), \theta^c)]_{A^S, I}\} = 0 \tag{15}$$

for

$$\left|\theta^c - \theta^{c*}\right| = \varepsilon(\theta^{c*})$$

and

$$|\mathbf{x}^c(\theta^c) - \mathbf{x}^{c*}(\theta^{c*})| = \eta(\theta^{c*}),$$

here, * refers to the whole of θ^c and $\mathbf{x}^c$, as the case may be, in the probability limiting sense.
Secondly, for the 'de-knowledge' system,

$$\frac{d}{d\theta^c}\{(\cup_I \cap_{A^S})[W(\mathbf{x}^c(\theta^c), \theta^c)]_{A^S, I}\} = 0 \tag{16}$$

for $\left|\theta^c - \theta^{c*}\right| = 0$ and for $|\mathbf{x}^c(\theta^c) - \mathbf{x}^{c*}(\theta^{c*})| = \eta(\theta^{c*})$.

In other words, in the language of social sciences in the case of 'de-knowledge' methodology the methods of optimization and steady-state equilibriums pertaining to the problem under investigation replace the simulation (continuously learning) methodology of the knowledge-induced *IIE*-process. The concept of 'de-knowledge' conveyed in the optimal and steady-state equilibrium case is the end of learning in (15) and evolutionary non-convergence and hence a de-unified system in (16). All these conditions are taken in the probabilistic sense of evolutionary equilibrium into order or disorder, respectively. The symbol *p*lim is not explicitly repeated.

Only expression (13) conveys complexity with knowledge in the probabilistic sense. This is distinct from the non-learning though convergent order of expression (16). In this regard Stewart (1989, p. 55) writes:
"…. Simple systems behave in simple ways, complicated systems behave in complicated ways. Between simplicity and complexity there can be no common ground."

14. Conclusion

From the knowledge-centered methodology a vast number of conceptions and possibilities can be investigated in cases where complementary relations of pervasive types replace the neoclassical postulate of marginal substitution. The knowledge-centered worldview and methodology thus mark a substantive revolutionary scientific shift in terms of the epistemology of unity of knowledge and its induced relational forms as vectors, matrices and tensors of variables and their complex transformations. Daly (1992) has incisively argued that the concept of complementarities between human capital and natural capital against the orthodox neoclassical concept of 'marginal substitution' between goods, resources and productive factors marks the rise of a new and powerful economic paradigm.

The reality in socio-scientific matters that we see today is that of extensive interdependence between various domains of human activity (Sztompka, 1991). When we address the topic of human ecology (Hawley, 1986), constitutional economics (Buchanan, 1999, reprinted), political economy of globalization (Henderson, 1999), poverty alleviation and equality (Sen, 1986), money and real economic activities (Choudhury, 1997) and we can go on in this list, we essentially address the topic of interaction, integration and their further evolution (*IIE*) by learning in unity of knowledge. This forms the body of complexity phenomena in which the human world realistically dwells. We have addressed such a problem of complexity but with probabilistic determination in generalizing the *IIE*-process model that can be cogently summarized in the knowledge-centered operator.

The operator is shown to summarize the entire model and implications of the relationally learning processes called the *IIE*-processes. These are evaluated for the attained degree of unity of systemic knowledge by reference to the fundamental epistemology of unity of knowledge. We have introduced the critical evaluation function for this criterion and called it substantively as the Social Wellbeing Function. This has been developed at length.

While in this paper we have formulated our methodology and the model of relationally learning systems of knowledge-flows and their knowledge-induced variables and forms, the same epistemology of unity of knowledge is capable of explaining the mathematical complementation (opposite) of knowledge – 'de-knowledge'. The uniqueness and universality of T is shown to explain both truth (knowledge) and falsehood ('de-knowledge') due to the topological property of completeness of both of these in T. T thereby contains the set $\{\theta\}$ and its mathematical complementation (opposite) $\{\theta^c\}$. Likewise, the methodology of $\{\theta^c\}$ is extended to its cognitive forms and relational functions. The primal difference between these two evolutionary kinds of systems is this: While $\{\theta\}$ generates complementary relations out of diversity, $\{\theta^c\}$ generates marginal substitution, independence and methodological pluralism along its evolutionary path of Darwinian mutations. 'Global' complementarities and hence a relational precept of reality becomes increasingly impossible in the latter case.

Technical appendix 1: mathematical results relating to knowledge-induced operator

Some results related with the operator of the knowledge-centered methodology can be derived as follows. For mathematical details see Maddox (1970):

Let θ_1^* be the limiting value of the sequence of interactions, $\{1,2,\ldots N_1\}$ and θ_2^* be the limiting value of the sequence of interactions, $\{N_1 + 1,\ldots N_2\}$, etc. Thus,

$$\cup_I = \{1, 2, \ldots, N_1,\ N_1+1, \ldots, N_2, \ldots\} \tag{A1}$$

Consequently, we obtain for $\{1, 2,\ldots, N_1\}$, $\{N_1+1\ldots, N_2\}$………

$\downarrow \qquad\qquad \downarrow$

the values of $\theta(I, A, S)$, such as, $\theta^*(1, A, S)$, $\theta^*(2, A, S)$…….. (A2)

That is,

$$p\lim_{I \to \infty} \cup_I \{\theta(I, A, S)\} = \{\theta^*(A, S)\} \tag{A3}$$

Likewise,

$$p\lim_{(A,S)} \cap_{A^S} \{\theta^*(A, S)\} = \theta^* \tag{A4}$$

over all possible A and S. We can re-write expression (A3) and (A4) as,

$$p\lim_{(A,S)} \; p\lim_{I \to \infty} \cup_I \cap_{A^S} \{\theta(I, A, S)\} = \theta^* \tag{A5}$$

In this way, expression (12) in the text is established in the limiting sense.

Furthermore, it can be shown that,

$$p\lim_{I \to \infty} \; p\lim_{(A,S)} \{\cup_I \cap_{A^S} \{\theta(I, A, S)\} = \theta^* \tag{A6}$$

This can be readily proven by starting in the same way as (A1) - (A5) but commencing with the limits over A and S and then taking the limit over I.

Note however that, expression (A6) although mathematically correct, it does not convey the correct picture on the methodology pertaining to how knowledge-flows are formed first by interaction, which then lead to integration (intersection) across A, S.

Finally, from the result that $\mathbf{x}(\theta^*)$ is monotonically related with θ^*-values and all these categories form topologies of their own, we extend the results (A5) and (A6) to the case of $\{\mathbf{x}(\theta^*)\}$ and their functional relation like $\{W(\mathbf{x}(\theta^*), \theta^*)\}$.

Technical appendix 2: A simple estimation of the learning model

The simple example given below was done by Hossain *et al*. (2006) using the θ-induced model for the problem of effects of globalization on the health illness of the labor force in the United States. The empirical summary is given below.

Table 1 shows how the ordinal values have been assigned discursively in respect to the ratio (X_i/X) of specific ith-industry injury and illness (X_i) compared to the total incidence across all industries (X).

With the assigned values on social wellbeing determined by noting the spread of the incidence of total illness and injury by industries and the given industrial incidence of total illness and injury the cross-section regression involving seven industries is run for the years 1992 and 2002 separately. The estimated slope coefficient α of the estimated equation $\theta = A + \alpha \log X$, signifies the sensitivity of a percentage change in the incidence value of illness and injury X, on the change in worker wellbeing ranked by numerically assigned values of θ. The log in the estimated equation above is taken to base.

Table 1. Assigned ordinal worker wellbeing values by industry-specific illness and injury, U.S. 1992, 2002 (1000 of workers under incidence).

1992

Industries (i)	Ordinal values of worker wellbeing (θ_i)	Total illness & injury by industries (X_i thousands)	(X_i / X) in %
Agriculture Forestry & Fishing	10.2	47.116	2.0212
Mining	11.5	22.972	0.9986
Construction	8.2	209.564	8.9800
Manufacturing	2.0	623.568	26.7500
Transportation & Public Utilities	8.0	224.654	9.6373
Wholesale & Retail Trade	2.1	590.144	25.3161
Finance, Insurance & Real Estate	10.0	60.415	2.5917
Services	2.5	552.665	23.7083

Total Incidence X 2331.098

e. A higher negative value of α corresponds to a higher wellbeing level and vice-versa. Table 2 gives the estimation results.

2002

Industries (i)	Ordinal values of worker wellbeing (θ)	Total illness & injury by industries (X_i thousands)	(X_i /X) in %
Agriculture Forestry & Fishing	10.1	31.52	2.1947
Mining	8.5	11.355	0.7906
Construction	7.1	163.641	11.3941
Manufacturing	4.0	280.005	19.4963
Transportation & Public Utilities	7.0	168.632	11.7416
Wholesale & Retail Trade	2.0	372.192	25.9152
Finance, Insurance & Real Estate	10.0	36.689	2.5546
Services	2.0	372.159	0.2591

Total Incidence X 1436.193

Source: Bureau of Labor Statistics, U.S.A., 1992, 2002

Table 2. Sensitivity of ordinal worker wellbeing values to incidence of illness and injury by industries, U.S.A. 1992, 2002.

1992

Industries (i)	Ordinal values of worker wellbeing (θ_i)	Estimated values of Worker wellbeing ($\hat{\theta}_i$}
Agriculture Forestry & Fishing	10.2	10.53
Mining	11.5	12.64
Construction	8.2	6.16
Manufacturing	2.0	2.96
Transportation & Public Utilities	8.0	5.96
Wholesale & Retail Trade	2.1	3.13
Finance, Insurance & Real Estate	10.0	9.80
Services	2.5	3.32

2002

Industries (i)	Ordinal values of worker wellbeing (θ_i)	Estimated values of Worker wellbeing ($\hat{\theta}_i$}
Agriculture Forestry & Fishing	10.1	8.89
Mining	8.5	11.07
Construction	7.1	5.37
Manufacturing	4.0	4.23
Transportation & Public Utilities	7.0	5.31
Wholesale & Retail Trade	2.0	3.62
Finance, Insurance & Real Estate	10.0	8.57
Services	2.0	3.62

Source: Bureau of Labor Statistics, U.S.A., 1992, 2002

The estimated equations relating the levels of worker wellbeing θ_i by industrial sectors to total incidence of illness and injury by industries in cross-sectional regression are given below for the years 1992, 2002 separately. The α-sensitivity parameters are shown in the estimated equations.

Estimated Equations

Year 1992: The estimated regression equation on cross-sectional data on illness and injury over industries is,

$$\theta = 21.820 - 2.930 \log X \tag{A7}$$

t-stats (9.754) (-6.890)
R-square = 0.888 ; DW = 3.280

The A and α estimates are confident at the 5% level of significance. The sensitivity indicator is

$$\hat{\alpha} = -2.930 = \frac{d\theta}{d(\log X)} = d\theta / \% \Delta X$$

That is, with one percentage decline in the rate of incidence the worker wellbeing increases by 2.930.

Year 2002: The estimated regression equation on cross-sectional data on illness and injury over industries as shown:

$$\theta = 16.263 - 2.136 \log X \quad \text{(A8)}$$

t-stats (6.247) (-3.944)
R-square = 0.777 ; DW = 3.285

The A and α estimates are confident at the 5% level of significance. The sensitivity indicator is

$$\hat{\alpha} = -2.136 = \frac{d\theta}{d(\log X)} = d\theta / \% \Delta X$$

That is with one percentage decline in the rate of incidence the Worker wellbeing increases by 2.136.

Comparison of the estimated values of α between 1992 and 2002 shows a decrease in worker wellbeing between these years. The decrease in this value of α implies that a decline in incidence of illness and injury did not sufficiently increase the ordinal value of worker wellbeing in the year 2002 compared to that in 1992. The resulting effect of this slack in the α-sensitivity values from a potential one is measured in the following way.

The increase in incidence as a result of the deterioration of the sensitivity parameter α between 1992 and 2002 equals 0.794. That is, the sensitivity parameter could have been potentially improved by this value for the sensitivity parameter to be at least equal to the 1992-estimated α-value (this is the slack). Because of this slack in the sensitivity parameter, worker wellbeing declined (absolute value) from –2.930 in 1992 to –2.136 in 2002. This adversely affected the ordinal value of worker wellbeing by an estimated $0.794 \times \log 1463.193$ (= X) = 5.7869 in the year 2002 based on the logarithmic value of the total incidence over all industries, which equals log 1463.193.

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