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A Fluid Film Squeezed Between Two Parallel Plane Surfaces Subject to Normal Oscillations

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فلم مائع مضغوط بين سطحين متوازيين في حركة تذبذبية عمودية

الصادق أحمد حمزة

خلاصة : يهتم هذا البحث بدراسة الحركة الناتجة من ضغط طبقة رقيقة مائعة بين سطحين متوازيين وفي حركة نسبية. ركزت الدراسة على الحالة الخاصة الناتجة من تثبيت أحد السطحين وتحريك السطح الآخر بدفع مفاجئ من حالة سكون إلي تذبذب متموج في اتجاه عمودي على مستواه. وبالإضافة إلى التوصل لإيجاد حلول تحليلية باستعمال طريقة التشويش المنتظم والتحليل الخطي للمعادلة التفاضلية الجزئية غير الخطية التي تحكم الحركة بصورة عامة ، فقد تم إيجاد حل عددي للمعادلة التفاضلية الجزئية غير الخطية كاملة بطريقة الفروق المحددة. وباستعمال هذه الحلول تم بحث تأثير الحركة المتموجة على أشكال سرعة الفلم المائع وقوته الضاغطة على السطحين.

ABSTRACT: We study the motion which results when a fluid film is squeezed between two parallel surfaces in relative motion. Particular attention is given to the special case where one surface is fixed and the other is impulsively accelerated from a state of rest to a state of sinusoidal oscillations in a direction normal to its plane. In addition to the presentation of analytic solutions which are based on the regular perturbation technique and on the linearised analysis of the resulting nonlinear partial differential equation, a numerical solution of the full nonlinear equation based on a finite-difference scheme is obtained. The effects of the sinusoidal motion on the velocity profiles and on the normal forces which the fluid exerts on the surfaces are investigated.

KEYWORDS: Squeezed film, Impulsive acceleration, Normal oscillations.

1. Introduction

The earliest attempts at the problem of the behavior of a thin film of liquid squeezed between opposing surfaces can be traced to Stefan (1874) and to Reynolds (1886), both of whom confined their attention to the special case where inertial forces are negligible in comparison to viscous forces. Their work is considered as the foundation of hydrodynamic lubrication analysis and later became known as the classical lubrication theory. Interesting and useful studies of the importance of inertia effects have been motivated by the increased machine speeds and low viscosity lubricants. Among authors who have studied the role played by fluid inertia are Ishizawa (1966), Kuzma (1967), Tichy and Winer (1972), Jones and Wilson (1975), and Hamza and MacDonald (1981). The mathematical analysis, when inertia terms are included, is basically based on an iteration or perturbation scheme. The last two authors presented an initial condition that describes the manner in which squeezing is initiated and discussed the length of the transition period during which the regular perturbation solution fails to approximate the exact solution accurately.

The case when one of the surfaces undergoes sinusoidal oscillation in squeezing film flows has received considerable attention due to the important roles it plays in many industrial application, especially in conditions of unsteady loading in machines which is often oscillatory in nature. Fuller (1956) was first to treat this problem. He obtained a solution for the pressure of the fluid between the surfaces by neglecting inertia terms in the Navier-Stokes equation. Kuhn and Yates (1964) extended Fuller's solution by including inertia terms. Their solution agrees with their experimental results. However, they did not take account of the time-dependent boundary condition and their result appears to be in error. Hunt (1966) obtained a similar solution to the one given by Fuller but he allowed for variation in the boundary point. He also performed some experimental work which satisfactorily agrees with his theoretical solution. Terrill (1969) obtained an analytic solution that depends on two parameters, the nondimensional amplitude of the oscillation of the surface and a Reynolds number that is related to the maximum velocity of the vibrating surface. Different cases depending on the magnitude of the two parameters were investigated. A similar solution for the case of oscillating squeeze film with arbitrarily varying surface geometry was presented by Tichy and Modest (1978). They included inertia forces in the equations of motion. In the case of the thrust bearing of fixed inclination, they found that the classical lubrication solution for the load and pressure fluctuations is in error by over 100 percent for Reynolds numbers as low as 5.

In this study we examine the motion of an incompressible viscous flow between two parallel plane disks where one disk is fixed and the other is rapidly accelerated from a state of rest to a state of normal sinusoidal oscillations. The nondimensionalization used by Terrill (1969) will be employed here. The resulting nonlinear partial differential equation that describes the flow is solved subject to boundary and initial conditions. In section (3) analytic solutions through a use of a regular perturbation technique and Laplace transformation are presented and in section (4) a numerical solution to the full nonlinear equation is given. The perturbation solution is in full agreement with Terrill's results. The objective of the study is to investigate the effects of oscillations on the load-carrying capacity and on the velocity profiles.

2. Equations of Motion

We consider the motion of a thin film of fluid squeezed between two parallel coaxial disks which are spaced a distance $h(t^*)$ apart, where $h(0) = H$. We choose cylindrical polar coordinates (r^* , θ^* , z^*) , in terms of which the lower fixed disk is described by $z^* = 0$ and the upper disk by $z^* = h(t^*)$. The corresponding velocity components are (u^* , v^* , w^*) . We shall assume that the fluid is at rest for $t^* < 0$ and that at $t^* = 0$ the upper disk moves impulsively with steady normal oscillations of frequency ω and amplitude V. The Navier-Stokes equations of motion are transformed to nondimensional form by referring all lengths to H , all velocities to V, time to $1/\omega$ and pressure to ρV^2, where ρ denotes density. The corresponding dimensionless variables are those without the asterisks. The configuration is sketched in Figure 1.

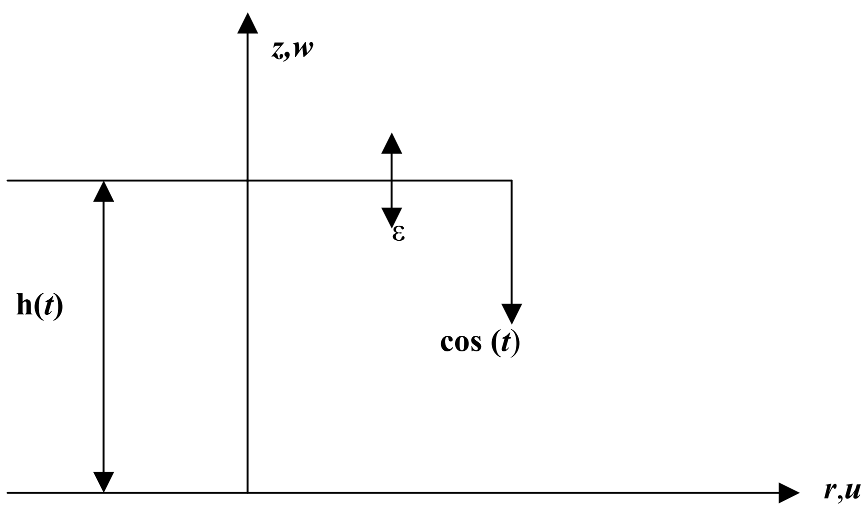

Figure 1. System configuration at time t.

The nondimensional equations governing the flow are:

$$\frac{\partial u}{\partial t}+\varepsilon\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\varepsilon\frac{\partial p}{\partial r}+\frac{1}{R}\left(\frac{\partial^2 u}{\partial^2 z}+\frac{\partial^2 u}{\partial^2 r}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right),$$

$$\frac{\partial w}{\partial t}+\varepsilon\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\varepsilon\frac{\partial p}{\partial z}+\frac{1}{R}\left(\frac{\partial^2 w}{\partial^2 z}+\frac{\partial^2 w}{\partial^2 r}+\frac{1}{r}\frac{\partial w}{\partial r}\right),$$

$$\frac{\partial}{\partial r}(ru)+r\frac{\partial w}{\partial z}=0,$$

where $R = H^2\omega/\nu$ and $\varepsilon = V/H\omega$ (ν denotes kinematic viscosity and ε is the nondimensional amplitude). The boundary conditions are:

$u = 0$ at $z = 0$ and at $z = 1 - \varepsilon \sin t$,

$w = 0$ at $z = 0$ and $w = -\cos t$ at $z = 1 - \varepsilon \sin t$.

The boundary conditions on w suggest that a solution in which w is independent of r should be sought. If we choose a stream function $F(z, t)$ which is such that

$$u=rF_z, \qquad w=-2F,$$

the mass conservation equation will be satisfied. Hence the above nondimensional momentum equations may be expressed in the form

$$F_{zt}+\varepsilon\left(F_z^2-2FF_{zz}\right)=-\frac{\varepsilon}{r}p_r+\frac{1}{R}F_{zzz}, \quad F_t-2\varepsilon FF_z=\frac{\varepsilon}{2}p_z+\frac{1}{R}F_{zz}. \tag{1), (2}$$

Furthermore equations (1) and (2) show that p is of the form

$$p=\frac{1}{2}r^2P_1(t)+P_2(z,t),$$

whence differentiation of equation (1) with respect to z and then use of the change of variables

$$z = (1-\varepsilon\sin t)y, \quad t = t,$$

leads to the equation

$$\varepsilon\cos t\,(1-\varepsilon\sin t)\left(y\,F_{yyy}+2F_{yy}\right)+(1-\varepsilon\sin t)^2F_{yyt}-2(1-\varepsilon\sin t)\varepsilon F\,F_{yyy}=\frac{1}{R}F_{yyyy}. \tag{3}$$

Equation (3) is the same equation given by Terrill (1969). The transformed boundary conditions are

$$F = 0,\quad F_y = 0,\quad y = 0;\quad F = \cos t/2,\quad F_y = 0,\quad y = 1. \tag{4}$$

The radial pressure gradient, $P_1(t)=\dfrac{1}{r}\dfrac{\partial p}{\partial r}$, is given by

$$\varepsilon P_1 = \left[\frac{1}{R(1-\varepsilon \sin t)^3} F_{yyy} - \frac{1}{(1-\varepsilon \sin t)} F_{yt} \right]_{y=0} \tag{5}$$

The initial condition, which states that for $y \neq 0,1$, the vorticity is zero at time $t = 0^+$ is

$$F = \frac{1}{2} y \cos t, \quad \text{when} \quad t = 0^+, \tag{6}$$

Thus at $t = 0^+$ the radial velocity u outside the infinitesimally thin sheets of vorticity which are formed on the surfaces is the inviscid velocity given by the continuity equation

$$u / r = \cos t / 2.$$

If the upper disk is assumed to be of radius c and of negligible thickness, the resultant normal force, or load W is given by

$$W = 2\pi \int_0^c r \left[p\left(r,h,t\right) - p_0 \right] dr,$$

where p_0 is the nondimensional pressure at $r = c$, $z = h$. Thus the above result may be expressed in the form

$$W = -\pi \int_0^c r^3 P_1 \, dr \tag{7}$$

In general, to solve equation (3) subject to conditions (4) and (6) a numerical approach is needed and this will be discussed in section (4). However, analytic approximate results will be considered first.

3. Analytic Solutions

The nonlinear partial differential equation (3) with conditions (4) and (6) can be solved for special practical cases when εR, R and t are small.

3.1 Solution for Small *R*.

The parameter ε is the ratio of the amplitude of the oscillation of the upper disk to the distance apart of the disks and is, therefore, less than one. Thus if R is small then εR will also be small.

By ignoring the initial condition (6), we can obtain from equations (3) and (4) the terms of the perturbation expansion

$$F = \sum_{n=0}^{\infty} \left(\varepsilon R\right)^n f_n\left(y\right) + \sum_{n=1}^{\infty} \left(R\right)^n g_n\left(y\right). \tag{8}$$

The first few terms are

$$f_0 = y^2 \left(\frac{3}{2} - y \right) \cos t, \quad f_1 = -\frac{1}{140} y^2 \left(2y-1\right)\left(y-1\right)^2 \left[\left(y^2 - y - 1\right) \cos^2 t + 7 \right],$$

$$f_2 = -\frac{1}{3880800} y^2 \left(2y-1\right)\left(y-1\right)^2 \left(q \cos^2 t + s\right) \cos t, \quad g_1 = \frac{1}{40} y^2 \left(2y-1\right)\left(y-1\right)^2 \sin t,$$

$$g_2 = \frac{1}{16800} y^2 \left(2y-1\right)\left(y-1\right)^2 \left(10y^2 - 10y - 3\right) \cos t, \tag{9}$$

where

$$q = 1512y^6 - 4536y^5 - 1659y^4 + 10878y^3 - 6207y^2 + 12y + 558,$$
$$s = 14245y^4 - 28490y^3 + 20867y^2 - 6622y - 3234.$$

The radial pressure gradient P_1 is given by

$$P_1 = \frac{1}{r}p_r = -6\left(\left(1-\varepsilon\sin t\right)^3 \varepsilon R\right)^{-1}\left\{\cos t + \frac{\varepsilon R}{140}\left(28 - 3\cos t\right) - \frac{(\varepsilon R)^2}{1940400}\left(3157 - 1110\cos^2 t\right)\cos t - \frac{R}{10}\sin t + \frac{R^2}{8400}\cos t\right\} \tag{10}$$

This is the forced solution in which the initial condition (6) is neglected and so we expect it not to be valid in the region of $t = 0$. However, for t not small and $R = o(1)$ and $\varepsilon R = o(1)$, the solution is a good approximation to the motion of the fluid. The particular case $R = 0$, which has the simple solution $f_0 = y^2\left(3/2 - y\right)\cos t$, has been discussed by Fuller (1956), Kuhn and Yates (1964) and Hunt (1966). Terrill (1969), correctly obtained f_0 , f_1 , g_1 and g_2 through using a perturbation scheme. When cos t = 1 and sin t = 0, the solution (the zero and first orders) reduces to the solution given by Ishizawa (1966) and Jones and Wilson (1975). They obtained the solution correct to $O((\varepsilon R)^2)$, (note that $\varepsilon R = HV / \nu$ is the usual squeezing Reynolds number) . For this case, good agreement with experiment, even when the Reynolds number was an order of magnitude greater than unity, was reported by Tichy and Winer (1972) and Kuzma (1967). The good agreement between theory and experiment at values of $\varepsilon R > 1$ can be explained by reference to the remarkable decrease in magnitude of the functions f_i, $i > 0$.

3.2 Solution for Small εR and Small t.

The perturbation solution described in section (3.1) cannot satisfy the initial condition which states that the vorticity is zero at $t = 0^+$. Here we look for a solution satisfying the initial condition. For t and εR small equation (3) becomes

$$RF_{yyt} = F_{yyyy} \tag{11}$$

To solve equations (11), (4) and (6), we shall employ Laplace transformation (Hamza and MacDonald (1981)) technique to get a solution which is rapidly convergent for $t \to 0$.

3.2.1 Laplace Transform Solution

The outline of the solution starts by integrating equation (11) with respect to y twice to get,

$$F_{yy} + A(t)y + B(t) = RF_t, \tag{12}$$

where $A(t)$, $B(t)$ are constants of integration. Multiplying equation (12) by e^{-st} and then integrating with respect to t over $[0,\infty)$, we obtain on using the initial condition (6),

$$\bar{F}_{yy}(y,s) + a(s)y + b(s) = Rs\bar{F}(y,s) - \frac{Ry}{2}, \tag{13}$$

where

$$\bar{F} = \int_0^\infty e^{-st}F(y,t)\,dt, \qquad a(s) = \int_0^\infty e^{-st}A(t)\,dt, \qquad b(s) = \int_0^\infty e^{-st}B(t)\,dt. \tag{14}$$

The transformed boundary conditions are

$$\bar{F} = 0, \quad \bar{F}_y = 0, \quad y = 0; \qquad \bar{F} = \frac{s}{2(1+s^2)}, \quad \bar{F}_y = 0, \quad y = 1 \tag{15}$$

The general solution of equation (13) is

$$\bar{F}(y,s) = C_1 e^{-dy} + C_2 e^{dy} + \left(\frac{1}{2s} + \frac{a(s)}{d^2}\right) y + \frac{b(s)}{d^2}, \tag{16}$$

where C_1, C_2 are arbitrary constants and $d = \sqrt{Rs}$. Applying conditions (15) we find that $\bar{F}(y,s)$ can be expressed as

$$\bar{F}(y,s) = \frac{I(y,s)}{4(1-d/2)} \sum_{n=0}^{\infty} \left[\frac{1+d/2}{1-d/2}\right]^n e^{-nd}, \tag{17}$$

where

$$I(y,s) = \frac{1}{s}\left[1 - dy\left(1+e^{-d}\right) - e^{-dy} + e^{-d(1-y)} - e^{-d}\right]. \tag{18}$$

For small t

$$\bar{F}(y,s) = \frac{1}{s}\frac{\left[1 - dy - e^{-dy} + e^{-d(1-y)} - de^{-d}y - e^{-d}\right]}{4(1-d/2)} + O\left(\frac{e^{-d}}{s}\right), \tag{19}$$

so that

$$F(y,t) = \frac{1}{4}\left[\begin{array}{l} 1 + 2e^{4\tau}(y-1/2)\operatorname{erfc}\left(-2\tau^{1/2}\right) + \operatorname{erfc}\dfrac{1-y}{2\tau^{1/2}} - \operatorname{erfc}\dfrac{y}{2\tau^{1/2}} \\ +e^{-2y+4\tau}\operatorname{erfc}\left(\dfrac{y}{2\tau^{1/2}} - 2\tau^{1/2}\right) - e^{-2(1-y)+4\tau}\operatorname{erfc}\left(\dfrac{1-y}{2\tau^{1/2}} - 2\tau^{1/2}\right) \\ +e^{-2+4\tau}\operatorname{erfc}\left(\dfrac{1}{2\tau^{1/2}} - 2\tau^{1/2}\right) + 2ye^{-2+4\tau}\operatorname{erfc}\left(\dfrac{1}{2\tau^{1/2}} - 2\tau^{1/2}\right) - \operatorname{erfc}\left(\dfrac{1}{2\tau^{1/2}}\right) \end{array}\right] \tag{20}$$
$$+O\left(\tau^{1/2}e^{-1/4\tau}\right),$$

where $\tau = t/R$.
The radial pressure gradient, on using the conditions $F = F_y = 0$ on $y = 0$, is given by

$$P_1 = \frac{1}{\varepsilon R(1-\varepsilon\sin t)^3}\left[F_{yyy}(0,t) - RF_{yt}(0,t) + O(\varepsilon R)\right], \tag{21}$$

i.e.

$$\mathrm{P}_1 = \left((1-\varepsilon\sin t)^3\varepsilon\mathrm{R}\right)^{-1}\left[\begin{array}{l} -2\left\{e^{4\tau}\operatorname{erfc}\left(-2\tau^{1/2}\right) + e^{-2+4\tau}\operatorname{erfc}\left(\dfrac{1}{2\tau^{1/2}} - 2\tau^{1/2}\right)\right\} \\ -(\pi\tau)^{-1/2}\left\{1 + e^{-1/4\tau}\left(\dfrac{1}{4\tau} + 1\right)\right\} \end{array}\right]. \tag{22}$$

The form (20) of the solution to equations (11), (4) and (6) should give accurate results for 0 < t < < 2π and it can be used to estimate the importance of the initial condition. However, for other ranges of interest, nonlinear terms must be taken into account and this necessitates a numerical solution of the full nonlinear equation.

4. Numerical Solution

To obtain satisfactory information on the nature of the flow for 0 < ε sin t < 1 and for values of R and ε which are not small a numerical solution of the governing nonlinear equations is necessary. To integrate equation (3) subject to conditions (4) and (6), we employ an implicit finite difference scheme of the Crank-Nicolson type. On the y-axis select uniformly spaced mesh points at $y_i = i\,h$,, i= 1,2,3, ..., $(m+1)$ where $(m + 1)h = 1$ and denote by F_i^j the value of F at $y = ih$ and $t = jk$, where j = 1,2,3, ..., $(n$ -1), and sin nk = $(\varepsilon)^{-1}$. Equation (3) is replaced by finite difference approximations in which central difference formulae are used for approximation of the derivatives in the y-direction and forward difference formulae are employed for the derivative in the t-direction. For example we find

$$\left.\frac{\partial^4 F}{\partial^4 y}\right|_i^j = \frac{F_{i+2}^j - 4F_{i+1}^j + 6F_i^j - 4F_{i-1}^j + F_{i-2}^j}{h^4} + O\left(h^2 \left.\frac{\partial^6 F}{\partial^6 y}\right|_i^j\right)$$
$$\left.\frac{\partial^3 F}{\partial^2 y\,\partial t}\right|_i^j = \frac{F_{i+1}^{(j+1)} - 2F_i^{(j+1)} + F_{i-1}^{(j+1)} - F_{i+1}^j + 2F_i^j - F_{i-1}^j}{kh^2} + O\left(h^2 \left.\frac{\partial^5 F}{\partial^4 y\,\partial t}\right|_i^j\right) \tag{23}$$

On replacing all terms of the equations other than F_{yyt}, by the mean of their values on the j-th and $(j + 1)$-th line rows, we obtain from (3)

$$F_i^{(j+1)} \sum_{s=1}^{5} a_{si}^{(j+1)} F_{i+3-s}^{(j+1)} + \sum_{s=1}^{5} b_{si}^{(j+1)} F_{i+3-s}^{(j+1)} = c_i^{(j+1)} \qquad \mathrm{i} = 1,2,3,\ldots,\mathrm{m} \tag{24}$$

where the coefficients $\mathrm{a}_{\mathrm{si}}^{(\mathrm{j+1})}, \mathrm{b}_{\mathrm{si}}^{(\mathrm{j+1})}, \mathrm{c}_{\mathrm{i}}^{(\mathrm{j+1})}$ are defined in terms of h, k, R, ε, $(t_j + t_{j+1})/2$, and the values of $F_{-1}^j, F_0^j, \ldots, F_{m+2}^j$.

From the boundary conditions we obtain

$$F_1^j = F_{-1}^j,\quad F_{m+2}^j = F_m^j,\quad F_0^j = 0,\quad F_{m+1}^j = \cos t/2,\quad j = 0,1,\ldots,(n-1). \tag{25}$$

We select $10h = 1/2^l$, l = 0,1,2, ...so that (10 x 2^l -1) nonlinear algebraic equations must be solved. The algebraic equations were solved by use of the Newton-Raphson iterative technique.

4.1 Computational Details

The program was so written that it could be used to give results for wide ranges of parameters R and ε. The calculations were performed by using double precision for the values of R and ε in the ranges $0.5 \le R \le 500$, and $0.1 \le \varepsilon \le 0.8$. For fixed R and ε, accuracy was checked by comparing the results for two consecutive l values. The range of l varies from 1 at the lower values of R and ε to 3 at the higher values. For l = 1, k was selected to be 0.000625 so that the stability parameter k/h^2 = 0.25. For l = 2, k was taken to be 0.000078125 and the stability parameter was 0.125. For l = 3, k was taken to be 0.9765625 x 10^{-5} and the stability parameter was 0.0625.

Equation (5) which gives the radial pressure gradient in the transformed coordinates, contains a number of higher order derivatives with respect to y. For small values of t these give rise to unsatisfactory finite difference results for the radial pressure gradient. Thus it is preferable, since $(\partial p/\partial r)/r$ is independent of y, to integrate (5) with respect to y over [0,1] to obtain the equation

$$(1-\varepsilon\sin t)^3 P_1 = \frac{\sin t\,(1-\varepsilon\sin t)^2\,\varepsilon^{-1}}{2} + (\varepsilon R)^{-1} F_{yy}\Big|_{y=0}^{1} - 3(1-\varepsilon\sin t)\int_0^1 (F_y)^2\,dy \tag{26}$$

where the integrals are evaluated by use of Simpson's rule.

5. Results and Discussion

We discuss the case of an upper surface which is rapidly accelerated from a state of rest to a state of steady oscillations normal to the lower surface which is at rest. Close to the start of the motion the vorticity layers adjacent to both boundaries are thin and the flow is largely inviscid, the inviscid velocity distribution being specified, when sin t is not large, by

$$u / r = \partial F / \partial z = \cos t / h(t) \tag{27}$$

where $\cos t$ and $h(t)$ respectively denote the speed of the moving surface and its distance of separation from the lower surface. Impulsive movement of the upper surface results in rapid acceleration of the fluid, the driving force being the radial pressure gradient, which does work to overcome fluid inertia and frictional resistance. In the early stage of motion the inertial terms dominate the flow, as vorticity diffuses the contribution due to the frictional resistance will be equal in importance to that due to fluid inertia and as the surfaces approach one another inertial resistance becomes negligible in comparison to frictional resistance since the vorticity layers adjoining the surfaces will merge and the stream function $F(z,t)$ will tend to the classical lubrication value f_0 (the manner in which $F \rightarrow f_0$ is, of course dependent on R).

Figures 2 and 3 present the load variation with time t for a range of Reynolds number extending to $R = 500$ for values of $\varepsilon = 0.1$ and $\varepsilon = 0.8$, respectively. The Figures indicate that for a

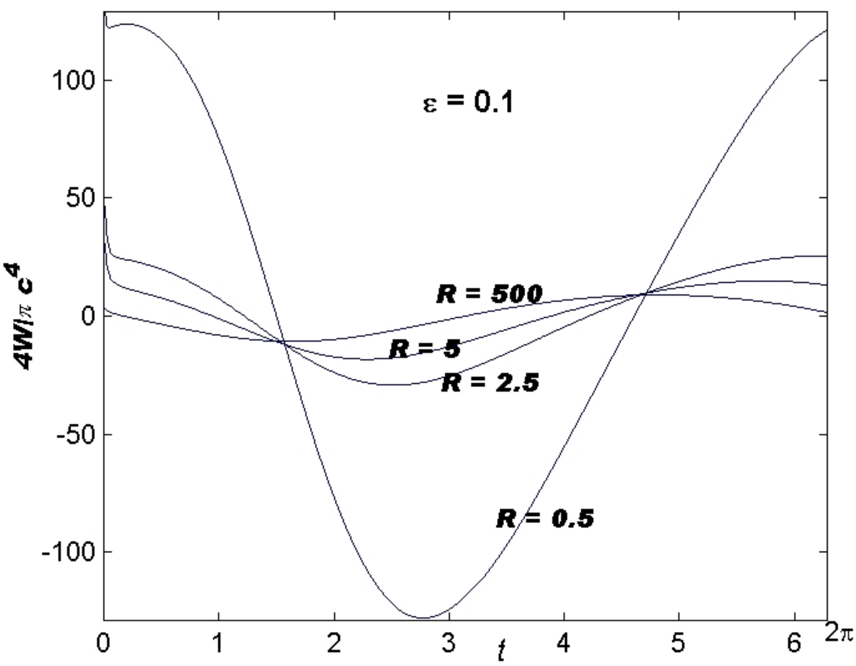

Figure 2. Normal force, or load, variation with R when $\varepsilon = 0.1$.

fixed value of ε the magnitude of the load on the disk decreases with increase of R, or equivalently with increase of the squeezing Reynolds number HV/ν. The result states that if V and H are held constant, a decrease in kinematic viscosity will result in a decrease in the magnitude of the load on the disk. We also notice that for fixed ε and for $t = \pi/2$ or $t = 3\pi/2$, the magnitude of the load is the same for all values of R in this range. (This can be seen, for small values of R and ε, from equation (10)). For small ε and near the vicinity of $t = 0$ the load is large (this large force is necessary in the early stages of motion to overcome inertial resistance). In fact as $t \rightarrow 0$, it can be shown from equation (22) that the load behaves like $(\pi R t)^{-1/2} / \varepsilon$.

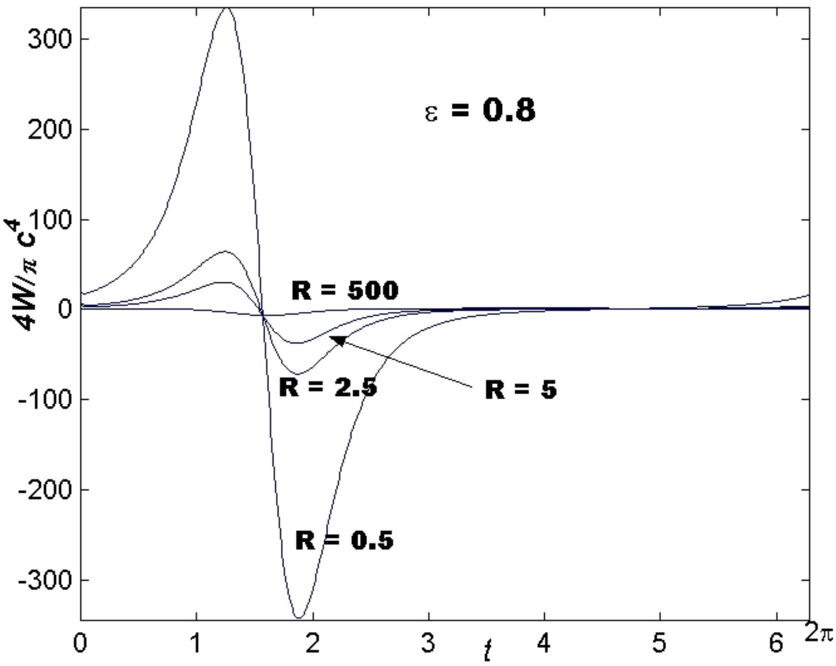

Figure 3. Normal force, or load, variation with R when ε = 0.8.

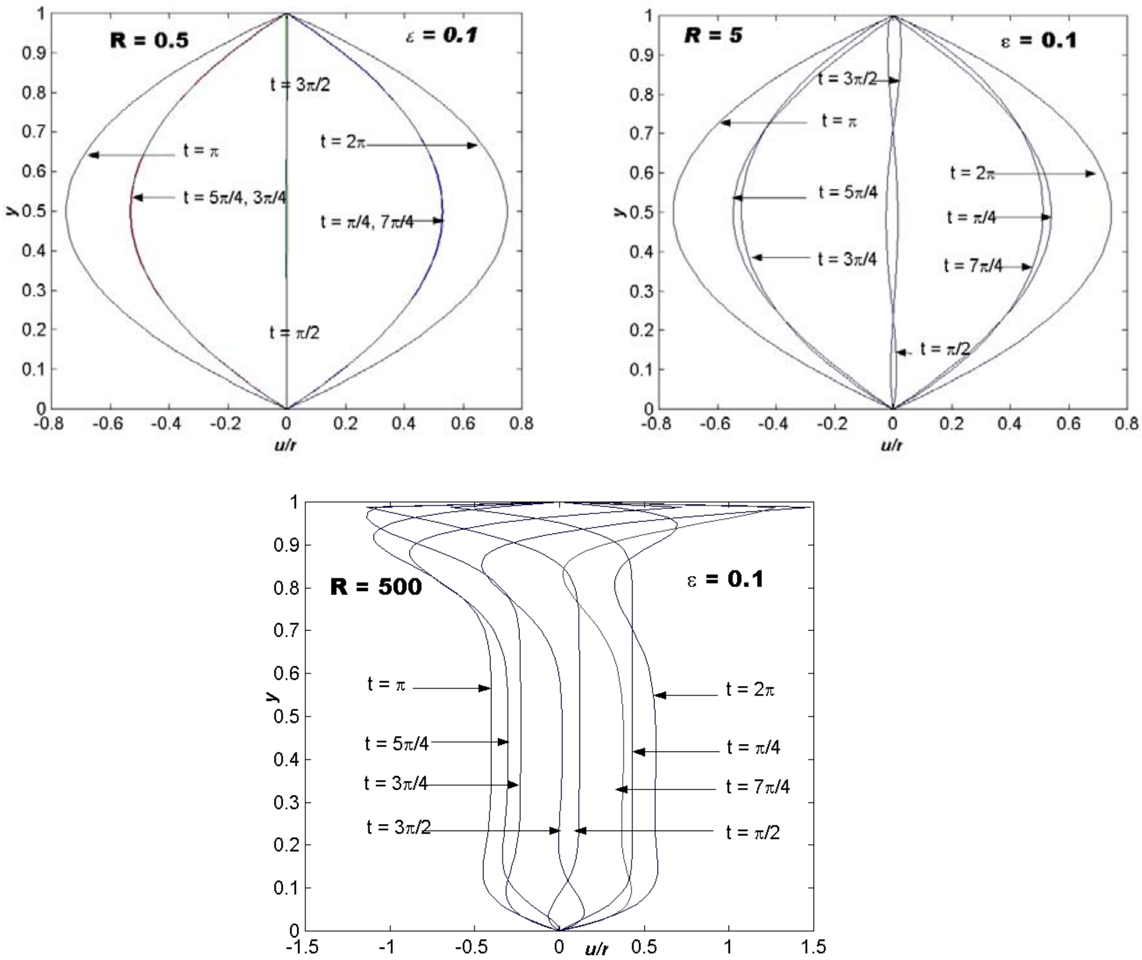

Figure 4. Radial velocity profile development with time t when ε = 0.1 and R = 0.5, 5.0,500.

The maximum and minimum values of the load (or equivalently the radial pressure gradient) are of particular interest in connection with surface wear and cavitation. For $\varepsilon = 0.8$ and $R = 0.5$ the maximum and minimum values of the radial pressure gradient are 335.2 and -342.4 while for $\varepsilon = 0.1$, $R = 0.5$ the corresponding values are 123.8 and -128. For fixed values of ε these maximum and minimum values of the radial pressure gradient decrease with increase of R, while for fixed values of R they increase with increase of ε.

It is of interest to compare the results obtained when the radial pressure gradient in equation (7) is obtained from the numerical solution and (i) the first-order regular perturbation solution, (ii) the solution to the linearised equation (11). For $R = 0.01, 0.025, 0.1, 0.5, 5, 10$ and for values of t in the range $0.0005 \le t \le 0.105$ this comparison is made in Table 1 (the upper values correspond to

Table 1: Comparison of (i) the numerical solution *(N)* and the first-order perturbation results *(P)* and (ii) the numerical solution *(N)* and the results of the linearised analysis based on equation (11) *(L)* for the radial pressure gradient: percent error $= \left|100\left(N - P/L\right)/N\right|$ when $\varepsilon = 0.1$ and $0 < t \ll 1$.

	R					
t	0.01	0.025	0.1	0.5	5	10
.0005	01.151 00.012	12.193 00.013	42.026 00.101	69.911 00.454	89.781 08.394	92.198 09.513
.0050	00.032 00.014	00.032 00.011	01.130 00.137	26.750 00.520	68.212 02.350	75.480 03.910
.0100	00.032 00.015	00.033 00.009	00.016 00.129	12.370 00.603	57.570 02.591	65.850 03.975
.0200	00.033 00.017	00.035 00.004	00.047 00.111	02.411 00.286	44.034 02.882	55.134 04.294
.0400	00.035 00.021	00.039 00.005	00.064 00.075	00.089 00.497	28.927 02.641	41.634 03.932
.0500	00.035 00.022	00.041 00.009	00.071 00.057	00.210 00.410	23.850 02.270	36.830 03.385
.0650	00.037 00.025	00.044 00.016	00.084 00.030	00.288 00.279	17.872 01.508	30.806 02.238
.0850	00.038 00.029	00.048 00.025	00.100 00.006	00.370 00.100	11.803 00.184	24.215 00.183
.0900	00.038 00.029	00.049 00.027	00.103 00.014	00.390 00.055	10.520 00.200	22.760 00.420
.1000	00.039 00.031	00.051 00.032	00.112 00.033	00.431 00.037	08.170 00.020	19.910 01.745
.1050	00.040 00.032	00.052 00.034	00.116 00.041	00.450 00.078	07.168 00.452	18.647 02.386

the first-order perturbation solution). The table shows that in the case of the first-order solution and for $R < 0.5$ the agreement for values of t in the range $t \ge 0.005$ is very good. For $R = 0.5$ the agreement is acceptable, but for $R > 0.5$ the numerical and the perturbation solutions differ appreciably even at higher values of t. On the other hand the table demonstrates the remarkable agreement between the numerical solution and the solution based on equation (11).

The radial velocity profiles for t in the range $\pi/2 \le t \le 2\pi$, for values of $R = 0.5, 5$ and $R = 500$ and for $\varepsilon = 0.1$ and $\varepsilon = 0.8$ are shown respectively in Figures 4 and 5. In general the magnitude

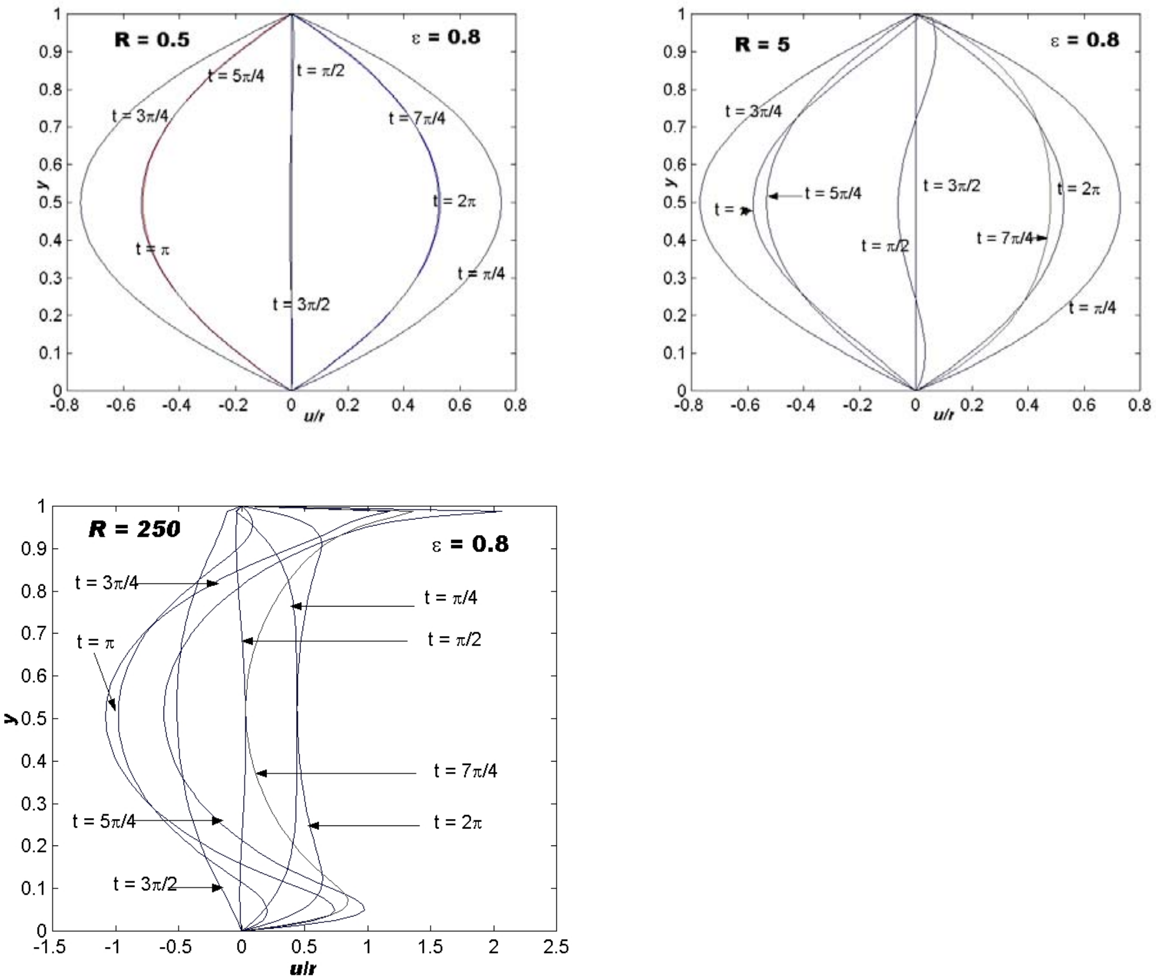

Figure 5. Radial velocity profile development with time t when $\varepsilon = 0.8$ and $R = 0.5, 5, 250$.

of the radial velocity profile increases with increase of amplitude. We also notice that as R increases the radial velocity profiles are beginning to experience oscillations with respect to time and position.

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