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Connected Components of $\mathcal{H}_{r,g}^{A}(G)$

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ABSTRACT: The Hurwitz space $\mathcal{H}_{r,g}^{A}(G)$ is the space of genus g covers of the Riemann sphere $\mathbb{P}^1$ with r branch points and the monodromy group G. In this paper, we enumerate the connected components of the Hurwitz spaces $\mathcal{H}_{r,g}^{A}(G)$ for a finite primitive group G of degree 7 and genus zero except S_7. We achieve this with the aid of the computer algebra system GAP and the MAPCLASS package.

Keywords: Monodromy Groups; Braid orbits; Connected Components.

المكونات المتصلة ل $\mathcal{H}_{r,g}^{A}(G)$

هافال م. محمد صالح

الملخص: الفضاء الهوروتيزي $\mathcal{H}_{r,g}^{A}(G)$ هي نوع من غلاف في الفضاء الريماني $\mathbb{P}^1$ مع نقاط تفرع والزمرة المونودرومية G . نحن في هذا البحث عددنا المكونات المترابطة لمجموعات البدائية ل $\mathcal{H}_{r,g}^{A}(G)$ من الدرجة والجنس صفرى عدا S_7. و أحرز هذا الغرض من خلال النظام الحاسوبي الجبري GAP و حزمة MAPCLASS

الكلمات المفتاحية: زمرة المنودرومية، مدارات بريد والمكونات المتصلة.

1. Introduction

Let Ω be a finite set and $|\Omega| = n$. Define a genus g system to be a triple $(G, \Omega, (x_1, \ldots, x_r))$. G is a transitive subgroup of S_n such that $G =< x_1, \ldots, x_r >$, $x_1 \cdot x_2 \cdot \ldots \cdot x_r = 1$ and $x_i \in G\backslash\{1\}$

$$2(n + g - 1) = \sum_{i=1}^{r} ind x_i$$

where $indx_i$ is the minimal number of transpositions need to express x_i as a product [6]. This condition is equivalent to the existence of the branched covering $f: X \to \mathbb{P}^1$ where $\mathbb{P}^1 = \mathbb{C} \cup \{\infty\}$ [8]. If f is an irreducible, then G is primitive.

Let $C_1, \ldots, C_r$ be non-trivial conjugacy classes of a finite group G. The set of generating systems $(x_1, \ldots, x_r)$ of G with $x_1 \ldots x_r = 1$ and such that there is a permutation $\pi \in S_r$ with $x_i \in S_{\pi(i)}$ for $i = 1, \ldots, r$ is called a *Nielsen class* and denoted by $\mathcal{N}(C)$, where $C = (C_1, \ldots, C_r)$.

Each Nielsen class is the disjoint union of braid orbits, which are defined as the smallest subsets of the Nielsen class closed under the braid operations [10]

$$(x_1, \ldots, x_r)^{Q_i} = (x_1, \ldots, x_{i+1}, x_{i+1}^{-1} x_i x_{i+1}, \ldots, x_r) \quad (1)$$

for $i = 1, \ldots, r$.

We denote by O_r, the space of subsets of $\mathbb{C}$ of cardinality r. The following definitions can be found in [10].

Definition 1.1

Let $B \in O_r$ and $b_0 \in \mathbb{P}^1 \setminus B$, we call a map $\varphi: \pi_1(\mathbb{P}^1 \setminus B, b_0) \to G$ admissible if it is a surjective homomorphsim, and $\varphi(\theta_b) \neq 1$ for each $b \in B$. Here θ_b is the conjugacy class of $\pi_1(\mathbb{P}^1 \setminus B, b_0)$.

Definition 1.2.

Let $B \in O_r$ and $\varphi: \pi_1(\mathbb{P}^1 \setminus B, \infty) \to G$ be admissible. Then we say that two pairs (B, φ) and $(\overline{B}, \overline{\varphi})$ are A-equivalent if and only if $B = \overline{B}$ and $\overline{\varphi} = a \circ \varphi$ for some $a \in A$.
Let $[B, \varphi]_A$ denote the A-equivalence class of (B, φ). The set of equivalence classes $[B, \varphi]_A$ is denoted by $\mathcal{H}^A_r(G)$ and is called the *Hurwitz space* of G-covers.

Here we enumerate the connected components of $\mathcal{H}^A_r(G)$ and then we show to which number of branch points r, it is connected. The MAPCLASS package of James, Magaard, Shpectorov and Volklein, is designed to perform braid orbit computations for a given finite group and given type.

2. Preliminary

As usual $Inn(G)$ and $Aut(G)$ denote the inner-automorphism and automorphism groups of a group G respectively. A denotes a subgroup of $Aut(G)$. In particular if $A = Inn(G)$, then the Hurwitz space $\mathcal{H}^A_r(G)$ is denoted by $\mathcal{H}^{in}_r(G)$.The details of the following results and concepts can be found in [10] and [8].

Lemma 2.1. The map $\Psi_A: \mathcal{H}^A_r(G) \to O_r$, $\Psi_A([B, \varphi]) = B$ is *covering*.

The fiber $\Psi^{-1}_A(B_0) = \{[B_0, \varphi]_A: \varphi: \pi_1(\mathbb{P}^1 \setminus B, \infty) \to G$ is admissible$\}$. This φ gives a product one generating tuple $(x_1, \dots, x_r)$ of G. Define $\mathcal{E}_r(G) = \{(x_1, \dots, x_r): G = \langle x_1, \dots, x_r \rangle,\ x_1 \cdot \dots \cdot x_r = 1,\ x_i \in G^{\#}, i = 1,2, \dots, r\}$. Let $A \leq Aut(G)$. Then the subgroup A acts on $\mathcal{E}_r(G)$ via sending $(x_1, \dots, x_r)$ to $(a(x_1), \dots, a(x_r))$, for $a \in A$. Let $\mathcal{E}^A_r(G) = \mathcal{E}_r(G)/A$ be the set of A-orbits. In particular, if $A = Inn(G)$, then we have $Inn(G) = G/Z(G)$. Therefore $\mathcal{E}^{in}_r(G)$ is the set of G-orbits.

Lemma 2.2. We obtain a bijection $\Psi^{-1}_A(B_0) \to \mathcal{E}^A_r(G)$ by sending $[B_0, \varphi]_A$ to the generators $(x_1, \dots, x_r)$ where $x_i = \varphi([\gamma_i])$ for $i = 1, \dots, r$.

Proposition 2.3. Let C be a fixed ramifcation type in G, and the subset $\mathcal{H}^{in}_r(C)$ of $\mathcal{H}^{in}_r(G)$ consists of all $[B, \emptyset]_A$ with $B = \{b_1, \dots, b_r\}$, $\emptyset: \pi_1(\mathbb{P}^1 \backslash B, \infty) \to G$ and $\emptyset(\theta_{b_i}) \in C_i$ for $i = 1, \dots, r$. Then $\mathcal{H}^A_r(C)$ is a union of connected components in $\mathcal{H}^A_r(G)$. Under the bijection from Lemma 2.2, the fiber in $\mathcal{H}^A_r(C)$ over B_0 corresponds the set $\mathcal{N}^A(C)$. This yields a one to one correspondence between components of $\mathcal{H}^A_r(C)$ and the braid orbits on $\mathcal{N}^A(C)$. In particular, $\mathcal{H}^{in}_r(C)$ is connected if and only if $B_r$acts transitively on $\mathcal{N}^{in}(C) = \mathcal{N}(C)$.

Lemma 2.4. Let G be a group and X be a G-set. Then G acts transitively on X if and only if there is only one orbit.

Proof. Straightforward.

Corollary 2.5. Let C be a fixed ramifcation type in G, and the subset $\mathcal{H}^{in}_r(C)$ of $\mathcal{H}^{in}_r(G)$ consists of all $[B, \emptyset]_A$ with $B = \{b_1, \dots, b_r\}$, $\emptyset: \pi_1(\mathbb{P}^1 \backslash B, \infty) \to G$ and $\emptyset(\theta_{b_i}) \in C_i$ for $i = 1, \dots, r$. Then $\mathcal{H}^A_r(C)$ is a union of connected components in $\mathcal{H}^A_r(G)$. Under the bijection from Lemma 2.2, the fiber in $\mathcal{H}^A_r(C)$ over B_0 corresponds the set $\mathcal{N}^A(C)$. This yields a one to one correspondence between components of $\mathcal{H}^A_r(C)$ and the braid orbits on $\mathcal{N}^A(C)$. In particular, $\mathcal{H}^{in}_r(C)$ is connected if and only if there is only one braid orbit.

Proof. It follows from Proposition 2.3 and Lemma 2.4.

3. Computing Indices and Labeling Conjugacy Classes

The classifications of all the primitive groups of degree 7 except S_7 for genus zero are given in this paper. Before, we discuss computing the indices, we give an alternative formula to compute an index of an element in a group. Let G be a group acting on a finite set Ω and $|\Omega| = n$. If $x \in G$, define the index of x by $ind\ x = n - orb\ x$, where orb x is the number of orbits of $< x >$ on Ω. Also $Fix\ x = \{w \in \Omega \mid xw = w\}$, $f(x) = |Fix\ x|$. Furthermore, $orb\ x = \frac{1}{d}\sum_{i=0}^{d-1} f(x^i)$ where x has order d [6]. From the character table of A_7, we see the elements of orders 2,3,4,5,6 and 7, then we compute fixed points, which are equal to 1a+2a of the elements of given orders.

The Character Table of A_7.

Alternating group A_7

Order $=2520=2^3.3^2.5.7$ mult =6 out = 2

Constructions

Alternating $S_7 \cong G.2$: all permutations of 7 letters;

$A_7 \cong G$: the even permutations; $2.G$ and $2.G.2$: the schur double covers

Lattice $2A_7 \cong 2.G$: the symmetries of the lattice Λ_{4,b_7} whose minimal vectors are obtained from

$\pm(i7;0,0,0) \pm (0;i7,0,0) \pm (1;,x,y,z) \pm (-x;1,y,-z) \pm (-xy;,1,1,1) \pm (1;xy,1,1) \pm (xy;0,z,1)$

$\pm(-x;yz,0,1) \pm (-1;0,xy,z) \pm (0;1,-x,yz)$ by replacing each of x,y,z by one of b_7 or b_7^{**}, and

Cyclically permuting the last 3 coordinates

Vectors $3A_7$: symmetries of the 21 (w) vectors obtained from (200 00 00), (00 11 11), (01 01 $w\bar{w}$) by

Bodiy permuting the 3 couples, and reversing any 2 couples (see A_6 (hexacode));

The lattice these generate has auomorphism group $6U_4(3).2$ (see $U_4(3)$)

Unitary $3A_7$ has a 3-dimensional unitary representation over F_{25} (see $U_3(5)$)

Presentations $G \cong < A,B \mid A^4 = B^5 = (AB)^3 = (A^{-1}BA^2B^2)^2 = 1 > \cong < x_1,\dots,x_5 | x_i^3 = (x_ix_j)^2 = 1 >$;

$G \cong < A,B \mid A^2 = [A,B^2]^2 = [A,B^3]^2 = 1; B^7 = (AB)^6 >$

Maximal subgroups specifications

Order	Index	Structure	$G.2$	Character	Abstract	Alternating
360	7	A_6	$:S_6$	$1a+2a$	$N(2A,3A,3B,4A,5A)$	point
168	15	$L_2(7)$	$7:6$	$1a+14b$	$N(2A,3B,4A,7AB)$	$S(2,3,7)$
168	15	$L_2(7)$	$7:6$	$1a+14b$	$N(2A,3B,4A,7AB)$	$S(2,3,7)$
120	21	S_5	$:S_5\times 2$	$1a+6a+14a$	$N(2A,3A,5A), C(2B)$	dual
72	35	$(A_4\times 3):2$	$:S_4\times S_3$	$1a+6a+14ab$	$N(3A), N(2A^2)$	triad

	;	@	@	@	@	@	@	@	@	@	:	:	@	@	@	@	@	@	@
	p p' ind	2520 power part 1A	24 A A 2A	36 A A 3A	9 A A 3B	4 A A 4A	5 A A 5A	12 AA AA 6A	7 A A 7A	7 A A B^{**}	fus	ind	120 A A 2B	24 A A 2C	12 A A 4B	6 AB AB 6B	3 BC BC 6C	5 AB AB 10A	6 AB AB 12A
$\mathcal{X}_1$	+	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1
$\mathcal{X}_2$	+	6	2	3	0	0	1	-1	-1	-1	:	++	4	0	2	1	0	-1	-1
$\mathcal{X}_3$	O	10	-2	1	1	0	0	1	b_7	**	|	+	0	0	0	0	0	0	0
$\mathcal{X}_4$	O	10	-2	1	1	0	0	1	**	b_7									
$\mathcal{X}_5$	+	14	2	2	-1	0	-1	2	0	0	:	++	6	2	0	0	-1	1	0
$\mathcal{X}_6$	+	14	2	-1	2	0	-1	-1	0	0	:	++	4	0	-2	1	0	-1	1
$\mathcal{X}_7$	+	15	-1	3	0	-1	0	-1	1	1	:	++	5	-3	1	-1	0	0	1
$\mathcal{X}_8$	+	21	1	-3	0	-1	1	1	0	0	:	++	1	-3	-1	1	0	1	-1
$\mathcal{X}_9$	+	35	-1	-1	-1	1	0	-1	0	0	:	++	5	1	-1	-1	1	0	-1

If x is an element of order 2, then
$ind\ x = n - \frac{1}{2}\sum_{i=0}^{1} f(x^i) = n - \frac{1}{2}[f(x^0) + f(x)] = 7 - \frac{1}{2}[7+3] = 2.$
If x is an element of order 3 of type $3A$, then
$ind\ x = n - \frac{1}{3}\sum_{i=0}^{2} f(x^i) = n - \frac{1}{3}[f(x^0) + f(x) + f(x^2)] = 7 - \frac{1}{3}[7+4+4] = 2.$
If x is an element of order 3 of type $3B$, then
$ind\ x = n - \frac{1}{3}\sum_{i=0}^{2} f(x^i) = n - \frac{1}{3}[f(x^0) + f(x) + f(x^2)] = 7 - \frac{1}{3}[7+1+1] = 4.$
If x is an element of order 4, then
$ind\ x = n - \frac{1}{4}\sum_{i=0}^{3} f(x^i) = n - \frac{1}{4}[f(x^0) + f(x) + f(x^2) + f(x^3)] = 7 - \frac{1}{4}[7+1+3+1] = 4.$
If x is an element of order 5, then
$ind\ x = n - \frac{1}{5}\sum_{i=0}^{4} f(x^i) = n - \frac{1}{5}[f(x^0) + f(x) + f(x^2) + f(x^4)] = 7 - \frac{1}{5}[7+2+2+2+2] = 4.$
If x is an element of order 6, then
$ind\ x = n - \frac{1}{6}\sum_{i=0}^{5} f(x^i) = n - \frac{1}{6}[f(x^0) + f(x) + f(x^2) + f(x^4) + f(x^5)] = 7 - \frac{1}{6}[7+0+4+3+4+0] = 4.$
If x is an element of order 7 of type $7A$ or $7B$, then
$ind\ x = n - \frac{1}{7}\sum_{i=0}^{6} f(x^i) = n - \frac{1}{7}[f(x^0) + f(x) + f(x^2) + f(x^4) + f(x^5) + f(x^6)] = 7 - \frac{1}{7}[7+0+0+0+0+0+0] = 6.$

4. Algorithm and Main Results

To obtain Tables 2 and 3, we need to perform the following steps:

1- We extract all primitive permutation groups G by using the GAP function [4] AllPrimitiveGroups (DegreeOperation,n).
2- For given degree, genus and G we compute all possible ramification types satisfying the Riemann-Hurwitz formula which is given in section 3.
3- We compute the character table of G and remove those types which have zero structure constant.
4- We obtain all generating types by GAP Codes which exist in Appendix C [8].
5- For each of the remaining generating types of length greater than or equal to 4, we use the MAPCLASS package to compute braid orbits. For tuples of length 3, we determine braid orbits via double cosets in [8].

We now give our main results as follows:

Lemma 4.1. The Hurwitz spaces, $\mathcal{H}_r^{in}(C)$ are connected if $G = D(2*7)$ or $G = AGL(1,7)$.

Proof. It follows from the fact that the Nielsen classes $\mathcal{N}(C)$ are the disjoint union of braid orbits but we have only one braid orbit for $r \geq 3$ and $n = 7$. From Corollary 2.5, we obtain the Hurwitz spaces $\mathcal{H}_r^{in}(C)$ are connected.

Lemma 4.2. The Hurwitz spaces, $\mathcal{H}_r^{in}(C)$ are connected if $r \geq 4$ and $G = L(3,2)$.

Proof. It follows from the fact that the Nielsen classes $\mathcal{N}(C)$ are the disjoint union of braid orbits but we have only one braid orbit for $r \geq 4$ and $n = 7$. From Corollary 2.5, we obtain the Hurwitz spaces $\mathcal{H}_r^{in}(C)$ are connected.

Lemma 4.3. The Hurwitz spaces, $\mathcal{H}_r^{in}(C)$ are disconnected if $G = A_7$ and $G = C_7$.

Proof. It follows from the fact that the Nielsen classes $\mathcal{N}(C)$ are the disjoint union of braid orbits but for these groups we have at least two braid orbits for some type C as given in Table 2. From Corollary 2.5, we obtain the Hurwitz spaces $\mathcal{H}_r^{in}(C)$ are disconnected.

Finally, we enumerate the connected components of $\mathcal{H}_r^{in}(G)$ in the cases where $g = 0$ and G is a primitive group of degree 7. The total numbers of connected components of $\mathcal{H}_r^{in}(G)$ is summarized in Table 1.

Table 1. Primitive Genus Zero Systems: Number of Components.

Degree	# Group Iso types	#RTs	# comp's r=3	# comp's r=4	# comp's r=5	# comp's r=6	# comp's Total
7	5	154	179	61	67	10	317

Example 4.4

First of all, read the file in GAP program which exists in [8] and then choose the group and the specific tuple.

For instance let $G = D(2 * 7)$ be the dihedral group and take the tuple $t = [(2,5)(3,6)(4,7), (2,5)(3,6)(4,7), (2,5)(3,6)(4,7), (2,5)(3,6)(4,7)]$. The run of the program which finds the braid orbits is shown below:

```
gap> Read("qu1.g");
----------------------------------------------------------------
Loading  MapClass 1.2
by Adam James (http://www.mat.bham.ac.uk/~jamesa)
Kay Magaard (http://mat.bham.ac.uk/staff/magaardk.shtml)
Sergey Shpectorov (http://web.mat.bham.ac.uk/S.Shpectorov/index.html)
Helmut Volklein (http://www.iem.uni-due.de/algebra/people/voelklein.html)
For help, type: ?MapClass:
----------------------------------------------------------------
gap> LL:=AllPrimitiveGroups(DegreeOperation,7);
[ C(7), D(2*7), 7:3, AGL(1, 7), L(3, 2), A(7), S(7) ]
gap> k:=LL[2];
D(2*7)
gap> CheckingTheGroup(k);
gap> GT:=GeneratingType(k,7,0);
Checking the ramification type 10 with 0 remaining
[ [ 2, 2, 5 ], [ 2, 2, 4 ], [ 2, 2, 3 ], [ 2, 2, 2, 2 ] ]
gap> t:=List(GT[4],x->CC[x]);
[ (2,5)(3,6)(4,7), (2,5)(3,6)(4,7), (2,5)(3,6)(4,7), (2,5)(3,6)(4,7) ]
gap> orb:=GeneratingMCOrbits(k,0,t);;
The current date is: Mon 07/10/2017
Enter the new date: (mm-dd-yy)
Total Number of Tuples: 336

Collecting 20 generating tuples .. done
Cleaning done; 20 random tuples remaining

Orbit1:
Length=24
Generating Tuple  =[ (1,4)(3,5)(6,7), (1,6)(2,3)(5,7), (1,7)(2,6)(3,4), (1,3)(2,4)(5,6) ]
Centralizer size=1
0 tuples remaining
Cleaning a list of 20 tuples
Random Tuples Remaining: 0
Cleaning done; 0 random tuples remaining
Computation complete: 1 orbits found.
```

Table 2. Primitive Groups of Degree 7.

Group	Ramification Type	# of orbits	Length of largest orbit	Ramification Type	# of orbits	Length of largest orbit
A_7	$(5A,5A,5A)$	2	1	$(4A,5A,5A)$	8	1
	$(4A,4A,5A)$	22	1	$(4A,4A,4A)$	24	1
	$(3B,5A,5A)$	6	1	$(3B,4A,5A)$	10	1
	$(3B,4A,4A)$	8	1	$(3B,3B,5A)$	2	1
	$(5A,5A,6A)$	8	1	$(4A,5A,6A)$	8	1
	$(4A,4A,6A)$	12	1	$(3B,5A,6A)$	6	1
	$(3B,4A,6A)$	6	1	$(3B,3B,6A)$	2	1
	$(6A,6A,6A)$	2	1	$(3B,6A,6A)$	2	1
	$(4A,6A,6A)$	4	1	$(5A,6A,6A)$	2	1
	$(3A,5A,7A)$	1	1	$(3A,5A,7B)$	1	1
	$(3A,4A,7A)$	2	1	$(3A,5A,7B)$	2	1
	$(3A,3B,7A)$	1	1	$(3A,3B,7B)$	1	1
	$(3A,6A,7A)$	1	1	$(3A,6A,7B)$	1	1
	$(2A,5A,7A)$	2	1	$(2A,5A,7B)$	2	1
	$(2A,4A,7A)$	2	1	$(2A,4A,7B)$	2	1
	$(2A,6A,7A)$	2	1	$(2A,6A,7B)$	2	1
	$(3A,3A,5A,5A)$	1	30	$(3A,3A,4A,5A)$	2	40
	$(3A,3A,4A,4A)$	2	92	$(3A,3A,3B,5A)$	1	40
	$(3A,3A,3B,4A)$	2	44	$(3A,3A,3B,3B)$	1	26
	$(3A,3A,6A,5A)$	1	60	$(3A,3A,6A,4A)$	1	72
	$(3A,3A,3B,6A)$	1	40	$(3A,3A,6A,6A)$	1	22
	$(3A,3A,3A,7A)$	1	7	$(3A,3A,3A,7B)$	1	7
	$(2A,3A,5A,5A)$	1	80	$(2A,3A,4A,5A)$	1	170
	$(2A,3A,4A,4A)$	1	300	$(2A,3A,3B,5A)$	1	90
	$(2A,3A,3B,4A)$	1	126	$(2A,3A,3B,3B)$	1	62
	$(2A,3A,6A,5A)$	1	80	$(2A,3A,4A,6A)$	1	118
	$(2A,3A,3B,6A)$	1	62	$(2A,3A,6A,6A)$	1	44
	$(2A,3A,3A,7A)$	1	14	$(2A,3A,6A,7B)$	1	14
	$(2A,2A,5A,5A)$	3	70	$(2A,2A,4A,5A)$	3	120
	$(2A,2A,4A,4A)$	3	168	$(2A,2A,3B,5A)$	1	150
	$(2A,2A,3B,4A)$	1	192	$(2A,2A,3B,3B)$	1	44
	$(2A,2A,6A,5A)$	3	60	$(2A,2A,6A,4A)$	3	76
	$(2A,2A,3B,6A)$	1	90	$(2A,2A,6A,6A)$	3	36
	$(2A,2A,3A,7A)$	1	28	$(2A,2A,3A,7B)$	1	28
	$(2A,2A,2A,7A)$	2	21	$(2A,2A,2A,7B)$	2	21
	$(2A,2C,3A,3A,4A)$	1	168	$(2A,2A,3A,3A,6A)$	1	300
	$(2A,2C,3A,3A,2B)$	1	42	$(2A,3A,3A,3A,4A)$	1	96
	$(2A,3A,3A,3A,6A)$	1	216	$(2A,2A,3A,3A,2B)$	1	44
	$(2A,2C,2C,3A,4A)$	1	240	$(2A,2C,2C,3A,6A)$	1	384
	$(2A,2C,2C,2C,3A)$	1	57	$(2A,2C,2C,2C,4A)$	1	312
	$(2A,2C,2C,2C,6A)$	1	486	$(2A,2C,2C,2C,2B)$	1	60
	$(2A,2A,3A,4A,4A)$	1	89	$(2A,2A,3A,4A,6A)$	1	202
	$(2A,2A,3A,6A,6A)$	1	336	$(2A,2A,3A,3A,5A)$	1	75
	$(2A,2A,3A,3A,4A)$	1	80	$(2A,2A,3A,3A,3B)$	1	39
	$(2A,2A,2B,3A,4A)$	1	36	$(2A,2A,2B,3A,6A)$	1	52
	$(2A,2A,2C,4A,4A)$	1	158	$(2A,2A,2C,4A,6A)$	1	273
	$(2A,2A,2C,6A,6A)$	1	426	$(2A,2A,2C,3A,5A)$	1	125
	$(2A,2A,2C,3A,4A)$	1	100	$(2A,2A,2C,3A,3B)$	1	48
	$(2A,2A,2C,2B,4A)$	1	40	$(2A,2A,2B,2C,6A)$	1	60

	$(2A,2A,2C,2C,5A)$	1	175	$(2A,2A,2C,2C,4A)$	1	128
	$(2A,2A,2C,2C,3B)$	1	54	$(2A,2A,2A,4A,5A)$	1	75
	$(2A,2A,2A,4A,5A)$	1	72	$(2A,2A,2A,4A,4A)$	1	36
	$(3A,3A,3A,3A,5A)$	1	300	$(3A,3A,3A,3A,4A)$	2	384
	$(3A,3A,3A,3A,3B)$	1	312	$(3A,3A,3A,3A,6A)$	1	480
	$(2A,3A,3A,3A,5A)$	1	750	$(2A,3A,3A,3A,4A)$	1	1392
	$(2A,3A,3A,3A,3B)$	1	744	$(2A,3A,3A,3A,6A)$	1	690
	$(2A,2A,3A,3A,5A)$	1	1550	$(2A,2A,3A,3A,4A)$	1	2704
	$(2A,2A,3A,3A,3B)$	1	1234	$(2A,2A,3A,3A,6A)$	1	1112
	$(2A,2A,2A,3A,5A)$	1	3000	$(2A,2A,2A,3A,4A)$	1	4584
	$(2A,2A,2A,3A,3B)$	1	2214	$(2A,2A,2A,3A,5A)$	1	1896
	$(2A,2A,2A,2A,5A)$	3	1800	$(2A,2A,2A,2A,4A)$	3	2880
	$(2A,2A,2A,2A,3B)$	1	3240	$(2A,2A,2A,2A,6A)$	3	1080
	$(2A,2A,3A,3A,3A,3A)$	1	13764	$(2A,3A,3A,3A,3A,3A)$	1	7280
	$(3A,3A,3A,3A,3A,3A)$	1	2870	$(2A,2A,2A,2A,3A,3A)$	1	45692
	$(2A,2A,2A,2A,2A,3A)$	1	79560	$(2A,2A,2A,2A,2A,2A)$	3	45360
	$(2A,2A,2A,3A,3A,3A)$	1	26210			

Table 3. Primitive Groups of Degree 7.

Group	Ramification Type	# of orbits	Length of largest orbit	Ramification Type	# of orbits	Length of largest orbit
$AGL(1,7)$	$(2A,3B,6B)$	1	1	$(2A,3A,6A)$	1	1
$L(3,2)$	$(3A,3A,4A)$	4	1	$(3A,4A,4A)$	2	1
	$(4A,4A,4A)$	4	1	$(2A,3A,7B)$	1	1
	$(2A,3A,7A)$	1	1	$(2A,4A,7B)$	1	1
	$(2A,4A,7A)$	1	1	$(2A,2A,3A,3A)$	1	30
	$(2A,2A,3A,4A)$	1	24	$(2A,2A,4A,4A)$	1	24
	$(2A,2A,2A,7A)$	1	1	$(2A,2A,2A,7B)$	1	7
	$(2A,2A,2A,2A,3A)$	1	216	$(2A,2A,2A,2A,4A)$	1	192
	$(2A,2A,2A,2A,2A)$	1	1680			
$D(2*7)$	$(2A,2A,7A)$	1	1	$(2A,2A,7B)$	1	1
	$(2A,2A,7C)$	1	1	$(2A,2A,2A,2A)$	1	24
$C_7:C_3$	$(3B,3B,3B)$	2	1	$(3A,3A,3A)$	2	1

5. Conclusion

In this paper, we use the algorithm in [8] to compute braid orbits on Nielsen class. An application of the algorithm is the classification of the primitive genus zero systems of degree 7. That is we find the connected components $\mathcal{H}_r^{in}(G)$ of G-curves X, such that $g=0$. In our situation, the computation shows that there are exactly 307 braid orbits of primitive genus 0 systems of degree 7.

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