International journal of
Sustainable Energy Planning and Management

Scaling laws and electricity consumption in cities: a sectoral view

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URL: dx.doi.org/10.5278/ijsepm.2014.2.3

ABSTRACT

With the use of electricity being increasingly concentrated in urban areas it becomes important to understand the influence of cities, and their size, on patterns of consumption. We tested the application of the scaling law to the Portuguese urban system, across time and municipalities, with special focus on the sectoral consumption of electricity from 1994 until 2009. Results showed that the scaling law is not suitable to describe a city's electricity consumption throughout the years. In the cross-sectional results, the scaling law proved to be applicable for all cases, although the scaling exponent varies both in time and across sectors. For the residential sector the decrease of the scaling exponent might be related with the electrification of the energy system and with the increase of average income. For the service sector the scaling exponent was fairly constant, above 1, during the 16 years of the study. The largest variation was found for the industrial sector whose scaling exponent decreased 15–20% in the time frame analyzed, though in this sector electricity consumption appeared to be the one with the weakest relation with city size.

Abbreviations

DGEG – *Direcção-Geral de Energia e Geologia*, Portuguese Energy and Geology Agency
HDD – Heating Degree Days
INE – *Instituto Nacional de Estatística*, National Institute of Statistics
NUTS – Nomenclature of Territorial Units for Statistics
OLS – Ordinary Least Squares

1. Introduction

Since the energy crisis in the 70' s energy demand has been on the agenda of researchers in economy, planning and engineering. In 2006, 76% of world electricity consumption was concentrated in urban areas [1] when cities comprised less than half of the total population [2]. With the foreseen growth of urbanization it becomes imperative to study the dynamics of cities and their impact on energy use.

There are many models of energy or electricity consumption in cities, most of them relating it with income and price, typically, through the calculation of elasticities and their significance level. Applying econometric methods has been a frequent choice in the literature using, for example, a multiple first-order linear model [3], general-to-specific modeling, or co-integration analysis using time series or panel data [4–7].

Here our aim is to find possible patterns linking city growth and energy use. Therefore, this work focusses on the application of scaling laws to the specific case of electricity consumption in urban areas of continental Portugal. Although this application has been made to other countries such as China [8–9], Germany [8] and Spain [10] this study come as the first one, as far as we

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know, that explores the time dynamics of the coefficients of the scaling law. Furthermore, we perform a sectoral analysis to identify possible differences in the observed patterns.

2. Urban scaling laws

Inspired by the connection between physiological characteristics of some biological organisms and their body mass, Bettencourt *et al*. [8] tested the existence of a scale relation between city size and a set of indicators. This relation is described by Equation (1):

$$I = \alpha S^{\beta} \quad (1)$$

In this equation, I is the indicator that we are trying to relate with city size (S) using a scaling relation of exponent β and a normalization constant (α) equal to the value of the indicator per capita when city size is 1, i.e., when there is no scaling effect. In Equation (1) the scaling exponent β also represents the elasticity of the indicator in relation to population. This elasticity gives the proportional variation of the indicator associated to a proportional variation of the population.

For the case of energy related variables Bettencourt *et al*. tested total and residential electricity consumption obtaining for the former a β of 1.07 (for Germany) and, for the latter, (β = 1.00 for Germany and β = 1.05 for China).

3. Data and methodology

The source of data for annual electricity consumption at the municipal level between 1994 and 2009 was the Portuguese Energy Agency - DGEG (Direcção-Geral de Geologia e Energia).

In order to use the same unit of data collection municipalities were used as the minimum scale for the demographic data which was taken from INE's (Instituto Nacional de Estatística) database. To identify which municipalities correspond to urban areas we defined two conditions based on the parishes' classification (*urban*, *semi-urban* and *rural*), which was set using the official thresholds of total population and population density for the year 2001. This was the only year for which we had demographic data of parishes' population and municipalities' electricity consumption. The conditions used are described in Table 1.

Table 1: Criteria of urban municipalities (percentage of population).

	Urban	Semi-urban	# Municipalities
Condition 1	15%	50%	24
Condition 2	40%	–	99

Equation (1) applied to the consumption of electricity in municipalities takes the form:

$$El_{mun,t} = \alpha S_{mun,t}{}^{\beta} \quad (2)$$

Where the indicator is now the consumption of electricity El in municipality mun in year t ($El_{mun,t}$) and city size S relates to that same municipality and year.

To use Ordinary Least Squares (OLS) regression we applied the logarithm to Equation (2).

$$\ln (El_{mun,\,t}) = \ln (\alpha) + \beta \ln(S_{mun,\,t}) \quad (3)$$

When necessary, the following tests were run to verify the conditions for the validity of the application of OLS:

- For heteroscedasticity (the variance of the error of each observation varies with, at least, one explanatory variable: Graphical and Breush-Pagan/Cook-Weisberg tests [11–12];
- For error autocorrelation with time series data: ACF (Autocorrelation function) and Durbin-Watson test [13–14];

For both the regressions and tests, we used two software packages for statistical analysis, Stata 11 and R 2.15.2.

4. Time-series approach

One alternative to the traditional cross-sectional analysis of the scaling law is to interpret is as the evolution throughout time of one entity, in this case, of one city. To better understand this concept we can make a parallel with the studies of ecology. When the scaling law is applied to cross section urban data, its equivalent in ecology is finding common scaling relations between different species of mammals; whereas an application to a time series (one city across time) finds its parallel in the understanding of the dynamics of growth of one specific species.

The application of scaling laws to the urban consumption of energy has already been tested for cross-section sets of data for different countries with surprising results. However, although it has been hypothesized by Bettencourt *et al.* [8] that scaling laws can be used to describe the relation between population growth and the use of energy evolution (among other indicators) for a single city, such relation has never been empirically tested.

In this section we test this approach in the Portuguese urban system, more precisely, to electricity consumption in Portuguese cities as defined in section 3.

For this test, we applied Equation (3) where in each regression we used electricity consumption and population for a city between 1994 and 2009. For reasons of simplicity, we will refer, from now on, to the scaling coefficients resulted from these regressions (that use time series data) dynamic scaling coefficients.

As a methodological note, it is important to mention that all regressions made within this section showed the presence of autocorrelation after the application of the Durbin-Watson test. To correct the autocorrelation we used the Cochrane-Orcutt method already included in the *R* software in the 'bstats' package.

4.1. Total consumption

The dynamic scaling coefficients obtained for total electricity consumption can be found in Figure 1 where the results have been divided into NUTS II regions (which correspond to 5 continental regional coordination commissions) and the diameter of the circle is proportional to the R^2 value of the corresponding linear regression. In the literature, the values of the scaling coefficient, obtained in cross-sectional analyses, are confined to the interval [0, 2], being close to 1 for most of the studies [15–16]. However, the dynamic scaling coefficients found have a very large range of values, going from −14 to + 25. Some regressions with a more extreme β value have a considerably high R^2 (larger than 0.80), however there are many cities for which the R^2 value is very low (close to 0) which indicates a weak correlation between the size of a city and its electricity consumption, along time. In these cases, the resulting scaling coefficient is not significant at the 5% level.

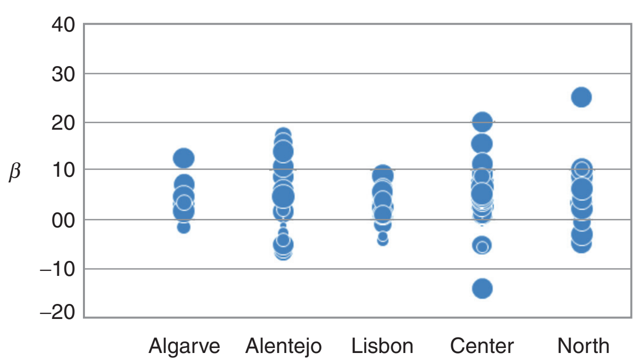

Figure 1: Dynamic scaling coefficient results for total electricity consumption divided by NUTS II regions.

To better understand the meaning of these results we show in Figure 2 and Figure 3 the example of some specific cities: Lisbon and Porto, the largest Portuguese cities, Montemor-o-Velho, Figueira da Foz and Penafiel which are some of the cities with the lowest and highest values of β and Santarém as an example of a municipality with low R^2 and non-significant scaling coefficients. The graphs in these figures show the evolution of electricity consumption and population between 1994 and 2009 in two different ways.

In Figure 2 we can observe the relation of the two variables throughout the years. Every time there is an identifiable trend in the points presented, an arrow with the direction of the evolution in time was added. As an example, in Lisbon, between 1994 and 2009 population decreased and electricity use monotonically increased. The absence of an arrow signifies that in the time period analyzed there was no identifiable trend between the two variables as it is the case of Santarém.

Figure 3 shows the relative values (with 1994 as the base year) and evolution of both population and total electricity consumption for the mentioned municipalities.

As it is possible to observe in Figure 2, there are cases (such as Santarém) where the scattering of the points led to a poor linear regression with a very low R^2. For such cases it is very difficult to identify the scaling relation tested. Some of the cities that show a high correlation factor have negative scaling coefficients. This is due to an inversion of the relation between electricity consumption and population, i.e., the population decreases whilst electricity consumption increases (see the examples of Lisbon and Porto).

One of the most interesting aspects of Figure 3 is that, whatever the population growth (or de-growth) trend is, total electricity use has a generally increasing evolution. This tendency was followed not only by the six

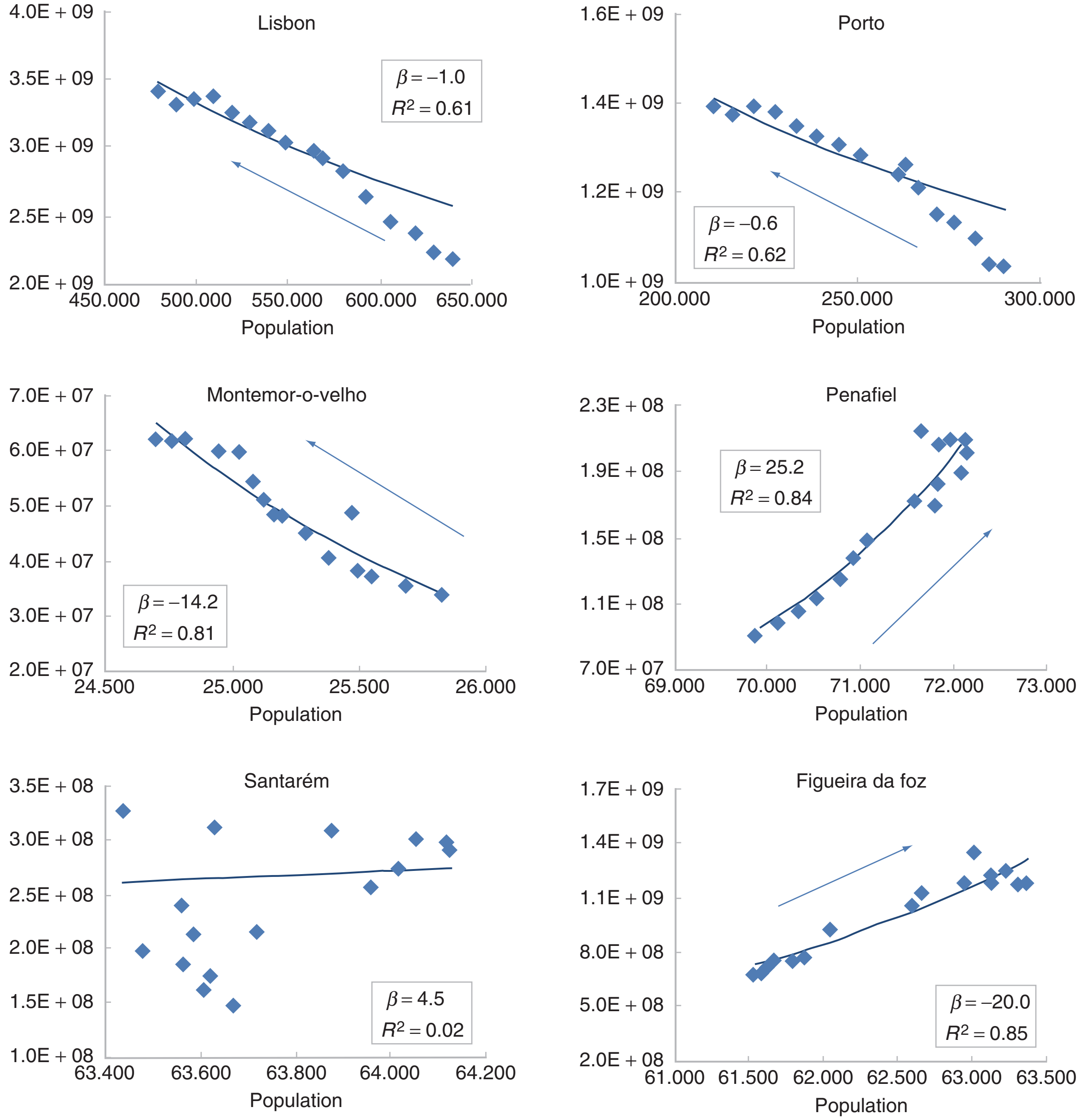

Figure 2: Total electricity consumption (kWh) versus population (points) and the regression fit (line) between 1994 and 2009 for six Portuguese municipalities.

municipalities that we use here as examples but in a wide majority of the municipalities (the only two exceptions, Barreiro and Santo Tirso, had a falling electricity consumption mainly due to reductions in the industrial sector). Beyond that, on average, electricity use doubled over the 16 years of the analysis. On the other hand, population trends seem to be more uneven. Including Porto and Lisbon, 24% of the municipalities had fewer inhabitants in 2009 when compared to 1994. In the case of our examples of Fig. 3 it is clear that the negative values of $\beta{-}1$ obtained correspond to those municipalities with an increasing consumption of electricity despite a population decrease (e.g. Lisbon, Porto and Montemor-o-Velho), as already mentioned.

If we divide both sides of Equation (2) by city size, we see that $\beta{-}1$ is the scaling exponent for per capita electricity consumption. So, in the cases, such as those of Porto or Lisbon, where $\beta{-}1$ is positive or zero, per

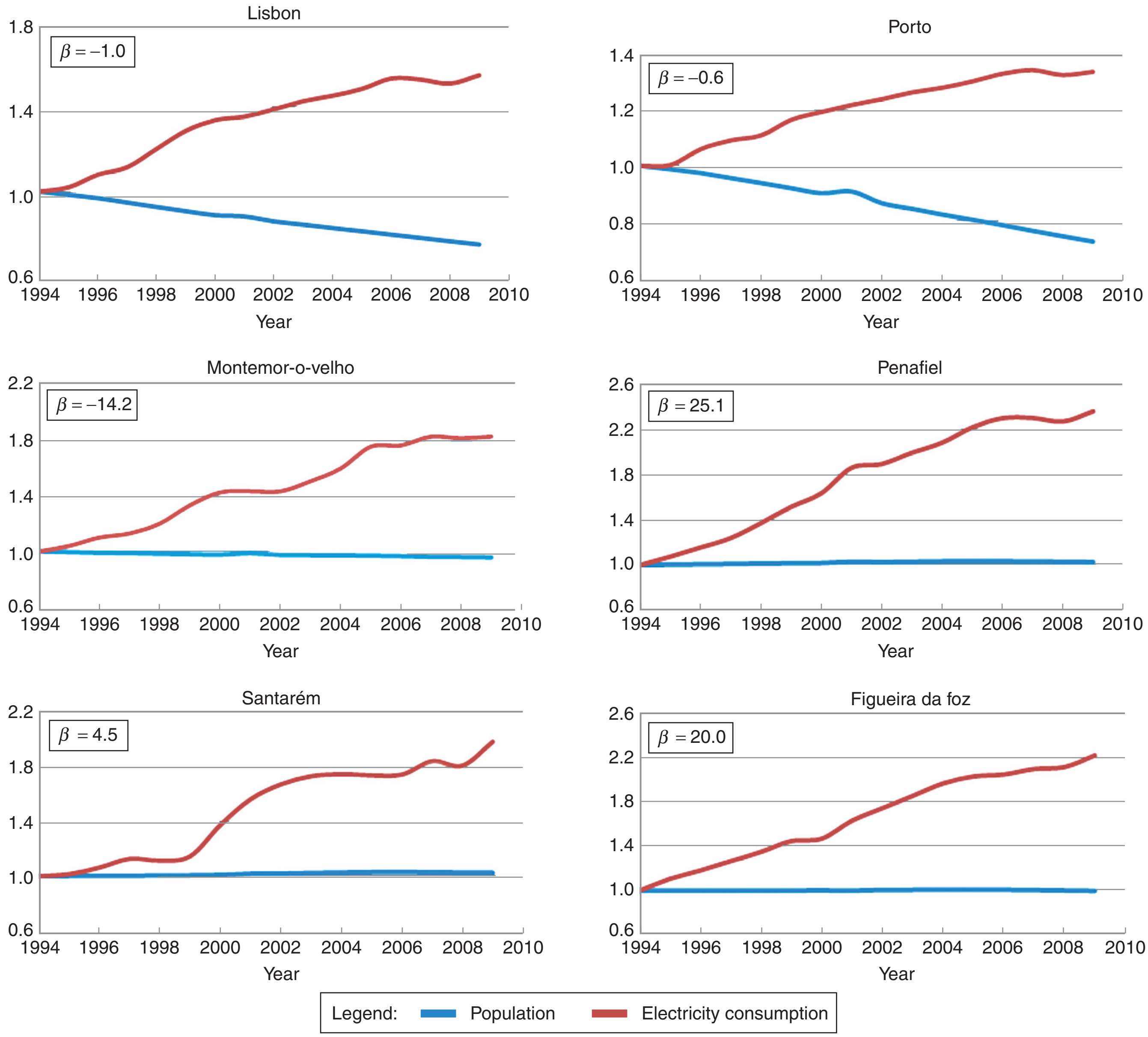

Figure 3: Relative values of total electricity consumption versus population between 1994 and 2009 for six Portuguese municipalities (base year 1994).

capita consumption is growing or constant. The cases where β–1 is negative are the odder ones, with per capita consumption decreasing along time; these cases are now being further investigated in order to understand what might be the cause of this decrease.

The disparity of relative growth of electricity consumption and population and the differences encountered, from city to city, in the scaling exponent seem to indicate that the scaling law, as stated in Equation (1), should not be applied as a general description of the electricity use in Portugal.

4.2. Total consumption's scaling exponent analysis

To finalize this analysis we tried to understand what influenced the scaling coefficients obtained. As discussed above, urban municipalities' electricity consumption seemed to increase in a similar way even

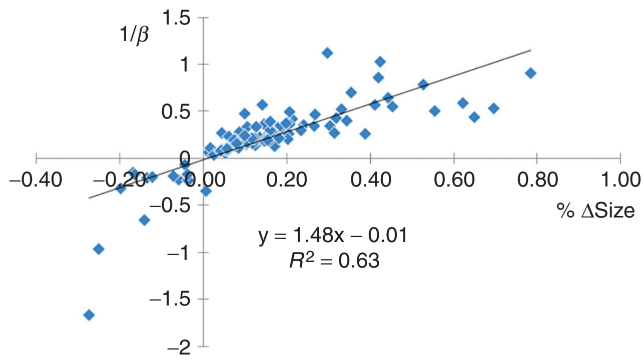

Figure 4: Relation between β coefficient for total electricity consumption and the relative variation of city's size.

for those with very different β values. Hence, the disparity of the scaling coefficient was more strongly linked with the evolution of the municipalities' population. Figure 4 shows the relation between the inverse of β and the relative variation of municipalities' size and the resulting linear regression. Relative variation of size was calculated as:

$$\%\Delta\text{Size} = \frac{S_{2009} - S_{1994}}{S_{1994}} \tag{4}$$

In this graph were included only the municipalities whose regressions had a scaling coefficient significant at the 5% level. With this condition, a total of 21 municipalities were excluded from the graph of Figure 4.

Notwithstanding the reduced number of municipalities, it is possible to observe a pattern in Figure 4. As hypothesized before, the variability of the scaling coefficients obtained seems to be correlated to the variation of population, even if it is not a strong relation.

In conclusion, the various analyses point to the fact that, although in some cases the regression shows a good fit (with significant scaling coefficients and high R^2), the scaling law should not be applied with time series data as there is a huge range of values encountered. Some of these values are below 0 which fall very far from the usual range found in the literature or in well-known scaling relations in nature. In addition, for many urban municipalities, the data points cannot be described by the scaling law equation as we could see in Figure 2 and the case of Santarém.

4.3. Sectoral analysis

Results for a sectoral analysis are very similar to those for total consumption (Figure 5). The main differences can be found for the industrial sector where a pattern between the inverse of β and the relative population growth is less evident and there are fewer municipalities with coefficient significant at the 5% level (74 out of 123). The conclusion that a scaling law, on its own, should not be employed to a time series without correction factors, is also true for each sector.

5. Cross-section analysis

After concluding that scaling laws may not be relevant to describe a single city's energy use throughout time it becomes essential to study how the cross-section scaling relation evolves within a certain period of time. Therefore, in this section, beyond testing the applicability of the scaling law to the total and sectoral consumption of electricity in Portuguese cities, we also present a discussion about the time dynamics of its coefficients. The time frame used was between 1994 and 2009.

Another important aspect to explore is the calculation and analysis of the differences between the real and regressed values of electricity consumption for each city. Identifying the urban municipalities that show a larger or smaller consumption than the one resulting from the direct application of the scaling law can provide insightful information on other drivers of the distribution of electricity use.

5.1. Total consumption

Given that in this work we only used one explanatory variable, one of the best tools to test for heteroscedasticity is the direct visualization of the relation of the residuals and the variable itself. We used this test for all years, but as the graphical results were similar in all cases we only present the results for 2001 (Figure 6). In this graph we can see the presence of a clear outlier, the municipality of Sines, with the other observations having no clear pattern. Another test for heteroscedasticity used was the Breusch-Pagan/Cook-Weisberg test that failed when used for the whole set of municipalities.

Sines is an industrial center and the location of the biggest oil refinery in Portugal, remaining, however, a quite small city. It is a clear exception, especially in terms of total and industrial energy consumption and, for that reason, will be excluded from these analyses. Performing

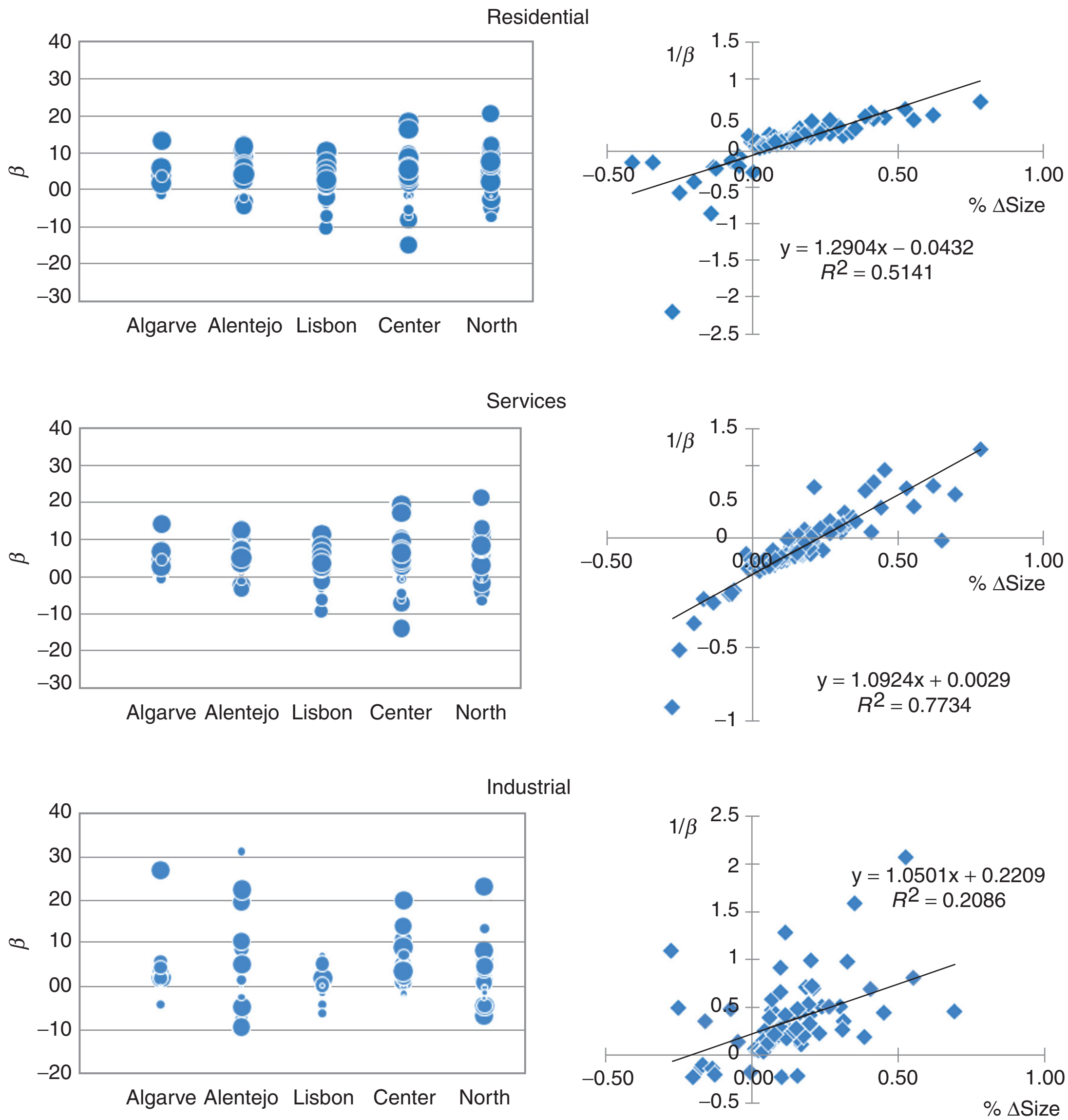

Figure 5: Dynamic scaling law results for different sectors of economic activity.

the Breusch-Pagan/Cook-Weisberg test again, excluding Sines, we could not reject the hypothesis of homoscedasticity with, a range of χ^2 values between 0.31 and 1.03 which are below the threshold of 3.84 for 122 observations and the 95% interval level [12].

Regressed coefficients (β and α) were found to be significant at the 5% level for all years. Their values are presented in Figure 7. The most intriguing results is the decreasing trend of β showing that electricity consumption started to follow more closely the distribution of population over the years, e.g., evolved towards a linear scaling law. Another fact to take into consideration is the increase of α. This may be explained by two facts: the growth of electricity consumption per capita along the years (around 50% increase between 1994 and 2009) and a compensation for the decrease of the β coefficient observed.

If we look to the literature [8] we can see the β's obtained in this work are consistent with the one found for Germany for 2002 (1.07). However this comparison does not seem to be of great relevance due to the large range of values obtained for both sets of municipalities.

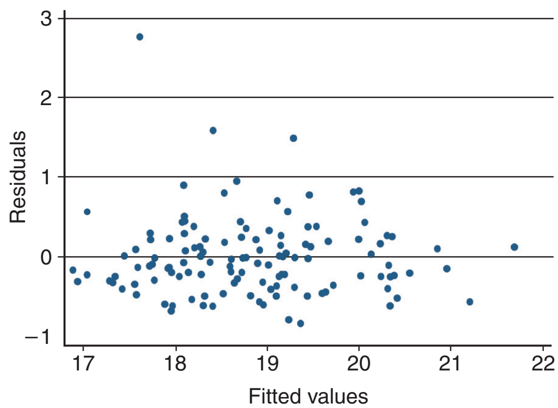

Figure 6: Graphical heteroscedasticity test for total electricity consumption in 2001 (Stata graphic output).

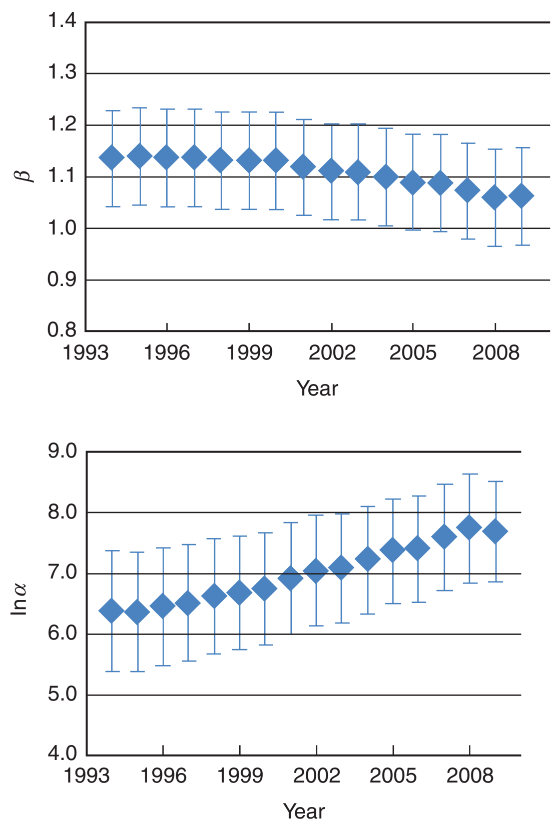

Figure 7: Parameters of total electricity scaling laws between 1994 and 2009 with 95% confidence intervals error bars.

A possible explanation for the trend in the values found is related with the sectoral composition of electricity consumption. The structure of consumption remained relatively constant and is very similar for

Table 2: Structure of urban electricity consumption divided by sectors for both urban criteria.

Year	Residential	Services	Industry	Others
1994	25.4%	20.6%	47.5%	6.4%
1995	24.8%	21.1%	47.5%	6.6%
1996	25.5%	21.8%	46.1%	6.7%
1997	24.9%	22.4%	46.2%	6.6%
1998	24.4%	23.1%	45.6%	6.8%
1999	25.2%	24.0%	44.8%	6.0%
2000	24.8%	24.2%	44.0%	6.9%
2001	24.5%	26.8%	41.8%	6.8%
2002	26.1%	24.0%	42.2%	7.6%
2003	26.1%	24.5%	41.5%	7.9%
2004	26.4%	24.4%	41.1%	8.2%
2005	27.3%	24.8%	39.7%	8.2%
2006	26.8%	25.6%	39.6%	8.1%
2007	27.0%	25.6%	39.3%	8.1%
2008	26.5%	26.2%	39.1%	8.3%
2009	28.4%	26.9%	36.4%	8.3%

both urban criteria used. Households show only a slight increase in its share, whereas the service sector had an increase of around 6 percentage points. This was reflected by the share decrease of industry that went from around 48% in 1994 to around 37% in 2009 (Table 2).

Furthermore, it is important to remember that, included in total electricity, there are several types of consumption from the different sectors of activity and the overall pattern may hide the behavior of electricity consumption of each one separately. The dynamics of total electricity consumption can be influenced by the dynamics of each sector in particular and by the increase of services share/reduction of the industrial share in total consumption.

5.2. Residential sector

Bettencourt *et al.* (2007) found two different values for the scaling parameter β, 1.05 and 1.00, for electricity consumption of households in China and Germany, respectively, although the relation was considered linear in both cases. Following this work, Horta-Bernús *et al.* (2010) did a similar study for the region of Andalucía in 2005 obtaining a value of 1.04 for the scaling factor of residential electricity consumption. Thinking of a set of cities in the same social context (i.e., similar average income, educational levels and respective distributions), household consumption represents individual needs that, rationally, should be

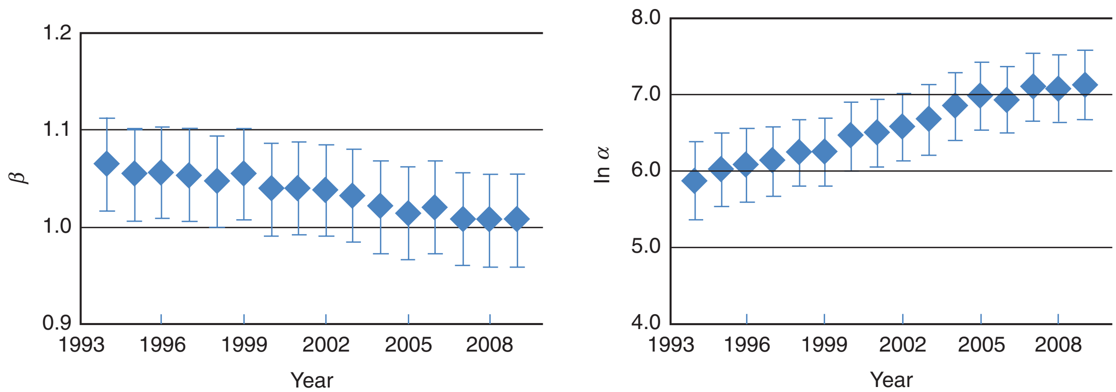

Figure 8: Parameters of residential electricity scaling laws between 1994 and 2009 with 95% confidence intervals error bars.

alike, whatever the size of the city, which is in line with the results obtained in the literature.

For Portugal, the fit of residential electricity consumption distribution to a scaling law is very good (R^2 around 0.94), as can be observed in Figure 9, with the parameters found to be significant at the 5% level. Nonetheless, once again, β coefficient results show a temporal dynamics that goes against the notion of this being a simple linear scaling relation (Figure 8).

One possible explanation for the non-linear behavior in earlier years might be the growing electrification of energy consumption, especially in thermal heating and cooking [17–19]. In rural areas and smaller cities, the use of gas and/or wood as the source of heating and cooking was the standard choice until recent years. During the 1990s, electrical thermal devices (both for temperature control and cooking) started to become more common. In fact, the proportion of electricity in the total energy spent for these uses more than tripled between 1996 and 2010 [18–19]. This led to an increase of electricity in the energy use of households affecting mostly the consumption of wood and bottled gas (Table 3).

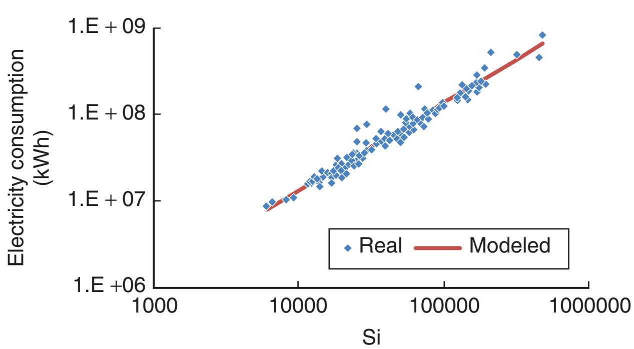

Figure 9: Comparison between the regressed and real values of residential electricity consumption in 2009.

As technological transitions are usually faster in larger cities where innovations are more easily accessible and innovators are concentrated [20], it can be expected that the spread of these electrical devices was not even within Portugal. As the dissemination reached smaller cities, the electrification of heating became more uniform and, with it, the values of electricity consumption as we saw in Fig. 8. This rationale could also explain the difference in the values obtained by Bettencourt *et al.* (2007). Germany is a country where residential heating technology is very mature and so similar in all regions. On the other hand, China is a developing country with large inequalities in life style between larger and smaller cities that, most probably, are also reflected in the type energy use of their inhabitants.

Distribution of electricity use is usually attributed to a number of variables linked with the characteristics of the dwellings [21–22]. As we have seen, at the level of municipality, this distribution can, apparently, be attributed solely to the number of inhabitants with little

Table 3: Percentage of energy carriers' contribution to domestic final energy consumption [17–19].

Energy Carrier	1989	1996	2010
Electricity	17%	28%	46%
Natural Gas	2%	2%	10%
Bottled Gas	20%	27%	14%
Wood	60%	43%	25%
Others	1%	1%	5%
TOTAL	100%	100%	100%

error. However, the scaling coefficient that describes this relation showed a temporal trend that we intend to study here.

Here we test the significance of electrification of the energetic system, the equality of income (given by Gini coefficient) and total income. We also include a proxy to characterize weather and consequent heating needs, HDD (Heating Degree Days).

The analysis of this section was run using national data, which, in this case, refers only to Continental Portugal as only the municipalities in the continent were considered in the regressions made in the previous section. Table 4 describes in more detail all data used.

In relation to the electrification coefficient of the residential energy system (*ecf*) the only data we found for specific residential breakdown of energy use was that collected by the national energy surveys made in 1989, 1996 and 2010 by both INE and DGGE. To estimate the missing values (for 1991–95 and 1997–2009) we used the share of electricity in total households' energy consumption in the years of the surveys and did linear interpolations. The negative impacts of using a low number of points can be relativized because it is not expected that electrification of the residential sector would have a high yearly variability.

Table 4: Variables for the econometric analysis of residential scaling coefficient parameterization.

Variable	Year	Symbol	Source
Residential scaling coefficient	1994–2009	β	Own calculations
HDD	1994–2009	*hdd*	Eurostat
Average income	1994–2009	*inc*	PorData
Gini coefficient	1994–2009	*gini*	INE
Electrification of the energy system	1989;1996;2010	*ecf*	DGEG & INE

A test for collinearity showed that average income and electrification coefficient are correlated. To decrease the error of the regression and its interpretation, we used these variables in separate models (Table 5).

Results confirm what has been previously hypothesized. Taken individually, both income and the electrification coefficient are highly significant in the trend of β. In both cases, an increase of their value leads to a decrease of the scaling coefficient.

Gini coefficient and HDD, the weather proxy used, are not significant in the characterization of the deviations of residential electricity use.

5.3. Service sector

Regarding the service sector, the only previous study is for the Andalucía region in Spain in 2005 [10], where a scaling factor of 1.21 was obtained.

Table 5: Results for the parameterization of the residential scaling coefficient.

	(1)		(2)		(3)		(4)	
(Intercept)	1.14	***	1.34	***	0.80	**	–0.32	
	(1.14)		(0.09)		(0.26)		(0.27)	
hdd	1.34E-05		1.10E-05		8.35E-03		–1.22E-03	
	(1.71E-05)		(2.09E-05)		(1.94E-02)		(2.14E-02)	
gini	–4.63E-04		–1.98E-03		5.82E-02		2.41E-02	
	(1.79E-03)		(2.23E-03)		(1.71E-05)		(7.52E-02)	
inc	–4.23E-06	***			–1.02E-01	***		
	(5.26E-07)				(1.33E-02)			
ecf			–0.83	***			–0.23	***
			(0.13)				(0.03)	
Adjusted R^2	0.86	***	0.79	***	0.86	***	0.82	***

Note: *** $p < 0.001$; ** $p< 0.01$; * $p< 0.05$; . $p < 0.1$

(1) $\beta_t = c_0 + c_1 hdd_t + c_2 gini_t + c_3 inc_t + \varepsilon$

(2) $\beta_t = c_0 + c_1 hdd_t + c_2 gini_t + c_3 ecf_t + \varepsilon$

(3) $\text{In}(\beta_t) = c_0 + c_1 \text{In}(hdd_t) + c_2 \text{In}(gini_t) + c_3 \text{In}(inc_t) + \varepsilon$

(4) $\text{In}(\beta_t) = c_0 + c_1 \text{In}(hdd_t) + c_2 \text{In}(gini_t) + c_3 \text{In}(ecf_t) + \varepsilon$

The coefficients obtained for Portugal were lower than the coefficients in Horta-Bernús *et al.* work but are still larger than 1 which implies that larger cities have higher services' electricity consumption. Furthermore, in contrast with what happened in Subsections 5.1 and 5.2, the value of β remained relatively constant (Figure 10). It is also relevant to mention that the R^2 values obtained were between 0.74 and 0.83, a seemingly good fit showed in Figure 11 and with coefficients significant at the 5% level.

Once again, α showed an upwards trend, although not monotonic. As β values for this sector are relatively constant (especially when comparing directly the first and last years), this increase is only explained by the rise of per capita consumption which was the largest of all sectors (more than 100%).

Looking at these results it seems plausible to hypothesize that this scaling relation is related with specific characteristics of cities. Due to the nature of services companies' businesses, location and distance to the client is, usually, more important than for other sectors. For example, the location of a restaurant, supermarket and/or bank branch is crucial for the success of the business, whilst for a metallurgical or toy factory it is much more important to maintain the overall production costs low. Thinking on the basics of urban economics that reports the importance of transportation needs in terms of city structure [23–24], urban environment seems to be well suited for services in general, and, the larger the city is, the better. Therefore, it seems logical to conclude that services are more concentrated in cities and that, the larger cities are, the larger this effect is.

5.4. Industrial sector

For the industrial sector, the heteroscedasticity visual test showed a presence of a clear outsider (Sines) as happened in sub-section 5.1 (Figure 12). Again, we disregarded this municipality in the analysis performed and, afterwards, the values of χ^2 obtained in the Breusch-Pagan/Cook-Weisberg test were below the 95% threshold (a range of 0.02–0.77).

The results obtained were different from the ones obtained in previous sectors (Figure 13). Values of R^2 for the industrial sector were the lowest of all, being, approximately 0.6 and the 95% confidence intervals obtained were considerably larger than for the other sectors (an average deviation of 16%, 10% and 5% for the industrial, services and residential sectors, respectively). These facts indicate that, in case of industry, city size and electricity consumption do not have such a strong correlation as the one observed for households and services, which

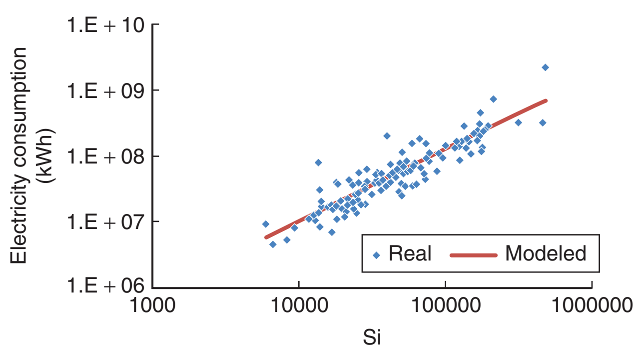

Figure 11: Comparison between the regressed and real values of services' electricity consumption in 2009.

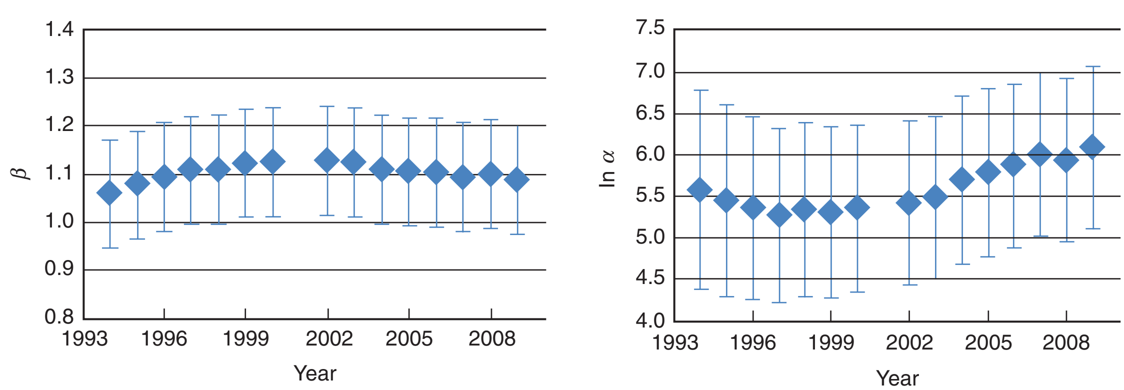

Figure 10: Parameters of services electricity scaling laws in between 1994 and 2009 with 95% confidence intervals error bars.

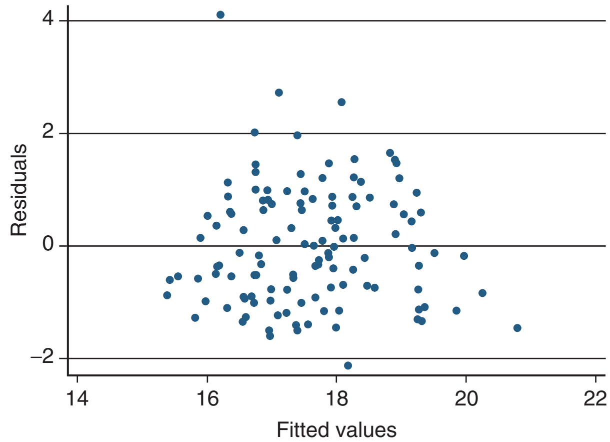

Figure 12: Graphical heteroscedasticity test for industry sector in 2001.

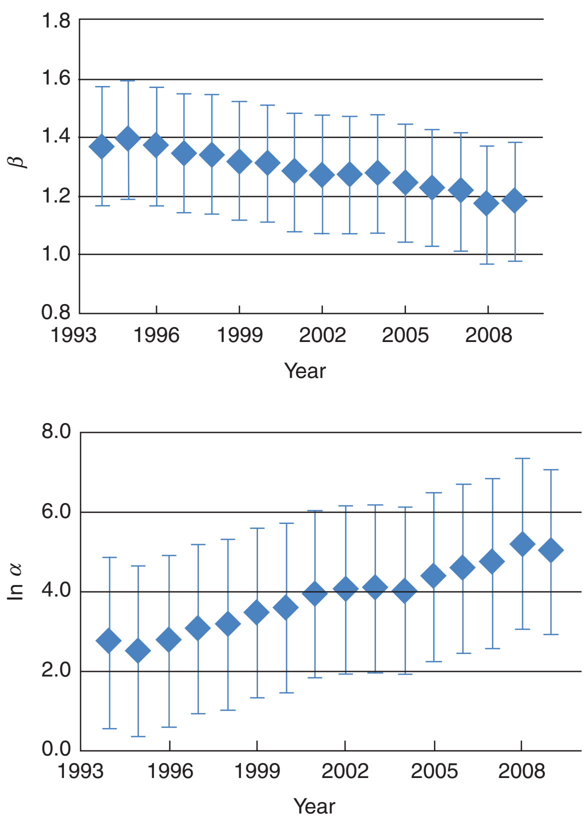

Figure 13: Parameters of industrial electricity scaling laws between 1994 and 2009 with 95% confidence intervals error bars.

can also be observed through the larger dispersion of values in Figure 14.

Nonetheless, looking at Figure 13 we can observe a decrease of the scaling coefficient (15% for the first criterion and 20% for the second) along the years, yet

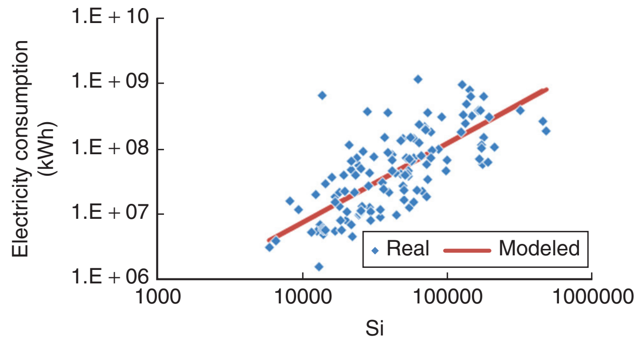

Figure 14: Comparison between the regressed and real values of industrial electricity consumption in 2009.

always above 1. Even with a lower accuracy of the results it is possible to conclude that industries were highly concentrated in larger urban areas and, although this concentration has diminished, it still exists.

The little information available together with the observation of a weaker correlation between city size and electricity consumption prevent us from explaining these observations.

A comparison with the value found in the literature [10] seems to be counterproductive as, in this paper, the value of the Adjusted R^2 was even lower (0.28) than the ones obtained in our study.

6. Conclusions

To study the distribution of electricity consumption is one of the most relevant issues about urban modeling, especially due to a higher energy use in cities than in rural areas. In this work we studied how can simple scaling laws describe electricity use in Portugal.

First, we observed that the scaling law is not suitable to describe a city's electricity consumption growth as some results are not significant and the range of scaling coefficients is very large.

In relation to a cross sectional analysis it was important to study the different sectors separately. The residential sector is the sector for which the urban scaling law obtained better correlation coefficients. In this case, the shape of the scaling law changed through time evolving towards a linear relation. A parameterization study showed that this evolution was linked with the progressive electrification of the residential energy system in the last two decades and with the overall increase of families' income. The set of both these analyses showed that it is possible to create scenarios for future electricity consumption of this sector distribution using a simple scaling law, which coefficient can be parameterized, if needed, using two variables.

Services, on the other hand, showed a relatively constant scaling exponent. Technology shifts that influenced the scaling law for households do not apply for this sector as fireplaces and small size gas heaters (the traditional forms of heating) are only used by residential consumers. We could assess that there is a clear concentration of services electricity consumption in the larger cities which we attributed to the attraction that large urban areas provide to markets.

The industry sector comes out, in the structural analysis of electricity demand, as the one with larger share. In fact, the time dynamics shown is similar to that of total consumption with an accentuated decrease of the scaling exponent. However, it is also the sector with the lowest accuracy and worst correlation indicating that city size is not as relevant as it is for services and households. Given the share of industry in total consumption it would be important to find an explanation for the decrease in β and the low accuracy obtained. The lack of data at the municipality scale largely contributes to the existing difficulties of finding these answers.

It should be referred that, even after the validation of the possibility of creating models for urban electricity use only with cities' population, it is also important to assess the relevance of this variable when considered together with other variables that might affect electricity consumption.

As future work we will focus on determining a model that could answer these questions and help us understand the mechanisms behind energy consumption, with special emphasis to the industrial sector.

Although there are still a few questions left to be studied in more detail, the results obtained were surprising, especially regarding the time evolution of the scaling exponent for total, residential and industrial electricity consumption. Furthermore, we observed the relevance of technology shifts in the distribution of residential electricity consumption explaining the deviation from a linear relation in the first years of the study.

7. Acknowledgments

This work was only possible due to the financial support given by Fundação para a Ciência e Tecnologia through PhD grant SFRH/BD/40941/2007 and project PTDC/SEN-ENR/111710/2009 (MeSUr - Metrics framework for Urban Metabolism Sustainability).

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