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THE BELL CURVE REVISITED: A SOUTH AFRICAN PERSPECTIVE

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ABSTRACT

The principal objective of the study was to examine some of the scientific premises of *The Bell Curve* by Herrnstein and Murray (1996). It was shown that the hypothetical construct referred to as 'g' is nothing more than the aggregate of the various cognitive tests contained in the battery of tests that is subjected to factor analysis. The nature of g, therefore, depends on the composition of the test battery. Furthermore, it was shown that the predictive validity of multiple factors is always better than that of g alone. The validity of Spearman's hypothesis was investigated and found wanting. The critical role of formal schooling in the development of intellect (both fluid and crystallised) was discussed with reference to illiterate adults in South Africa. The importance of the interaction of heredity and environment was stressed. The best genetic endowment can only come to fruition in a stimulating environment. The relationship between locus of control and intelligence was discussed.

OPSOMMING

Die hoofdoelstelling van die studie was om die wetenskaplike aannames waarop *The Bell Curve* deur Herrnstein en Murray (1996) gebaseer is, te ondersoek. Dit is aangetoon dat die hipotetiese konstruk, bekend as 'g', niks meer is as die somtotaal van die verskillende kognitiewe toetse wat in die battery ingesluit is en aan faktorontleding onderwerp word. Hierbenewens is getoon dat die voorspellingsgeldigheid van veelvuldige faktore altyd hoër is as dié van g alleen. Die geldigheid van Spearman se hipotese is ondersoek en gebrekkig bevind. Die kritieke rol van formele skoolopleiding in die ontwikkeling van intellik (sowel vloeibaar as gekristalliseerd) is bespreek met verwysing na ongeletterde volwassenes in Suid-Afrika. Die belangrikheid van interaksie tussen oorerwing en omgewing is beklemtoon. Die beste genetiese potensiaal kan slegs tot verwekliking kom in 'n stimulerende omgewing. Die verband tussen lokus van beheer en intelligensie is bespreek.

The Bell Curve by Herrnstein and Murray was first released by the Free Press in October 1994. According to Murray the initial reaction to the book was quite positive, but soon thereafter there was an avalanche of publications that varied from moderate to virulently hostile. The book was characterised by some as pseudoscience – a racist screed designed to promote a radical political agenda (Herrnstein & Murray, 1996, p. 553). However, despite the negative reception of the book, its scientific premises deserve careful examination.

The Bell Curve is so wide in scope that it is virtually impossible to summarise it in a paragraph or two, let alone evaluate all the research on which it is based. However, a very brief account will be given here of the most important findings and conclusions reached by Herrnstein and Murray (1996). Where relevant their basic assumptions and scientific premises will be highlighted.

Part I of their book deals with the emergence of a cognitive elite in the US. The growth in the proportion of people obtaining college degrees from 1900 to 1990 is given, as well as the percentage of students in the top quartile of intellectual ability who continue to college after high school. The influx of people with high IQs to graduate schools, high-IQ professions, executive and managerial positions, is described.

The principal conclusion reached in Part I of *The Bell Curve* is that general intelligence, defined as g, determines success in both the academic and occupational realm.

Part II of *The Bell Curve* deals with the role of socioeconomic background and intelligence in the etiology of poverty, unemployment, and crime. The main conclusion reached in Part II of *The Bell Curve* is that the highest risk factor in poverty, unemployment, and crime, is not socioeconomic background or education, but low levels of intelligence.

Part III of *The Bell Curve* deals with ethnic differences in cognitive ability, and social problems in relation to intelligence.

As the present study relates to Herrnstein and Murray's findings concerning ethnic differences in intelligence, Part III of their book will be dealt with in more detail. In particular, their scientific premises will be critically examined.

To start off with, Herrnstein and Murray compared the IQs of whites and East Asians in the US. A difference of approximately three IQ points was found (p. 276).

In comparing the verbal and non-verbal IQs of Asian-Americans, a marked discrepancy was observed: According to an estimate by Vernon (1982) the mean IQ of Asian-Americans is about 97 in respect of verbal tests, and about 110 in respect of visuospatial tests (p. 301). In contrast with this, the mean verbal IQ and mean performance IQ of whites are very similar (pp. 299–300).

Herrnstein and Murray explained that the similarity in verbal and performance IQ of whites is due to the fact that the tests of intelligence have been standardised on predominantly white populations (pp. 299–300). If this is true, then it implies that both blacks and East-Asians have been excluded from the standardisation sample of the tests that were used for drawing the comparisons.

In contrasting the IQs of blacks and whites, Herrnstein and Murray reviewed 156 studies and found the mean difference to be 1,08 standard deviations, or about 16 IQ points in favour of whites (p. 276). In the National Longitudinal Survey of Youth (NLSY), conducted in 1980, the B/W difference in IQ was estimated at 1,2 standard deviations, or about 18 IQ points. A

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1. *All page numbers refer to Herrnstein and Murray (1996) unless otherwise indicated.*

total of 6 502 whites and 3 022 blacks were involved in the survey (pp. 277-278). In contrasting the mean IQs of blacks and whites at every level of socioeconomic status (SES), it was found that there are differences at every level, but the biggest differences were found at the high levels of SES (p. 269).

Furthermore, Herrnstein and Murray maintain that races differ not only in average intellectual capacity, but also in the pattern of their abilities (p. 299).

To support their claim, they cited a study by Jensen and Reynolds (1982) who compared the profiles of white and black children, with identical overall scores on the Wechsler Intelligence Scale for Children (WISC-R), and found that whites typically have higher scores than blacks on the subtests involving spatial-perceptual ability, and blacks typically have higher scores than whites in subtests such as arithmetic and immediate memory (p. 426). However, no evidence is given to support this finding in respect of adult subjects.

Jensen (1985, 1987) linked the above-mentioned findings to an observation made by Charles Spearman (1927), and subsequently formulated what has become known as Spearman's hypothesis. Spearman's observation relates to a study by Pressey and Teter (1919) who applied ten [cognitive] tests to 120 black American children, aged 10 – 14 years, and to 2 000 white American children. On the average the black children lagged about two years behind the white children. In reviewing the results of this study Spearman made the observation that 'their inferiority extended through all ten tests, but it was most marked in just those [tests] which are known to be most saturated with g' (p. 379).

Following in the footsteps of Jensen, Herrnstein and Murray formulated the Spearman hypothesis as follows: 'if the B/W difference on test scores reflects a real underlying difference in the general mental ability, g, then the size of the B/W difference will be related to the degree to which the test is saturated with g' (p. 301). The hypothesis therefore implies that 'the better a test measures g, the larger the black-white difference will be' (pp. 301-302).

Jensen (1993) subsequently reviewed 13 studies pertaining to Spearman's hypothesis, and concluded that his hypothesis has been substantiated by all of them, and that no appropriate data set has yet been found that refutes his hypothesis (p. 48). In order to obviate the usual objections to the use of intelligence tests in the exploration of ethnic differences, he also made use of reaction time tests. Again he found support for Spearman's hypothesis: 'The size of the standardized mean W-B difference (or ES) on the chronometric variables is directly related to the variables' loadings on psychometric g' (p. 70). But he reported no studies with adult subjects.

From the foregoing, it should be apparent that Spearman's hypothesis, and g, defined as general intelligence, are the two cornerstones on which Herrnstein and Murray's theory of ethnic differences is based.

Herrnstein and Murray (1996, p. 561) maintain that the loading of a test is associated with its heritability – the higher the g loading, the higher the degree of inbreeding depression. They are cautious not to state explicitly that ethnic differences have a genetic basis, but they do intimate that Spearman's hypothesis 'undercuts many of the environmental explanations' of the black-white difference (p. 303).

They also investigated the relationship of intelligence to various social problems. Marked ethnic differences were observed, but most of these differences were reduced if the groups were equated in terms of intelligence. For blacks and whites two of the most important indicators of success were in fact reversed: After controlling for intelligence it was found that more blacks than whites graduate from college and enter into prestigious occupations. Furthermore, the wage gap for year-round workers shrinks from several thousand to a few hundred dollars (p. 320-3-23).

In a nutshell, the main conclusion reached in Part III of *The Bell Curve* is that there is a highly significant difference in mean IQ between blacks and whites, in favour of whites. It is hinted that the difference might have genetic roots.

The following issues will be critically examined in this article:

(a) The psychometric interpretation of the construct, g.
(b) Spearman's hypothesis.
(c) Standardisation of tests of intelligence.
(d) The predictive power of g versus the predictive power of rotated factor scores.
(e) Factors that influence the g-loadings of tests in a factor battery.
(f) The nature-nurture controversy: Brain changes in response to experience.
(g) Locus of control and intelligence.

ETHNIC DIFFERENCES IN COGNITIVE ABILITY

The psychometric interpretation of the construct, g

In order to evaluate ethnic differences in measured intelligence properly a full understanding of the construct, g, is essential.

It will be shown here that if the variables in a test battery are standardised with a mean of zero and a standard deviation of one, then the correlation of each of the variables with the total score will be equal to its loading on the first centroid of the test battery. The implications of this from an interpretative point of view will be dealt with. A formal proof of the stated relationship is given in Appendices 1 and 2.

In order to facilitate the arguments in this section, use will be made of real data.

Table 1 shows the intercorrelations of the subtests of the *General Scholastic Aptitude Test* (GSAT) and *Senior Ability Tests* (SAT) jointly with the standardised total score across all the variables. The intercorrelations are based on a sample of 1598 freshmen from the Rand Afrikaans University.

The correlations of the various subtests with the total score appear in the last row and last column of the intercorrelation matrix (17 x 17). The first centroid loadings appear in the last row of Table 1.

From an inspection of Table 1 it should be obvious that the loadings of the first centroid are identical to the correlations of the various tests in the battery with the total standardised score.

The correlations of the various tests with the total score, as well as the loadings of the tests on the first principal component, first principal factor and first centroid are given in Table 2. The intercorrelations of the four constructs mentioned are given at the bottom of the table. These correlations range from 0.9955 to 1.0000.

From the foregoing it should be clear that the hypothetical construct referred to as g is nothing more than the aggregate of the various tests contained in the factor test battery. Furthermore, the g-loadings or saturations of the various tests in the battery represent the correlations of the respective tests with the total standardised score. Each of the correlations are, in fact, part-whole correlations, as each of the tests are represented in the total score. The magnitude of these correlations will, therefore, also be a function of the number of tests in the battery.

TABLE 1
MATRIX OF INTERCORRELATIONS OF GSAT AND SAT JOINTLY WITH Z-TOTAL SCORE

Variables	GSAT1	GSAT2	GSAT3	GSAT4	GSAT5	GSAT6	SAT1	SAT2	SAT3	SAT4	SAT5	SAT6	SAT7	SAT8	SAT9	SAT10	Z-total
1. GSAT1: Word analogies	1.000	0.542	0.582	0.454	0.663	0.484	0.446	0.239	0.381	0.176	0.355	0.301	0.300	0.323	0.331	0.289	0.660
2. GSAT2: Number series	0.542	1.000	0.646	0.570	0.552	0.600	0.465	0.485	0.323	0.306	0.489	0.455	0.433	0.450	0.291	0.308	0.761
3. GSAT3: Verbal reasoning	0.582	0.646	1.000	0.571	0.625	0.605	0.500	0.409	0.373	0.249	0.481	0.440	0.434	0.467	0.344	0.343	0.776
4. GSAT4: Pattern completion	0.454	0.570	0.571	1.000	0.509	0.630	0.377	0.305	0.200	0.203	0.496	0.414	0.435	0.475	0.268	0.308	0.694
5. GSAT5: Word pairs	0.663	0.552	0.625	0.509	1.000	0.541	0.445	0.248	0.405	0.194	0.377	0.368	0.336	0.359	0.313	0.336	0.699
6. GSAT6: Figure analogies	0.484	0.600	0.605	0.630	0.541	1.000	0.397	0.338	0.255	0.277	0.491	0.461	0.468	0.529	0.246	0.286	0.732
7. SAT1: Verbal comprehension	0.446	0.465	0.500	0.377	0.445	0.397	1.000	0.417	0.493	0.358	0.464	0.499	0.401	0.406	0.403	0.345	0.713
8. SAT2: Calculations	0.239	0.485	0.409	0.305	0.248	0.338	0.417	1.000	0.264	0.409	0.393	0.343	0.402	0.319	0.274	0.181	0.580
9. SAT3: Disguised words	0.381	0.323	0.373	0.200	0.405	0.255	0.493	0.264	1.000	0.266	0.281	0.312	0.254	0.260	0.323	0.283	0.545
10. SAT4: Comparison	0.176	0.306	0.249	0.203	0.194	0.277	0.358	0.409	0.266	1.000	0.333	0.320	0.273	0.211	0.307	0.271	0.496
11. SAT5: Pattern completion	0.355	0.489	0.481	0.496	0.377	0.491	0.464	0.393	0.281	0.333	1.000	0.497	0.456	0.496	0.258	0.292	0.689
12. SAT6: Figure series	0.301	0.455	0.440	0.414	0.368	0.461	0.499	0.343	0.312	0.320	0.497	1.000	0.477	0.545	0.302	0.287	0.675
13. SAT7: Spatial 2D	0.300	0.433	0.434	0.435	0.336	0.468	0.401	0.402	0.254	0.273	0.456	0.477	1.000	0.643	0.227	0.265	0.654
14. SAT8: Spatial 3D	0.323	0.450	0.467	0.475	0.359	0.529	0.406	0.319	0.260	0.211	0.496	0.545	0.643	1.000	0.209	0.278	0.670
15. SAT9: Memory (paragraph)	0.331	0.291	0.344	0.268	0.313	0.246	0.403	0.274	0.323	0.307	0.258	0.302	0.227	0.209	1.000	0.398	0.528
16. SAT10: Memory (symbols)	0.289	0.308	0.343	0.308	0.336	0.286	0.345	0.181	0.283	0.271	0.292	0.287	0.265	0.278	0.398	1.000	0.526
17. Z-total	0.660	0.761	0.776	0.694	0.699	0.732	0.713	0.580	0.545	0.496	0.689	0.675	0.654	0.670	0.528	0.526	1.000
Column sums (excluding variable 17)	6.865	7.915	8.069	7.214	7.271	7.607	7.414	6.027	5.671	5.153	7.160	7.021	6.803	6.969	5.493	5.469	
α_{j1} First centroid loadings	0.660	0.761	0.776	0.694	0.699	0.732	0.713	0.580	0.545	0.496	0.689	0.675	0.654	0.670	0.528	0.526	

Note GSAT: General Scholastic Aptitude Test; SAT: Senior Ability Tests

It should therefore be obvious that the g-loadings depend upon the unique composition of the test battery that is subjected to factor analysis. If, for example, the majority of the tests are memory tests, then g would represent general memory and not general intelligence. How, then should a test battery be constituted in order to represent general intelligence?

The structure of any test battery is a function of both the sample of subjects to whom the tests are applied, and the composition of the test battery. If the sample of subjects to whom the test battery (containing a variety of cognitive tests) is applied, is psychologically undifferentiated, a one-factor-structure would result. By contrast, multiple factors would arise from a highly differentiated sample of subjects.

A well designed factor analysis calls for a certain degree of over-determination of every factor postulated (Ledermann, 1937). For a factor to exist there should be three or more high loadings. Furthermore, to ensure stability of the outcome, samples of 500 or more should be used (Browne, 1965).

In most studies the general factor (unrotated first principal factor) is represented by the loadings of a hodgepodge of tests. Thus, the nature of the general factor depends on the particular mix of tests included in the test battery.

Herrnstein and Murray 'accept that there is such a thing as a general factor of cognitive ability on which human beings differ: the famous g' (1996, p. 557). It is indeed one of the cornerstones on which their theory of ethnic differences in intelligence rests. However, they are rather vague in specifying the exact nature of g.

There are many different factor analytical models, but most of them can account equally well for the interrelationships between tests in a test battery. The models vary in mathematical sophistication, but not so much in explanatory power. Similarly, there are many different rotational procedures, both orthogonal and oblique. Again, the rotational procedures vary in mathematical sophistication, but not so much in explanatory power.

The British factor analytical school (Holzinger & Harman, 1941; Spearman, 1927; Vernon, 1961) has typically favoured solutions where the first principal axis (or so-called general factor) receives prominence. In contrast to this, the American factor analytical school (Thurstone, 1947; Tucker, 1944) has preferred rotation to simple structure. In the latter case the variance of the general factor is redistributed over all the factors that have been extracted from the intercorrelation or intercovariance matrix. Accordingly the general factor disappears.

Guttman (1954, 1957) developed a non-metrical factor analytical model, known as the radex. It is quite different from conventional multiple factor analysis (Thurstone, 1947), yet both models can account equally well for the underlying structure of an intercorrelation matrix (Guttman, 1965). Guttman (1957) re-analysed the data used by Thurstone and Thurstone (1941) and found that two facets, viz. 'language of communication', and 'operation', could account for all the interrelationships between the measures. Thus, a very economic description of the underlying structure (of the interrelationships) of the variables, was obtained.

In a nutshell he found that:

> The three 'languages of communication' [verbal, numerical and figural] seem to provide three poles in the plane, and the tests radiate outward toward these poles from an inner 'analytical ability' belt to an outer 'achievement' belt (Guttman, 1966, p. 2).

TABLE 2
LOADINGS OF GSAT AND SAT ON Z-TOTAL, FIRST PRINCIPAL COMPONENT, FIRST PRINCIPAL FACTOR, AND FIRST CENTROID

	Z-total score	First principal component	First principal factor	First centroid
1. GSAT1: Word analogies	0.660	0.670	0.657	0.660
2. GSAT2: Number series	0.761	0.776	0.756	0.761
3. GSAT3: Verbal reasoning	0.776	0.792	0.780	0.776
4. GSAT4: Pattern completion	0.694	0.711	0.691	0.694
5. GSAT5: Word pairs	0.699	0.711	0.703	0.699
6. GSAT6: Figure analogies	0.732	0.750	0.735	0.732
7. SAT1: Verbal comprehension	0.713	0.707	0.685	0.713
8. SAT2: Calculations	0.580	0.567	0.531	0.580
9. SAT3: Disguised words	0.545	0.525	0.493	0.545
10. SAT4: Comparison	0.496	0.462	0.429	0.496
11. SAT5: Pattern completion	0.689	0.693	0.664	0.689
12. SAT6: Figure series	0.675	0.676	0.647	0.675
13. SAT7: Spatial 2D	0.654	0.657	0.633	0.654
14. SAT8: Spatial 3D	0.670	0.679	0.662	0.670
15. SAT9: Memory (paragraph)	0.528	0.499	0.466	0.528
16. SAT10: Memory (symbols)	0.526	0.501	0.462	0.526

MATRIX OF INTERCORRELATIONS

Variables	1	2	3	4
1. Z-total score	1.000	0.998	0.996	1.000
2. First principal component	0.998	1.000	0.999	0.998
3. First principal factor	0.996	0.999	1.000	0.996
4. First centroid	1.000	0.998	0.996	1.000

TABLE 3
MATRIX OF INTERCORRELATIONS: COOPER UNION DATA (1959)

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. Addition	1.000	0.628	0.764	0.112	-0.065	0.074	0.491	0.238	0.425	0.103	0.190	-0.006	0.168	0.216	0.151	0.280	0.360
2. Division	0.628	1.000	0.665	0.159	0.014	0.037	0.448	0.199	0.466	-0.016	0.143	0.037	0.185	0.209	0.097	0.226	0.398
3. Subtraction & Multiplication	0.764	0.665	1.000	0.076	-0.040	0.039	0.432	0.183	0.431	0.176	0.273	0.078	0.134	0.245	0.156	0.184	0.324
4. Paper Form Board	0.112	0.159	0.076	1.000	0.561	0.706	0.296	0.329	0.190	0.211	0.228	0.353	0.172	0.238	0.329	0.231	0.203
5. Surface Development	-0.065	0.014	-0.040	0.561	1.000	0.584	0.133	0.274	0.089	-0.050	0.063	0.297	0.041	0.128	0.291	0.153	0.065
6. Block Counting	0.074	0.037	0.039	0.706	0.584	1.000	0.318	0.348	0.091	0.147	0.151	0.300	0.133	0.231	0.428	0.217	0.274
7. SCAT-quantitative	0.491	0.448	0.432	0.296	0.133	0.318	1.000	0.299	0.428	0.233	0.344	0.255	0.417	0.614	0.260	0.408	0.349
8. Marks	0.238	0.199	0.183	0.329	0.274	0.348	0.299	1.000	0.344	0.109	0.172	0.120	0.135	0.285	0.374	0.431	0.373
9. Lettergrouping	0.425	0.466	0.431	0.190	0.089	0.091	0.428	0.344	1.000	0.200	0.205	0.107	0.069	0.306	0.369	0.424	0.387
10. Vocabulary	0.103	-0.016	0.176	0.211	-0.050	0.147	0.233	0.109	0.200	1.000	0.838	0.161	0.264	0.265	0.317	0.314	0.306
11. SCAT-verbal	0.190	0.143	0.273	0.228	0.063	0.151	0.344	0.172	0.205	0.838	1.000	0.263	0.387	0.343	0.242	0.357	0.368
12. Plane Geometry	-0.006	0.037	0.078	0.353	0.297	0.300	0.255	0.120	0.107	0.161	0.263	1.000	0.591	0.518	0.197	0.183	0.154
13. Plane Trigonometry	0.168	0.185	0.134	0.172	0.041	0.133	0.417	0.135	0.069	0.264	0.387	0.591	1.000	0.565	0.032	0.078	0.093
14. Intermediate Algebra	0.216	0.209	0.245	0.238	0.128	0.231	0.614	0.285	0.306	0.265	0.343	0.518	0.565	1.000	0.382	0.407	0.231
15. Concept Attainment: spatial	0.151	0.097	0.156	0.329	0.291	0.428	0.260	0.374	0.369	0.317	0.242	0.197	0.032	0.382	1.000	0.620	0.446
16. Concept Attainment: verbal	0.280	0.226	0.184	0.231	0.153	0.217	0.408	0.431	0.424	0.314	0.357	0.183	0.078	0.407	0.620	1.000	0.698
17. Concept Attainment: numerical	0.360	0.398	0.324	0.203	0.065	0.274	0.349	0.373	0.387	0.306	0.368	0.154	0.093	0.231	0.446	0.698	1.000
Column sums*	4.213	3.980	4.257	4.907	3.315	4.668	–	4.568	–	4.145	5.017	4.245	3.977	5.263	5.063	5.380	5.293
α^*_{ji} First centroid loadings	0.510	0.482	0.515	0.594	0.401	0.565	–	0.553	–	0.502	0.607	0.514	0.481	0.637	0.613	0.651	0.641
α_{jI} First centroid loadings	0.541	0.516	0.540	0.569	0.373	0.535	0.709	0.549	0.583	0.483	0.587	0.486	0.471	0.651	0.600	0.655	0.636

*Note**: Excluding Variables 7 and 9.

TABLE 4:
FIRST PRINCIPAL COMPONENT LOADINGS AS A FUNCTION OF BATTERY COMPOSITION

Variables	First principal component				Changes in loadings: initial value – final value
	Including all variables	Excluding Variable 7	Excluding Variable 9	Excluding Variables 7 & 9	
1. Addition	0.559	0.531	0.530	0.492	0.559 – 0.492 = +0.067
2. Division	0.528	0.503	0.492	0.458	0.528 – 0.458 = +0.070
3 Subtraction & Multiplication	0.551	0.532	0.522	0.492	0.551 – 0.492 = +0.059
4. Paper Form Board	0.533	0.551	0.556	0.580	0.533 – 0.580 = – 0.047
5. Surface Development	0.320	0.343	0.342	0.372	0.320 – 0.372 = – 0.052
6. Block Counting	0.502	0.516	0.536	0.557	0.502 – 0.557 = – 0.055
7. SCAT quantitative	0.728	-	0.723	-	
8. Marks	0.549	0.565	0.544	0.559	0.549 – 0.559 = – 0.010
9. Lettergrouping	0.603	0.597	-	-	
10. Vocabulary	0.482	0.502	0.496	0.518	0.482 – 0.518 = – 0.036
11. SCAT-verbal	0.583	0.593	0.602	0.614	0.583 – 0.614 = – 0.031
12. Plane Geometry	0.451	0.460	0.482	0.495	0.451 – 0.495 = – 0.044
13. Plane Trigonometry	0.449	0.425	0.479	0.457	0.449 – 0.457 = – 0.008
14. Intermediate Algebra	0.660	0.631	0.672	0.642	0.660 – 0.642 = 0.018
15. Concept Attainment: spatial	0.609	0.642	0.608	0.642	0.609 – 0.642 = – 0.033
16. Concept Attainment: verbal	0.686	0.701	0.678	0.692	0.686 – 0.692 = – 0.006
17. Concept Attainment: numerical	0.662	0.681	0.654	0.670	0.662 – 0.670 = – 0.008

In many respects Guttman's radex model offers a better description of the 'operation' involved in cognitive tests than the construct, g does. The various tests in the battery can be ordered along a continuum that stretches from heuristic (analytical) to ostensive (achievement).

From a cognitive development point of view it would be very illuminating to study the mediation processes of particular ethnic groups (Tzuriel, 1997). This brings us to the next point, namely ethnic differences in patterns of ability.

Spearman's hypothesis

As mentioned in the introduction, Spearman's hypothesis concerns ethnic differences in patterns of cognitive ability. More specifically the hypothesis states that the biggest black/white differences exist in respect of those measures with the highest loadings on the general factor.

Jensen (1993) is very dogmatic about Spearman's hypothesis and maintains that as far as conventional psychometric tests are concerned it 'is really no longer an hypothesis but an empirical fact' (p. 48). Herrnstein and Murray (1996, p. 304) agree with Jensen in this regard: 'Our own appraisal of the situation is that Jensen's main contentions regarding Spearman's hypothesis are intact and constitute a major challenge to purely environmental explanations of the B/W difference'.

From a psychometric point of view the matter is not so simple as it might appear. For Spearman's hypothesis to hold exactly, the g-loadings of cognitive tests have to be very stable indeed. However, it is a well known fact that numerous factors have an influence on the g-loadings of tests. So the invariance of the g-loadings cannot be guaranteed. In this section only one of the factors will be examined, namely the composition of the test battery.

To illustrate the point data collected at Cooper Union in New York will be used.

The following tests were applied to the full intake of first year engineering students:

(a) *The Addition, Division, Subtraction* and *Multiplication* tests from John French's *Reference Kit.*

(b) Thurstone's *Paper Form Board, Surface Development, Marks, Letter Grouping,* and *Vocabulary test.*

(c) *The School and College Ability Test,* and the *Block Counting Test* of the Educational Testing Service (ETS).

(d) *The Plane Geometry, Plane Trigonometry* and *Intermediate Algebra* tests of the Coopera-tive Test Division of ETS.

(e) The three *Concept Attainment Tests* designed by the present author.

The tests were intercorrelated and the first centroid was calculated. Next, the composition of the test battery was changed by excluding Variables 7 and 9. The first centroid loadings were then recalculated.

The matrix of intercorrelations is given in Table 3. The centroid loadings are given in the last row of the table which is indicated as α_{jI}. The centroid loadings after the exclusion of Variables 7 and 9 appear in the second last row which is indicated as as α^*_{jI}.

From an inspection of Table 3 it should be clear that the loadings of the three arithmetic tests have decreased after the elimination of Variables 7 and 9. But the loadings of the three spatial tests have increased.

The picture becomes even clearer from an inspection of Table 4. Here the first principal components are given for the full battery; then after exclusion of Variable 7; then after exclusion of Variable 9, and lastly after exclusion of Variables 7 and 9.

From an inspection of the last column of Table 4 it should be clear that the loadings of the three arithmetic tests have decreased systematically, whereas the loadings of the three spatial tests have increased systematically. The opposite effect can be produced by including more arithmetic tests in the battery.

From the foregoing it should be clear that the loadings on the general factor can be changed by changing the composition of the test battery. This complicates the decision as to which tests in a battery are the best measures of g, or general intelligence. Moreover it also casts doubt on the validity of Spearman's

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[2] *The full statistical analysis is available from the author on request.*

hypothesis.

In contrasting the IQs of blacks and whites, Herrnstein and Murray (1996, pp. 299-300) found mean differences of 16 to 18 IQ points in favour of the whites. But the tests of intelligence used were standardised on predominantly white populations. The question thus arises as to how fair such comparisons are? This brings us to the next point, namely the standardisation of tests of intelligence.

Standardisation of tests of intelligence

As far as tests of intelligence are concerned, use is commonly made of national norms, and the scores are expressed as deviation IQs (M = 100 and SD = 15). In the preparation of norms the particular test of intelligence is applied to a well stratified random sample of test subjects that is representative of the population for which the test is intended.

In stratifying a sample for normative purposes, the various strata should be represented proportionately according to size. For example, if a single set of norms is to be prepared for whites and blacks in the US, then 85% of the test subjects in the sample should be white and 15% should be black. If, however, the purpose is to compare the scores of blacks and whites, then equal numbers of the two ethnic groups should be included in the sample. If the latter approach is followed, the B/W difference in mean IQ would be halved.

Herrnstein and Murray's statement that the tests of intelligence used were standardised on predominantly white populations, is too vague to evaluate. Proportionate representation of blacks in the sample will, however, exaggerate the B/W difference.

In his *Afterword,* Murray takes the psychometricians to task:

> Suppose you give a psychometrician the chance to extract g and leave you with all the remaining factors in a given mental test. You cannot manipulate any one or any combination of those factors so as to produce the relationships I just listed. Only g, that supposedly arbitrary creation of the psychometricians can do so (Herrnstein & Murray, 1996, p. 561).

The relationships he refers to concern links of g with neurophysiological functioning and a genetic basis.

In the next section his assertions will be critically examined.

The predictive power of g versus the predictive power of rotated factor scores

Rotation to simple structure (orthogonal and oblique) never ignores the first principal factor (referred to as g by Murray). It simply redistributes the variance across all the factors. The total variance accounted for is, therefore, always more for multiple factors than for g alone.

The superiority of the predictive validity of multiple factors, as opposed to g alone, can best be illustrated with reference to real data: Consider the prediction of the standardised total score of the GSAT (dependent variable) with the aid of the rotated factor scores of the SAT (independent variables), as opposed to the unrotated first principal factor score of the SAT (cf. Table 1).

The SAT was subjected to a principal factor analysis and yielded two factors. The first factor accounted for 36.53% of the total variance and the second for 6.45%. Next, a Varimax rotation and a Direct Oblimin rotation of the factor matrix was done. Following the Varimax rotation the first factor accounted for 22.06% of the total variance and the second for 20.91%. The Direct Oblimin rotation is an oblique rotation, and resulted in the two factors being correlated to the extent of 0.60.

Next, a regression analysis was done using the Varimax factor scores as predictors and the GSAT-total score as criterion. A multiple correlation of 0.712; $p(F) < 0.0001$ was obtained. This correlation accounts for 50.65% of the variance of the dependent variable. The correlation of the unrotated first factor score of the SAT with the GSAT-total score is 0.656; $p < 0.0001$. This correlation accounts for 43.07% of the variance of the dependent variable.

Following this the regression of the Direct Oblimin factors scores on the GSAT-total score, was done. The outcome was identical to that of the Varimax factor scores only the regression weights were different.[2]

From the foregoing it should be clear that the rotational procedure does not affect the outcome. But the discarding of meaningful factors does. Therefore Murray's claim as regards the superiority of the predictive validity of g does not stand on firm ground.

Factors that influence the g-loadings of tests in a factor test battery

As indicated under the first point, the g-loadings or saturations of tests in a factor test battery represent the correlations of the respective tests with the total standardised score. Several factors can influence the magnitude of these correlations. The following are the most important:

- The composition of the test battery: If the various tests measure essentially the same mental ability, the correlations with the standardised total score will tend to be high. The converse will be true if the tests are largely unrelated.
- The part-whole effect, previously mentioned, will be large for small batteries of tests and small for large batteries.
- The magnitude of the correlations will be low if the tests contained in the test battery are basically unreliable. The opposite is not necessarily true.
- Restriction of range will lower the correlations with the total score.
- Differential skewness of the variables will influence the g-saturations of the measures, even to the extent of creating more than one factor of general intelligence.
- Subjects with an undifferentiated intellect will generally produce one-factor-structures with high g-loadings. The opposite will tend to be true for subjects with a highly differentiated intellect.

In the light of the foregoing the following statement by Murray appears unfounded: 'The higher the g loading of a subtest is, the higher is its heritability. The higher the g loading of a subtest is, the higher is the degree of inbreeding depression (an established genetic phenomenon)' (Herrnstein & Murray, 1996, p. 561).

The nature-nurture controversy: Brain changes in response to experience

Rosenzweig, Bennett and Diamond (1972) conducted a series of experiments with rats of a given strain in order to determine whether there were any changes in brain anatomy and chemistry as a function of experience.

Littermates of the same strain were selected at random, and assigned to one of three experimental conditions:

(a) A standard laboratory condition, with three rats per cage.
(b) An impoverished environment, with one rat per cage.
(c) An enriched environment, with 12 rats in a large cage furnished with playthings that were changed daily.

Food and water were freely available in all three environments. The rats remained in the same environment for 30 days or more. At the end of a predetermined experimental period, the rats were sacrificed and their brains were removed.

The experiments were replicated 16 times, and the ratio of the weight of the cortex to the weight of the subcortex was determined for each rat. Next, the brains of the rats from the enriched environment were compared with those from the impoverished environment. The main findings were as follows:

(a) In 14 of the 16 experiments there were statistically significant differences between the brains of the rats from the two conditions (enriched vs. impoverished). These differences were all significant at the 0.01 level. 'It appears that the cortex increases in weight quite readily in response to an enriched environment, whereas the weight of the rest of the brain changes little' (Rosenzweig et al., 1972, p. 120).

(b) 'Measurement of the synaptic junctions revealed that rats from enriched environments had junctions that averaged approximately 50 percent larger in cross section than similar junctions in littermates from impoverished environments' (Rosenzweig et al., 1972, p. 122).

Although it is dangerous to extrapolate from animal behaviour to human behaviour, the findings of the above-mentioned study indicate very dramatically indeed – that brain changes occur in response to a stimulating environment.

Neuroscientists have been quick to build on the findings of Rosenzweig et al. (1972) and are rapidly coming up with similar findings pertaining to the human brain.

At birth a baby's brain contains virtually all the nerve cells it will ever have, but the pattern of 'wiring' between the cells is still incomplete. Appropriate circuits for vision, language, etc. have been laid down, but these need to be progressively refined. Neural activity, driven by a flood of sensory experiences, is responsible for the refinement of the rough blueprint. Connections, or synapses, that are seldom or never used are eliminated, and new circuits are laid out (Nash, 1997, p. 34).

From the foregoing the importance of a stimulating environment, in the development of the brain, should be apparent.

There are at least two important sources of stimulation that play a role in the cognitive development of the child: namely the parents who act as mediators, and the school (Tzuriel, 1997). The effect of formal schooling (or the lack of it) can best be illustrated by studying the cognitive abilities of illiterate adults.

Schepers (1974) studied the cognitive abilities of 246 black industrial workers with the aid of six cognitive tests that have essentially the same rationale as those of Cattell's Culture-Fair Intelligence Tests. The difficulty levels of the tests are pitched at six or seven year-olds.

The subjects ranged from 18 to 64 years in age, with a mean of 35.1 years, and a standard deviation of 10 years. Their educational level ranged from no education to 12 years of formal schooling. However, as their mean level of education was 2.9 years of formal schooling, and the standard deviation was 2.7 years of schooling, the sample can at best be described as semi-literate.

The six tests were intercorrelated, and the matrix of intercorrelations was subjected to a principal factor analysis. One factor was obtained, and all the residuals were close to zero. The loadings ranged from 0.65 to 0.86. The loading in respect of Raven's Coloured Progessive Matrices was 0.84.

The fact that a unitary factor structure was obtained with an adult sample, and the fact that the difficulty levels of the tests are at a six or seven year-old level, stresses the importance of formal schooling in the differentiation process of cognitive abilities.

In an illiterate society the opportunity for acquiring what Gardner (1983) calls 'linguistic intelligence' and Carroll (1993) calls 'crystallised intelligence' is almost totally lacking. And the opportunity for acquiring Gardner's 'logical-mathematical intelligence' or Carroll's 'fluid intelligence' is extremely limited.

Adult illiterates, tested in South Africa with Raven's Coloured Progressive Matrices and numerous other tests of fluid intelligence, appear to function at the level of six to seven year-olds. But these subjects are not mentally defective. They are highly responsible citizens, and most of them are gainfully employed in the Gold Mining Industry in the Republic of South Africa.

From the foregoing the important role of formal schooling in the development of linguistic and logical-mathematical intelligence should be obvious. Furthermore, the importance of the interaction of heredity and environment should be clear. The best genetic endowment can only come to fruition in a stimulating environment. The heritability of 'g or general intelligence should therefore also take into account the quality of formal schooling received by the test subjects.

It is the present author's contention that the heritability indices used by Herrnstein and Murray yield gross overestimates of the contribution of genetic factors to intelligence. Be that as it may. What really matters is the interaction between genetics on the one hand and environment on the other.

The following approach is suggested to gauge the relative importance of genetic factors and environmental factors, notably formal schooling, in the development of intellect:

A sample of monozygotic twins, in the age range 18 to 24 years, should be selected. These twins should have been raised apart, but with the following important proviso – the one member must be illiterate, and the other member should have received the best quality of formal schooling.

Next, a sample of monozygotic twins, in the same age-range as those in the first sample, should be selected. These twins should have been raised together, and must have received a good quality of formal schooling.

A battery of culture-fair tests, with a broad band of difficulty values, such as Raven's Progressive Matrices, could be used to estimate the IQ's of the subjects.

Then the following index should be computed:

$HI = \frac{\overline{D}_T}{D_A} \times 100$, where

DT = the mean intrapair IQ difference of the twins raised together, and
DA = the mean intrapair IQ difference of the twins raised apart.

A high index (percentage) signifies a strong genetic contribution, and a low index a strong environmental contribution.

Locus of control and intelligence

A promising new lead that might be used in raising the

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3. *The full analysis is available from the author on request.*

intelligence of groups who were suppressed in the past, has been established, namely a strong link between locus of control and intelligence.

In a study conducted by Schepers (1995) at the Rand Afrikaans University, the *General Scholastic Aptitude Test*, the *Senior Ability Test*, and *Locus of Control Inventory* (LCI) (Schepers, 1995) were applied to 1 662 freshmen. A cluster analysis was then done on the full sample, using the three scores of the LCI as input. Four meaningful clusters were obtained. Next, the four clusters were compared on the GSAT and SAT, using Tukey's Studentised Range test. Only the results in respect of Clusters 3 and 4 will be given here.[3]

The three scores of the LCI were standardised with a mean of 50 and a standard deviation of 10. The scores of Clusters 3 and 4 in respect of the LCI are given in Table 5.

TABLE 5
MEAN SCORES OF CLUSTERS 3 AND 4 ON LCI

	External control	Internal control	Autonomy	N
Cluster (HLL)	58,42	37,75	37,73	322
Cluster 4 (LHH)	39,80	58,83	61,30	399

Note: LCI Locus of Control Inventory

Cluster 3 (HLL) has high scores on external control and low scores on internal control and autonomy. Cluster 4 (LHH) has low scores on external control and high scores on internal control and autonomy. It was found that Cluster 3 has a mean total IQ of 107.91 and Cluster 4 a mean total IQ of 113.26. Furthermore, there were highly significant differences (in favour of Cluster 4) on all the subtests of the SAT, with the exception of Pattern Completion and Figure Series.

These findings are all the more interesting if one keeps in mind that the sample consisted of first year university students. Restriction of range must, therefore, have played a role in reducing the observed relationships.

Locus of control is also highly relevant in the work situation:

Bothma and Schepers (1997) studied the work performance of 102 Black male managers in the chemical industry. They used a scale of achievement motivation (yielding five scores) jointly with the LCI (three scores), to predict the work performance of the managers. The work performance of the managers was assessed by their superiors using a work performance scale that yielded three scores, viz. volume and quality of work produced, initiative and creativity in the work situation, and managerial skills.

Two significant canonical correlations were obtained: The first canonical variate had high loadings on three of the achievement motivation scores, and on volume and quality of work produced, yielding a canonical correlation of 0.62 ($p < 0.001$). The second canonical variate had high loadings on internal control and autonomy, and on all three the work performance measures, yielding a canonical correlation of 0.54 ($p < 0.0001$).

From the foregoing it should be clear that intelligence does not operate in isolation. The total personality should be considered when assessing work performance. In this regard the approach of Herrnstein and Murray is rather naïve.

I therefore propose that large scale therapeutic interventions with young people, aimed at shifting their locus of control from external to internal would result in an increase in their intelligence.

In conclusion, I suggest that Herrnstein and Murray have ascribed to g qualities that are unfounded and misleading. And I also have serious misgivings about the validity of the so-called Spearman hypothesis. The use of tests that have not been standardised for blacks, in comparing B/W differences, is completely unjustified.

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APPENDIX 1

Derivation of centroid model
It will be shown in Appendix 2 that if the variables in a test battery are standardised with a mean of zero and a standard deviation of one, then the correlation of each of the variables with the total score will be equal to its loading on the first centroid of the test battery. However, before this can be done it will be necessary to state the centroid model of Thurstone (1947, pp. 149-153) very briefly in matrix notation.

Notation. For the sake of clarity the following matrices are defined:

R = Marix of intercorrleations of vaiables.
A = Orthogonal factor matrix containing the factor loadings, a_{jm}, of the various variables, j, m, = 1, 2, ... , n.

Fundamental equality. Central to the derivation of the centroid model is the fundamental equation of multiple-factor analysis:

$R = AA'$,
where R and A are defined as above.

Consider the matrix product WRW', where W' represents a unit-row vector and W a unit-column vector:

$$\underset{\substack{w\\1\times n}}{[1\ 1\ 1\ \ldots\ 1]}\,.\,\underset{\substack{R\\n\times n}}{\begin{bmatrix} 1 & r_{12} & r_{13} & \cdot & \cdot & \cdot & r_{1n}\\ r_{12} & 1 & r_{23} & \cdot & \cdot & \cdot & r_{2n}\\ r_{31} & r_{32} & 1 & \cdot & \cdot & \cdot & r_{3n}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ r_{n1} & r_{n2} & r_{n3} & \cdot & \cdot & \cdot & 1 \end{bmatrix}}\,.\,\underset{\substack{w'\\n\times 1}}{\begin{bmatrix}1\\1\\1\\ \cdot\\ \cdot\\ \cdot\\ 1\end{bmatrix}'}$$

$$WRW' = \sum_{j=1}^{n}\sum_{k=1}^{n} r_{jk}$$

$$WRW' = W(AA')W'$$
$$= (WA)(WA)'$$

Consider the matrix product WA:

$$\underset{\substack{w\\1\times n}}{[1\ 1\ 1\ \ldots\ 1]}\,.\,\underset{\substack{A\\n\times n}}{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdot & \cdot & \cdot & a_{1n}\\ a_{21} & a_{22} & a_{23} & \cdot & \cdot & \cdot & a_{2n}\\ a_{31} & a_{32} & a_{33} & \cdot & \cdot & \cdot & a_{3n}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ a_{n1} & a_{n2} & a_{n3} & \cdot & \cdot & \cdot & a_{nn} \end{bmatrix}}$$

$$WA = [\sum a_1 \sum a_2 \sum a_3 \ldots \sum a_n]$$

WA represents the colmun totals of *A*.

If the axes are placed such that the first axis contains the centroid, then

$\sum a_2 = \sum a_3 \ldots \sum a_n = 0$ (Thursstone, 1947, p. 151).

$$\text{Then } (WA)(WA)' = (\textstyle\sum a_1)^2$$
$$= WRW'$$
$$= \textstyle\sum_{j=1}^{n}\sum_{k=1}^{n} r_{jk}$$

Therefore $WA = \sqrt{WRW'}$

We also have

$$WR = (WA)A' \text{ because } R = AA'$$

$$= \sqrt{WRW'}\,A'$$

$$RW' = A\sqrt{WRW'} \text{ (transpose and } R' = R)$$

$$RW'[\sqrt{WRW'}]^{-1} = A\sqrt{WRW'}[\sqrt{WRW'}]^{-1}$$
$$= A$$

Thus there are two important equations:

$A = RW'[\sqrt{WRW'}^{\,-1}$ .... (1) where RW' represents the row totals of R, and $[\sqrt{WRW'}]^{-1}$ represents the reciprocal of the square root of the grand total of R, and

$[\sum a_1]^2 = WRW' = \sum_{j=1}^{n}\sum_{k=1}^{n} r_{jk}$ (2) i.e. the sum of the centroid loadings, all square, is equal to the grand total of R.

From equation (1) it is clear that the first centroid loadings can be obtained by multiplying the row totals of R with thereciprocal of the square root of the grand total of R.

The second equation can be used to check the accuracy of the computation as the sum of the first centroid loadings, allsquared, equals the grand total of R.

APPENDIX 2

Part-whole correlations expressed as first centroid loadings

It will be shown here that if the variables in a test battery are standardised with a mean of zero and a standard deviation of one, then the correlation of each variable with the total score will be equal to its loading on the first centroid of the test battery.

To start off with, a brief exposition will be given of how to calculate the correlation of a variable with a total score of which it forms part. For the sake of clarity the procedure will be illustrated with reference to a test battery comprising five variables. Thereafter the general principle will be given.

Consider the variance-covariance matrix of a test battery comprising five variables, that together constitute a total score, jointly with one of the five variables – say the first variable. Variable 1 is duplicated in the first row and first column of the matrix, but it could just as well have been any one of the other variables. We refer to this matrix as K where K is a 6×6 matrix.

The covariance of the first variable with the total score can readily be obtained by pre- and post-multiplying matrix K with appropriate vectors of weights, say V and L', respectively (Cliff, 1987, pp. 60–61).

The vector V must be chosen such that the product matrix, VK, is equal to the first row of K. This can be obtained by setting the first element of V equal to one and the rest of the elements equal to zero. The vector L must be chosen such that the product VKL' summates the first row of K with the exclusion of the first element. This can be obtained by setting the first element of L equal to zero and the rest of the elements equal to unity.

Thus $V = [\,1\,0\,0\,0\,0\,0\,]$ and $L = [\,0\,1\,1\,1\,1\,1\,]$.

The covariance of the first variable with the total score is given by VKL', i.e. the sum of the elements of the first row of K with the exclusion of the first element.

Variance-covariance matrix

$$\underset{V}{[1\,0\,0\,0\,0\,0]} \cdot \underset{K}{\begin{array}{c|cccccc} & 1 & 1 & 2 & 3 & 4 & 5 \\ 1 & s_1^2 & s_1^2 & r_{12}s_1s_2 & r_{13}s_1s_3 & r_{14}s_1s_4 & r_{15}s_1s_5 \\ 1 & s_1^2 & s_1^2 & r_{12}s_1s_2 & r_{13}s_1s_3 & r_{14}s_1s_4 & r_{15}s_1s_5 \\ 2 & r_{21}s_2s_1 & r_{21}s_2s_1 & s_2^2 & r_{23}s_2s_3 & r_{24}s_2s_4 & r_{25}s_2s_5 \\ 3 & r_{31}s_3s_1 & r_{31}s_3s_1 & r_{33}s_3s_2 & s_3^2 & r_{34}s_3s_4 & r_{35}s_3s_5 \\ 4 & r_{41}s_4s_1 & r_{41}s_4s_1 & r_{42}s_4s_2 & r_{43}s_4s_3 & s_4^2 & r_{45}s_4s_5 \\ 5 & r_{51}s_5s_1 & r_{51}s_5s_1 & r_{52}s_5s_2 & r_{53}s_5s_3 & r_{45}s_4s_5 & s_5^2 \end{array}} \cdot \underset{L'}{\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}'}$$

$$VKL' = [s_1^2 + r_{12}s_1s_2 + r_{13}s_1s_3 + r_{14}s_1s_4 + r_{15}s_1s_5]$$

The first element of K represents the variance of the first variable. The variance of the total score, s_t^2, is given by the sum of the elements in the variance-covariance matrix K, excluding the first row and first column.

Therefore $s_t^2 = s_1^2 + r_{12}s_1s_2 + \ldots + s_5^2$

The correlation of the first variable with the total score is given by

$$r_{1t} = \frac{s_1^2 + r_{12}s_1s_2 + r_{13}s_1s_3 + r_{14}s_1s_4 + r_{15}s_1s_5}{\sqrt{s_1^2 s_t^2}}$$

If the variables have been standardised with a variance equal to one, the above equation reduces to

$$\frac{r_{1t} = 1 + r_{12} + r_{13} + r_{14} + r_{15}}{\sqrt{s_t^2}}$$

The numerator here represents the total of the first row of the intercorrelation matrix of the five variables, and the denominator represents the standard deviation of the total score. In similar fashion the totals of all the rows can be obtained.

In Appendix 1 the row totals of R were represented by RW', and the grand total of R was represented by WRW'. As mentioned earlier on in this appendix, the variance of the total score is given by the sum of all the elements of the variance-covariance matrix or by the sum of all the elements of the intercorrelation matrix if the variables have been standardised with a variance equal to one.

Thus, the standard deviation of the total score is given by $\sqrt{WRW'}$.

Let F be defined as a column vector which contains the part-whole correlations, then

$$F = RW'[\sqrt{WRW'}]^{-1} = A,$$

where A represents the first centroid of R.

It should, therefore, be clear that the loadings of the first centroid of a battery of tests is nothing more than the part-whole correlations of the various variables in the test battery with the total score, given that the variables have been standardised with a variance equal to one.