Ratio Mathematica Volume 45, 2023

Monophonic Distance Energy for Join of Some Graphs

Sinju Manohar V S[1]
Binu Selin T[2]

Abstract

Let G be a connected graph with n vertices and m edges. Let $\mu_1, \mu_2, \ldots, \mu_n$ be the Eigen values of distance matrix of G. The distance energy of a graph $E_D(G) = \sum_{i=1}^{p} |\mu_i|$,was already studied. We now define and investigate the monophonic distance energy as $E_M(G) = \sum_{i=1}^{n} |\mu_i^M|$, where $\mu_1^M, \mu_2^M \ldots, \mu_n^M$ are the eigen values of monophonic distance matrix of graphs. In this paper we find the monophonic distance energy for join of some graphs.

Keywords: Join of graphs; Monophonic distance matrix; Monophonic distance energy.

2010 Mathematics Subject Classification: 05C12, 05B20[3].

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[1] Research Scholar, Reg No: 20113162092016, Department of Mathematics, Scott Christian College (Autonomous), Nagercoil - 629 003, India. (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, Tamil Nadu, India.). Email: sinjusinju2124@gmail.com

[2] Assistant Professor, Department of Mathematics, Scott Christian College (Autonomous), Nagercoil - 629 003, India. (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, Tamil Nadu, India.). Email: binuselin@gmail.com

[3] Received on July 19, 2022. Accepted on October 15, 2022. Published on January 30, 2023. doi: 10.23755/rm.v45i0.979. ISSN: 1592-7415. eISSN: 2282-8214.

1. Introduction

In this paper we considered simple, connected and undirected graphs. The concept of energy of a graph was introduced by I. Gutman [2] in the year 1978. Let G be a connected graph with n vertices and m edges. Let $A = (a_{ij})$ be the adjacency matrix of the graph. The eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of A, assumed in non-increasing order, are the eigen values of the graph G. The energy $E(G)$ of G is defined to be the sum of the absolute values of the eigen values of G. ie., $E(G) = \sum_{i=1}^{n} |\lambda_i|$. Also, Distance energy of a graph was introduced by I. Gutman and others [4] in the year 2008. A. P. Santhakumaran and others introduced the monophonic number of a graph in 2014 [8]. For any two vertices u and v in a connected graph$G$, a $u - v$ path is a monophonic path if it contains no chords, and the monophonic distance $d_m(u, v)$ is the length of a longest $u - v$ monophonic path in G. Based on these we introduce a new concept monophonic distance energy of a graph. Based on these we introduce a new concept monophonic distance energy of a graph. In this paper we investigate the monophonic distance energy of $K_{1,n} + K_{1,n}$, $K_{n,n} + K_n$, $K_{n,n} + K_{n,n}$ and $K_n + K_{1,n}$.

2. Definitions

Definition 2.1. Let G be a connected graph with vertex set $v_1, v_2, \ldots, v_n$. The monophonic distance matrix of G is defined as

$$M = M[G] = (d_{m_{ij}})_{n\times n} \quad \text{Where } d_{m_{ij}} = \begin{cases} d_m(v_i, v_j), & if\ i \neq j \\ 0, & otherwise \end{cases}.$$

Here $d_m(v_i, v_j)$ is the monophonic distance of v_i to v_j. The eigen values of Monophonic distance matrix $M(G)$ are denoted by $\mu_1^M, \mu_2^M \ldots, \mu_n^M$ and are said to be M-Eigen values of G and to form the M-spectrum of G, denoted by $spec_M(G)$. We note that since the Monophonic distance matrix is symmetric, its eigen values are real and can be ordered as $\mu_1^M \leq \mu_2^M \leq \ldots \leq \mu_n^M$. We can define the monophonic distance energy of a graph as $E_M(G) = \sum_{i=1}^{n} |\mu_i^M|$.

3. Main Results

Definition 3.1. [3] The join $G = G_1 + G_2$ of graphs G_1 and G_2 with disjoint point sets V_1 and V_2 and edge sets X_1 and X_2 is the graph union $G_1 \cup G_2$ together with all the edges joining V_1 and V_2.

Theorem 3.2. For the Star graph $K_{1,n}$, $E_M\left(K_{1,n} + K_{1,n}\right) = 5(n-1) + \sqrt{9n^2 - 2n + 9}$.

Proof. From the definition of join, the monophonic distance matrix of $K_{1,n} + K_{1,n}$ can be written as

$$M(K_{1,n}+K_{1,n}) \; = \begin{bmatrix} M(K_{1,n}) & J_n \\ J_n & M(K_{1,n}) \end{bmatrix}$$

where $M(K_{1,n})$ be a monophonic distance matrix of $K_{1,n}$.

We have $M(K_{1,n}+K_{1,n}) = (d_{m_{ij}})_{2(n+1)\times 2(n+1)}$.

The monophonic distance spectrum $Spec_{\mathrm{M}}\,(K_{1,n}+K_{1,n})$ is

$$\begin{pmatrix} -2 & -1 & \dfrac{(3n-1)-\sqrt{9n^2-2n+9}}{2} & (n-2) & \dfrac{(3n-1)+\sqrt{9n^2-2n+9}}{2} \\ (n-1) & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Thus the monophonic distance energy of $K_{1,n}+K_{1,n}$ is

$E_{\mathrm{M}}(K_{1,n}+K_{1,n}) \; = \; \sum_{i=1}^{2(n+1)} |\mu_i{}^M|$,where $\mu_1{}^M, \mu_2{}^M, \ldots, \mu_{2(n+1)}{}^M$ are the eigen values of monophonic distance matrix $M(K_{1,n}+K_{1,n})$.

For $n \; > \; 1$,

$$\begin{aligned} E_{\mathrm{M}}\big(K_{1,n}+K_{1,n}\big) = \; & |-2|+|-2|+\cdots+|-2|+|-1| \\ & +\left|\frac{(3n-1)-\sqrt{9n^2-2n+9}}{2}\right| + |(n-2)| \\ & +\left|\frac{(3n-1)+\sqrt{9n^2-2n+9}}{2}\right| \\ = \; & 2\,(2(n-1)+1-\frac{(3n-1)+\sqrt{9n^2-2n+9}}{2}+(n-2) \\ & +\frac{(3n-1)+\sqrt{9n^2-2n+9}}{2} \\ = \; & 5(n-1)+\sqrt{9n^2-2n+9}. \end{aligned}$$

Theorem 3.3. For the complete bipartite graph $K_{n,n}$ and complete graph K_n,

$$E_M\big(K_{n,n}+K_n\big) = \begin{cases} (6n-7)+\sqrt{12n^2-4n+1} & if\ 1 \le n \le 4 \\ 10(n-1) & if\ n \; \ge 5 \end{cases}.$$

Proof. From the definition of join, the monophonic distance matrix of $K_{n,n}+K_n$ can be written as

$$M(K_{n,n}+K_n) \; = \begin{bmatrix} M(K_{n,n}) & J_{2n,n} \\ J_{n,2n} & M(K_n) \end{bmatrix}$$

where $M(K_{n,n})$ be a monophonic distance matrix of $K_{n,n}$ and $\mathrm{M}(K_n)$ be a monophonic distance matrix of K_n.

We have $M(K_{n,n}+K_n) = (d_{m_{ij}})_{3n\times 3n}$.

The monophonic distance spectrum $Spec_{\mathrm{M}}\,(K_{n,n}+K_n)$ is

$$\begin{pmatrix} -2 & -1 & \dfrac{(4n-3)-\sqrt{12n^2-4n+1}}{2} & \dfrac{(4n-3)+\sqrt{12n^2-4n+1}}{2} & n-2 \\ 2(n-1) & (n-1) & 1 & 1 & 1 \end{pmatrix}.$$

Thus the monophonic distance energy of $K_{n,n}+K_n$ is

$E_{\mathrm{M}}(K_{n,n}+K_{,n}) = \sum_{i=1}^{3n}|\mu_i{}^M|$,where $\mu_1{}^M, \mu_2{}^M, \ldots, \mu_{3n}{}^M$ are the eigen values of monophonic distance matrix $M(K_{n,n}+K_n)$.

For $1 \le n \le 4$,

$$E_{\mathrm{M}}\big(K_{n,n}+K_n\big) = |-2|+|-2|+\cdots+|-2|+|-1|+|-1|+\cdots+|-1|$$

$$+\left|\frac{(4n-3)-\sqrt{12n^2-4n+1}}{2}\right|+\left|\frac{(4n-3)+\sqrt{12n^2-4n+1}}{2}\right|+(n-2)$$

$$= 2\,(2(n-1)+(n-1)-\frac{(4n-3)+\sqrt{12n^2-4n+1}}{2}$$

$$+\frac{(4n-3)+\sqrt{12n^2-4n+1}}{2}+(n-2)$$

$$= (6n-7)+\sqrt{12n^2-4n+1}$$

For $n \ge 5$,

$$E_{\mathrm{M}}\big(K_{n,n}+K_n\big) = |-2|+|-2|+\cdots+|-2|+|-1|+|-1|+\cdots+|-1|$$

$$+\left|\frac{(4n-3)-\sqrt{12n^2-4n+1}}{2}\right|+\left|\frac{(4n-3)+\sqrt{12n^2-4n+1}}{2}\right|+(n-2)$$

$$= 2\,(2(n-1)+(n-1)+\frac{(4n-3)-\sqrt{12n^2-4n+1}}{2}$$

$$+\frac{(4n-3)+\sqrt{12n^2-4n+1}}{2}+(\mathrm{n}-2)$$

$$= 10(n-1).$$

Theorem 3.4. For the complete bipartite graph $K_{n,n}$,

$$E_M\big(K_{n,n}+K_{n,n}\big) = \begin{cases} 2(5n-2) & if\ n=1 \\ 16(n-1) & if\ n\ \ge 2 \end{cases}.$$

Proof. From the definition of join, the monophonic distance matrix of $K_{n,n}+K_{n,n}$ can be written as

$$M(K_{n,n}+K_{n,n}) = \begin{bmatrix} M(K_{n,n}) & J_{2n} \\ J_{2n} & M(K_{n,n}) \end{bmatrix}$$

where $M(K_{n,n})$ be a monophonic distance matrix of $K_{n,n}$

We have $M(K_{n,n}+K_{n,n}) = (d_{m_{ij}})_{4n\times 4n}$.

The monophonic distance spectrum $Spec_{\mathrm{M}}(K_{n,n}+K_{n,n})$ is

$$\begin{pmatrix} -2 & (n-2) & (5n-2) \\ 4(n-1) & 3 & 1 \end{pmatrix}.$$

Thus the monophonic distance energy of $K_{n,n}+K_{n,n}$ is

$E_{\mathrm{M}}(K_{n,n}+K_{n,n}) = \sum_{i=1}^{4n}|\mu_i{}^M|$,where $\mu_1{}^M, \mu_2{}^M, \ldots, \mu_{4n}{}^M$ are the eigen values of monophonic distance matrix $M(K_{n,n}+K_{n,n})$.

For $n = 1$,

$E_{\mathrm{M}}\big(K_{n,n} + K_{n,n}\big) = |-2| + |-2| + \cdots + |-2| + |n-2| + \cdots + |n-2| + |5n-2| =$
$2\,(4(n-1) + 3(-n+2) + 5n - 2 = 8n - 8 - 3n + 6 + 5n - 2$
$= 10n - 4$
$= 2(5n-2)$

For $n \geq 2$,

$E_{\mathrm{M}}\big(K_{n,n} + K_{n,n}\big) = |-2| + |-2| + \cdots + |-2| + |n-2| + \cdots + |n-2| + |5n-2| =$
$2\,(4(n-1) + 3(n-2) + 5n - 2 \;\; = 8n - 8 + 3n - 6 + 5n - 2$
$= 16n - 16$
$= 16(n-1)$.

Theorem 3.5. For the star graph $K_{1,n}$ and complete graph K_n,

$$E_M\big(K_n + K_{1,n}\big) = \begin{cases} (3n-2) + \sqrt{5n^2+4} & if\ n \leq 2 \\ 2(3n-2) & if\ n \geq 3 \end{cases}.$$

Proof. . From the definition of join, the monophonic distance matrix of $K_n + K_{1,n}$ can be written as

$$M(K_n + K_{1,n}) = \begin{bmatrix} M(K_n) & J_{n,n+1} \\ J_{n+1,n} & M(K_{1,n}) \end{bmatrix}$$

where $M(K_{1,n})$ be a monophonic distance matrix of $K_{1,n}$ and $\mathrm{M}(K_n)$ be a monophonic distance matrix of K_n.

We have $M(K_n + K_{1,n}) = (d_{m_{ij}})_{(2n+1)\times(2n+1)}$.

The monophonic distance spectrum $Spec_{\mathrm{M}}\,(K_{n,n} + K_n)$ is

$$\begin{pmatrix} -2 & -1 & \dfrac{(3n-2) - \sqrt{5n^2+4}}{2} & \dfrac{(3n-2) + \sqrt{5n^2+4}}{2} \\ (n-1) & n & 1 & 1 \end{pmatrix}.$$

Thus the monophonic distance energy of $K_n + K_{1,n}$ is

$E_{\mathrm{M}}(K_n + K_{1,n}) = \sum_{i=1}^{2n+1} |\mu_i{}^M|$,where $\mu_1{}^M, \mu_2{}^M, \ldots, \mu_{2n+1}{}^M$ are the eigen values of monophonic distance matrix $M(K_n + K_{1,n})$.

For $n \leq 2$,

$$E_{\mathrm{M}}\big(K_n + K_{1,n}\big) = |-2| + |-2| + \cdots + |-2| + |-1| + |-1| + \cdots + |-1|$$
$$+ \left|\frac{(3n-2) - \sqrt{5n^2+4}}{2}\right| + \left|\frac{(3n-2) + \sqrt{5n^2+4}}{2}\right|$$
$$= 2\,(n-1) + n - \frac{(3n-2) + \sqrt{5n^2+4}}{2} + \frac{(3n-2) + \sqrt{5n^2+4}}{2}$$
$$= 2n - 2 + n + \sqrt{5n^2+4}$$
$$= (3n-2) + \sqrt{5n^2+4}.$$

For $n \geq 5$,

$$E_{\mathrm{M}}\big(K_n + K_{1,n}\big) = |-2| + |-2| + \cdots + |-2| + |-1| + |-1| + \cdots + |-1|$$

$$+\left|\frac{(3n-2)-\sqrt{5n^2+4}}{2}\right|+\left|\frac{(3n-2)+\sqrt{5n^2+4}}{2}\right|$$

$$= 2\,(n-1)+n+\frac{(3n-2)-\sqrt{5n^2+4}}{2}+\frac{(3n-2)+\sqrt{5n^2+4}}{2}$$

$$= 2n-2+n+3n-2$$

$$= 6n-4$$

$$= 2(3n-2).$$

4. Conclusions

In this paper we discussed the concept of monophonic distance energy and obtained some results on monophonic distance energy for certain Join graphs. In future we planned to extend our research for various graph operations.

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