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The Extension Of Generalized Intuitionistic Topological Spaces

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Abstract

In this paper, irresolute functions in generalized intuitionistic topological spaces were introduced. Regarding these functions, we attempted to unveil the notions of some minimal and maximal irresolute functions. In addition, the generalized intuitionistic topological spaces were extended by using their open sets which are finer than of it and their basic characterizations were investigated. Some continuous functions in the extension of generalized intuitionistic topological spaces are also been discussed in this paper.
Keywords: mn-μ_I-ops, mx-μ_I-ops, Pμ_I-ops, Sμ_I-ops, mn-μ_I-cts, mx-μ_I-cts, mn-μ_I-irresolute, mx-$\mu_I$irresolute.
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1 Introduction

The concept of an intuitionistic set which is a generalization of an ordinary set and the specialization of an intuitionistic fuzzy set was given by Coker[2]. After that time, intuitionistic topological spaces were introduced [3]. A.Csaszar[1] introduced many closed sets in generalized topological spaces based on their basics. In 2019 [9], some new generalized closed sets in ideal nano topological spaces were developed. In 2022 [6], we have introduced a new type of topology called generalized intuitionistic topological spaces with the help of intuitionistic closed sets. After that time we introduced and studied μ_I-maps in generalized intuitionistic topological spaces. In addition we have introduced and defined a new structure of minimal and maximal μ_I-open sets in generalized intuitionistic topological spaces. In 2011 [10], the subject like minimal and maximal continuous, minimal and maximal irresolute, T-min space etc. were investigated on basic topological spaces.
In 2022 [7], the characterizations of nIαg-closed sets are proved. In that paper authors has been used Kuratowski's closure operator. Taking it as an inspiration we introduce μ_I-irresolute functions in generalized intuitionistic topological spaces throughout this paper. Also, some minimal and maximal μ_I-irresolute functions were introduced and studied in detail.
The aim of this paper is, to introduce the μ_I(A)-topology which is finer than μ_I-topology by using the formula $U \cup (V \cap A)$, where U and V are μ_I-open. In addition, some important and interesting results were discussed by using μ_I-continuous maps on the extension of μ_I-topology. Also, some counterexamples are given to support this work.

2 Preliminaries

Definition 2.1 (6). *A μ_I topology on a non-empty set X is a family of intuitionistic subsets of X satisfying the following axioms:*

1. $\emptyset \in \mu_I$

2. *Arbitrary union of elements of μ_I belongs to μ_I.*

For a GITS (X,μ_I), the elements of μ_I are called μ_I-open sets(briefly μ_I-ops) and the complement of μ_I-open sets are called μ_I-closed sets(briefly μ_I-cds).

Note:[6] $C_{\mu_I}(\emptyset) \neq \emptyset$, $C_{\mu_I}(X) = X$, $I_{\mu_I}(\emptyset) = \emptyset$ and $I_{\mu_I}(X) \neq X$.

Definition 2.2 (6). *Let (X,μ_I) be a GITS.*

1. *A proper non-null μ_I-ops G of (X, μ_I) is said to be a mn-μ_I-ops if any μ_I-ops which is contained in G is $\emptyset$ or G.*

2. *A proper non-null μ_I-ops G($\neq M_{\mu_I}$) of (X,μ_I) is said to be a mx-μ_I-ops set if any μ_I-ops which contains G is M_{μ_I} or G.*

Definition 2.3 (6). *Let (X,μ_I) and (Y,σ_I) be the topological spaces. A map f: (X,μ_I) $\to$ (Y,σ_I) is called,*

1. *mn-μ_I-cts if $f^{-1}(G)$ is a μ_I-ops in (X, μ_I) for every mn-μ_I-ops G in (Y,σ_I).*

2. *mx-μ_I-cts if $f^{-1}(G)$ is a μ_I-ops in (X, μ_I) for every mx-μ_I-ops set G in (Y,σ_I).*

Results: [6]

1. Every μ_I-cts map is mn-μ_I-cts.
2. Every μ_I-cts map is mx-μ_I-cts.
3. Mn-μ_I-cts and mx-μ_I-cts maps are independent of each other.
4. If f: (X,μ_I) $\to$ (Y,σ_I) is μ_I-cts and g: (Y,σ_I) $\to$ (Z,ρ_I) is mn-μ_I-cts then g$\circ$f: (X,μ_I) $\to$ (Z,ρ_I) is mn-μ_I-cts.
5. f: (X,μ_I) $\to$ (Y,σ_I) is μ_I-cts and g: (Y,σ_I) $\to$ (Z,ρ_I) is mx-μ_I-cts then g$\circ$f: (X,μ_I) $\to$ (Z,ρ_I) is mx-μ_I-ops.

Definition 2.4 (4). *Let X be a μ_I-topological spaces. A subset A of X is said to be μ_I-dense if $C_{\mu_I}(A) = X$. Clearly, X is the only μ_I-closed set dense in (X,μ_I).*

Theorem 2.1. *Let (X,μ_I) be a GITS with closed under intersection property. Then $C_{\mu_I}(A \cup B) = C_{\mu_I}(A) \cup C_{\mu_I}(B)$.*
Proof: *Since $A \subset A \cup B$ and $B \subset A \cup B$, $C_{\mu_I}(A) \subset C_{\mu_I}(A \cup B)$ and $C_{\mu_I}(B) \subset C_{\mu_I}(A \cup B)$. Now we have to prove the second part, Since $A \subseteq C_{\mu_I}(A)$ and $B \subseteq C_{\mu_I}(B)$, $A \cup B \subseteq C_{\mu_I}(A) \cup C_{\mu_I}(B)$ which is μ_I-closed. Then $C_{\mu_I}(A \cup B) \subseteq C_{\mu_I}(A) \cup C_{\mu_I}(B)$. Hence the theorem.*

3 μ_I-irresolute in GITS

Definition 3.1. *A mapping $\Bbbk$: (X,μ_I) $\to$ (Y,σ_I) is said to be a*

1. *semi μ_I-irresolute function(briefly Sμ_I-irresolute) if the inverse image of semi μ_I-open sets(briefly Sμ_I-ops) in (Y,σ_I) is Sμ_I-op in (X,μ_I).*

2. *pre μ_I-irresolute function(briefly Pμ_I-irresolute) if the inverse image of pre μ_I-open sets(briefly Pμ_I-ops) in (Y,σ_I) is Pμ_I-op in (X,μ_I).*

3. *$\alpha\mu_I$-irresolute function if the inverse image of $\alpha\mu_I$-ops in (Y,σ_I) is $\alpha\mu_I$-open in (X, μ_I).*

4. *$\beta\mu_I$-irresolute function if the inverse image of $\beta\mu_I$-ops in (Y,σ_I) is $\beta\mu_I$-open in (X,μ_I).*

Theorem 3.1. *Let $\Bbbk$: (X,μ_I) $\to$ (Y,σ_I) be a semi μ_I-irresolute function if and only if the inverse image of semi μ_I-cds in (Y,σ_I) is semi μ_I-closed in (X,μ_I).*
Proof:
Necessary part: Let $\Bbbk$: (X,μ_I) $\to$ (Y,σ_I) be a semi μ_I-irresolute function and A be a semi μ_I-cds in (Y,σ_I). Since f is Sμ_I-irresolute, $\Bbbk^{-1}(Y - A) = X - \Bbbk^{-1}(A)$ is Sμ_I-open in (X,μ_I). Hence $\Bbbk^{-1}(A)$ is Sμ_I-closed in (X,μ_I).
Sufficient part: Assume that $\Bbbk^{-1}(A)$ is Sμ_I-closed in (X,μ_I) for each Sμ_I-closed set in (Y,σ_I). Let V be a Sμ_I-ops in (Y,σ_I) which yields that $Y - V$ is Sμ_I-cds in (Y,σ_I). Then we get $\Bbbk^{(-1)}(Y - V) = X - \Bbbk^{(-1)}(V)$ is Sμ_I-closed in (X,μ_I) this implies $\Bbbk^{-1}(V)$ is Sμ_I-open in (X,μ_I). Hence $\Bbbk$ is Sμ_I-irresolute.

Theorem 3.2. *If $\Bbbk$ is Sμ_I-irresolute then $\Bbbk$ is Sμ_I-cts.*
Proof: *Suppose $\Bbbk$ is Sμ_I-irresolute. Let A be any Sμ_I-ops in (Y,σ_I). Since every μ_I-ops is Sμ_I-open and since A is Sμ_I-open, $\Bbbk^{-1}(A)$ is Sμ_I-open in (X,μ_I). Hence $\Bbbk$ is Sμ_I-cts.*

Remark 3.1. *Since every Sμ_I-ops need not be μ_I-open, we cannot deduce the reversal concept of the above statement.*

Theorem 3.3. *Let (X,μ_I), (Y,σ_I) and (Z,ρ_I) be three μ_I-topological spaces. For any Sμ_I-irresolute map $\Bbbk$: (X,μ_I) $\to$ (Y,σ_I) and any Sμ_I-cts $\hbar$: (Y,σ_I) $\to$ (Z,ρ_I) the composition $\hbar \circ \Bbbk$: (X,μ_I) $\to$ (Z,ρ_I) is Sμ_I-cts.*
Proof: *Let A be any μ_I-ops in (Z,ρ_I). Since $\hbar$ is Sμ_I-cts, $\hbar^{-1}(A)$ is Sμ_I-open in (Y,σ_I). By using $\Bbbk$ is semi μ_I-irresolute, we get $\Bbbk^{-1}[\hbar^{-1}(A)]$ is Sμ_I-open in (X,μ_I).*

But $\mathbb{k}^{-1}[\hbar^{-1}(A)] = (\hbar \circ \mathbb{k})^{-1}(A)$. *Therefore, inverse image of* μ_I*-ops in (Z,*ρ_I*) is S*μ_I*-open in (X,*μ_I*). Hence* $\hbar \circ \mathbb{k}$*: (X,*μ_I*)* $\rightarrow$ *(Z,*ρ_I*) is S*μ_I*-cts.*

Theorem 3.4. *If* $\mathbb{k}$*: (X,*μ_I*)* $\rightarrow$ *(Y,*σ_I*) and* $\hbar$*: (Y,*σ_I*)* $\rightarrow$ *(Z,*ρ_I*) are both S*μ_I*-irresolute then* $\hbar \circ \mathbb{k}$*: (X,*μ_I*)* $\rightarrow$ *(Z,*ρ_I*) is also S*μ_I*-irresolute.*
Proof: *Let A be any S*μ_I*-ops in (Z,*ρ_I*). Since* $\mathbb{k}$ *and* $\hbar$ *are S*μ_I*-irresolute,* $\hbar^{-1}(A)$ *is S*μ_I*-open in (Y,*σ_I*) and* $\mathbb{k}^{-1}[\hbar^{-1}(A)]$ *is S*μ_I*-open in (X,*μ_I*). Hence* $(\hbar \circ \mathbb{k})^{-1}(A) = \mathbb{k}^{-1}[\hbar^{-1}(A)]$ *is S*μ_I*-open and so* $\hbar \circ \mathbb{k}$*: (X,*μ_I*)* $\rightarrow$ *(Z,*ρ_I*) is S*μ_I*-irresolute.*

Theorem 3.5. *Let* $\mathbb{k}$*: (X,*μ_I*)* $\rightarrow$ *(Y,*σ_I*) be a P*μ_I*-irresolute(resp.* $\alpha\mu_I$*-irresolute and* $\beta\mu_I$*-irresolute) function if and only if the inverse image of P*μ_I*-closed(resp.* $\alpha\mu_I$*-closed and* $\beta\mu_I$*-closed) sets in (Y,*σ_I*) is P*μ_I*-closed(resp.* $\alpha\mu_I$*-closed and* $\beta\mu_I$*-closed) in (X,*μ_I*).*
Proof: *We can prove this theorem as we have done in the theorem 3.2.*

Theorem 3.6. *If f is P*μ_I*-irresolute(resp.* $\alpha\mu_I$*-irresolute and* $\beta\mu_I$*-irresolute) then f is P*μ_I*-continuous(resp.* $\alpha\mu_I$*-cts and* $\beta\mu_I$*-cts).*
Proof: *We can prove this theorem as we have done in the theorem 3.3.*

Remark 3.2. *Since every P*μ_I*-open(resp.* $\alpha\mu_I$*-open and* $\beta\mu_I$*-open) set need not be* μ_I*-open, we cannot deduce the reversal concept of the above statement.*

Theorem 3.7. *Let (X,*μ_I*), (Y,*σ_I*) and (Z,*ρ_I*) be three* μ_I*-topological spaces. For any P*μ_I*-irresolute(resp.* $\alpha\mu_I$*-irresolute and* $\beta\mu_I$*-irresolute) map* $\mathbb{k}$*: (X,*μ_I*)* $\rightarrow$ *(Y,*σ_I*) and any P*μ_I*-cts(resp.* $\alpha\mu_I$*-cts and* $\beta\mu_I$*- cts)* $\hbar$*: (Y,*σ_I*)* $\rightarrow$ *(Z,*ρ_I*) the composition* $\hbar \circ \mathbb{k}$*: (X,*μ_I*)* $\rightarrow$ *(Z,*ρ_I*) is P*μ_I*-cts(resp.* $\alpha\mu_I$*-cts and* $\beta\mu_I$*-cts).*
Proof: *We can prove this theorem as we have done in the theorem 3.5.*

Theorem 3.8. *If* $\mathbb{k}$*: (X,*μ_I*)* $\rightarrow$ *(Y,*σ_I*) and* $\hbar$*: (Y,*σ_I*)* $\rightarrow$ *(Z,*ρ_I*) are both P*μ_I*-irresolute(resp.* $\alpha\mu_I$*-irresolute and* $\beta\mu_I$*-irresolute) then* $\hbar \circ \mathbb{k}$*: (X,*μ_I*)* $\rightarrow$ *(Z,*ρ_I*) is also P*μ_I*-irresolute(resp.* $\alpha\mu_I$*-irresolute and* $\beta\mu_I$*-irresolute).*
Proof: *We can prove this theorem as we have done in the theorem 3.6*

4 Minimal and Maximal μ_I-irresolute

Definition 4.1. *Let (X,*μ_I*) and (Y,*σ_I*) be the topological spaces. A map* $\mathbb{k}$*: (X,*μ_I*)* $\rightarrow$ *(Y,*σ_I*) is called,*

1. *mn-*μ_I*-irresolute if the inverse image of every mn-*μ_I*-ops in (Y,*σ_I*) is mn-*μ_I*-open in (X,*μ_I*).*

2. *mx-μ_I-irresolute if the inverse image of every mx-μ_I-ops in (Y,σ_I) is mx-μ_I-open in (X,μ_I).*

Example 4.1. *Let X = $\{a, b, c, d\}$ and Y = $\{t, u, v, w\}$ with μ_I = $\{\emptyset$, $< X, \emptyset, \{b\} >$, $< X, \emptyset, \{d\} >$, $< X, \{a, d\}, \emptyset >$, $< X, \{a\}, \emptyset >$, $< X, \emptyset, \emptyset >$, $< X, \emptyset, \{c, d\} >$, $< X, \emptyset, \{c\} >$, $< X, \{d\}, \emptyset >$, $< X, \{d\}, \{b\} >\}$ and σ_I = $\{\emptyset$, $< X, \emptyset, \{v\} >$, $< X, \emptyset, \{w\} >$, $< X, \emptyset, \{u, v\} >$, $< X, \emptyset, \emptyset >$, $< X, \{v\}, \emptyset >$, $< X, \{v\}, \{w\} >\}$. Define $\mathbb{k}$: (X,μ_I) $\rightarrow$ (Y,σ_I) by $\mathbb{k}$(a) = t, $\mathbb{k}$(b) = w, $\mathbb{k}$(c) = u and $\mathbb{k}$(d) = v. Hence $\mathbb{k}$ is a mn-μ_I-irresolute map.*

Theorem 4.1. *Every mn-μ_I-irresolute map is mn-μ_I-cts.*
Proof: *Let $\mathbb{k}$: (X,μ_I) $\rightarrow$ (Y,σ_I) be a mn-μ_I-irresolute map. Let G be any mn-μ_I-ops in (Y,σ_I). Since $\mathbb{k}$ is mn-μ_I-irresolute, $\mathbb{k}^{-1}(A)$ is a mn-μ_I-ops in (X,μ_I). That is $\mathbb{k}^{-1}(A)$ is a μ_I-ops in (X,μ_I) Hence $\mathbb{k}$ is mn-μ_I-cts.*

Remark 4.1. *The reversal statement of the above theorem is not necessarily true. In example 4.3, $\mathbb{k}$ is mn-μ_I-cts but not mn-μ_I-irresolute. Since $\mathbb{k}^{-1}$(¡X,w,$\emptyset$¿) = ¡X,b,$\emptyset$¿ which is not minimal μ_I-open in (X,μ_I).*

Theorem 4.2. *Every mx-μ_I-irresolute map is mx-μ_I-cts.*
Proof: *We can prove this theorem as we have done in the theorem 4.4.*

Remark 4.2. *The reversal statement of the above theorem is not necessarily true. In example 4.2, $\mathbb{k}$ is mx-μ_I-cts but not mx-μ_I-irresolute. Since $\mathbb{k}^{-1}$(¡X,v,w¿ = ¡X,d,b¿ which is not mx-μ_I-open in (X,μ_I).*

Remark 4.3. *In example 4.2, $\mathbb{k}$ is a mn-μ_I-irresolute map but not mx-μ_I-irresolute. In example 4.3, $\mathbb{k}$ is a mx-μ_I-irresolute map but not mn-μ_I-irresolute. That is mn-μ_I-irresolute maps and mx-μ_I-irresolute maps are independent of each other.*

Remark 4.4. *Since mn-μ_I-ops and mx-μ_I-ops are independent of each other,*

1. *mn-μ_I-irresolute and mx-μ_I-cts are independent of each other.*

2. *mx-μ_I-irresolute and mn-μ_I-cts are independent of each other.*

Theorem 4.3. *Let $\mathbb{k}$: (X,μ_I) $\rightarrow$ (Y,σ_I) be a mn-μ_I-irresolute map if and only if the inverse image of each mx-μ_I-closed in (Y,σ_I) is a mx-μ_I-closed in (X,μ_I).*
Proof: *We can prove this theorem by using the result, if G is a mn-μ_I-ops if and only if G^c is a mx-μ_I-closed set.*

Theorem 4.4. *If $\mathbb{k}$: (X,μ_I) $\rightarrow$ (Y,σ_I) and $\hbar$: (Y,σ_I) $\rightarrow$ (Z,ρ_I) are mn-μ_I-irresolute then $\hbar \circ \mathbb{k}$: (X,μ_I) $\rightarrow$ (Z,ρ_I) is a mn-μ_I-irresolute map.*
Proof: *Let G be any mn-μ_I-ops in (Z,ρ_I). Since $\hbar$ is mn-μ_I-irresolute, $\hbar^{-1}(G)$ is a mn-μ_I-ops in (Y,σ_I). Also since $\mathbb{k}$ is mn-μ_I-irresolute, $\mathbb{k}^{-1}[\hbar^{-1}(G)]$ = $(\hbar \circ \mathbb{k})^{-1}(G)$ is a mn-μ_I-ops in (X,μ_I). Hence $\hbar \circ \mathbb{k}$ is mn-μ_I-irresolute.*

Theorem 4.5. *Let $\mathbb{k}$: (X,μ_I) $\rightarrow$ (Y,σ_I) be a mx-μ_I-irresolute map if and only if the inverse image of each mn-μ_I-closed in (Y,σ_I) is a mn-μ_I-closed in (X,μ_I).*
Proof: *We can prove this theorem by using the result, if G is a mx-μ_I-ops if and only if G^c is a mn-μ_I-cds.*

Theorem 4.6. *If $\mathbb{k}$: (X,μ_I) $\rightarrow$ (Y,σ_I) and $\hbar$: (Y,σ_I) $\rightarrow$ (Z,ρ_I) are mx-μ_I-irresolute then $\hbar \circ \mathbb{k}$: (X,μ_I) $\rightarrow$ (Z,ρ_I) is a mx-μ_I-irresolute map.*
Proof: *Similar to that of theorem 4.11.*

5 The Simple Extension of μ_I-topology over a μ_I-set

In (X,μ_I) a subset A of X, we denote by $\mu_I(A)$ the simple extension of μ_I over A, that is the collection of sets U∪(V∩A), where U,V $\in \mu_I$. Note that $\mu_I(A)$ is finer than μ_I.

Theorem 5.1. *If A is μ_I-dense subset of the space (X,μ_I), then A is also μ_I-dense in (X,$\mu_I(A)$).*
Proof: *Since $\mu_I(A)$ is finer than μ_I, $\mu_I \subset \mu_I(A)$. This gives $C_{\mu_I(A)}(A) \subset C_{\mu_I}(A)$. To prove $C_{\mu_I}(A) \subset C_{\mu_I(A)}(A)$. Let $x \in C_{\mu_I}(A)$ and let G be a μ_I-ops of x in $\mu_I(A)$. Then x∈G = H∪(J∩A) where H,J $\in \mu_I$. If x∈H then H∩A $\neq \emptyset$ and G∩A $\neq \emptyset$. If x∈J∩A then J∩A $\neq \emptyset$ and G∩A $\neq \emptyset$. Hence $x \in C_{\mu_I(A)}(A)$. Therefore $C_{\mu_I(A)}(A)$ = $C_{\mu_I}(A)$.*

Theorem 5.2. *Let (X,μ_I) be a μ_I-topological space with closed under intersection property. Let A be a μ_I-dense subset of the space (X,μ_I). Then for every μ_I-open subset G of the space (X,$\mu_I(A)$) we have $C_{\mu_I}(G)$ = $C_{\mu_I(A)}(G)$ and for every μ_I-closed subset F of the space (X,$\mu_I(A)$) we have $I_{\mu_I}(F)$ = $I_{\mu_I(A)}(F)$.*
Proof: *Let V $\in \mu_I$. Since $\mu_I(A)$ is finer than μ_I, $C_{\mu_I(A)}(V) \subset C_{\mu_I}(V)$. Now to prove, $C_{\mu_I}(V) \subset C_{\mu_I(A)}(V)$. Let $x \in C_{\mu_I}(V)$ and let G be a μ_I-open neighborhood of x in (X,$\mu_I(A)$). Then x∈G = H∪(J∩A) where H,J $\in \mu_I$. If x∈H then H∩V $\neq \emptyset$. Again if x $\in$ J∩A⊂J then J∩V $\neq \emptyset$ and hence J∩V∩A $\neq \emptyset$, since J∩V $\in \mu_I$ and since A is μ_I-dense. Thus also in this case G∩V $\neq \emptyset$ and hence $x \in C_{\mu_I(A)}(V)$. This implies $C_{\mu_I}(V) \subset C_{\mu_I(A)}(V)$. Henceforth $C_{\mu_I}(V)$ = $C_{\mu_I(A)}(V)$ for each V $\in \mu_I$. Let G $\in \mu_I(A)$ then G = H∪(J∩A) where H,J $\in \mu_I$. Clearly $C_{\mu_I}(H)$ = $C_{\mu_I(A)}(H)$. Since J $\in \mu_I(A)$ and since A is a μ_I-dense subset of (X,$\mu_I(A)$), $C_{\mu_I(A)}$(J∩A) = $C_{\mu_I(A)}(J)$ = $C_{\mu_I}(J)$ = C_{μ_I}(J∩A). Thus $C_{\mu_I(A)}(G)$ = $C_{\mu_I}(H) \cup$ C_{μ_I}(J∩A) =C_{μ_I}(H∪(J∩A)) = $C_{\mu_I}(G)$. Proceeding like this we can prove $I_{\mu_I}(F)$ = $I_{\mu_I(A)}(F)$.*

Corolary 5.1. *Let (X,μ_I) be a GITS with closed under intersection property. If A is a μ_I-dense subset of the space (X,μ_I). Then for every V $\in \mu_I(A)$ we have $I_{\mu_I}(C_{\mu_I}(V))$ = $I_{\mu_I(A)}(C_{\mu_I(A)}(V))$. Hence the set V is a regular μ_I-open subset of*

(X,μ_I) if and only if it is regular μ_I-open in (X,$\mu_I(A)$).
Proof: *From the previous theorem we have $I_{\mu_I}(C_{\mu_I}(V)) = I_{\mu_I}(C_{\mu_I(A)}(V)) = I_{\mu_I(A)}(C_{\mu_I(A)}(V))$.*

6 The characterization of extension on μ_I-topology

Remark 6.1. *If $\Bbbk$: (X,$\mu_I(A)$) $\rightarrow$ (Y,σ_I) is μ_I-cts. Then the restriction of $\Bbbk$ on (X,μ_I) [Shortly, $\Bbbk_|$(X,μ_I)] need not be μ_I-cts.*

Example 6.1. *Let X = $\{a, b, c\}$ and Y = $\{u, v, w\}$ with μ_I = $\{\emptyset, < X, \emptyset, \{a\} >, < X, \emptyset, \{b\} >, < X, \emptyset, \emptyset >, < X, \emptyset, \{a, b\} >, < X, \{a, b\}, \emptyset >\}$, $\mu_I(A)$ = $\{ \emptyset, < X, \emptyset, \{a\} >, < X, \emptyset, \{b\} >, < X, \emptyset, \emptyset >, < X, \emptyset, \{a, b\} >, < X, \{a, b\}, \emptyset >, < X, \{b\}, \emptyset >\}$ and σ_I = $\{\emptyset, < X, \emptyset, \{u\} >, < X, \emptyset, \{v\} >, < X, \emptyset, \emptyset >, < X, \{v\}, \emptyset >\}$. Define $\Bbbk$: (X,$\mu_I(A)$) $\rightarrow$ (Y,σ_I) by $\Bbbk$(a) = u, $\Bbbk$(b) = v and $\Bbbk$(c) = w. Hence $\Bbbk$ is $\mu_I(A)$-cts. But $\Bbbk_|$(X,$\mu_I(A)$) is not μ_I-cts, since $\Bbbk^{-1}(< X, \{v\}, \emptyset >)$ = $< X, \{b\}, \emptyset > \notin \mu_I$.*

Remark 6.2. *Since $\mu_I(A)$ is finer than μ_I, some elements of $\mu_I(A)$ does not belongs to μ_I and the elements of $\mu_I(A)$ which is not in μ_I need not be mn-μ_I-open in (X,μ_I). For, U $\subset$ U$\cup$(V$\cap$A) $\notin \mu_I$ and U $\in \mu_I(A)$, U$\cup$(V$\cap$A) should not be mn-μ_I-open in (X,$\mu_I(A)$). By the previous example, we may conclude that every mx-μ_I-ops in (X,$\mu_I(A)$) need not be μ_I-open in (X,μ_I).*

Remark 6.3. *A function $\Bbbk$ is mn-$\mu_I(A)$-cts in (X,$\mu_I(A)$) then $\Bbbk_|$(X,μ_I) is mn-μ_I-cts. In example 6.2, A function f is mx-$\mu_I(A)$-cts in (X,$\mu_I(A)$) then $f_|$(X,μ_I) need not be mx-μ_I-cts.*

7 Conclusions

In example 4.2, k is a mn-μ_I-irresolute map but not mx-μ_I-irresolute and in example 4.3, k is a mx-μ_I-irresolute map but not mn-μ_I-irresolute. This examples evinces mn-μ_I-irresolute maps and mx-μ_I-irresolute maps are independent of each other. Remark 6.1 propounded the restriction of the function K on (X,μ_I) need not be a μ_I-continuous function. In remark 6.3, we discussed the connections between minimal μ_I-open sets in (X,μ_I) and in (X,$\mu_I(A)$). We hope that we improved some results concerning $\mu_I(A)$-topological spaces. We will extend our research in kernel and contra continuous of μ_I-topological spaces.

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