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Some Graph Parameters of Clique graph of Cyclic Subgroup graph on certain Non-Abelian Groups

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Abstract

The aim of this paper is to examine various graph parameters of clique graph of cyclic subgroup graph on certain non-abelian groups and also we obtain some theorems and results in detail.

Keywords: Cyclic Subgroup graph, Clique graph, Hub number, Topological Indices

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1. Introduction

Algebraic Graph theory is a branch of mathematics in which graphs are constructed from the algebraic structures such as groups, rings etc. J. John Arul Singh and S. Devi [3] have introduced the notion of Cyclic Subgroup Graph of a finite group. The concept of Clique graphs was discussed at least as early as 1968 by Hamelink and Ronald. C. After that, Roberts and Spencer have given the concept of A Characterization of Clique Graphs [2] in 1971. Gutman introduced the concept of Energy in 1978. A topological index of a graph G is a numerical parameter mathematically derived from the graph structure. It is a graph invariant which does not depend on the labeling or pictorial representation of the graph and it is the graph invariant number calculated from a graph representing a molecule. Our present work is provoked by the above study. In section 2, we discuss some graph parameters for the clique graph of cyclic subgroup graph on certain non-abelian groups. In section 3, we examine some energies on clique graph of cyclic subgroup graph for dihedral group and generalised quaternion group. In section 4, we give some topological indices for the clique graph of cyclic subgroup graph on certain non-abelian groups. In this paper, p represents prime number and pq represents distinct primes where $q > p$. Before entering, let us look into some necessary definitions and notations. The *cyclic subgroup graph* $\Gamma_z(G)$ for a finite group G is a simple undirected graph in which the cyclic subgroups are vertices and two distinct subgroups are adjacent if one of them is a subset of the other. The *clique graph* $\mathcal{K}(G)$ of an undirected simple graph G, is a graph with a node for each maximal cliques in G. Two vertices in $\mathcal{K}(G)$ are adjacent when their corresponding maximal cliques in G share at least one vertex in common. For an integer $n \geq 3$, the *dihedral group* D_{2n} of order $2n$ is $D_{2n} = < r, f: r^n = f^2 = 1, frf = r^{-1} >$. The generalized quaternion group $Q_{4n} = < a, b: b^2 = a^n, a^{2n} = e, bab^{-1} = a^{-1} >$, where e is the identity element.A *hub set* in a graph G is a set H of vertices in G, such that any two vertices outside H are connected by a path whose all internal vertices lie in H. The *hub number* of G, denoted by $h(G)$, is the minimum cardinality of a hub set in G. Let G be a simple undirected graph, note that $u_i \sim u_j$ denotes that u_i is adjacent to u_j for $1 \leq i \neq j \leq n$. The *adjacency matrix* of G denoted by $A(G) = (a_{ij})$ is an $n \times n$ matrix defined as $a_{ij} = 1$ when $u_i \sim u_j$ and 0 otherwise. The sum of the absolute values of the eigen values of its *adjacency matrix* is called the energy i.e., $E(G) = \sum_{i=1}^{n} |\lambda_i|$. The *closed neighborhood matrix* $N = [n_{i,j}] = A + I_n$ has $n_{i,j} = 1$ if and only if $u_i \in N[u_j]$. The sum of the absolute values of the eigen values of its closed neighborhood matrix is called the *closed neighborhood energy*. The *Laplacian matrix* and *Signless Laplacian matrix* is defined as $L(G) = D(G) - A(G)$ and $SL(G) = D(G) + A(G)$ where $A(G)$ is the adjacency matrix and $D(G)$ is the diagonal matrix of vertex degrees.

2. Some Graph Parameters of Clique graph of Cyclic Subgroup graph on certain Non-Abelian Groups

Theorem 2.1: If $\mathcal{K}(\Gamma_z(G))$ be a clique graph of cyclic subgroup graph on a non-abelian group, then $\mathcal{K}(\Gamma_z(G))$ is biconnected.

Proof: Let $\mathcal{K}(\Gamma_z(G))$ be the clique graph of cyclic subgroup graph on a non-abelian group. Now, $W_1, W_2, \dots, W_n$ be the maximal cliques of $\Gamma_z(G)$. By the definition of clique graph, $V(U) = W_1, W_2, \dots, W_n$ and $(W_i, W_j) \in E(U)$ if only if $i \neq j$ and $W_i \cap W_j \neq \emptyset$ and take $U = \mathcal{K}(\Gamma_z(G))$. For $\Gamma_z(G)$, there exists a universal vertex which is adjacent to rest of its vertices. Now, any two vertices in $\mathcal{K}(\Gamma_z(G))$ are adjacent only when their corresponding maximal cliques in $\Gamma_z(G)$ have atleast one vertex in common. It is clear that, there is a path in between every starting vertex and ending vertex. Even after removing any vertex, the graph remains connected. Now concluding that G is connected and it does not contain any articulation point which results to a biconnected graph.

Theorem 2.2: For a clique graph of cyclic subgroup graph on any non-abelian group, the hub number is 0.

Proof: Based on the proof of 2.1, for $\mathcal{K}\big(\Gamma_z(G)\big)$, every pair of vertices are adjacent. Hence, there does not exist an intermediate vertex lies in the hub set. In this case, the minimum hub set is a null set. Hence, $h\Big(\mathcal{K}\big(\Gamma_z(G)\big)\Big) = 0$.

Theorem 2.3: Let $\mathcal{G} = \mathcal{K}(\Gamma_z(G))$ be a clique graph of cyclic subgroup graph on any non-abelian group and $|V(\mathcal{G})| = m$. Then $\mathscr{g}(\mathcal{G}) = \mathscr{m}(\mathcal{G}) = m$.

Proof: Let $\mathcal{G} = \mathcal{K}(\Gamma_z(G))$ be a clique graph of cyclic subgroup graph on any non-abelian group. The geodetic closure of a vertex set $S \subset \mathcal{V}$ is the set of all vertices $y \in \mathcal{V}$which lies in some geodesic in $\mathcal{G}$ joining two vertices u and v of S. Clearly for $\mathcal{K}\big(\Gamma_z(G)\big)$, there exists m maximal cliques which is connected by at least one vertex in common. Now, the resulting graph consists of m independent vertices in it. Hence $\mathscr{g}(\mathcal{G}) = m$.

Consider, a set D of vertices of $\mathcal{K}(\Gamma_z(G))$ is a monophonic set of $\mathcal{K}\big(\Gamma_z(G)\big)$, if each vertex $v \in \mathcal{K}(\Gamma_z(G))$ lies on an $u - w$ monophonic path in $\mathcal{K}\big(\Gamma_z(G)\big)$ for some $u, w \in D$ and the minimum cardinality of a monophonic set of $\mathcal{K}\big(\Gamma_z(G)\big), \mathscr{m}(\mathcal{G}) = m$.

Theorem 2.4: For any non-abelian group, $\mathcal{K}\big(\Gamma_z(G)\big)$ is non-planar.

Proof directly follows from theorem 2.1.

Theorem 2.5: Let $\mathcal{G} = \mathcal{K}(\Gamma_z(G))$ be a clique graph of cyclic subgroup graph on any non-abelian group and $|V(\mathcal{G})| = m$. Then $\kappa(\mathcal{G}) = m - 1$.

Proof: For $\mathcal{K}\big(\Gamma_z(G)\big)$, by removing $m-1$ vertices which makes the graph disconnected.

Theorem 2.6: The independence number of $\mathcal{K}(\Gamma_z(G))$ is 1.
Proof follows from direct computation.

Theorem 2.7: For any non-abelian group, $h\left(\mathcal{K}(\Gamma_z(G))\right) \neq \gamma\left(\mathcal{K}(\Gamma_z(G))\right)$.

Proof: Let $\mathcal{K}(\Gamma_z(G))$ be a clique graph of cyclic subgroup graph on any non-abelian group. Here, every pair of vertices are adjacent. Choose any one vertex $u_1 \in \mathcal{K}(\Gamma_z(G))$ as a dominating set, which is adjacent to all other vertices. Hence, the domination number is 1. By theorem 2.2, this theorem can be proved.

3.Various Graph Energies on $\mathcal{K}(\Gamma_z(D_{2n}))$ and $\mathcal{K}(\Gamma_z(Q_{4n}))$

Theorem 3.1: The adjacency energy on clique graph of cyclic subgroup graph for a dihedral group of order $2n, n \in \mathbb{N}$ and $n > 2$ is

$$E\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = \begin{cases} 2n & if\ n = p, p^2 \\ 2(n+1) & if\ n = pq \end{cases}$$

Proof: Case i: For $n = p, p^2$
The vertex set of $\mathcal{K}(\Gamma_z(D_{2n})) = \{u_1, u_2, \dots, u_{n+1}\}$.
The Adjacency matrix can be written as $A = J_{n+1} - I_{n+1}$.
The obtained characteristic polynomial is $(x-n)(x+1)^n$.
The spectrum of $\mathcal{K}(\Gamma_z(D_{2n}))$will be written as, $spec\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = \begin{Bmatrix} n & -1 \\ 1 & n \end{Bmatrix}$.
For $n = p, p^2, E\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = 2n$

Case ii: For $n = pq$
The vertex set of $\mathcal{K}(\Gamma_z(D_{2n})) = \{u_1, u_2, \dots, u_{n+2}\}$.
The Adjacency matrix can be written as $A = J_{n+2} - I_{n+2}$.
The obtained characteristic polynomial is $(x-(n+1))(x+1)^{n+1}$.
The spectrum of $\mathcal{K}(\Gamma_z(D_{2n}))$will be written as, $spec\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = \begin{Bmatrix} n+1 & -1 \\ 1 & n+1 \end{Bmatrix}$.
For $n = pq, E\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = 2(n+1)$.

Theorem 3.2: If $n = p,$ then the eigen values of $A\left(\mathcal{K}(\Gamma_z(Q_{4n}))\right)$ are $n+1$ with multiplicity 1 &-1 with multiplicity $n+1$ and $E\left(\mathcal{K}(\Gamma_z(Q_{4n}))\right) = 2(n+1)$.

Proof: The vertex set of $\mathcal{K}(\Gamma_z(Q_{4n})) = \{u_1, u_2, \dots, u_{n+2}\}$.
The Adjacency matrix can be written as $A = J_{n+2} - I_{n+2}$.
The obtained characteristic polynomial is $-(x-(n+1))(x+1)^{n+1}$.
The spectrum of $\mathcal{K}(\Gamma_z(Q_{4n}))$will be written as, $spec\left(\mathcal{K}(\Gamma_z(Q_{4n}))\right) = \begin{Bmatrix} n+1 & -1 \\ 1 & n+1 \end{Bmatrix}$.
For $n = p, E\left(\mathcal{K}(\Gamma_z(Q_{4n}))\right) = 2(n+1)$.

Theorem 3.3: The closed neighborhood energy on clique graph of cyclic subgroup graph for a dihedral group of order $2n, n \in \mathbb{N}$ and $n > 2$ is $E_N\left(\mathcal{K}\left(\Gamma_z(D_{2n})\right)\right) = \begin{cases} n+1 & if\ n = p, p^2 \\ n+2 & if\ n = pq \end{cases}$

Proof: Case i: For $n = p, p^2$

The vertex set of $\mathcal{K}\left(\Gamma_z(D_{2n})\right) = \{u_1, u_2, \dots, u_{n+1}\}$.

The Closed Neighborhood matrix can be written as $N = J_{n+1}$.

The obtained characteristic polynomial is $(x - (n+1))x^n$.

The Closed Neighborhood spectrum of $\mathcal{K}\left(\Gamma_z(D_{2n})\right)$will be written as,

$spec\left(\mathcal{K}\left(\Gamma_z(D_{2n})\right)\right) = \begin{Bmatrix} n+1 & 0 \\ 1 & n \end{Bmatrix}$.

For $n = p, p^2, E_N\left(\mathcal{K}\left(\Gamma_z(D_{2n})\right)\right) = n + 1$.

Case ii: For $n = pq$

The vertex set of $\mathcal{K}\left(\Gamma_z(D_{2n})\right) = \{u_1, u_2, \dots, u_{n+2}\}$.

The Closed Neighborhood matrix can be written as $N = J_{n+2}$.

The obtained characteristic polynomial is $(x - (n+2))x^{n+1}$.

The Closed Neighborhood spectrum of $\mathcal{K}\left(\Gamma_z(D_{2n})\right)$will be written as,

$spec\left(\mathcal{K}\left(\Gamma_z(D_{2n})\right)\right) = \begin{Bmatrix} n+2 & 0 \\ 1 & n+1 \end{Bmatrix}$.

For $n = pq, E_N\left(\mathcal{K}\left(\Gamma_z(D_{2n})\right)\right) = n + 2$.

Theorem 3.4: If $n = p$, then the eigen values of $N\left(\mathcal{K}\left(\Gamma_z(Q_{4n})\right)\right)$ are $n + 2$ with multiplicity 1 &0 with multiplicity $n + 1$ and $E_N\left(\mathcal{K}\left(\Gamma_z(Q_{4n})\right)\right) = n + 2$.

Proof: The vertex set of $\mathcal{K}\left(\Gamma_z(Q_{4n})\right) = \{u_1, u_2, \dots, u_{n+2}\}$.

The Closed Neighbourhood matrix can be written as $N = J_{n+2}$.

The obtained characteristic polynomial is $-(x - (n+2))x^{n+1}$.

The Closed Neighborhood spectrum of $\mathcal{K}\left(\Gamma_z(Q_{4n})\right)$will be written as, $spec\left(\mathcal{K}\left(\Gamma_z(Q_{4n})\right)\right) = \begin{Bmatrix} n+2 & 0 \\ 1 & n+1 \end{Bmatrix}$.

For $n = p, E_N\left(\mathcal{K}\left(\Gamma_z(Q_{4n})\right)\right) = n + 2$.

Theorem 3.5: The Laplacian spectrum on clique graph of cyclic subgroup graph for a dihedral group of order $2n, n \in \mathbb{N}$ and $n > 2$ is

(i) For $n = p, p^2, spec\left(\mathcal{K}\left(\Gamma_z(D_{2n})\right)\right) = \begin{Bmatrix} n+1 & 0 \\ n & 1 \end{Bmatrix}$

(ii) For $n = pq, spec\left(\mathcal{K}\left(\Gamma_z(D_{2n})\right)\right) = \begin{Bmatrix} n+2 & 0 \\ n+1 & 1 \end{Bmatrix}$.

Proof: Case i: For $n = p, p^2$

The vertex set of $\mathcal{K}\left(\Gamma_z(D_{2n})\right) = \{u_1, u_2, \dots, u_{n+1}\}$.

The Laplacian matrix can be written as $L = (n+1)I_{n+1} - J_{n+1}$.

The obtained characteristic polynomial is $(x - (n+1))^n x$.

The Laplacian spectrum of $\mathcal{K}(\Gamma_z(D_{2n}))$will be written as, $spec\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = \begin{Bmatrix} n+1 & 0 \\ n & 1 \end{Bmatrix}$.

Case ii: For $n = pq$

The vertex set of $\mathcal{K}(\Gamma_z(D_{2n})) = \{u_1, u_2, \dots, u_{n+2}\}$.

The Laplacian matrix can be written as $L = (n+2)I_{n+2} - J_{n+2}$.

The obtained characteristic polynomial is$\left(x - (n+2)\right)^{n+1} x$.

The Laplacian spectrum of $\mathcal{K}(\Gamma_z(D_{2n}))$will be written as, $spec\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = \begin{Bmatrix} n+2 & 0 \\ n+1 & 1 \end{Bmatrix}$.

Theorem 3.6: If $n = p,$ then the eigen values of $L\left(\mathcal{K}(\Gamma_z(Q_{4n}))\right)$ are $n+2$ with multiplicity $n+1$ &0 with multiplicity 1.

Proof: The vertex set of $\mathcal{K}(\Gamma_z(Q_{4n})) = \{u_1, u_2, \dots, u_{n+2}\}$.

The Laplacian matrix can be written as $L = (n+2)I_{n+2} - J_{n+2}$.

The obtained characteristic polynomial is $-\left(x - (n+2)\right)^{n+1} x$.

The Laplacian spectrum of $\mathcal{K}(\Gamma_z(Q_{4n}))$will be written as, $spec\left(\mathcal{K}(\Gamma_z(Q_{4n}))\right) = \begin{Bmatrix} n+2 & 0 \\ n+1 & 1 \end{Bmatrix}$.

Theorem 3.7: The signless Laplacian spectrum on clique graph of cyclic subgroup graph for a dihedral group of order $2n, n \in \mathbb{N}$ and $n > 2$ is

$(i) For\ n = p, p^2, spec\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = \begin{Bmatrix} 2n & n-1 \\ 1 & n \end{Bmatrix}$

$(ii) For\ n = pq, spec\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = \begin{Bmatrix} 2n+2 & n \\ 1 & n+1 \end{Bmatrix}$

Proof: Case i: For $n = p, p^2$

The vertex set of $\mathcal{K}(\Gamma_z(D_{2n})) = \{u_1, u_2, \dots, u_{n+1}\}$.

The Signless Laplacian matrix can be written as $SL = J_{n+1} + (n-1)I_{n+1}$.

The obtained characteristic polynomial is $(x - 2n)(x - (n-1))^n$.

The Signless Laplacian spectrum of $\mathcal{K}(\Gamma_z(D_{2n}))$will be written as,

$spec\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = \begin{Bmatrix} 2n & n-1 \\ 1 & n \end{Bmatrix}$.

Case ii: For $n = pq$

The vertex set of $\mathcal{K}(\Gamma_z(D_{2n})) = \{u_1, u_2, \dots, u_{n+2}\}$.

The Signless Laplacian matrix can be written as $SL = J_{n+2} + nI_{n+2}$.

The obtained characteristic polynomial is $(x - (2n+2))(x - n)^{n+1}$.

The Signless Laplacian spectrum of $\mathcal{K}(\Gamma_z(D_{2n}))$will be written as,

$spec\left(\mathcal{K}(\Gamma_z(D_{2n}))\right) = \begin{Bmatrix} 2n+2 & n \\ 1 & n+1 \end{Bmatrix}$.

Theorem 3.8: If $n = p$, then the eigen values of $SL\left(\mathcal{K}\left(\Gamma_z(Q_{4n})\right)\right)$ are $2n+2$ with multiplicity 1 &n with multiplicity $n+1$.
Proof: The vertex set of $\mathcal{K}\left(\Gamma_z(Q_{4n})\right) = \{u_1, u_2, \dots, u_{n+2}\}$.
The Signless Laplacian matrix can be written as $SL = J_{n+2} + nI_{n+2}$.
The obtained characteristic polynomial is $(x-(2n+2))(x-n)^{n+1}$.
The Signless Laplacian spectrum of $\mathcal{K}\left(\Gamma_z(Q_{4n})\right)$will be written as,
$spec\left(\mathcal{K}\left(\Gamma_z(Q_{4n})\right)\right) = \left\{\begin{matrix} 2n+2 & n \\ 1 & n+1 \end{matrix}\right\}$.

4.Some Topological Indices on Clique graph of Cyclic Subgroup graph for certain Non-Abelian Groups

Theorem 4.1: If $\mathcal{K}\left(\Gamma_z(G)\right)$ be a clique graph of cyclic subgroup graph on a non-abelian group and $\left|V\left(\mathcal{K}\left(\Gamma_z(G)\right)\right)\right| = m$, then the balaban index is $J\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = \frac{m^3-m^2}{2(m^2-3m+4)}$

Proof:$J\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = \frac{y}{y-x+2}\sum_{uv\in E\left(\mathcal{K}\left(\Gamma_z(G)\right)\right)} \frac{1}{\sqrt{w(u).w(v)}}$, where the sum is taken over all edges of a connected graph G, x and y are the cardinalities of the vertex and the edge set of $G, w(u)$ and $w(v)$ denoted the sum of distances from u (resp.v) to all other vertices of G. $J\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = \frac{m^2-m}{m^2-m-2m+4}\left(\frac{m(m-1)}{2(m-1)}\right)$

$$= \frac{m^2-m}{m^2-3m+4}\left(\frac{m}{2}\right) = \frac{m^3-m^2}{2(m^2-3m+4)}$$

Theorem 4.2: If $\mathcal{K}\left(\Gamma_z(G)\right)$ be a clique graph of cyclic subgroup graph on a non-abelian group and $\left|V\left(\mathcal{K}\left(\Gamma_z(G)\right)\right)\right| = m$, then atom bond connectivity status index is $ABCS\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = \frac{1}{\sqrt{2}}m\sqrt{m-2}$.

Proof: $ABCS\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = \sum_{uv\in E\left(\mathcal{K}\left(\Gamma_z(G)\right)\right)} \sqrt{\frac{\sigma(u)+\sigma(v)-2}{\sigma(u)\sigma(v)}}$

$$= \left(\sqrt{\frac{(m-1)+(m-1)-2}{(m-1)(m-1)}}\right)\frac{m(m-1)}{2}$$

$$= \frac{1}{\sqrt{2}}m\sqrt{m-2}.$$

Theorem 4.3: If $\mathcal{K}\left(\Gamma_z(G)\right)$ be a clique graph of cyclic subgroup graph on a non-abelian group and $\left|V\left(\mathcal{K}\left(\Gamma_z(G)\right)\right)\right| = m$, then the arithmetic-geometric status index is $AGS\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = \frac{m(m-1)}{2}$.

Proof: $AGS\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = \sum_{uv\in E\left(\mathcal{K}\left(\Gamma_z(G)\right)\right)} \frac{\sigma(u)+\sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}$

$$= \frac{m-1+m-1}{2\sqrt{(m-1)(m-1)}} \times \frac{m(m-1)}{2}$$

$$= \frac{m(m-1)}{2}$$

Theorem 4.4: If $\mathcal{K}\left(\Gamma_z(G)\right)$ be a clique graph of cyclic subgroup graph on a non-abelian group and $\left|V\left(\mathcal{K}\left(\Gamma_z(G)\right)\right)\right| = m,$ then

(i)The First Zagrebdegree eccentricityindex, $DE_1\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = m^3$

(ii)The Second Zagreb degree eccentricity index, $DE_2\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = \frac{m^3(m-1)}{2}$.

Proof: (i) $DE_1\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = \sum_{v_i\in V\left(\mathcal{K}\left(\Gamma_z(G)\right)\right)} (e_i + d_i)^2$ (where e_i be the eccentricity and d_i be the degree)

$$= m(1+m-1)^2 = m^3$$

$$(ii) DE_2\left(\mathcal{K}\left(\Gamma_z(G)\right)\right) = \sum_{v_iv_j\in E\left(\mathcal{K}\left(\Gamma_z(G)\right)\right)} (e_i + d_i)(e_j + d_j)$$

$$= \frac{m(m-1)}{2}(1+m)(1+m)$$

$$= \frac{m^3(m-1)}{2}.$$

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