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Odd Fibonacci Stolarsky-3 Mean Labeling of Some Special Graphs

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Abstract

Let G be a graph with p vertices and q edges and an injective function $f: V(G) \rightarrow \{1,3,5,13,21,55,89,\ldots\}$ where each f_i is a odd Fibonacci number and the induced edge labeling $f: V(G) \rightarrow N$ are defined by $f(e = uv) = \left\lceil \sqrt{\frac{f(u)^2 + f(u)f(v) + f(v)^2}{3}} \right\rceil \ or \ \left\lfloor \sqrt{\frac{f(u)^2 + f(u)f(v) + f(v)^2}{3}} \right\rfloor$ and all these edge labeling are distinct is called Odd Fibonacci Stolarsky-3 Mean Labeling. A graph which admits a Odd Fibonacci Stolarsky-3 Mean Labeling is called a Odd Fibonacci Stolarsky-3 mean graph.

Keywords: Stolarsky-3 Mean Labeling of Graphs, Odd Fibonacci Stolarsky-3 Mean Labeling of Graphs, Bull graph, Wheel graph, (m, n)-tadpole graph, Fire Cracker graph, Pan graph, Gear graph, Star graph.

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1. Introduction

The graph considered here will be finite, undirected and simple graph with p vertices and q edges. For all detailed survey of graph labeling, we refer to Galian [1]. For all other standard terminology and notations, we follow Harary [2]. S. S. Sandhya, S. Somasundaram and S. Kavitha introduced the concept of Stolarsky 3 Mean labeling of graphs in [3]. In this paper, we introduced a new concept namely Odd Fibonacci Mean Labeling of graphs.

Definition: 1.1 The Fibonacci numbers can be defined by linear recurrence $\boldsymbol{F_n} = \boldsymbol{F_{n-1}} + \boldsymbol{F_{n-2}}\,; (\boldsymbol{n \geq 2})$. This generates the infinite sequence of integer beginning 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,144, 233, ….

Definition: 1.2 Let G be a graph with p vertices q edges. An injective function $f: V(G) \to \{1,3,5,13,21,55,...\}$ where each f_i isa odd Fibonacci number and the induced edge labeling $f: E(G) \to N$ defined by edge e =uv is labeled with f=(e=uv)= $\left\lceil \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rceil$ (or) $\left\lfloor \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rfloor$, then all the edge labels are distinct and are from Odd Fibonacci

number $F_1, F_2, ..., F_q$ where $F_1 = 1, F_2 = 3, F_3 = 5, ...$. A graph that admits Odd Fibonacci Stolarsky-3 Mean labeling is called Odd Fibonacci Stolarsky-3 Mean graph.

Definition: 1.3 The Bull graph is a planar undirected graph with 5 vertices and 5 edges in the form of a triangle with two disjoint pendent edges.

Definition: 1.4 The Wheel graph $W_n, n \geq 3$ is join of the graphs $C_n \& K_1$, $W_n = C_n + K_1$.

Definition: 1.5 The (m, n)-tadpole graph is a graph is a special consisting of a graph on m (at least 3) vertices and a path graph on n vertices connected with a bridge.

Definition: 1.6 An Firecracker is a graph obtained by the concatenation of stars by linking one leaf.

Definition: 1.7 Gear graph is obtained from the wheel by adding a vertex between every pair of adjacent vertices of the cycle. The gear graph G_n has 2n+1 vertices and 3n edges.

Definition:1.8 The pan graph is the graph obtained by joining a cycle graph to a singleton graph with a bridge.

Definition: 1.9A Star graph with n vertices is a tree with one vertex having degree n-1 and other n-1 vertices having degree 1. A star graph with n+1 vertices $K_{1,n}$.

2. Main Results

Theorem 2.1: The Bull graph is a Odd Fibonacci Stolarsky-3 mean labeling.
Proof. Let G be a Bull graph.
Here $|V| = n,\ |E| = n - 1$
Define a function $f: V(G) \to \{1,3,5,13,21,55, \dots\}$ where each f_i is a odd Fibonacci number.
$f(u_i) = 2i - 1, 1 \le i \le 3$
$f(u_4) = 3i + 4, i = 4$
$f(u_5) = 4i + 1, i = 5$
Then the induced edge labeling is all distinct.
Hence, we proved a Bull graph admits Odd Fibonacci Stolarsky-3 mean labeling.

Example 2.2: Odd Fibonacci Stolarsky-3 mean labeling of Bull graph is shown below.

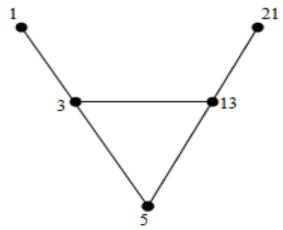

Figure 2.1: Bull graph

Theorem 2.3: The Wheel graph W_n admits Odd Fibonacci Stolarsky-3 mean labeling.
Proof. Let $G = W_n$ be a Wheel graph.
Here $|V| = n,\ |E| = n - 1$
Define a function $f: V(G) \to \{1,3,5,13,21,55, \dots\}$ where each f_i is a odd Fibonacci number.
$f(u_i) = 2i - 1, 1 \le i \le 3$
$f(u_4) = 3i + 4, i = 4$
$f(u_5) = 4i + 1, i = 5$
$f(u_6) = 9i + 1, i = 6$
Then the induced edge labeling are all distinct.
Hence, we proved a wheel graph W_n admits Odd Fibonacci Stolarsky-3 mean labeling.

Example 2.4: Odd Fibonacci Stolarsky-3 mean labeling of Wheel graph W_5 is shown below.

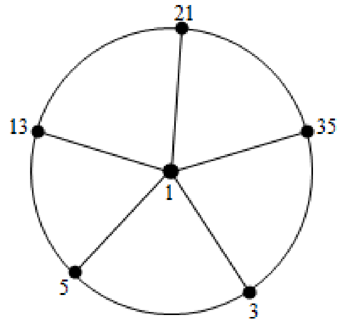

Figure 2.2 Wheel graph W_5

Theorem 2.5: The (m, n)- tadpole graph admits Odd Fibonacci Stolarsky-3 mean labeling.
Proof. Let G be a (m, n)- tadpole graph.
Here $|V| = n,\ |E| = n - 1$
Define a function $f: V(G) \to \{1,3,5,13,21,55, \dots\}$ where each f_i is a odd Fibonacci number.
$f(u_i) = 2i - 1, 1 \leq i \leq 3$
$f(u_4) = 3i + 4, i = 4$
$f(u_5) = 4i + 1, i = 5$
$f(u_6) = 9i + 1, i = 6$
Then the induced edge labeling is all distinct.
Hence, we proved a (m, n)- tadpole graph admits Odd Fibonacci Stolarsky-3 mean labeling.

Example 2.6: Odd Fibonacci Stolarsky-3 mean labeling of (5,1)- tadpole graph is shown below.

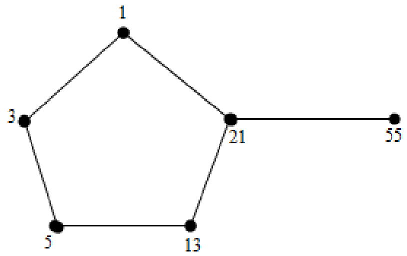

Figure 2.3 (5, 1)- tadpole graph

Theorem 2.7: A Fire Cracker graph admits Odd Fibonacci Stolarsky-3 mean labeling.
Proof. Let G be a Fire Cracker graph.
Here $|V| = n,\ |E| = n - 1$
Define a function $f: V(G) \to \{1,3,5,13,21,55, \dots\}$ where each f_i is a odd Fibonacci number and assignment of vertex labeling are
$f(u_i) = 2i - 1, 1 \leq i \leq 3$
$f(u_4) = 3i + 4, i = 4$
$f(u_5) = 4i + 1, i = 5$

$f(u_6) = 9i + 1, i = 6$
$f(u_7) = 12i + 5, i = 7$
$f(u_8) = 29i + 1, i = 8$
Then the induced edge labeling is all distinct.
Hence, we proved a Fire Cracker admits Odd Fibonacci Stolarsky-3 mean labeling.

Example 2.8: Odd Fibonacci Stolarsky-3 mean labeling of Fire Cracker graph is shown below.

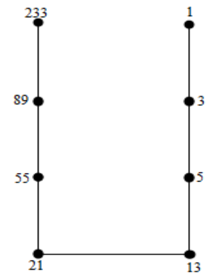

Figure 2.4: Fire Cracker graph

Theorem 2.9: The Pan graph admits an Odd Fibonacci Stolarsky-3 mean labeling.
Proof. Let G be a Pan graph.
Here $|V| = n,\ |E| = n - 1$
Define a function $f: V(G) \to \{1,3,5,13,21,55, ...\}$ where each f_i is a odd Fibonacci number and assignment of vertex labeling are
$f(u_i) = 2i - 1, 1 \le i \le 3$
$f(u_4) = 3i + 4, i = 4$
$f(u_5) = 4i + 1, i = 5$
Then the induced edge labeling are all distinct.
Hence, we proved a Pan admits Odd Fibonacci Stolarsky-3 mean labeling.

Example 2.10: Odd Fibonacci Stolarsky-3 mean labeling of Pan graph is shown below.

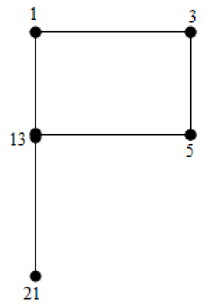

Figure 2.5: Pan graph

Theorem 2.11: The Gear graph G_n admits Odd Fibonacci Stolarsky-3 mean labeling.
Proof. Let $G = G_n$ be Gear graph.

Here $|V| = n,\ |E| = n - 1$
Define a function $f: V(G) \rightarrow \{1,3,5,13,21,55, \dots\}$ where each f_i is a odd Fibonacci number.
$f(u_i) = 2i - 1, 1 \leq i \leq 3$
$f(u_4) = 3i + 4, i = 4$
$f(u_5) = 4i + 1, i = 5$
$f(u_6) = 9i + 1, i = 6$
$f(u_7) = 12i + 1, i = 7$
$f(u_8) = 29i + 1, i = 8$
$f(u_9) = 42i - 1, i = 9$
Then the induced edge labeling is all distinct.
Hence, we proved a Gear graph G_n admits Odd Fibonacci Stolarsky-3 mean labeling.

Example 2.12: Odd Fibonacci Stolarsky-3 mean labeling of Gear graph G_4 is shown below.

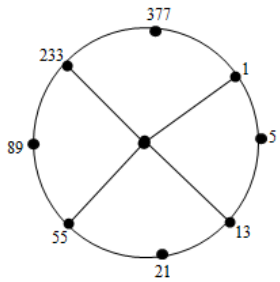

Figure 2.6 Gear graph

Theorem 2.13: The Star graph admits an Odd Fibonacci Stolarsky-3 mean labeling.
Proof. Let G be a Star graph.
Here $|V| = n,\ |E| = n - 1$
Define a function $f: V(G) \rightarrow \{1,3,5,13,21,55, \dots\}$ where each f_i is a odd Fibonacci number and assignment of vertex labeling are
$f(u_i) = 2i - 1, 1 \leq i \leq 3$
$f(u_4) = 3i + 4, i = 4$
$f(u_5) = 4i + 1, i = 5$
$f(u_6) = 9i + 1, i = 6$
$f(u_7) = 12i + 5, i = 7$
Then the induced edge labeling is all distinct.
Hence, we proved a Star graph admits Odd Fibonacci Stolarsky-3 mean labeling.

Example 2.14: Odd Fibonacci Stolarsky-3 mean labeling of Star graph is shown below.

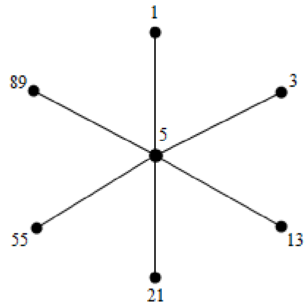

Figure 2.7 Star graph

3. Conclusion

we have introduced a new labeling namely Odd Fibonacci Stolarsky-3 Mean Labeling of graphs. We prove that Bull graph, Wheel graph, (m,n)-tadpole graph, Fire Cracker graph, Pan graph, Gear graph, Star graph. Extending the study to other families of graphs is an open area of research.

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