Ratio Mathematica Volume 44, 2022

Various Product on Multi Fuzzy Graphs

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Abstract

In this paper, the definition of complement of multi fuzzy graph, direct sum of two multi fuzzy graphs are given and derived some theorems related to them. Also, we examine the different product on multi fuzzy graphs such as Direct product, Cartesian product, Strong product, Composition, Corona product and some properties are analyzed.

Key Words: Multi fuzzy graph, complement of multi fuzzy graph, direct sum, direct product, cartesian product, strong product, composition, corona product,

AMS Subject Classification: 03E72, 05C72, 05C07[4]

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[4] Received on June 26 th, 2022. Accepted on Sep 1st, 2022. Published on Nov 30th, 2022. doi: 10.23755/rm.v44i0.911. ISSN: 1592-7415. eISSN: 2282-8214.

1. Introduction

The notion of fuzzy set and fuzzy relations were proposed by L.A Zadeh [18] in 1965 for representing uncertainty. The concept of fuzzy graph was first introduced by Kauffman [2] from the concept fuzzy relation introduced by L.A Zadeh in 1973. In 1975, Rosenfeld [14] developed the theory of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. Thereafter in 1987, Bhattacharya [1] defined some remarks on fuzzy graphs. The operations of union, join, cartesian product and composition of two fuzzy graphs were defined by Mordeson. J.N, and Prem Chand S. Nair, [3] in 2000. After that M.S. Sunitha and A. Vijayakumar [17] extended the concept of operations on fuzzy graph in 2002.Sebu Sebastian, T.V. Ramakrishnan [15] defined Multi fuzzy set in 2010. Radha. K and Arumugam. S [11, 12] defined the direct sum of two fuzzy graphs in 2013 and strong product of two fuzzy graphs in 2014.OzgeColakogluHavare and Hamza Menken [10] defined the Corona Product of Two Fuzzy Graphs in 2016.In 2020R.Muthuraj and S. Revathi [5] introduced the concept of multi fuzzy graph which is the extension of a fuzzy graph with single phenomenon into a multi-phenomenon which suits to describe the real-life problems in a better manner than fuzzy graph. Later on, and Multi anti fuzzy graph defined by Muthuraj. Ret.al [6]. In this paper complement of multi fuzzy graph, direct sum of two multi fuzzy graphs and various product on multi fuzzy graphs are defined and proved some theorems related to them.

2. Preliminaries

Definition 2.1 [2] A fuzzy graph $G=(\sigma,\mu)$ defined on the underlying crisp graph $G^*=(V,E)$ where $E\subseteq V\times V$ is a pair of functions $\sigma : V\to[0,1]$ and $\mu : V\times V\to[0,1]$, μ is a symmetric fuzzy relation on σ such that $\mu(uv)\le\min\{\sigma(u),\sigma(v)\}$ for $u,v\in V$

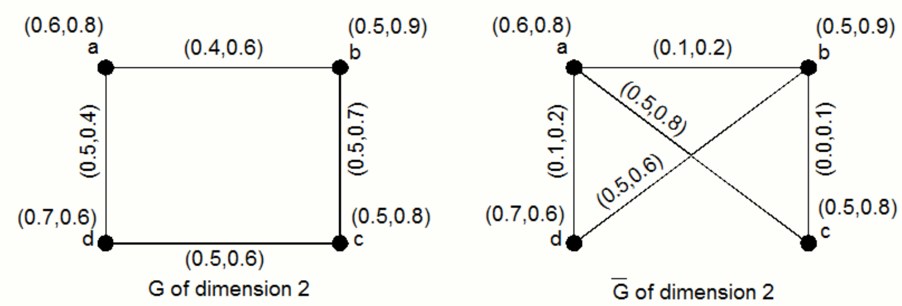
Definition 2.2 [15] Let X be a non-empty set. A Multi Fuzzy Set A in X is defined as a set of ordered sequences: $A=\{(x,\mu_1(x),\mu_2(x),......\mu_i(x)...) : x\in X\}$ where $\mu_i : X\to[0,1]$ for all i.

Definition 2.3 [5] A Multi fuzzy Graph (MFG) of dimension m defined on the underlying crisp graph $G^*=(V,E)$ where $E\subseteq V\times V$, is denoted as $G=((\sigma_1,\sigma_2,...\sigma_m),(\mu_1,\mu_2,...\mu_m))$ and $\sigma_i : V\to[0,1]$ and $\mu_i : V\times V\to[0,1]$, μ_i is a symmetric fuzzy relation on σ_i such that $\mu_i(uv)\le\min\{\sigma_i(u),\sigma_i(v)\}$ for all $i=1,2,3....m$ where $u,v\in V$ and $uv\in E$

3. Complement of Multi Fuzzy Graph

Definition 3.1 The complement of a multi-fuzzy graph $G=((\sigma_1,\sigma_2,....\sigma_m),(\mu_1,\mu_2,....\mu_m))$ of dimension m is a multi-fuzzy graph $\overline{G}=((\overline{\sigma_1},\overline{\sigma_2},...\overline{\sigma_m}),(\overline{\mu_1},\overline{\mu_2},...\overline{\mu_m}))$ of dimension m where $\overline{\sigma_i}=\sigma_i$ and $\overline{\mu_i}(u,v)=(\sigma_i(u)\wedge\sigma_i(v))-\mu_i(u,v)$ for all $u,v\in V$ and for all $i=1,2,3....m$

Example 3.2



Theorem 3.3 If G is a strong multi fuzzy graph then $\overline{G}$ is also strong multi fuzzy graph.

Proof: Let $u,v\in E$. Then

$\overline{\mu_i}(u,v)=(\sigma_i(u)\wedge\sigma_i(v))-\mu_i(u,v)$

$=(\sigma_i(u)\wedge\sigma_i(v))-(\sigma_i(u)\wedge\sigma_i(v))=0$ since G is strong.

Let $u,v\notin E$. Then

$\overline{\mu_i}(u,v)=(\sigma_i(u)\wedge\sigma_i(v))-\mu_i(u,v)=(\sigma_i(u)\wedge\sigma_i(v))-0=(\sigma_i(u)\wedge\sigma_i(v)$

Theorem 3.4 The complement of complete multi fuzzy graph is a null graph.

Proof: Let $G=(V,E)$ be a multi-fuzzy graph with the underlying crisp graph $G^*=(V,E)$ is complete. ie., $\mu_i(u,v)=(\sigma_i(u)\wedge\sigma_i(v))\ \forall u,v\in V\ \&\ uv\in E$

Let $u,v\in E$

$\overline{\mu_i}(u,v)=(\sigma_i(u)\wedge\sigma_i(v))-\mu_i(u,v)$

$=(\sigma_i(u)\wedge\sigma_i(v))-(\sigma_i(u)\wedge\sigma_i(v))=0$ since G is complete.

So, we have the edge set of $\overline{G}$ is empty when G is a complete multi fuzzy graph. Hence the complement of complete multi fuzzy graph is a null graph.

4. Various Product On Multi Fuzzy Graphs

In this section $G_1=((\sigma_1,\sigma_2,...\sigma_m),(\mu_1,\mu_2,...\mu_m))$ denotes the multi fuzzy graph with dimension m with the underlying crisp graph $G_1^*=(V_1,E_1)$ and $G_2=((\alpha_1,\alpha_2,...\alpha_n),(\beta_1,\beta_2,...\beta_n))$ denotes the multi fuzzy graph with dimension n with the underlying crisp graph $G_2^*=(V_2,E_2)$

Definition 4.1 The operation **Direct sum** between two MFG G_1 and G_2 is defined as follows, $G_1 \oplus G_2 = ((\sigma_1 \oplus \alpha_1, \sigma_2 \oplus \alpha_2, ...\sigma_k \oplus \alpha_k), (\mu_1 \oplus \beta_1, \mu_2 \oplus \beta_2, ...\mu_k \oplus \beta_k))$ with the underlying crisp graph $G_1^* \oplus G_2^* = (V_1 \oplus V_2, E_1 \oplus E_2)$, $(\sigma_i \oplus \alpha_i)(u) = \begin{cases} \sigma_i(u) \ if \ u \in V_1 - V_2 \\ \alpha_i(u) \ if \ u \in V_2 - V_1 \\ \max\{\sigma_i(u), \alpha_i(u)\} \ if \ u \in V_1 \cap V_2 \end{cases}$

for all $i = 1,2,3....k$ and

$(\mu_i \oplus \beta_i)(u,v) = \begin{cases} \mu_i(u,v) & if \ (u,v) \in E_1 \\ \beta_i(u,v) & if \ (u,v) \in E_2 \end{cases}$ for all $i = 1,2,3....k$

If m ≠ n, let k = max (m, n). Suppose $m < n$, then let us introduce n – m membership values of multi fuzzy graph G_1 into 0 so as to convert the multi fuzzy graphs G_1 and G_2 have the same dimension as k.

Theorem 4.2 The direct sum of two multi fuzzy graph is also a multi-fuzzy graph,

Proof: Let $G_1 = ((\sigma_1, \sigma_2, ...\sigma_m), (\mu_1, \mu_2, ...\mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, ...\alpha_n), (\beta_1, \beta_2, ...\beta_n))$ be the multi fuzzy graph with dimension m and n respectively

To prove: $G = G_1 \oplus G_2$ is also multi fuzzy graph with dimension k where k=max (m, n)

$$(\sigma_i \oplus \alpha_i)(u) = \begin{cases} \sigma_i(u) \ if \ u \in V_1 - V_2 \\ \alpha_i(u) \ if \ u \in V_2 - V_1 \\ \max\{\sigma_i(u), \alpha_i(u)\} \ if \ u \in V_1 \cap V_2 \end{cases}$$

Case (i): Let $(u,v) \in E_1$

$(\mu_i \oplus \beta_i)(u,v) = \mu_i(u,v)$

$\le \min(\sigma_i(u), \sigma_i(v))$

$= \min((\sigma_i \oplus \alpha_i)(u), (\sigma_i \oplus \alpha_i)(v))$

$\therefore (\mu_i \oplus \beta_i)(u,v) \le \min((\sigma_i \oplus \alpha_i)(u), (\sigma_i \oplus \alpha_i)(v))$

Case (ii): Let $(u,v) \in E_2$

$(\mu_i \oplus \beta_i)(u,v) = \beta_i(u,v)$

$\le \min(\alpha_i(u), \alpha_i(v))$

$= \min((\sigma_i \oplus \alpha_i)(u), (\sigma_i \oplus \alpha_i)(v))$

$\therefore (\mu_i \oplus \beta_i)(u,v) \le \min((\sigma_i \oplus \alpha_i)(u), (\sigma_i \oplus \alpha_i)(v))$

Definition4.3 The operation **Direct Product** between two MFG G_1 and G_2 is defined as follows, $G_1 * G_2 = ((\sigma_1 * \alpha_1, \sigma_2 * \alpha_2, ...\sigma_k * \alpha_k), (\mu_1 * \beta_1, \mu_2 * \beta_2, ...\mu_k * \beta_k))$ with the underlying crisp graph $G_1^* * G_2^* = (V, E)$ where $V = V_1 \times V_2$ and $E = \{(u_1, v_1), (u_2, v_2) / (u_1, u_2) \in E_1 \ \& \ (v_1, v_2) \in E_2\}$ with $(\sigma_i * \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1), \ \alpha_i(v_1)\} \ for \ all \ u_1 \in V_1, v_1 \in V_2 \ \& \ (u_1, v_1) \in V_1 \times V_2$

$(\mu_i * \beta_i)((u_1, v_1), (u_2, v_2)) = \min\{\mu_i(u_1, u_2), \beta_i(v_1, v_2)\} \ for \ all \ (u_1, u_2) \in E_1 \ \& \ (v_1, v_2) \in E_2$ for all I = 1, 2, 3, … k. If $m \ne n$, let k = min (m, n). Suppose $m < n$, then we take first m dimensions for G_2 so as to convert the MFG G_1 and G_2 have the same dimension k.

Example 4.4

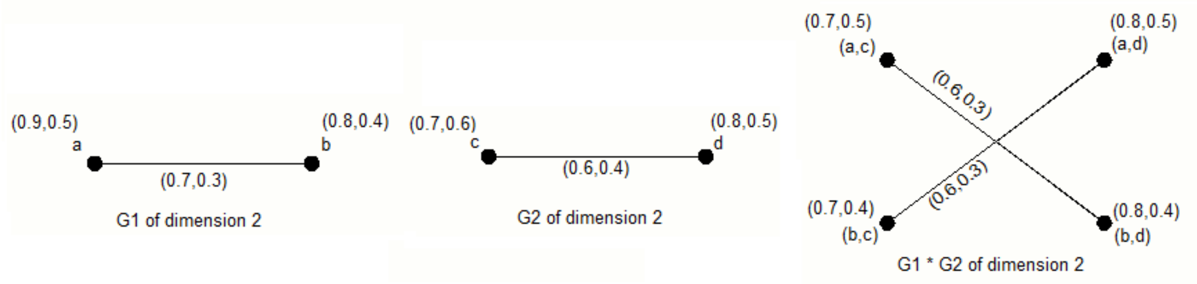


Figure1

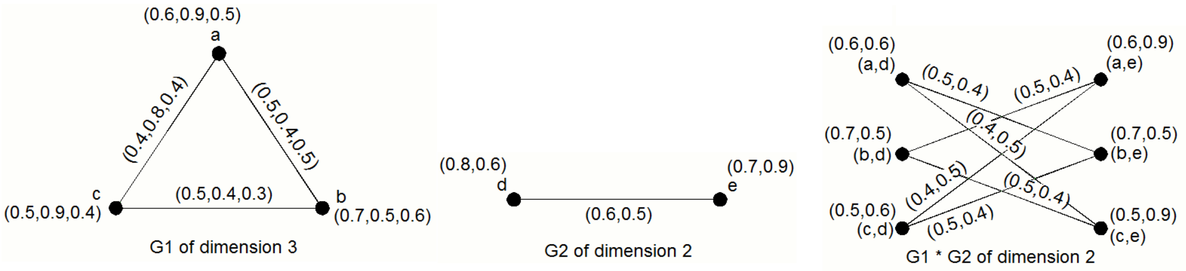


Figure2

Theorem 4.5 Direct product of two multi fuzzy graph is also a multi-fuzzy graph.

Proof: Let $G_1 = ((\sigma_1, \sigma_2, \ldots \sigma_m), (\mu_1, \mu_2, \ldots \mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, \ldots \alpha_n), (\beta_1, \beta_2, \ldots \beta_n))$ be the multi fuzzy graph with dimension m and n respectively

To Prove: $G = G_1 * G_2$ is a multi-fuzzy graph of dimension k where k=min (m, n)

$(\sigma_i * \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1),\ \alpha_i(v_1)\}$

$(\mu_i * \beta_i)((u_1, v_1), (u_2, v_2)) = \min\{\mu_i(u_1, u_2), \beta_i(v_1, v_2)\}$

$\leq \min\{\min(\sigma_i(u_1), \sigma_i(u_2)), \min(\alpha_i(v_1), \alpha_i(v_2)\}$

$= (\sigma_i(u_1) \wedge \sigma_i(u_2)) \wedge ((\alpha_i(v_1) \wedge \alpha_i(v_2))$

$= (\sigma_i(u_1) \wedge \alpha_i(v_1)) \wedge ((\sigma_i(u_2) \wedge \alpha_i(v_2))$

$= ((\sigma_i * \alpha_i)(u_1, v_1)) \wedge ((\sigma_i * \alpha_i)(u_2, v_2))$

$\therefore (\mu_i * \beta_i)((u_1, v_1), (u_2, v_2)) \leq \min((\sigma_i * \alpha_i)(u_1, v_1), (\sigma_i * \alpha_i)(u_2, v_2))$.

Theorem 4.6 If G_1 and G_2 are strong multi fuzzy graphs then $G_1 * G_2$ is also a strong multi fuzzy graph.

Proof:

$(\sigma_i * \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1),\ \alpha_i(v_1)\}$

$(\mu_i * \beta_i)((u_1, v_1), (u_2, v_2)) = \min\{\mu_i(u_1, u_2), \beta_i(v_1, v_2)\}$

$= \min\{\min(\sigma_i(u_1), \sigma_i(u_2)), \min(\alpha_i(v_1), \alpha_i(v_2)\}$

$= (\sigma_i(u_1) \wedge \sigma_i(u_2)) \wedge ((\alpha_i(v_1) \wedge \alpha_i(v_2))$

$= (\sigma_i(u_1) \wedge \alpha_i(v_1)) \wedge ((\sigma_i(u_2) \wedge \alpha_i(v_2))$

$= ((\sigma_i * \alpha_i)(u_1, v_1)) \wedge ((\sigma_i * \alpha_i)(u_2, v_2))$

$\therefore (\mu_i * \beta_i)((u_1,v_1),(u_2,v_2)) = \min((\sigma_i * \alpha_i)(u_1,v_1),(\sigma_i * \alpha_i)(u_2,v_2))$

Remark: If G_1 and G_2 are complete multi fuzzy graphs then $G_1 * G_2$ is not a complete multi fuzzy graph.

Definition 4.7 The operation **Cartesian Product** between two MFG G_1 and G_2 as follows, $G_1 \times G_2 = ((\sigma_1 \times \alpha_1, \sigma_2 \times \alpha_2, \ldots \sigma_k \times \alpha_k),(\mu_1 \times \beta_1, \mu_2 \times \beta_2, \ldots \mu_k \times \beta_k))$ with the underlying crisp graph $G_1^* \times G_2^* = (V,E)$ where $V = V_1 \times V_2$ and $E = \{(u_1,v_1)(u_2,v_2) / u_1 = u_2, (v_1,v_2) \in E_2 \, or \, v_1 = v_2, (u_1,u_2) \in E_1\}$ with

$(\sigma_i \times \alpha_i)(u_1,v_1) = \min\{\sigma_i(u_1), \alpha_i(v_1)\}$ *for all* $u_1 \in V_1$ *and* $v_1 \in V_2$ & $(u_1,v_1) \in V_1 \times V_2$

$$(\mu_i \times \beta_i)((u_1,v_1),(u_2,v_2)) = \begin{cases} \min\{\sigma_i(u), \beta_i(v_1,v_2)\} & u_1 = u_2 = u, \text{ for all } u \in V_1 \ \& (v_1,v_2) \in E_2 \\ \min\{\mu_i(u_1,u_2), \alpha_i(v)\} & v_1 = v_2 = v, \text{ for all } v \in V_2 \ \& (u_1,u_2) \in E_1 \end{cases}$$

for all i= 1, 2, 3, ... k

If $m \neq n$ let k = min (m, n). Suppose $m < n$ then we take first m dimensions for G_2 so as to convert the MFG G_1 and G_2 have the same dimension k.

Example 4.8

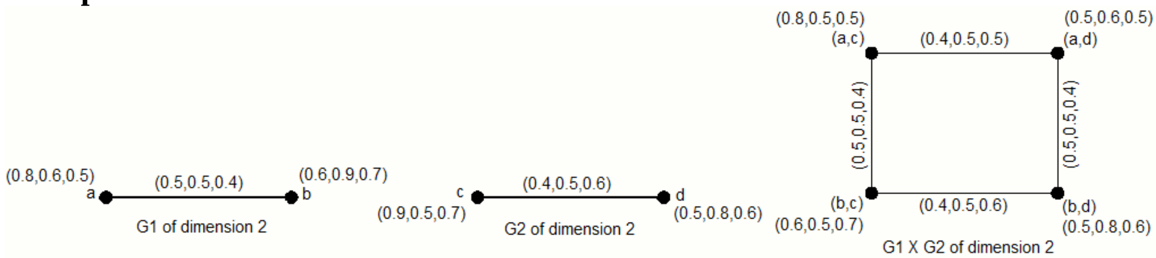


Figure 3

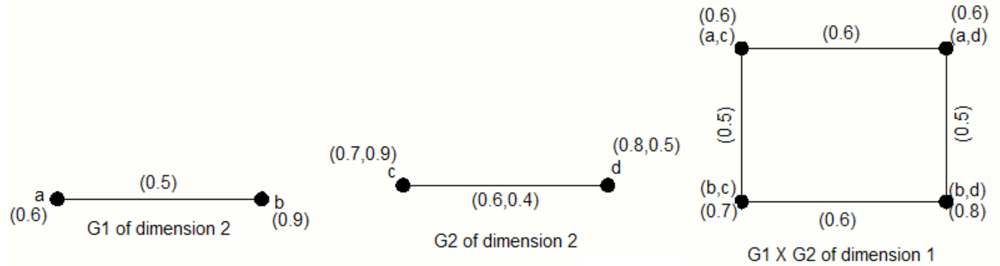


Figure 4

Theorem 4.9 Cartesian product of two multi fuzzy graph is also a multi-fuzzy graph.

Proof: Let $G_1 = ((\sigma_1, \sigma_2, \ldots \sigma_m),(\mu_1, \mu_2, \ldots \mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, \ldots \alpha_n),(\beta_1, \beta_2, \ldots \beta_n))$ be the multi fuzzy graph with dimension m and n respectively

To Prove: $G = G_1 \times G_2$ is a multi-fuzzy graph of dimension k where k=min (m, n)

$(\sigma_i \times \alpha_i)(u_1,v_1) = \min\{\sigma_i(u_1), \alpha_i(v_1)\}$

$(\mu_i \times \beta_i)((u,v_1),(u,v_2)) = \min\{\sigma_i(u), \beta_i(v_1,v_2)\}$

$\leq \min\{\sigma_i(u), \min\{\alpha_i(v_1), \alpha_i(v_2)\}\}$
$= \min\{\min\{\sigma_i(u), \alpha_i(v_1)\}, \min\{\sigma_i(u), \alpha_i(v_2)\}\}$
$= \min\{(\sigma_i \times \alpha_i)(u, v_1), (\sigma_i \times \alpha_i)(u, v_2)\}$
$(\mu_i \times \beta_i)((u_1, v), (u_2, v)) = \min\{\mu_i(u_1, u_2), \alpha_i(v)\}$
$\leq \min\{\min\{\sigma_i(u_1), \sigma_i(u_2)\}, \alpha_i(v)\}$
$= \min\{\min\{\sigma_i(u_1), \alpha_i(v)\}, \min\{\sigma_i(u_2), \alpha_i(v)\}\}$
$= \min\{(\sigma_i \times \alpha_i)(u_1, v), (\sigma_i \times \alpha_i)(u_2, v)\}$

Theorem 4.10 Cartesian product of two strong multi fuzzy graph is also a strong multi fuzzy graph.
Proof: Let $G_1 = ((\sigma_1, \sigma_2, ...\sigma_m), (\mu_1, \mu_2, ...\mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, ...\alpha_n), (\beta_1, \beta_2, ...\beta_n))$ be the multi fuzzy graph with dimension m and n respectively
To Prove: $G = G_1 \times G_2$ is a multi-fuzzy graph of dimension k where k=min (m,n)
$(\sigma_i \times \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1), \alpha_i(v_1)\}$
$(\mu_i \times \beta_i)((u, v_1), (u, v_2)) = \min\{\sigma_i(u), \beta_i(v_1, v_2)\}$
$= \min\{\sigma_i(u), \min\{\alpha_i(v_1), \alpha_i(v_2)\}\}$
$= \min\{\min\{\sigma_i(u), \alpha_i(v_1)\}, \min\{\sigma_i(u), \alpha_i(v_2)\}\}$
$= \min\{(\sigma_i \times \alpha_i)(u, v_1), (\sigma_i \times \alpha_i)(u, v_2)\}$
$(\mu_i \times \beta_i)((u_1, v), (u_2, v)) = \min\{\mu_i(u_1, u_2), \alpha_i(v)\}$
$= \min\{\min\{\sigma_i(u_1), \sigma_i(u_2)\}, \alpha_i(v)\}$
$= \min\{\min\{\sigma_i(u_1), \alpha_i(v)\}, \min\{\sigma_i(u_2), \alpha_i(v)\}\}$
$= \min\{(\sigma_i \times \alpha_i)(u_1, v), (\sigma_i \times \alpha_i)(u_2, v)\}$

Remark: If G_1 and G_2 are complete multi fuzzy graphs then $G_1 \times G_2$ is not a complete multi fuzzy graph.

Theorem 4.11 If $G_1 \times G_2$ is a strong multi fuzzy graph then at least one G_1 or G_2 is a strong multi fuzzy graph.
Proof: Suppose assume that the contrary that G_1 and G_2 are not strong fuzzy graphs.
$\mu_i(u_1, v_1) < \sigma_i(u_1) \wedge \sigma_i(v_1)$ and $\beta_i(u_2, v_2) < \alpha_i(u_2) \wedge \alpha_i(v_2)$ (1)
Without loss of generality, we assume that
$\beta_i(u_2, v_2) \leq \mu_i(u_1, v_1) < \sigma_i(u_1) \wedge \sigma_i(v_1) \leq \sigma_i(u_1)$
Let $E = \{(u_1, v_1)(u_2, v_2) / u_1 = u_2, (v_1, v_2) \in E_2 \; or \; v_1 = v_2, (u_1, u_2) \in E_1\}$
Consider $(u_1, v_1)(u_2, v_2) \in E$, by definition of $G_1 \times G_2$ & inequality (1)
$(\mu_i \times \beta_i)((u_1, v_1), (u_1, v_2)) = \sigma_i(u_1) \wedge \beta_i(v_1, v_2) < \sigma_i(u_1) \wedge \alpha_i(v_1) \wedge \alpha_i(v_2)$ (2)
$(\sigma_i \times \alpha_i)(u_1, v_1) = \sigma_i(u_1) \wedge \alpha_i(v_1)$ & $(\sigma_i \times \alpha_i)(u_1, v_2) = \sigma_i(u_1) \wedge \alpha_i(v_2)$
$(\sigma_i \times \alpha_i)(u_1, v_1) \wedge (\sigma_i \times \alpha_i)(u_1, v_2) = \sigma_i(u_1) \wedge \alpha_i(v_1) \wedge \sigma_i(u_1) \wedge \alpha_i(v_2)$
$= \sigma_i(u_1) \wedge \alpha_i(v_1) \wedge \alpha_i(v_2)$ (3)

From (2) and (3),
$(\mu_i \times \beta_i)((u_1,v_1),(u_1,v_2)) < \sigma_i(u_1) \wedge \alpha_i(v_1) \wedge \alpha_i(v_2) = (\sigma_i \times \alpha_i)(u_1,v_1) \wedge (\sigma_i \times \alpha_i)(u_1,v_2)$
$\therefore (\mu_i \times \beta_i)((u_1,v_1),(u_1,v_2)) < (\sigma_i \times \alpha_i)(u_1,v_1) \wedge (\sigma_i \times \alpha_i)(u_1,v_2)$
This implies that $G_1 \times G_2$ is not a strong multi fuzzy graph.
This gives a contradiction. So, if $G_1 \times G_2$ is a strong multi fuzzy graph then atleast one G_1 or G_2 is a strong multi fuzzy graph.

Definition 4.12 The operation **Strong Product** between two MFG G_1 and G_2 is defined as follows, $G_1 \bullet G_2 = ((\sigma_1 \bullet \alpha_1, \sigma_2 \bullet \alpha_2, \ldots \sigma_k \bullet \alpha_k), (\mu_1 \bullet \beta_1, \mu_2 \bullet \beta_2, \ldots \mu_k \bullet \beta_k))$ with the underlying crisp graph $G_1^* \bullet G_2^* = (V, E)$ where $V = V_1 \times V_2$ and
$E = \{(u_1,v_1)(u_2,v_2) / u_1 = u_2, (v_1,v_2) \in E_2 \; or \; v_1 = v_2, (u_1,u_2) \in E_1 \; or \; (u_1,u_2) \in E_1 \; \& (v_1,v_2) \in E_2\}$
with $(\sigma_i \bullet \alpha_i)(u_1,v_1) = \min\{\sigma_i(u_1), \alpha_i(v_1)\}$ *for all* $u_1 \in V_1$ *and* $v_1 \in V_2$ & $(u_1,v_1) \in V_1 \times V_2$

$$(\mu_i \bullet \beta_i)((u_1,v_1),(u_2,v_2)) = \begin{cases} \min\{\sigma_i(u), \beta_i(v_1,v_2)\} & u_1 = u_2 = u, \text{ for all } u \in V_1 \; \& (v_1,v_2) \in E_2 \\ \min\{\mu_i(u_1,u_2), \alpha_i(v)\} & v_1 = v_2 = v, \text{ for all } v \in V_2 \; \& (u_1,u_2) \in E_1 \\ \min\{\mu_i(u_1,u_2), \beta_i(v_1,v_2)\} & \text{for all } (u_1,u_2) \in E_1 \; \& (v_1,v_2) \in E_2 \end{cases}$$

for all i= 1, 2, 3, ... k
If $m \neq n$, let k = min (m, n). Suppose $m < n$, then we take first m dimensions for G_2 so as to convert the MFG G_1 and G_2 have the same dimension k.

Example 4.13

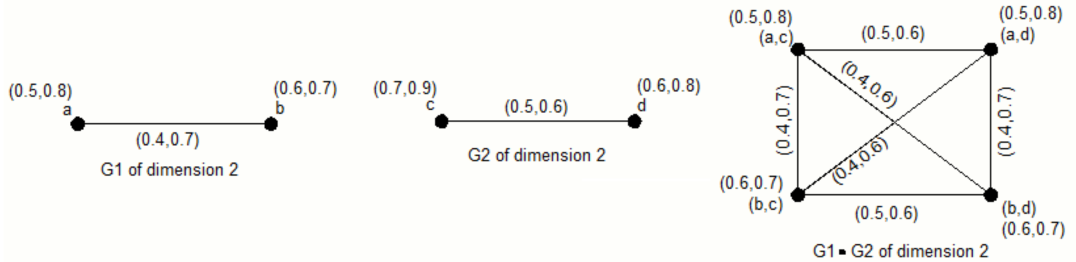


Figure 5

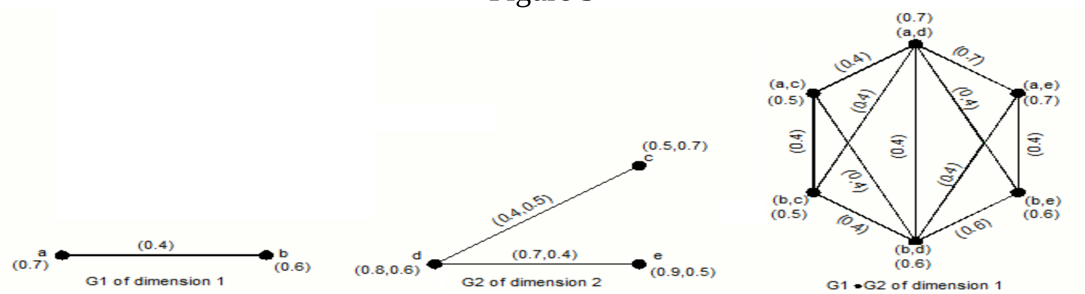


Figure 6

Theorem 4.14 Strong product of two multi fuzzy graph is also a multi-fuzzy graph.
Proof: Let $G_1 = ((\sigma_1, \sigma_2, \ldots \sigma_m), (\mu_1, \mu_2, \ldots \mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, \ldots \alpha_n), (\beta_1, \beta_2, \ldots \beta_n))$ be the multi fuzzy graph with dimension m and n respectively

To Prove: $G = G_1 \bullet G_2$ is a multi-fuzzy graph of dimension k where k=min (m, n)
$(\sigma_i \bullet \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1), \alpha_i(v_1)\}$ *for all* $u_1 \in V_1$ *and* $v_1 \in V_2$ & $(u_1, v_1) \in V_1 \times V_2$
$(\mu_i \bullet \beta_i)((u, v_1), (u, v_2)) = \min\{\sigma_i(u), \beta_i(v_1, v_2)\}$
$\le \min\{\sigma_i(u), \min\{\alpha_i(v_1), \alpha_i(v_2)\}\}$
$= \min\{\min\{\sigma_i(u), \alpha_i(v_1)\}, \min\{\sigma_i(u), \alpha_i(v_2)\}\}$
$= (\sigma_i(u) \wedge \alpha_i(v_1)) \wedge (\sigma_i(u) \wedge \alpha_i(v_2)) \quad = \min\{(\sigma_i \bullet \alpha_i)(u, v_1), (\sigma_i \bullet \alpha_i)(u, v_2)\}$
$(\mu_i \bullet \beta_i)((u_1, v), (u_2, v)) = \min\{\mu_i(u_1, u_2), \alpha_i(v)\}$
$\le \min\{\min\{\sigma_i(u_1), \sigma_i(u_2)\}, \alpha_i(v)\}$
$= \min\{\min\{\sigma_i(u_1), \alpha_i(v)\}, \min\{\sigma_i(u_2), \alpha_i(v)\}\} \quad = (\sigma_i(u_1) \wedge \alpha_i(v)) \wedge (\sigma_i(u_2) \wedge \alpha_i(v))$
$= \min\{(\sigma_i \bullet \alpha_i)(u_1, v), (\sigma_i \bullet \alpha_i)(u_2, v)\}$
$(\mu_i \bullet \beta_i)((u_1, v_1), (u_2, v_2)) = \min\{\mu_i(u_1, u_2), \beta_i(v_1, v_2)\}$
$\le \min\{\min\{\sigma_i(u_1), \sigma_i(u_2)\}, \min\{\alpha_i(v_1), \alpha_i(v_2)\}\}$
$= (\sigma_i(u_1) \wedge \sigma_i(u_2)) \wedge (\alpha_i(v_1) \wedge \alpha_i(v_2))$
$= (\sigma_i(u_1) \wedge \alpha_i(v_1)) \wedge (\sigma_i(u_2) \wedge \alpha_i(v_2))$
$= \min\{(\sigma_i \bullet \alpha_i)(u_1, v_1), (\sigma_i \bullet \alpha_i)(u_2, v_2)\}$

Theorem 4.15 If G_1 and G_2 are strong multi fuzzy graphs then $G_1 \bullet G_2$ is also a strong multi fuzzy graph.
Proof: Let $G_1 = ((\sigma_1, \sigma_2, ...\sigma_m), (\mu_1, \mu_2, ...\mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, ...\alpha_n), (\beta_1, \beta_2, ...\beta_n))$ be the multi fuzzy graph with dimension m and n respectively
To Prove: $G_1 \bullet G_2$ is a strong multi fuzzy graph of dimension k where k=min (m, n)
$(\sigma_i \bullet \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1), \alpha_i(v_1)\}$ *for all* $u_1 \in V_1$ *and* $v_1 \in V_2$ & $(u_1, v_1) \in V_1 \times V_2$
$(\mu_i \bullet \beta_i)((u, v_1), (u, v_2)) = \min\{\sigma_i(u), \beta_i(v_1, v_2)\}$
$= \min\{\sigma_i(u), \min\{\alpha_i(v_1), \alpha_i(v_2)\}\}$
$= \min\{\min\{\sigma_i(u), \alpha_i(v_1)\}, \min\{\sigma_i(u), \alpha_i(v_2)\}\} \quad = (\sigma_i(u) \wedge \alpha_i(v_1)) \wedge (\sigma_i(u) \wedge \alpha_i(v_2))$
$= \min\{(\sigma_i \bullet \alpha_i)(u, v_1), (\sigma_i \bullet \alpha_i)(u, v_2)\}$
$(\mu_i \bullet \beta_i)((u_1, v), (u_2, v)) = \min\{\mu_i(u_1, u_2), \alpha_i(v)\}$
$= \min\{\min\{\sigma_i(u_1), \sigma_i(u_2)\}, \alpha_i(v)\}$
$= \min\{\min\{\sigma_i(u_1), \alpha_i(v)\}, \min\{\sigma_i(u_2), \alpha_i(v)\}\} \quad = (\sigma_i(u_1) \wedge \alpha_i(v)) \wedge (\sigma_i(u_2) \wedge \alpha_i(v))$
$= \min\{(\sigma_i \bullet \alpha_i)(u_1, v), (\sigma_i \bullet \alpha_i)(u_2, v)\}$
$(\mu_i \bullet \beta_i)((u_1, v_1), (u_2, v_2)) = \min\{\mu_i(u_1, u_2), \beta_i(v_1, v_2)\}$
$= \min\{\min\{\sigma_i(u_1), \sigma_i(u_2)\}, \min\{\alpha_i(v_1), \alpha_i(v_2)\}\}$
$= (\sigma_i(u_1) \wedge \sigma_i(u_2)) \wedge (\alpha_i(v_1) \wedge \alpha_i(v_2))$
$= (\sigma_i(u_1) \wedge \alpha_i(v_1)) \wedge (\sigma_i(u_2) \wedge \alpha_i(v_2))$
$= \min\{(\sigma_i \bullet \alpha_i)(u_1, v_1), (\sigma_i \bullet \alpha_i)(u_2, v_2)\}$

Theorem 4.16 If G_1 and G_2 are complete multi fuzzy graphs then $G_1 \bullet G_2$ is also a complete multi fuzzy graph.
Proof: Let $G_1 = ((\sigma_1, \sigma_2, ...\sigma_m), (\mu_1, \mu_2, ...\mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, ...\alpha_n), (\beta_1, \beta_2, ...\beta_n))$ be the two complete multi fuzzy graphs with dimension m and n respectively. Then G_1 and G_2

are strong multi fuzzy graphs where $G_1^* \, and \, G_2^*$ are complete graphs. Therefore, $G_1 \bullet G_2$ is a strong multi fuzzy graph by the theorem (4.15) with $G_1^* \, and \, G_2^*$ are complete graphs. Hence $G_1 \bullet G_2$ is a complete multi fuzzy graph.

Theorem 4.17 The strong product of two multi fuzzy graphs G_1 and G_2 is the direct sum of the cartesian product of G_1 and G_2 and the direct product of G_1 and G_2.

Proof: Let $G_1 = ((\sigma_1, \sigma_2, ...\sigma_m), (\mu_1, \mu_2, ...\mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, ...\alpha_n), (\beta_1, \beta_2, ...\beta_n))$ be the multi fuzzy graph with dimension m and n respectively. Let $G_1 \times G_2$ and $G_1 * G_2$ be the cartesian product and direct product of G_1 and G_2 with dimension k where k = min(m,n)

To Prove: $G_1 \bullet G_2 = (G_1 \times G_2) \oplus (G_1 * G_2)$

$$(\sigma_i \times \alpha_i)(u_1, v_1) = (\sigma_i * \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1), \ \alpha_i(v_1)\} \ \forall (u_1, v_1) \in V_1 \times V_2$$

So, $(\sigma_i \times \alpha_i)(u_1, v_1) \oplus (\sigma_i * \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1), \ \alpha_i(v_1)\} \ \forall (u_1, v_1) \in V_1 \times V_2$

$$(\sigma_i \bullet \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1), \alpha_i(v_1)\}$$

$$\therefore (\sigma_i \bullet \alpha_i)(u_1, v_1) = (\sigma_i \times \alpha_i)(u_1, v_1) \oplus (\sigma_i * \alpha_i)(u_1, v_1)$$

$$((\mu_i \times \beta_i)(u_1, v_1), (u_2, v_2)) = \begin{cases} \sigma_i(u_1) \wedge \beta_i(v_1, v_2) & f \ u_1 = u_2 \ and \ (v_1, v_2) \in E_2 \\ \alpha_i(v_1) \wedge \mu_i(u_1, u_2) & if \ v_1 = v_2 \ and \ (u_1, u_2) \in E_1 \end{cases}$$

$$(\mu_i * \beta_i)(u_1, v_1), (u_2, v_2)) = \min\{\mu_i(u_1, u_2), \beta_i(v_1, v_2)\}, \ if \ (u_1, u_2) \in E_1 \ \& \ (v_1, v_2) \in E_2$$

$$((\mu_i \times \beta_i) \oplus (\mu_i * \beta_i))(u_1, v_1), (u_2, v_2)) = \begin{cases} \sigma_i(u_1) \wedge \beta_i(v_1, v_2) & if \ u_1 = u_2 \ and \ (v_1, v_2) \in E_2 \\ \alpha_i(v_1) \wedge \mu_i(u_1, u_2) & if \ v_1 = v_2 \ and \ (u_1, u_2) \in E_1 \\ \min\{\mu_i(u_1, u_2), \beta_i(v_1, v_2)\} & if \ (u_1, u_2) \in E_1 \ \& \ (v_1, v_2) \in E_2 \end{cases}$$

$$= (\mu_i \bullet \beta_i)((u_1, v_1), (u_2, v_2))$$

Result: Let $G_1 = ((\sigma_1, \sigma_2, ...\sigma_m), (\mu_1, \mu_2, ...\mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, ...\alpha_n), (\beta_1, \beta_2, ...\beta_n))$ be two strong multi fuzzy graph with dimension m and n respectively and $G_1 \times G_2$ & $G_1 * G_2$ be the cartesian product and direct product of G_1 and G_2 with dimension k where k = min(m,n) and $\overline{G_1 \times G_2}$ and $\overline{G_1 * G_2}$ be the complement of two multi fuzzy graphs then
$\overline{G_1 \times G_2} \oplus \overline{G_1 * G_2} = G_1 \times G_2 \oplus G_1 * G_2$.

Definition 4.18 The operation **Composition** between two MFG G_1 and G_2 as follows $G_1[G_2] = ((\sigma_1 \circ \alpha_1, \sigma_2 \circ \alpha_2, ...\sigma_k \circ \alpha_k), (\mu_1 \circ \beta_1, \mu_2 \circ \beta_2, ...\mu_k \circ \beta_k))$ with the underlying crisp graph $G_1^*[G_2^*] = (V, E)$ where $V = V_1 \times V_2$ and $E = \{(u_1, v_1)(u_2, v_2) / u_1 = u_2, (v_1, v_2) \in E_2 \ or \ v_1 = v_2, (u_1, u_2) \in E_1 \ or \ v_1 \neq v_2, (u_1, u_2) \in E_1\}$ with

$$(\sigma_i \circ \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1), \alpha_i(v_1)\} \ for \ all \ u_1 \in V_1 \ and \ v_1 \in V_2 \ \& \ (u_1, v_1) \in V_1 \times V_2$$

$$(\mu_i \circ \beta_i)((u_1, v_1), (u_2, v_2)) = \begin{cases} \min\{\sigma_i(u), \beta_i(v_1, v_2)\} & u_1 = u_2 = u, \ for \ all \ u \in V_1 \ \& \ (v_1, v_2) \in E_2 \\ \min\{\mu_i(u_1, u_2), \alpha_i(v)\} & v_1 = v_2 = v, \ for \ all \ v \in V_2 \ \& \ (u_1, u_2) \in E_1 \\ \min\{\alpha_i(v_1), \alpha_i(v_2), \mu_i(u_1, u_2)\} & for \ all \ (u_1, u_2) \in E_1 \end{cases}$$

for all i= 1, 2, 3, ... k.

If $m \neq n$, let k = min (m, n). Suppose $m < n$, then we take first m dimensions for G_2 so as to convert the MFG G_1 and G_2 have the same dimension k

Example 4.19

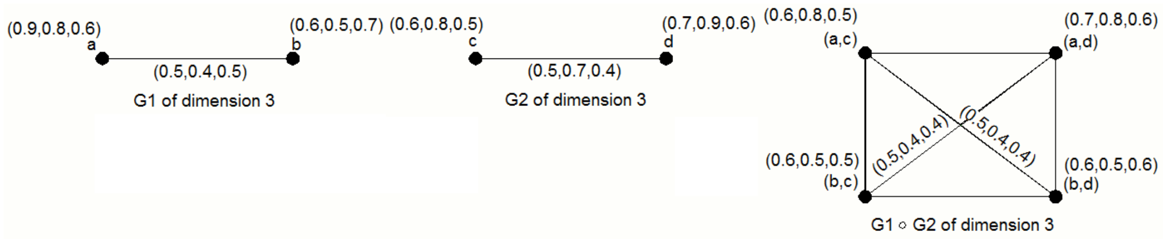


Figure 7

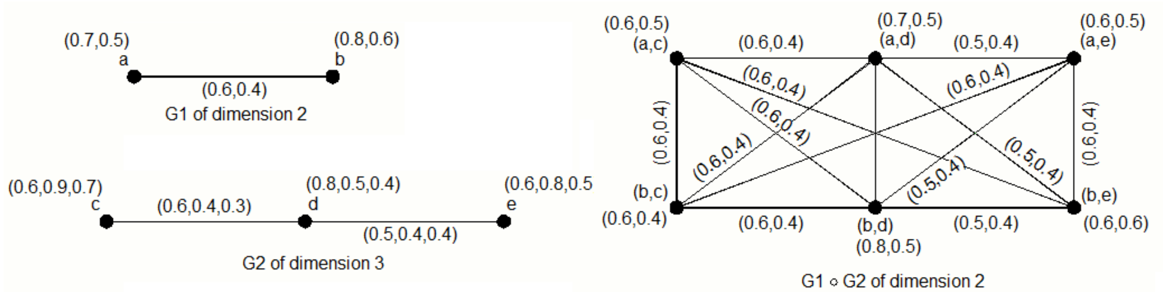


Figure 8

Theorem 4.20 Composition of two multi fuzzy graph is also a multi-fuzzy graph.
Proof: Let $G_1 = ((\sigma_1, \sigma_2, ...\sigma_m), (\mu_1, \mu_2, ...\mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, ...\alpha_n), (\beta_1, \beta_2, ...\beta_n))$ be the multi fuzzy graph with dimension m and n respectively

To Prove: $\boldsymbol{G = G_1 \circ G_2}$ is a multi-fuzzy graph of dimension k where k = min (m, n)

$(\sigma_i \circ \alpha_i)(u_1, v_1) = \min\{\sigma_i(u_1), \alpha_i(v_1)\}$ *for all* $u_1 \in V_1$ *and* $v_1 \in V_2$ & $(u_1, v_1) \in V_1 \times V_2$

$(\mu_i \circ \beta_i)((u, v_1), (u, v_2)) = \min\{\sigma_i(u), \beta_i(v_1, v_2)\}$

$\leq \min\{\sigma_i(u), \min\{\alpha_i(v_1), \alpha_i(v_2)\}\}$

$= \min\{\min\{\sigma_i(u), \alpha_i(v_1)\}, \min\{\sigma_i(u), \alpha_i(v_2)\}\} = (\sigma_i(u) \wedge \alpha_i(v_1)) \wedge (\sigma_i(u) \wedge \alpha_i(v_2))$

$= \min\{(\sigma_i \circ \alpha_i)(u, v_1), (\sigma_i \circ \alpha_i)(u, v_2)\}$

$(\mu_i \circ \beta_i)((u_1, v), (u_2, v)) = \min\{\mu_i(u_1, u_2), \alpha_i(v)\}$

$\leq \min\{\min\{\sigma_i(u_1), \sigma_i(u_2)\}, \alpha_i(v)\}$

$= \min\{\min\{\sigma_i(u_1), \alpha_i(v)\}, \min\{\sigma_i(u_2), \alpha_i(v)\}\} = (\sigma_i(u_1) \wedge \alpha_i(v)) \wedge (\sigma_i(u_2) \wedge \alpha_i(v))$

$= \min\{(\sigma_i \circ \alpha_i)(u_1, v), (\sigma_i \circ \alpha_i)(u_2, v)\}$

$(\mu_i \circ \beta_i)((u_1, v_1), (u_2, v_2)) = \min\{\alpha_i(v_1), \alpha_i(v_2), \mu_i(u_1, u_2)\}$

$\leq \min\{\alpha_i(v_1), \alpha_i(v_2), \min\{\sigma_i(u_1), \sigma_i(u_2)\}\}$

$= \min\{\alpha_i(v_1), \alpha_i(v_2), (\sigma_i(u_1) \wedge \sigma_i(u_2))\}$

$= (\sigma_i(u_1) \wedge \alpha_i(v_1)) \wedge (\sigma_i(u_2) \wedge \alpha_i(v_2))$

$= \min\{(\sigma_i \circ \alpha_i)(u, v_1), (\sigma_i \circ \alpha_i)(u, v_2)\}$

Theorem 4.21 If $G_1 = ((\sigma_1, \sigma_2, ...\sigma_m), (\mu_1, \mu_2, ...\mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, ...\alpha_n), (\beta_1, \beta_2, ...\beta_n))$ are two strong multi fuzzy graphs with dimension m and n respectively and $G_1 \circ G_2$ is a strong multi fuzzy graph of dimension k where k = min (m, n). Prove that $\overline{G_1 \circ G_2} = \overline{G_1} \circ \overline{G_2}$

Proof: Let $G = G_1 \circ G_2 = ((\sigma_1 \circ \alpha_1, \sigma_2 \circ \alpha_2, ...\sigma_k \circ \alpha_k), (\mu_1 \circ \beta_1, \mu_2 \circ \beta_2, ...\mu_k \circ \beta_k))$

$\overline{G_1 \circ G_2} = ((\sigma_1 \circ \alpha_1, \sigma_2 \circ \alpha_2, ...\sigma_k \circ \alpha_k), (\overline{\mu_1 \circ \beta_1}, \overline{\mu_2 \circ \beta_2}, ...\overline{\mu_k \circ \beta_k}))$

$\overline{G_1} = ((\sigma_1, \sigma_2, ...\sigma_m), (\overline{\mu_1}, \overline{\mu_2}, ...\overline{\mu_m}))\ \overline{G_2} = ((\alpha_1, \alpha_2, ...\alpha_n), (\overline{\beta_1}, \overline{\beta_2}, ...\overline{\beta_n}))$

$\overline{G_1} \circ \overline{G_2} = ((\sigma_1 \circ \alpha_1, \sigma_2 \circ \alpha_2, ...\sigma_k \circ \alpha_k), (\overline{\mu_1} \circ \overline{\beta_1}, \overline{\mu_2} \circ \overline{\beta_2}, ...\overline{\mu_k} \circ \overline{\beta_k}))$

To prove $\overline{G_1 \circ G_2} = \overline{G_1} \circ \overline{G_2}$ It is enough to prove $\overline{\mu_i \circ \beta_i} = \overline{\mu_i} \circ \overline{\beta_i}$ for all i= 1, 2, 3, ... k.

To prove the above result, there are different cases may arise depending upon the edges joining the vertices

Case(i): Consider the edge $e = ((u, v_1), (u, v_2)),\ (v_1, v_2) \in E_2$

Then $e \in E$ and G is a strong multi fuzzy graph, so $\overline{\mu_i \circ \beta_i}(e) = 0$

Also $(\overline{\mu_1} \circ \overline{\beta_1})(e) = 0$ since $(v_1, v_2) \notin \overline{E_2}$

If $e = ((u, v_1), (u, v_2))$, $v_1 \neq v_2\ and (v_1, v_2) \notin E_2$ then $e \notin E\ (\mu_i \circ \beta_i)((u, v_1), (u, v_2)) = 0$

Now $\overline{\mu_i \circ \beta_i}(e) = ((\sigma_i \circ \alpha_i)(u, v_1)) \wedge ((\sigma_i \circ \alpha_i)(u, v_2))$

$= (\sigma_i(u) \wedge \alpha_i(v_1)) \wedge ((\sigma_i(u) \wedge \alpha_i(v_2))$

$= \sigma_i(u) \wedge \alpha_i(v_1) \wedge \alpha_i(v_2)$

$(\overline{\mu_1} \circ \overline{\beta_1})(e) = \sigma_i(u) \wedge \overline{\beta_i}(v_1, v_2) = \sigma_i(u) \wedge \alpha_i(v_1) \wedge \alpha_i(v_2)$

$\therefore\ \overline{\mu_i \circ \beta_i} = \overline{\mu_i} \circ \overline{\beta_i}$ for all i= 1, 2, 3, ... k.

Case(ii): Consider the edge $e = ((u_1, v), (u_2, v)),\ (u_1, u_2) \in E_1$

Then $e \in E$ and G is a strong multi fuzzy graph, so $\overline{\mu_i \circ \beta_i}(e) = 0$

Also $(\overline{\mu_1} \circ \overline{\beta_1})(e) = 0$ since $(u_1, u_2) \notin \overline{E_1}$

If $e = ((u_1, v), (u_2, v)),\ (u_1, u_2) \notin E_1$ then $e \notin E\ (\mu_i \circ \beta_i)((u, v_1), (u, v_2)) = 0$

Now $\overline{\mu_i \circ \beta_i}(e) = ((\sigma_i \circ \alpha_i)(u_1, v)) \wedge ((\sigma_i \circ \alpha_i)(u_2, v))$

$= (\sigma_i(u_1) \wedge \alpha_i(v)) \wedge ((\sigma_i(u_2) \wedge \alpha_i(v))$

$= \sigma_i(u_1) \wedge \sigma_i(u_2) \wedge \alpha_i(v)$ since $(u_1, u_2) \in \overline{E_1}$

$(\overline{\mu_1} \circ \overline{\beta_1})(e) = \overline{\mu_i}(u_1, u_2) \wedge \alpha_i(v) = \sigma_i(u_1) \wedge \sigma_i(u_2) \wedge \alpha_i(v)$

$\therefore\ \overline{\mu_i \circ \beta_i} = \overline{\mu_i} \circ \overline{\beta_i}$ for all i= 1, 2, 3, ... k.

Case(iii): Consider the edge $e = ((u_1, v_1), (u_2, v_2)),\ (u_1, u_2) \in E_1\ \&\ v_1 \neq v_2$

Then $e \in E$ and G is a strong multi fuzzy graph, So $\overline{\mu_i \circ \beta_i}(e) = 0$

since $(u_1, u_2) \notin \overline{E_1}$, $(\overline{\mu_1} \circ \overline{\beta_1})(e) = 0$

If $e = ((u_1, v_1), (u_2, v_2)),\ (u_1, u_2) \notin E_1\ \&\ v_1 \neq v_2$

Then $e \notin E\ (\mu_i \circ \beta_i)(e) = 0$

$\overline{\mu_i \circ \beta_i}(e) = ((\sigma_i \circ \alpha_i)(u_1, v_1)) \wedge ((\sigma_i \circ \alpha_i)(u_2, v_2)) = (\sigma_i(u_1) \wedge \alpha_i(v_1)) \wedge ((\sigma_i(u_2) \wedge \alpha_i(v_2))$

Since $(u_1,u_2)\in\overline{E_1}$ we have

$\overline{(\mu_1 \circ \beta_1)}(e)=\overline{\mu_i}(u_1,u_2)\wedge\alpha_i(v_1)\wedge\alpha_i(v_2)=\sigma_i(u_1)\wedge\sigma_i(u_2)\wedge\alpha_i(v_1)\wedge\alpha_i(v_2)=\overline{\mu_i \circ \beta_i}(e)$

$\therefore\ \overline{\mu_i \circ \beta_i}=\overline{\mu_i}\circ\overline{\beta_i}$ for all i= 1, 2, 3, ... k.

Case(iv): Consider the edge $e=((u_1,v_1),(u_2,v_2)),\ (u_1,u_2)\notin E_1$ & $(v_1,v_2)\notin E_2$

Since $e\in E$, $(\mu_i \circ \beta_i)(e)=0$

$\overline{\mu_i \circ \beta_i}(e)=((\sigma_i \circ \alpha_i)(u_1,v_1))\wedge((\sigma_i \circ \alpha_i)(u_2,v_2))=(\sigma_i(u_1)\wedge\alpha_i(v_1))\wedge((\sigma_i(u_2)\wedge\alpha_i(v_2))$

If $(u_1,u_2)\in\overline{E_1}$ and if $v_1=v_2$ then we have case (ii)

If $(u_1,u_2)\notin\overline{E_1}$ and if $v_1\neq v_2$ then we have case (iii)

In all the cases we have, $\overline{\mu_i \circ \beta_i}=\overline{\mu_i}\circ\overline{\beta_i}$ for all i= 1, 2, 3, ... k.

Definition 4.22 The operation **Corona Product** between two MFG G_1 and G_2 is defined as follows, $G_1\otimes G_2=((\sigma_1\otimes\alpha_1,\sigma_2\otimes\alpha_2,\ldots\sigma_k\otimes\alpha_k),(\mu_1\otimes\beta_1,\mu_2\otimes\beta_2,\ldots\mu_k\otimes\beta_k))$ with the underlying crisp graph $G_1^*\otimes G_2^*=(V,E)=(V_1\cup V_2,E_1\cup E_2)$

$(\sigma_i\otimes\alpha_i)(u)=\begin{cases}\sigma_i(u), u\in V_1\\ \alpha_i(u), u\in V_2\end{cases}$ and $(\mu_i\otimes\beta_i)(u,v)=\begin{cases}\mu_i(u,v), & (u,v)\in E_1\\ \beta_i(u,v), & (u,v)\in E_2\\ \min\{\sigma_i(u),\alpha_i(v)\}, & uv\in E'\end{cases}$ where E' is the set of all edges joining by an edge the ith vertex of G_1 to every vertex in the ith copy of G_2

If $m\neq n$, let k = max (m, n). Suppose $m<n$ then let us introduce n – m membership values of multi fuzzy graph G_1 into 0 so as to convert the multi fuzzy graphs G_1 and G_2 have the same dimension as k.

Example 4.23

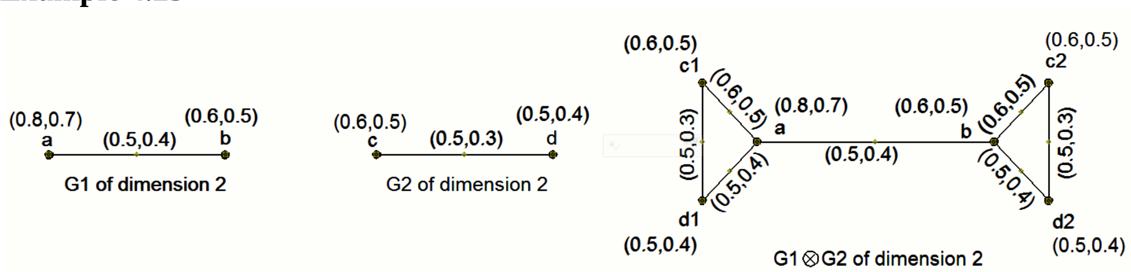


Figure 9

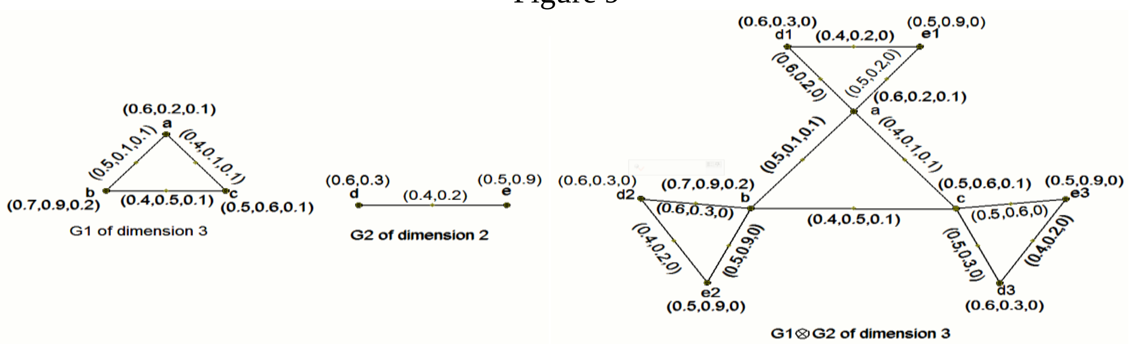


Figure 10

Theorem 4.24 Corona product of two multi fuzzy graph is also a multi-fuzzy graph.
Proof: Let $G_1 = ((\sigma_1, \sigma_2, ...\sigma_m), (\mu_1, \mu_2, ...\mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, ...\alpha_n), (\beta_1, \beta_2, ...\beta_n))$ be the multi fuzzy graph with dimension m and n respectively
To Prove: $G = G_1 \otimes G_2$ is a multi-fuzzy graph of dimension k where k= max (m, n)

$$(\sigma_i \otimes \alpha_i)(u) = \begin{cases} \sigma_i(u), u \in V_1 \\ \alpha_i(u), u \in V_2 \end{cases}$$

Case(i): If $(u, v) \in E_1$

$$(\mu_i \otimes \beta_i)(u, v) = \mu_i(u, v)$$
$$\leq \min\{\sigma_i(u), \sigma_i(v)\}$$
$$= \min\{(\sigma_i \otimes \alpha_i)(u), (\sigma_i \otimes \alpha_i)(v)\}$$

Case(ii): If $(u, v) \in E_2$

$$(\mu_i \otimes \beta_i)(u, v) = \beta_i(u, v)$$
$$\leq \min\{\alpha_i(u), \alpha_i(v)\}$$
$$= \min\{(\sigma_i \otimes \alpha_i)(u), (\sigma_i \otimes \alpha_i)(v)\}$$

Case(iii): If $(u, v) \in E'$

$$(\mu_i \otimes \beta_i)(u, v) = \min\{\sigma_i(u), \alpha_i(v)\} = \min\{(\sigma_i \otimes \alpha_i)(u), (\sigma_i \otimes \alpha_i)(v)\}$$

5. Conclusion

In this paper, the complement of multi fuzzy graph and direct sum of two multi fuzzy graphs are defined and proved some results connected to them. Also defined various product on multi fuzzy graphs such as direct product, strong product, cartesian product, composition, corona product and proved some properties related to them.

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