Ratio Mathematica Volume 44, 2022

Soft Pre*-Generalized Continuous Functions in Soft Topological Spaces

C. Reena[1]
M. Karthika[2]

Abstract

The aim of this paper is to define a new class of generalized continuous functions called soft pre^*-generalized continuous functions and soft pre^*-generalized irresolute functions in soft topological spaces. We discuss several characterizations of soft pre^*-generalized continuous and irresolute functions and also investigate their relationship with other soft continuous functions.

Keywords: soft pre^*-generalized continuous functions and soft pre^*-generalized irresolute functions.

2010 AMS subject classification: 54A05, 54A10[3]

---

[1]Assistant Professor, Department of Mathematics, St. Mary's College (Autonomous), (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli), Thoothukudi-1, TamilNadu, India. Mail Id: reenastephany@gmail.com
[2]Research Scholar, Reg.No. 21112212092004, Department of Mathematics, St. Mary's College (Autonomous), (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli), Thoothukudi-1, TamilNadu, India. Mail Id: karthikamarimuthu97@gmail.com
[3]Received on June 21st, 2022. Accepted on Aug 10th, 2022. Published on Nov30th, 2022. doi: 10.23755/rm.v44i0.901. ISSN: 1592-7415. eISSN: 2282-8214.

1. Introduction

In 1999 Molostsov [6] initiated the study of soft set theory as a new mathematical tool to deal with uncertainties. Muhammad Shabir [7] and Munazza Naz (2011) introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. Athar Kharal [2] and Ahmad introduced the concept of soft mapping. Aras [1] and Sonmez discussed the properties of soft continuous mappings. Akdag M [5] and Ozkan introduced soft pre-continuity in soft topological space. The authors [8] of this paper introduced a new glass of generalized closed set called soft pre^*-generalized closed sets in soft topological spaces. In this paper, we introduce soft pre^*-generalized continuous and soft pre^*-generalized irresolute functions in soft topological spaces. We investigate its fundamental properties and find its relation with other soft continuous functions.

2. Preliminaries

Throughout this paper, $(X, \tilde{\tau}, A)$, $(Y, \tilde{\sigma}, B)$ and $(Z, \tilde{\mu}, C)$ are soft topological spaces. Let (F, A) be a subset of a soft topological space. Then$S_t cl(F, A)$, $S_t int(F, A)$, $S_t cl^*(F, A)$ and $S_t int^*(F, A)$ denote the soft closure, soft interior, soft generalized closure and soft generalized interior respectively.

Definition 2.1: [6] Let X be an initial universe, E be a set of parameters, $P(X)$ denote the power set of X and$A$ be a non-empty set of E. A pair (F, A) is called **soft set** over$X$, where F is a mapping given by $F: A \to P(X)$.

Definition 2.2: [7] Let $\tilde{\tau}$ be a collection of soft sets over X. Then $\tilde{\tau}$ is called a **soft topology** on X if

i. $\tilde{\emptyset}, \tilde{X}$ belons to $\tilde{\tau}$.
ii. The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
iii. The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet $(X, \tilde{\tau}, A)$ is called soft topological space over X. The members of $\tilde{\tau}$ are called **soft open** and their complements are called **soft closed.**

Definition 2.3. A function $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ is said to be **soft continuous [1]** (respectively **soft semi continuous [4]**, **soft $semi^*$continuous**, **soft α continuous [5], soft regular continuous [3]**, **soft generalized continuous [9]**, **soft generalized α continuous** and **soft generalized pre continuous**) if inverse image of every soft closed set in $(Y, \tilde{\sigma}, B)$ is soft closed (respectively soft semi-closed, soft $semi^*$-closed, soft α closed, soft regular closed, soft generalized closed, soft generalized α closed and soft generalized pre closed) in $(X, \tilde{\tau}, A)$.

Definition 2.4. [8] A subset (F, A) of a soft topological space $(X, \tilde{\tau}, A)$ is said to be soft **pre^*- generalized closed** if $S_t pcl(F, A) \tilde{\subseteq} (U, A)$ whenever $(F, A) \tilde{\subseteq} (U, A)$ and (U, A)

is soft pre^*-open. The complement of soft pre^*-generalized closed is called **soft $\boldsymbol{pre^*}$-generalized open**.

Theorem 2.5. [8] In any topological space $(X, \tilde{\tau}, A)$,

i. Every soft closed set is soft pre^*-generalized closed.
ii. Every soft regular-closed set is soft pre^*-generalized closed.
iii. Every soft α-closed set is soft pre^*-generalized closed.
iv. Every soft generalized α-closed set is soft pre^*-generalized closed.
v. Every soft generalized pre-closed set is soft pre^*-generalized closed.

Remark 2.6: The above theorem is true for soft pre^*-generalized open.

3. Soft PRE^*-Generalized Continuous Functions

Definition 3.1. Let $(X, \tilde{\tau}, A)$ and $(Y, \tilde{\sigma}, B)$ be soft topological spaces. Let $u: X \to Y$and $p: A \to B$ be mappings. The function $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ is said tobe soft pre^*-generalized continuous function if the inverse image of every soft closed set in $(Y, \tilde{\sigma}, B)$ is soft pre^*-generalized closed in $(X, \tilde{\tau}, A)$.

The following soft sets are used in all the examples:

Let $X = \{a, b\}$ and $A = \{e_1, e_2\}$. Then the soft sets are

$F_1 = \{(e_1, \{\emptyset\}), (e_2, \{\emptyset\})\} = \tilde{\emptyset}$ $\quad$ $F_2 = \{(e_1, \{\emptyset\}), (e_2, \{a\})\}$
$F_3 = \{(e_1, \{\emptyset\}), (e_2, \{b\})\}$ $\quad$ $F_4 = \{(e_1, \{\emptyset\}), (e_2, \{a, b\})\}$
$F_5 = \{(e_1, \{a\}), (e_2, \{\emptyset\})\}$ $\quad$ $F_6 = \{(e_1, \{a\}), (e_2, \{a\})\}$
$F_7 = \{(e_1, \{a\}), (e_2, \{b\})\}$ $\quad$ $F_8 = \{(e_1, \{a\}), (e_2, \{a, b\})\}$
$F_9 = \{(e_1, \{b\}), (e_2, \{\emptyset\})\}$ $\quad$ $F_{10} = \{(e_1, \{b\}), (e_2, \{a\})\}$
$F_{11} = \{(e_1, \{b\}), (e_2, \{b\})\}$ $\quad$ $F_{12} = \{(e_1, \{b\}), (e_2, \{a, b\})\}$
$F_{13} = \{(e_1, \{a, b\}), (e_2, \{\emptyset\})\}$ $\quad$ $F_{14} = \{(e_1, \{a, b\}), (e_2, \{a\})\}$
$F_{15} = \{(e_1, \{a, b\}), (e_2, \{b\})\}$ $\quad$ $F_{16} = \{(e_1, \{a, b\}), (e_2, \{a, b\})\} = \tilde{X}$

Similarly, let $Y = \{x, y\}$ and $B = \{k_1, k_2\}$ then the soft sets $G_1, G_2, \ldots, G_{16}$are obtained by replacing a, b, e_1 and e_2 by x, y, k_1 and k_2 respectively in the above sets.

Example 3.2. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as $u(a) = x$, $u(b) = y$, p $(e_1) = k_1$ and $p(e_2) = k_2$. Consider the soft topologies$\tilde{\tau}^{\tilde{c}} = \{F_1, F_2, F_8, F_{10}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_4, G_8, G_{12}, G_{16}\}$. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_4) = F_4$ and $\tilde{f}^{-1}(G_8) = F_8$ and $\tilde{f}^{-1}(G_{12}) = F_{12}$ are soft pre^*- generalized closed, $\tilde{f}$ is soft pre^*-generalized continuous.

Theorem 3.3. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft continuous function. Then $\tilde{f}$ is soft pre^*-generalized continuous.

Proof: Let (G, B) be a soft closed set in Y. Since $\tilde{f}$ is soft continuous, $\tilde{f}^{-1}(G, B)$ is soft closed. Then by theorem 2.5(i), $\tilde{f}^{-1}(G, B)$ is soft pre^*-generalized closed. Hence $\tilde{f}$ is soft pre^*-generalized continuous.

Remark 3.4. The converse of the above theorem need not be true as shown in the following example.

Example 3.5. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as $u(a) = x$, $u(b) = y$, p $(e_1) = k_1$ and $p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau}^{\tilde{c}} = \{F_1, F_5, F_8, F_{13}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_9, G_{12}, G_{13}, G_{16}\}$. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_9) = F_5$, $\tilde{f}^{-1}(G_{12}) = F_8$ and $\tilde{f}^{-1}(G_{13}) = F_{13}$ are soft pre^*-generalized closed but not soft closed, $\tilde{f}$ is soft pre^*-generalized continuous but not soft continuous.

Theorem 3.6. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft function.

i. If $\tilde{f}$ is soft regular continuous, then $\tilde{f}$ is soft pre^*-generalized continuous.
ii. If $\tilde{f}$ is soft α continuous, then $\tilde{f}$ is soft pre^*-generalized continuous.
iii. If $\tilde{f}$ is soft generalized α continuous, then $\tilde{f}$ is soft pre^*-generalized continuous.
iv. If $\tilde{f}$ is soft generalized pre continuous, then $\tilde{f}$ is soft pre^*-generalized continuous.

Proof: The proofs are similar to theorem 3.3.

Remark 3.7. The converse of each of the statements in above theorem need not be true.

Example 3. 8. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as$u(a) = x$, $u(b) = y$, $p(e_1) = k_2$ and $p(e_2) = k_1$. Consider the soft topologies $\tilde{\tau}^{\tilde{c}} = \{F_1, F_5, F_9, F_{13}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_3, G_8, G_{11}, G_{16}\}$. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_3) = F_9$, $\tilde{f}^{-1}(G_8) = F_{14}$ and $\tilde{f}^{-1}(G_{11}) = F_{11}$are soft pre^*-generalized closed but not soft regular closed, $\tilde{f}$ is soft pre^*-generalized continuous but not regular continuous.

Example 3.9. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as$u(a) = x$, $u(b) = y$, $p(e_1) = k_2$ and $p(e_2) = k_1$. Consider the soft topologies$\tilde{\tau}^{\tilde{c}} = \{F_1, F_6, F_8, F_{14}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_3, G_{11}, G_{16}\}$. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_3) = F_9$ and $\tilde{f}^{-1}(G_{11}) = F_{11}$ are soft pre^*-generalized closed but not softα-closed, $\tilde{f}$ is soft pre^*-generalized continuous but not soft α continuous.

Example 3.10. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as $u(a) = x$, $u(b) = y$, $p(e_1) = k_2$ and $p(e_2) = k_1$. Consider the soft topologies$\tilde{\tau}^{\tilde{c}} = \{F_1, F_9, F_{10}, F_{12}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_5, G_9, G_{13}, G_{16}\}$. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_5) = F_2$, $\tilde{f}^{-1}(G_9) = F_3$ and $\tilde{f}^{-1}(G_{13}) = F_4$are soft pre^*-generalized closed but not soft generalized α-closed, $\tilde{f}$ is soft pre^*-generalized continuous but not soft generalized α continuous.

Example 3.11. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as $u(a) = y$, $u(b) = x$, $p(e_1) = k_1$ and $p(e_2) = k_2$. Consider the soft topologies$\tilde{\tau}^{\tilde{c}} = \{F_1, F_4, F_8, F_{12}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_{14}, G_{16}\}$. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_{14}) = F_{15}$is soft pre^*-generalized closed but not soft generalized pre-closed, $\tilde{f}$ is soft pre^*-generalized continuous but not soft generalized pre continuous.

Remark 3.12. The concept of soft pre^*-generalized continuity and soft generalized continuity are independent as shown in the following example.

Example 3.13. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as $u(a) = x$, $u(b) = y$, $p(e_1) = k_1$, $p(e_2) = k_2$. Consider the soft topologies$\tilde{\tau}^{\tilde{c}} = \{F_1, F_2, F_3, F_4, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_5, G_6, G_8, G_{16}\}$. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_5) = F_5$ and $\tilde{f}^{-1}(G_6) = F_6$are soft pre^*-generalized closed but not soft generalized closed,$\tilde{f}$ is soft pre^*-generalized continuous but not soft generalized continuous.

Example 3.14. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as $u(a) = x$, $u(b) = y$, $p(e_1) = k_2$, $p(e_2) = k_1$. Consider the soft topologies $\tilde{\tau}^{\tilde{c}} = \{F_1, F_3, F_{11}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_{14}, G_{16}\}$. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_{14}) = F_8$is soft generalized closed but not soft pre^*-generalized closed, $\tilde{f}$ is soft generalized continuous but not soft pre^*-generalized continuous.

Remark 3.15. The concept of soft pre^*-generalized continuity and soft $semi^*$-continuity are independent as shown in the following example.

Example 3.16. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as $u(a) = x$, $u(b) = y$, $p(e_1) = k_2$, $p(e_2) = k_1$. Consider the soft topologies $\tilde{\tau}^{\tilde{c}} = \{F_1, F_5, F_9, F_{13}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_4, G_8, G_{12}, G_{16}\}$.Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_8) = F_{14}$ and $\tilde{f}^{-1}(G_{12}) = F_{15}$ are soft pre^*-generalized closed but not soft $semi^*$-closed, $\tilde{f}$ is soft pre^*-generalized continuous but not soft $semi^*$-continuous.

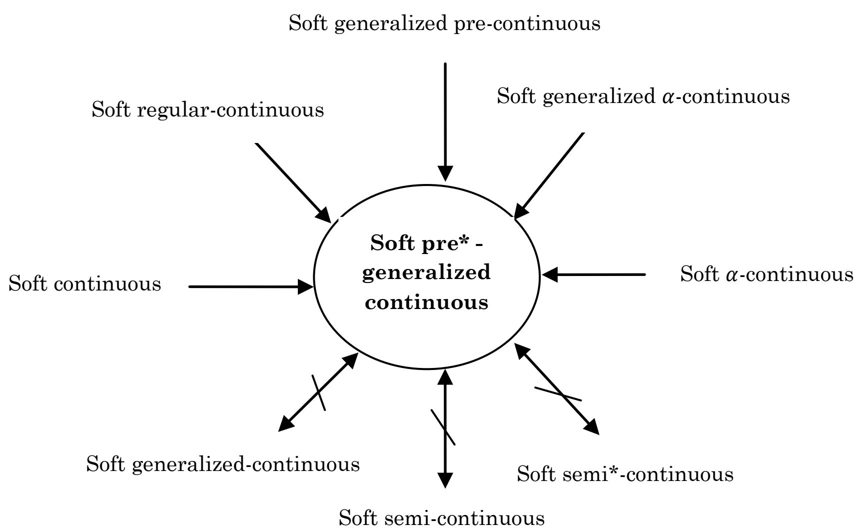
Example 3.17. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as $u(a) = y$, $u(b) = x$, $p(e_1) = k_1$, $p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau}^{\tilde{c}} = \{F_1, F_5, F_8, F_{13}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_4, G_9, G_{12}, G_{16}\}$. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_4) = F_4$ is soft $semi^*$-generalized closed but not soft pre^*-generalizedclosed, $\tilde{f}$ is soft $semi^*$-generalized continuous but not soft pre^*-generalized continuous.

Remark 3.18. The concept of soft pre^*-generalized continuity and soft semi continuity are independent as shown in the following example.

Example 3.19. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as $u(a) = y$, $u(b) = x$, $p(e_1) = k_1$, $p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau}^{\tilde{c}} = \{F_1, F_3, F_8, F_{11}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_2, G_9, G_{10}, G_{16}\}$. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_9) = F_5$ and $\tilde{f}^{-1}(G_{10}) = F_7$ are soft pre^*-generalized closed but not soft semi closed, $\tilde{f}$ is soft pre^*-generalized continuous but not soft semi continuous.

Example 3.20. Let $X = \{a, b\}$, $Y = \{x, y\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: A \to B$ as $u(a) = y$, $u(b) = x$, $p(e_1) = k_1$, $p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau}^{\tilde{c}} = \{F_1, F_6, F_8, F_{14}, F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_9, G_{10}, G_{12}, G_{16}\}$. Let$\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_{10}) = F_7$ is semi-closed but not soft pre^*-generalized closed, $\tilde{f}$ is soft semi continuous but not soft pre^*-generalized continuous.

From the above discussion we have the following diagram:



Theorem 3.21. Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be a function. Then the following are equivalent.

i. $\tilde{f}$ is soft pre^*-generalized continuous.

ii. The inverse image of every soft open set in $(Y, \tilde{\sigma}, B)$ is soft pre^*-generalized closed in $(X, \tilde{\tau}, A)$.

iii. For every subset (G, B) of Y, $S_t p^* gcl\left(\tilde{f}^{-1}(G, B)\right) \tilde{\subseteq} \tilde{f}^{-1}\left(S_t cl(G, B)\right)$.

iv. For every subset (F, A) of X, $\tilde{f}\left(S_t p^* gcl(F, A)\right) \tilde{\subseteq} S_t cl\left(\tilde{f}(F, A)\right)$.

Proof: (i)⇒(ii): Let $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ be soft pre^*-generalized continuous and (G, B) be a soft open set in Y. Then $Y \setminus (G, B)$is soft closed in Y. Since $\tilde{f}$ is soft pre^*-

generalized continuous $\tilde{f}^{-1}\big(Y \setminus (G,B)\big)$ is soft pre^*-generalized closed in X. But $\tilde{f}^{-1}(Y \setminus (G,B) = X \setminus \tilde{f}^{-1}(G,B)$. Hence $\tilde{f}^{-1}(G,B)$ is soft pre^*-generalized open in X.
(ii)⇒(i): Suppose the inverse image of every soft open set in Y is soft pre^*-generalized open in X. Let (G,B) be soft closed in Y. Then $Y \setminus (G,B)$ is open in Y. By assumption $\tilde{f}^{-1}(Y \setminus (G,B)$ is soft pre^*-generalized open. $\tilde{f}^{-1}(Y \setminus (G,B) = X \setminus \tilde{f}^{-1}(G,B)$. Therefore $\tilde{f}^{-1}(G,B)$ is soft pre^*-generalized closed in X. Hence $\tilde{f}$ is soft pre^*-generalized continuous.
(i)⇒(iii): Let (G,B) be a subset of Y. Since $\tilde{f}$ is soft pre^*-generalized continuous, $\tilde{f}^{-1}\big(S_tcl(G,B)\big)$ is soft pre^*-generalized closed in X. Then $S_tp^*gcl\left(\tilde{f}^{-1}\big(S_tcl(G,B)\big)\right) = \tilde{f}^{-1}\big(S_tcl(G,B)\big)$.
Now$S_tp^*gcl\left(\tilde{f}^{-1}(G,B)\right) \tilde{\subseteq} S_tp^*gcl\left(\tilde{f}^{-1}\big(S_tcl(G,B)\big)\right) = \tilde{f}^{-1}\big(S_tcl(G,B)\big)$. This proves (ii).
(iii)⇒(iv): Let (F,A) be a subset of X. Then $\tilde{f}(F,A)$ is a subset of Y. By our assumption,$S_tp^*gcl\left(\tilde{f}^{-1}\left(\tilde{f}(F,A)\right)\right) \tilde{\subseteq} \tilde{f}^{-1}\left(S_tcl\left(\tilde{f}(F,A)\right)\right)$.But
$\big(S_tp^*gcl(F,A)\big) \tilde{\subseteq} S_tp^*gcl\left(\tilde{f}^{-1}\left(\tilde{f}(F,A)\right)\right)$.
Thus$S_tp^*gcl(F,A) \tilde{\subseteq} \tilde{f}^{-1}\left(S_tcl\left(\tilde{f}(F,A)\right)\right)$. Hence $\tilde{f}(S_tp^*gcl(F,A) \tilde{\subseteq} S_tcl(F,A)$. This proves (iii).
(iv)⇒(i): Let (F,A) be soft subset of X. Then $\tilde{f}^{-1}(F,A) = \tilde{f}^{-1}\big(S_tcl(F,A)\big)$. By (iii)$\tilde{f}\big(S_tp^*gcl(F,A)\big) \tilde{\subseteq} S_tcl\left(\tilde{f}(F,A)\right) \tilde{\subseteq} S_tcl(F,A) = (F,A)$. That implies$S_tp^*gcl\left(\tilde{f}^{-1}(F,A)\right) \tilde{\subseteq} \tilde{f}^{-1}(F,A)$.But$\tilde{f}^{-1}(F,A) \tilde{\subseteq} S_tp^*gcl\left(\tilde{f}^{-1}(F,A)\right)$.
Thus$S_tp^*gcl\left(\tilde{f}^{-1}(F,A)\right) = \tilde{f}^{-1}(F,A)$ and so $\tilde{f}^{-1}(F,A)$ is soft pre^*-generalized closed. Hence $\tilde{f}$ is soft pre^*-generalized continuous.

Remark 3.22. The composition of two soft pre^*-generalized continuous functions need not be soft pre^*-generalized continuous as shown in the following example.

Example 3.23. Let$X = \{a,b\}$, $Y = \{x,y\}$, $Z = \{m,n\}$, $\tilde{\tau}^{\tilde{c}} = \{F_1, F_5, F_6, F_{15}, F_{16}\}$, $\tilde{\sigma}^{\tilde{c}} = \{G_1, G_3, G_8, G_{11}, G_{16}\}$ and $\tilde{\mu}^{\tilde{c}} = \{H_1, H_4, H_{16}\}$ where$H_4 = \big\{(l_1, \emptyset), \big(l_2, (m,n)\big)\big\}$. Define $u_1: X \to Y$ and $p_1: A \to B$ as $u_1(a) = y$, $u_2(b) = x$, $p_1(e_1) = k_2$, $p_2(e_2) = k_1$. Then the soft mapping $\tilde{f}: (X, \tilde{\tau}, A) \to (Y, \tilde{\sigma}, B)$ is soft pre^*-generalized continuous. Also, define $u_2: Y \to Z$ and $p_2: B \to C$ as $u_1(x) = m$, $u_2(y) = n$, $p_1(k_1) = l_2$, $p_2(k_2) = l_1$. Then the soft mapping $\tilde{g}: (Y, \tilde{\sigma}, B) \to (Z, \tilde{\mu}, L)$ is soft pre^*-generalized continuous. Now let $\tilde{g} \circ \tilde{f}: (X, \tilde{\tau}, A) \to (Z, \tilde{\mu}, L)$ be the composition of two soft pre^*-generalized continuous functions. Since $\big(\tilde{g} \circ \tilde{f}\big)^{-1}(H_4) = \tilde{f}^{-1}\big(\tilde{g}^{-1}(H_4)\big) = \tilde{f}^{-1}(G_{13}) = F_4$ is not soft pre^*-generalized closed, $\tilde{g} \circ \tilde{f}$ is not soft pre^*-generalized continuous.

Theorem 3.24. If $\tilde{f}:(X,\tilde{\tau},A)\to(Y,\tilde{\sigma},B)$ is soft pre^*-generalized continuous and $\tilde{g}:(Y,\tilde{\sigma},B)\to(Z,\tilde{\mu},C)$ is soft continuous then their composition $\tilde{g}\circ\tilde{f}:(X,\tilde{\tau},A)\to(Z,\tilde{\mu},C)$ is also soft pre^*-generalized continuous.

Proof: Let (H,C) be soft closed set in Z. Since g is soft continuous, $\tilde{g}^{-1}(H,C)$ is closed in Y and since $\tilde{f}$ is soft pre^*-generalized continuous, $\tilde{f}^{-1}\left(\tilde{g}^{-1}(H,C)\right)$ is soft pre^*-generalized closed in X. This implies $\left(\tilde{g}\circ\tilde{f}\right)^{-1}(H,C)$ is soft pre^*-generalized closed in X. Thus $(\tilde{g}\circ\tilde{f})^{-1}$ is soft pre^*-generalized closed in X for every soft closed subset (H,C) of Z. Hence $\tilde{g}\circ\tilde{f}$ is soft pre^*-generalized continuous.

4. Soft Pre*-Generalized Irresolute Functions

Definition 4.1. A function $\tilde{f}:(X,\tilde{\tau},A)\to(Y,\tilde{\sigma},B)$ is said to be soft pre^*-generalized irresolute if $\tilde{f}^{-1}(G,B)$ is soft pre^*-generalized closed in $(X,\tilde{\tau},A)$ for every soft pre^*-generalized closed set in $(Y,\tilde{\sigma},B)$.

Example 4.2. Let $X=\{a,b\}$, $Y=\{x,y\}$, $A=\{e_1,e_2\}$ and $B=\{k_1,k_2\}$. Define $u:X\to Y$ and $p:A\to B$ as $u(a)=y$, $u(b)=x$, p $(e_1)=k_1$ and $p(e_2)=k_2$. Consider the soft topologies $\tilde{\tau}^{\tilde{c}}=\{F_1,F_2,F_8,F_{10},F_{16}\}$ and $\tilde{\sigma}^{\tilde{c}}=\{G_1,G_4,G_8,G_{12},G_{16}\}$. Let $\tilde{f}:(X,\tilde{\tau},A)\to(Y,\tilde{\sigma},B)$ be a soft mapping. Since $\tilde{f}^{-1}(G_4)=F_4$ and $\tilde{f}^{-1}(G_8)=F_{12}$ and $\tilde{f}^{-1}(G_{12})=F_8$ are soft pre^*- generalized closed, $\tilde{f}$ is soft pre^*-generalized irresolute.

Theorem 4.3. If$\tilde{f}:(X,\tilde{\tau},A)\to(Y,\tilde{\sigma},B)$ is a soft pre^*-generalized irresolute function then $\tilde{f}$ is soft pre^*-generalized continuous.

Proof: Let (G,B) be soft closed in Y. By theorem 2.10(i), (G,B) is soft pre^*-generalized closed. Since $\tilde{f}$ is soft pre^*-generalized irresolute function, $\tilde{f}^{-1}(G,B)$ is soft pre^*-generalized closed in X.Hence $\tilde{f}$ is soft pre^*-generalized continuous.

Theorem 4.4. If$\tilde{f}:(X,\tilde{\tau},A)\to(Y,\tilde{\sigma},B)$ and $\tilde{g}:(Y,\tilde{\sigma},B)\to(Z,\tilde{\mu},C)$ are soft pre^*-generalized irresolute functions then $\tilde{g}\circ\tilde{f}$ is soft pre^*-generalized irresolute.

Proof: Let (H,C) be soft pre^*-generalized closed in Z. Since $\tilde{g}$ is soft pre^*-generalized irresolute,$\tilde{g}^{-1}(H,C)$ is soft pre^*-generalized closed in Y. Also, since$\tilde{f}$ is soft pre^*-generalized irresolute, $\tilde{f}^{-1}\left(\tilde{g}^{-1}(H,C)\right)=\left(\tilde{g}\circ\tilde{f}\right)^{-1}(H,C)$ is soft pre^*-generalized closed in X. Hence $\tilde{g}\circ\tilde{f}$ is soft pre^*-generalized irresolute.

Theorem 4.5. Let $\tilde{f}:(X,\tilde{\tau},A)\to(Y,\tilde{\sigma},B)$ is soft pre^*-generalized irresolute and $\tilde{g}:(Y,\tilde{\sigma},B)\to(Z,\tilde{\mu},C)$ is soft pre^*-generalized continuous. Then $\tilde{g}\circ\tilde{f}:(X,\tilde{\tau},A)\to(Z,\tilde{\mu},C)$ is soft pre^*-generalized continuous.

Proof: Let (H,C) be soft closed set in Z. Since $\tilde{g}$ is soft pre^*-generalized continuous, $\tilde{g}^{-1}(H,C)$ is soft pre^*-generalized closed in Y. Also, since$\tilde{f}$ is soft pre^*-generalized irresolute, $\tilde{f}^{-1}\left(\tilde{g}^{-1}(H,C)\right)=\left(\tilde{g}\circ\tilde{f}\right)^{-1}(H,C)$ is soft pre^*-generalized closed in X.Hence$\tilde{g}\circ\tilde{f}$ is soft pre^*-generalized continuous.

Theorem 4.6. Let $\tilde{f}:(X,\tilde{\tau},A)\to(Y,\tilde{\sigma},B)$ be a function. Then the following are equivalent.

i. $\tilde{f}$ is soft pre^*-generalized irresolute.
ii. The inverse image of every soft pre^*-generalized open set in $(Y,\tilde{\sigma},B)$ is soft pre^*-generalized open in $(X,\tilde{\tau},A)$.
iii. $S_tp^*gcl\left(\tilde{f}^{-1}(G,B)\right)\tilde{\subseteq}\tilde{f}^{-1}\left(S_tp^*gcl(G,B)\right)$ for every subset (G,B) of Y.
iv. $\tilde{f}\left(S_tp^*gcl(F,A)\right)\tilde{\subseteq}S_tp^*gcl\left(\tilde{f}(F,A)\right)$ for every subset (F,A) of X.

Proof: The proof is similar to theorem 3.21.

Theorem 4.7. A function $\tilde{f}:(X,\tilde{\tau},A)\to(Y,\tilde{\sigma},B)$ is soft pre^*-generalized iresolute if and only if $\tilde{f}^{-1}\left(S_tp^*gint(G,B)\right)\tilde{\subseteq}S_tp^*gint\left(\tilde{f}^{-1}(G,B)\right)$ for every subset (G,B) of Y.

Proof: Let $\tilde{f}:(X,\tilde{\tau},A)\to(Y,\tilde{\sigma},B)$ be soft pre^*-generalized irresolute. Let (G,B) be a subset of Y. Then $S_tp^*gint(G,B)$ is soft pre^*-generalized open in Y. Since $\tilde{f}$ is soft pre^*-generalized irresolute, $\tilde{f}^{-1}\left(S_tp^*gint(G,B)\right)$ is soft pre^*-gene- ralized open in$X$. Then

$\tilde{f}^{-1}\left(S_tp^*gint(G,B)\right)=S_tp^*\,gint\left(\tilde{f}^{-1}\left(S_tp^*gint(G,B)\right)\right)\tilde{\subseteq}S_tp^*gint\left(\tilde{f}^{-1}(G,B)\right)$.

Thus $\tilde{f}^{-1}\left(S_tp^*gint(G,B)\right)\tilde{\subseteq}S_tp^*gint\left(\tilde{f}^{-1}(G,B)\right)$.

Conversely, let (G,B) be soft pre^*-generalized open in Y. Then by (iv), $\tilde{f}^{-1}(G,B)=\tilde{f}^{-1}\left(S_tp^*gint(G,B)\right)\tilde{\subseteq}S_tp^*gint\left(\tilde{f}^{-1}(G,B)\right)$.But$S_tp^*gint\left(\tilde{f}^{-1}(G,B)\right)\tilde{\subseteq}\tilde{f}^{-1}(G,B$. Therefore $S_tp^*gint\left(\tilde{f}^{-1}(G,B)\right)=\tilde{f}^{-1}(G,B)$ and so $\tilde{f}^{-1}(G,B)$ is soft pre^*-generalized open. Hence $\tilde{f}$ is soft pre^*-generalized irresolute.

References

[1] Aras C.G and Sonmez A, On Soft Mappings, arXiv: 1305.4545, (2013).

[2] Athar Kharal and Ahmad. B, Mappings on Soft Classes, *New Math. Nat. Comput.* 7(3), 471-481 (2011)

[3] Janaki C and Jeyanthi V, On Soft πGr-Continuous in Soft Topological Spaces, *International Journal of Engineering Research and Technology*, ISSN:2278-0181, vol 3, (2014).

[4] Mahanta J and Das P.K., On Soft Topological Space Via Semiopen and Semiclosed Soft Sets, arXiv:1203.4133v1 (math.GN), (2012).

[5] Metin Akdag and Alkan Ozkan, Soft Alpha-Open Sets and Soft Alpha-Continuous Functions, *Hindawi Publishing Corporation, Abstract and Applied Analysis,* 2014, 7 pages.

[6] Molodtsov D.A., Soft Set Theory-First Results, *Computers and Mathematics with Applications*37 (1999) 19-31.

[7] Muhammad Shabir and Munazza Naz, On Soft Topological Spaces, *Computers and Mathematics with Applications* volume 61, Issue 7, April 2011, 1786-1799.

[8] Reena C and Karthika M, A New Class of Generalized Closed Sets In Soft Topological Spaces, *paper presented in the International Virtual Conference on Current Scenario in Modern Mathematics* (ICCSMM-2022).

[9] Sattar Hameed Hamzah and Samer Adnan Jubair, On Soft Generalized Continuous Mappings, *Journal of Al-Qadisiyah for Computer Science and Mathematics*, 8(1) 83-92, (2016).