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A New Class of Nano Generalized Closed Sets in Nano Topological Spaces

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Abstract

In this paper, we introduce a new class of nano generalized closed sets in nano topological spaces namely nano generalized α^*-closed sets. Then we discuss some of its properties and investigate their relation with many other nano closed sets. Also, we define nano generalized α^*-open set and discuss its relation with other open sets. Finally, we define the properties of nano generalized α^*-interior and nano generalized α^*-closure.

Keywords: $\mathbb{N}g\alpha^*$- closed sets, $\mathbb{N}g\alpha^*$- open sets, $\mathbb{N}g\alpha^*$- int, $\mathbb{N}g\alpha^*$- cl.

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1. Introduction

The theory of nano topology proposed by Lellis Thivagar [4] and Carmel Richard is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. The elements of a nano topological space are called the nano open set. The author has defined nano topological space in terms of lower and upper approximations and boundary region. He has defined nano closed sets, nano-interior and nano-closure of a set. He also introduced certain weak forms of nano open sets such as nano α-open set, nano semi-open sets and nano pre-open sets. Levine [5] introduced the class of g-closed sets in 1970. K. Bhuvaneswari introduced nano g-closed [1], nano gs-closed [3], nano αg-closed [6], nano gp-closed [2], nano gr-closed [12] sets and studied their properties. Nano g^*p-closed sets was introduced by Rajendran [10] and investigated. The aim of this paper is to introduce and study the properties of nano $g\alpha^*$-closed sets and nano $g\alpha^*$-open sets in nano topological spaces. Finally, we define the properties of nano generalized α^*-interior and nano generalized α^*-closure.

2. Preliminaries

Throughout this paper $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ represent nano topological spaces on which no separation axioms are assumed unless and otherwise mentioned. For a subset $\mathbb{S}$ of $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$, $Ncl(\mathbb{S})$ and $Nint(\mathbb{S})$ denote the nano closure of $\mathbb{S}$ and nano interior of $\mathbb{S}$ respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. [4] Let $\mathbb{U}$ be a non-empty finite set of objects called the universe and $\mathbb{R}$ be an equivalence relation on $\mathbb{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair ($\mathbb{U}$,$\mathbb{R}$) is said to be the approximation space. Let $\mathbb{X} \subseteq_{\mathbb{N}} \mathbb{U}$. Then

1) The lower approximation of $\mathbb{X}$ with respect to $\mathbb{R}$ is the set of all objects, which can be for certain classified as $\mathbb{X}$ with respect to $\mathbb{R}$ and it is denoted by $\mathbb{L}_{\mathbb{R}}(\mathbb{X})$.

$$\mathbb{L}_{\mathbb{R}}(\mathbb{X}) = \bigcup_{x\in\mathbb{U}} \{\mathbb{R}(x): \mathbb{R}(x) \subseteq_{\mathbb{N}} \mathbb{X}\}$$

2) The upper approximation of $\mathbb{X}$ with respect to $\mathbb{R}$ is the set of all objects, which can be possibly classified as $\mathbb{X}$ with respect to $\mathbb{R}$ and it is denoted by $\mathbb{U}_{\mathbb{R}}(\mathbb{X})$.

$$\mathbb{U}_{\mathbb{R}}(\mathbb{X}) = \bigcup_{x\in\mathbb{U}} \{\mathbb{R}(x): \mathbb{R}(x) \cap \mathbb{X} \neq \phi\}$$

3) The boundary region of $\mathbb{X}$ with respect to $\mathbb{R}$ is the set of all objects, which can be classified neither as $\mathbb{X}$ nor as not –$\mathbb{X}$ with respect to $\mathbb{R}$ and it is denoted by $\mathbb{B}_{\mathbb{R}}(\mathbb{X})$.

$\mathbb{B}_{\mathbb{R}}(\mathbb{X}). = \mathbb{U}_{\mathbb{R}}(\mathbb{X}) - \mathbb{L}_{\mathbb{R}}(\mathbb{X})$.

Definition 2.2. [4] Let $\mathbb{U}$ be the universe, $\mathbb{R}$ be an equivalence relation on $\mathbb{U}$ and $\tau_{\mathbb{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \mathbb{U}_{\mathbb{R}}(\mathbb{X}), \mathbb{L}_{\mathbb{R}}(\mathbb{X}), \mathbb{B}_{\mathbb{R}}(\mathbb{X})\}$ where $\mathbb{X} \subseteq_{\mathbb{N}} \mathbb{U}$. Then $\mathbb{R}(\mathbb{X})$ satisfies the following axioms:

1) $\mathbb{U}$ and $\phi \in \tau_{\mathbb{R}}(\mathbb{X})$,
2) The union of the elements of any sub collection of $\tau_{\mathbb{R}}(\mathbb{X})$ is in $\tau_{\mathbb{R}}(\mathbb{X})$,

3) The intersection of the elements of any finite sub collection of $\tau_{\mathbb{R}}(\mathbb{X})$ is in $\tau_{\mathbb{R}}(\mathbb{X})$.
That is, $\tau_{\mathbb{R}}(\mathbb{X})$is a topology on $\mathbb{U}$ called the nano topology on $\mathbb{U}$ with respect to $\mathbb{X}$. We call $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ as the nano topological space(NTS). The elements of $\tau_{\mathbb{R}}(\mathbb{X})$ are called as nano open sets. The complement of nano-open sets is called nano closed sets.

Definition 2.3. [4] If $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is a NTS with respect to $\mathbb{X}$ and if $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{U}$, then

- The nano interior of $\mathbb{S}$ is defined as the union of all nano open subsets of $\mathbb{S}$ and it is denoted by $\mathbb{N}int(\mathbb{S})$. That is, $\mathbb{N}int(\mathbb{S})$ is the largest open subset of $\mathbb{S}$.
- The nano closure of $\mathbb{S}$ is defines as the intersection of all nano closed sets containing $\mathbb{S}$ and it is denoted by $\mathbb{N}cl(\mathbb{S})$. That is, $\mathbb{N}cl(\mathbb{S})$ is the smallest nano closed set containing $\mathbb{S}$.

Definition 2.4. A subset $\mathbb{S}$ of a NTS $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is called;
1) Nano pre-open [4] if $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}int(\mathbb{N}cl(\mathbb{S}))$
2) Nano semi-open [4] if $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}cl(\mathbb{N}int(\mathbb{S}))$
3) Nano α-open [4] if $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}int(\mathbb{N}cl(\mathbb{N}int(\mathbb{S})))$
4) Nano β-open [11] if $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}cl(\mathbb{N}int(\mathbb{N}cl(\mathbb{S})))$
5) Nano regular-open [4] if $\mathbb{S} = \mathbb{N}int(\mathbb{N}cl(\mathbb{S}))$

The complements of the above-mentioned sets are called their respective closed sets.

Definition 2.5. A subset $\mathbb{S}$ of a NTS $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is called;
1) $\mathbb{N}g$-closed [1] if $\mathbb{N}cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$.
2) $\mathbb{N}gs$-closed [3] if $\mathbb{N}scl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$.
3) $\mathbb{N}\alpha g$-closed [6] if $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$.
4) $\mathbb{N}gp$-closed [2] if $\mathbb{N}pcl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$.
5) $\mathbb{N}g\beta$-closed [7] if $\mathbb{N}\beta cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$.
6) $\mathbb{N}gr$-closed [12] if $\mathbb{N}rcl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$.
7) $\mathbb{N}g^*$-closed [8] if $\mathbb{N}cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$.
8) $\mathbb{N}g^*s$-closed [9] if $\mathbb{N}scl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$.
9) $\mathbb{N}g^*p$-closed [10] if $\mathbb{N}pcl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$.
10) $\mathbb{N}g^*r$-closed [13] if $\mathbb{N}rcl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$.

Theorem 2.6. [1] Every nano open set is $\mathbb{N}g$-open.

3. Nano generalized α^*-closed sets

Definition 3.1. A Nano generalized α^*(in short, $\mathbb{N}g\alpha^*$) closed set is a subset $\mathbb{S}$ of a $NTS(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ if$\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}and\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$.

Instance 3.2. Let $\mathbb{U}$ = {p, q, r, s} with $\mathbb{U}/\mathbb{R}$ ={{p}, {r}, {q, s}} and $\mathbb{X}$ = {p, q} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_R(\mathbb{X})$ = {$\mathbb{U}$, ϕ, {p}, {q, s}, {p, q, s}}. Here $\mathbb{N}g\alpha^*$-closed = {$\mathbb{U}$, ϕ, {r}, {p, r}, {q, r}, {r, s}, {p, q, r}, {p, r, s}, {q, r, s}}.

Instance 3.3. Let $\mathbb{U}$ = {p, q, r} with $\mathbb{U}/\mathbb{R}$ = {{p}, {p, q}} and $\mathbb{X}$ = {p} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {$\mathbb{U}$, ϕ, {p}, {q}, {p, q}}. Here $\mathbb{N}g\alpha^*$- closed = {ϕ, $\mathbb{U}$, {r}, {p, r}, {q, r}}.

Theorem 3.4. Every nano closed set is $\mathbb{N}g\alpha^*$-closed.
Proof: Let $\mathbb{S}$ be a nano closed set in $NTS\left(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})\right)$. Then we have, $\mathbb{N}cl(\mathbb{S}) = \mathbb{S}$. Let $\mathbb{F}$ be a nano open set in $\mathbb{U}$ such that $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and$\mathbb{F} = \mathbb{N}int(\mathbb{F})$. By theorem 2.6, we have $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Then $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}cl(\mathbb{S}) = \mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F} = \mathbb{N}int(\mathbb{F}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Thus $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Therefore $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed.

Remark 3.5 The invert of the preceding theorem does not hold as witnessed in the succeeding instance.

Instance 3.6. Let $\mathbb{U}$ = {p, q, r, s} with $\mathbb{U}/\mathbb{R}$ = {{r}, {p, q, s}} and $\mathbb{X}$ = {q, s}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {ϕ, $\mathbb{U}$, {p, q, s}}. Here {$\mathbb{U}$, ϕ, {r}, {p, r}, {q, r}, {r, s}, {p, q, r}, {p, r, s}, {q, r, s}} is $\mathbb{N}g\alpha^*$- closed set but the set is not nano closed.

Theorem 3.7. Every $\mathbb{N}g\alpha^*$- closed set is $\mathbb{N}g$ - closed.
Proof. Let $\mathbb{S}$ be a $\mathbb{N}g\alpha^*$- closed set in $NTS\left(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})\right)$ and let $\mathbb{F}$ be a nano open set in $\mathbb{U}$ such that $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F} = \mathbb{N}int(\mathbb{F})$. By theorem 2.6, we have $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Also, $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed, then $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$.
Then, $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F}) \subseteq_{\mathbb{N}} \mathbb{F}$. Thus $\mathbb{N}cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$. Hence $\mathbb{S}$ is $\mathbb{N}g$-closed.

Remark 3.8. The invert of the preceding theorem does not hold as witnessed in the succeeding instance.

Instance 3.9. Let $\mathbb{U}$ = {p, q, r, s} with $\mathbb{U}/\mathbb{R}$ = {{p, q}, {r, s}} and $\mathbb{X}$ = {p, q}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {$\mathbb{U}$, ϕ, {p, q}}. Here {$\mathbb{U}$, ϕ, {r}, {s}, {p, r}, {p, s}, {q, r}, {q, s}, {r, s}, {p, q, r}, {p, q, s}, {p, r, s}, {q, r, s}} is $\mathbb{N}g$ – closed set but the set is not $\mathbb{N}g\alpha^*$- closed.

Theorem 3.10. Every $\mathbb{N}g\alpha^*$- closed set is $\mathbb{N}\alpha g$- closed.
Proof. Let $\mathbb{S}$ be a $\mathbb{N}g\alpha^*$-closed set in $NTS\left(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})\right)$ and let $\mathbb{F}$ be a nano open set in $\mathbb{U}$ such that $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and$\mathbb{F} = \mathbb{N}int(\mathbb{F})$. By theorem 2.6, we have $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Also, $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed, then $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Then $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F}) = \mathbb{F}$. Thus $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$. Hence $\mathbb{S}$ is $\mathbb{N}\alpha g$-closed.

Remark 3.11. The invert of the preceding theorem does not hold as witnessed in the succeeding instance.

Instance 3.12. Let $\mathbb{U}$ = {p, q, r} with $\mathbb{U}/\mathbb{R}$ = {{{r}, {p, q}} and $\mathbb{X}$ = {r}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {ϕ, $\mathbb{U}$, {r}}. Here {ϕ, {p}, {q}, {p, q}, {p, r}, {q, r}} is $\mathbb{N}\alpha g$ - closed but the set is not $\mathbb{N}g\alpha^*$- closed.

Theorem 3.13. Every $\mathbb{N}g\alpha^*$-closed set is $\mathbb{N}gs$-closed.
Proof. Let $\mathbb{S}$ be a $\mathbb{N}g\alpha^*$-closed set in $NTS\left(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X})\right)$. Let $\mathbb{F}$ be a nano open set in $\mathbb{U}$ such that $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F} = \mathbb{N}int(\mathbb{F})$. By theorem 2.6, $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Since $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed, $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Then $\mathbb{N}scl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F}) = \mathbb{F}$. Thus $\mathbb{N}scl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$. Hence $\mathbb{S}$ is $\mathbb{N}gs$-closed.

Remark 3.14. The invert of the former theorem does not holds as witnessed in the succeeding instance.

Instance 3.15. Let $\mathbb{U}$ = {p, q, r, s} with $\mathbb{U}/\mathbb{R}$ = {{p}, {q}, {r, s}} and $\mathbb{X}$ = {q, s}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {ϕ, $\mathbb{U}$, {q}, {r, s}, {q, r, s}. Here {ϕ, $\mathbb{U}$, {p}, {q}, {r}, {s}, {p, q}, {p, r}, {p, s}, {r, s}, {p, q, r}, {p, q, s}, {p, r, s}} is $\mathbb{N}gs$ - closed but it is not $\mathbb{N}g\alpha^*$- closed.

Theorem 3.16. Every $\mathbb{N}g\alpha^*$-closed set is $\mathbb{N}gp$ - closed.
Proof: Let $\mathbb{S}$ be a $\mathbb{N}g\alpha^*$-closed set in $NTS\left(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X})\right)$. Let $\mathbb{F}$ be a nano open set in $\mathbb{U}$ such that $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F} = \mathbb{N}int(\mathbb{F})$. By theorem 2.6, we have $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Since $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed, $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Then $\mathbb{N}pcl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F}) = \mathbb{F}$. Thus $\mathbb{N}pcl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$. Hence $\mathbb{S}$ is $\mathbb{N}gp$-closed.

Remark 3.17. The invert of the former theorem does not holds as witnessed in the succeeding instance.

Instance 3.18. Let $\mathbb{U}$ = {p, q, r, s} with $\mathbb{U}/\mathbb{R}$ = {(p}, {r}, {q, s}} and $\mathbb{X}$ = {r, s}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {ϕ, $\mathbb{U}$, {r}, {q, s}, {q, r, s}}. Here {ϕ, $\mathbb{U}$, {p}, {q}, {s}, {p, q}, {p, r}, {p, s}, {p, q, r}, {p, q, s} {p, r, s}} is $\mathbb{N}gp$-closed but it is not $\mathbb{N}g\alpha^*$- closed.

Theorem 3.19. Every $\mathbb{N}g\alpha^*$-closed set is $\mathbb{N}g\beta$-closed.
Proof. Let $\mathbb{S}$ be a $\mathbb{N}g\alpha^*$-closed set in $NTS\left(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X})\right)$ and let $\mathbb{F}$ be a nano open set in $\mathbb{U}$ such that $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F} = \mathbb{N}int(\mathbb{F})$. By theorem 2.6, we have $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Since $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed, $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Then $\mathbb{N}\beta cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F}) = \mathbb{F}$. Thus $\mathbb{N}\beta cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$. Hence $\mathbb{S}$ is $\mathbb{N}g\beta$-closed.

Remark 3.20. The invert of the former theorem does not hold as witnessed in the succeeding instance.

Instance 3.21. Let $\mathbb{U}$ = {p, q, r} with $\mathbb{U}/\mathbb{R}$ = {{p}, {q, r}} and $\mathbb{X}$ = {p, q}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {ϕ, $\mathbb{U}$, {p}, {q, r}}. Here {ϕ, $\mathbb{U}$, {p}, {q}, {r}, {p, q}, {p, r}, {q, r}} is $\mathbb{N}g\beta$-closed but it is not $\mathbb{N}g\alpha^*$-closed.

Theorem 3.22. Every $\mathbb{N}g\alpha^*$- closed set is $\mathbb{N}gr$ - closed.

Proof. Let $\mathbb{S}$ be a $\mathbb{N}g\alpha^*$-closed set in $NTS\left(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})\right)$. Let $\mathbb{F}$ be a nano open set in $\mathbb{U}$ such that $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F} = \mathbb{N}int(\mathbb{F})$. By theorem 2.6, we have $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Since $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed, $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Then $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}rcl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F}) = \mathbb{F}$. Thus $\mathbb{N}rcl(A) \subseteq_{\mathbb{N}} \mathbb{F}$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$. Hence $\mathbb{S}$ is $\mathbb{N}gr$-closed.

Remark 3.23. The transpose of the preceding theorem does not hold as witnessed in the succeeding instance.

Instance 3.24. Let $\mathbb{U}$ = {p, q, r} with $\mathbb{U}/\mathbb{R}$ = {{r}, {p, q}} and $\mathbb{X}$={r}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {ϕ, $\mathbb{U}$, {r}}. Here {ϕ, {p}, {q}, {p, q}, {p, r}, {q, r}} is $\mathbb{N}gr$-closed which is not $\mathbb{N}g\alpha^*$-closed.

Theorem 3.25. Every $\mathbb{N}g^*$-closed set is $\mathbb{N}g\alpha^*$-closed.

Proof: Let $\mathbb{S}$ be a $\mathbb{N}g^*$-closed set in $NTS\left(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})\right)$. Let $\mathbb{F}$ be a nano open set in $\mathbb{U}$ such that $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F} = \mathbb{N}int(\mathbb{F})$. Through theorem 2.6, we have $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Since $\mathbb{S}$ is $\mathbb{N}g^*$-closed, $\mathbb{N}cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$. Then $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F} = \mathbb{N}int(\mathbb{F}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Thus $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Hence $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed.

Remark 3.26. The transpose of the preceding theorem does not hold as witnessed in the succeeding instance.

Instance 3.27. Let $\mathbb{U}$ = {p, q, r} with $\mathbb{U}/\mathbb{R}$ = {{p}, {q, r}} and $\mathbb{X}$ = {p}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {ϕ, $\mathbb{U}$, {p}}. Here {ϕ, {q}, {r}, {q, r}} is $\mathbb{N}g\alpha^*$-closed which is not $\mathbb{N}g^*$-closed.

Theorem 3.28. Every $\mathbb{N}g\alpha^*$-closed set is $\mathbb{N}g^*s$-closed.

Proof. Let $\mathbb{S}$ be a $\mathbb{N}g\alpha^*$-closed set in $NTS\left(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})\right)$. Then $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$.Thus $\mathbb{N}scl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F}) \subseteq_{\mathbb{N}} \mathbb{F}$, we get $\mathbb{N}scl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Hence $\mathbb{S}$ is $\mathbb{N}g^*s$-closed.

Remark 3.29. The transpose of the preceding theorem does not hold as witnessed in the succeeding instance.

Instance 3.30. Let $\mathbb{U}$ = {p, q, r, s} with $\mathbb{U}/\mathbb{R}$ = {{p}, {r}, {q, s}} and $\mathbb{X}$ = {q, r}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {ϕ, $\mathbb{U}$, {r}, {q, s}, {q, r, s}}. Here {ϕ, $\mathbb{U}$, {p}, {r}, {p, q}, {p, r}, {p, s}, {q, s}, {p, q, r}, {p, q, s}, {p, r, s}} is $\mathbb{N}g^*s$-closed which is not $\mathbb{N}g\alpha^*$-closed.

Theorem 3.31. Every $\mathbb{N}g\alpha^*$-closed set is $\mathbb{N}g^*p$-closed.

Proof: Let $\mathbb{S}$ be a $\mathbb{N}g\alpha^*$-closed set in $NTS\left(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})\right)$. Then $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$.Thus $\mathbb{N}pcl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F}) \subseteq_{\mathbb{N}} \mathbb{F}$, we get $\mathbb{N}pcl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$ whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Hence $\mathbb{S}$ is $\mathbb{N}g^*p$-closed.

Remark 3.32. The polar statement of the preceding theorem does not hold as witnessed in the succeeding instance.

Instance 3.33. Let $\mathbb{U}$ = {p, q, r, s} with $\mathbb{U}/\mathbb{R}$ = {{p, q}, {r, s}} and $\mathbb{X}$ = {q, r, s}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {ϕ, $\mathbb{U}$, {p, q}, {r, s}}. Here {ϕ, $\mathbb{U}$, {p}, {q}, {r}, {s}, {p, q}, {p, r}, {p, s}, {q, r}, {q, s}, {r, s}, {p, q, r}, {p, q, s}, {p, r, s}, {q, r, s}} is $\mathbb{N}g^*p$-closed but it is not $\mathbb{N}g\alpha^*$-closed.

Theorem 3.34. Every $\mathbb{N}g^*r$-closed set is $\mathbb{N}g\alpha^*$-closed.
Proof: Let $\mathbb{S}$ be a $\mathbb{N}g^*r$-closed set in $NTS(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ and let $\mathbb{F}$ be a nano open set in $\mathbb{U}$ such that $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and$\mathbb{F} = \mathbb{N}int(\mathbb{F})$. Through theorem 2.6, we have $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Since $\mathbb{S}$ is $\mathbb{N}g^*r$-closed, $\mathbb{N}rcl(A) \subseteq_{\mathbb{N}} \mathbb{F}$. Then $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}rcl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F} = \mathbb{N}int(\mathbb{F}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Thus $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Hence $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed.

Remark 3.35. The polar statement of the preceding theorem does not hold as witnessed in the succeeding instance.

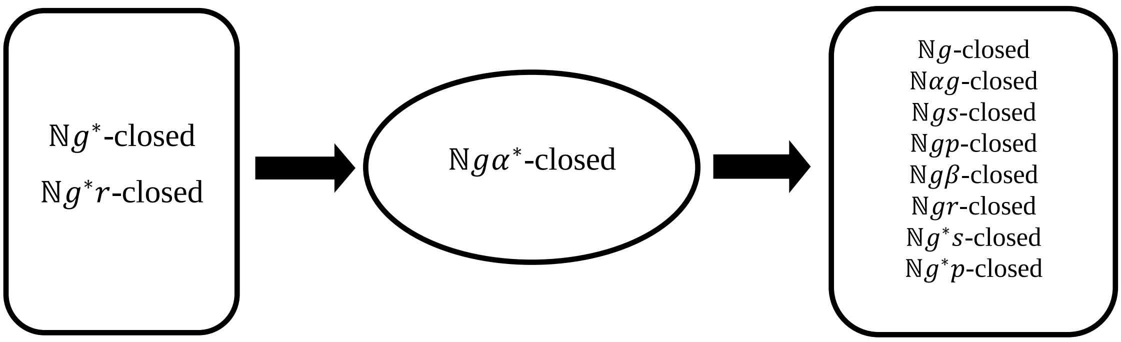
Instance 3.36. Let $\mathbb{U}$ = {p, q, r, s} with $\mathbb{U}/\mathbb{R}$ = {{r}, {q, s}} and $\mathbb{X}$ = {p, r}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {ϕ, $\mathbb{U}$, {r}}. Here {ϕ, $\mathbb{U}$, {p}, {q}, {s}, {p, q}, {p, s}, {q, s}, {p, q, s}} is $\mathbb{N}g\alpha^*$-closed but it is not $\mathbb{N}g^*r$-closed.

Remark 3.37. The concepts of nano semi closed and $\mathbb{N}g\alpha^*$-closed are independent as witnessed in the succeeding instance.

Instance 3.38. Let $\mathbb{U}$ = {p, q, r, s} with $\mathbb{U}/\mathbb{R}$ = {{p}, {r}, {q, s}} and $\mathbb{X}$ = {p, q} ⊆ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {$\mathbb{U}$, ϕ, {p}, {q, s}, {p, q, s}}. The set {ϕ, $\mathbb{U}$, {p}, {r}, {p, r}, {q, s}, {q, r, s}} is nano semi closed yet not $\mathbb{N}g\alpha^*$-closed. The set {ϕ, $\mathbb{U}$, {r}, {p, r}, {q, r}, {r, s}, {p, q, r}, {p, r, s}, {q, r, s}} is $\mathbb{N}g\alpha^*$-closed but the set is not nano semi closed.

Remark 3.39. $\mathbb{N}g\alpha^*$-closed set lies between $\mathbb{N}g^*$-closed set and $\mathbb{N}g$-closed set. That is, $\mathbb{N}g^*$-closed$\subseteq_{\mathbb{N}}$ $\mathbb{N}g\alpha^*$-closed$\subseteq_{\mathbb{N}}$ $\mathbb{N}g$-closed.

Remark 3.40. The diagram that follows exhibit the relation between $\mathbb{N}g\alpha^*$-closed sets and other closed sets.



Theorem 3.41. If $\mathbb{G}$ and $\mathbb{H}$ are $\mathbb{N}g\alpha^*$-closed sets in $NTS(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$, then $\mathbb{G} \cup \mathbb{H}$ is a $\mathbb{N}g\alpha^*$-closed set.

Proof: Let $\mathbb{G}$and $\mathbb{H}$ be $\mathbb{N}g\alpha^*$-closed sets in a $NTS(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ and let $\mathbb{F}$ be any $\mathbb{N}g$-open set in $\mathbb{U}$ containing $\mathbb{G}$ and $\mathbb{H}$. Then $\mathbb{G} \cup \mathbb{H} \subseteq_{\mathbb{N}} \mathbb{F}$. Then $\mathbb{G} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{H} \subseteq_{\mathbb{N}} \mathbb{F}$. Since $\mathbb{G}$ and $\mathbb{H}$ are $\mathbb{N}g\alpha^*$-closed sets, $\mathbb{N}\alpha cl(\mathbb{G}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$and $\mathbb{N}\alpha cl(\mathbb{H}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Now, $\mathbb{N}\alpha cl(\mathbb{G} \cup \mathbb{H}) = \mathbb{N}\alpha cl(\mathbb{G}) \cup \mathbb{N}\alpha cl(H) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Thus, $\mathbb{N}\alpha cl(\mathbb{G} \cup \mathbb{H}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$ whenever $\mathbb{G} \cup \mathbb{H} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Hence $\mathbb{G} \cup \mathbb{H}$ is a $\mathbb{N}g\alpha^*$-closed.

Theorem 3.42. If $\mathbb{G}$ and $\mathbb{H}$ are $\mathbb{N}g\alpha^*$-closed sets in $NTS\ (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$, then $\mathbb{G} \cap \mathbb{H}$ is a $\mathbb{N}g\alpha^*$-closed set.

Proof: Let $\mathbb{G}$ and $\mathbb{H}$ be $\mathbb{N}g\alpha^*$-closed sets in a $NTS(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ and let $\mathbb{F}$ be a $\mathbb{N}g$-open set in $\mathbb{U}$ such that $\mathbb{G} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{H} \subseteq_{\mathbb{N}} \mathbb{F}$. Then $\mathbb{G} \cap \mathbb{H} \subseteq_{\mathbb{N}} \mathbb{F}$. Since $\mathbb{G}$ and $\mathbb{H}$ are $\mathbb{N}g\alpha^*$-closed sets, $\mathbb{N}\alpha cl(\mathbb{G}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$and $\mathbb{N}\alpha cl(\mathbb{H}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Now,$\mathbb{N}\alpha cl(\mathbb{G} \cap \mathbb{H}) \subseteq_{\mathbb{N}} \mathbb{N}\alpha cl(\mathbb{G}) \cap \mathbb{N}\alpha cl(\mathbb{H}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$. Thus $\mathbb{N}\alpha cl(\mathbb{G} \cap \mathbb{H}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$ whenever $\mathbb{G} \cap \mathbb{H} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$. Hence $\mathbb{G} \cap \mathbb{H}$ is a $\mathbb{N}g\alpha^*$-closed.

Corollary 3.43. If $\mathbb{G}$ is $\mathbb{N}g\alpha^*$-closed and $\mathbb{H}$ is nano closed in $\mathbb{U}$, then $\mathbb{G} \cap \mathbb{H}$ is $\mathbb{N}g\alpha^*$-closed.

Proof: Let $\mathbb{H}$ be nano closed in $\mathbb{U}$. Then by theorem 3.4, $\mathbb{H}$ is $\mathbb{N}g\alpha^*$-closed. $\mathbb{G}$ is also $\mathbb{N}g\alpha^*$-closed. By theorem 3.42, $\mathbb{G} \cap \mathbb{H}$ is $\mathbb{N}g\alpha^*$-closed.

Corollary 3.44. If $\mathbb{G}$ is $\mathbb{N}g\alpha^*$-closed and $\mathbb{H}$ is nano open in $\mathbb{U}$, then $\mathbb{G} \backslash \mathbb{H}$ is $\mathbb{N}g\alpha^*$-closed.

Proof: Let $\mathbb{G} \backslash \mathbb{H} = \mathbb{G} \cap (\mathbb{U} \backslash \mathbb{H})$. Since $\mathbb{H}$ is nano open in $\mathbb{U}$, $\mathbb{U} \backslash \mathbb{H}$ is nano closed in $\mathbb{U}$. Since $\mathbb{G}$ is $\mathbb{N}g\alpha^*$-closed and $\mathbb{U} \backslash \mathbb{H}$ is nano closed in $\mathbb{U}$, by corollary 3.43, $\mathbb{G} \cap (\mathbb{U} \backslash \mathbb{H})$is $\mathbb{N}g\alpha^*$-closed. Hence $\mathbb{G} \backslash \mathbb{H}$ is $\mathbb{N}g\alpha^*$-closed.

4. Nano generalized α^*-open sets

Definition 4.1. A subset $\mathbb{S}$ of a $NTS(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is called nano generalized α^*(in short, $\mathbb{N}g\alpha^*$) open set if its complement is $\mathbb{N}g\alpha^*$-closed.

Instance 4.2. Let $\mathbb{U}$ = {p, q, r, s} with $\mathbb{U}/\mathbb{R}$ = {{q}, {r}, {p, s}} and $\mathbb{X}$ = {p, r}. Then $\tau_{\mathbb{R}}(\mathbb{X})$ = {$\mathbb{U}$, ϕ, {r}, {p, s}, {p, r, s}}. Here {ϕ, $\mathbb{U}$, {p}, {r}, {s}, {p, r}, {p, s}, {r, s}, {p, r, s}} is $\mathbb{N}g\alpha^*$-open sets.

Theorem 4.3. Every nano open set is $\mathbb{N}g\alpha^*$-open but the invert may not be true.

Theorem 4.4. Every $\mathbb{N}g\alpha^*$-open set is $\mathbb{N}g$-open but the invert may not be true.

Theorem 4.5. Every $\mathbb{N}g\alpha^*$-open set is $\mathbb{N}\alpha g$-open but the invert may not be true.

Theorem 4.6. Every $\mathbb{N}g\alpha^*$-open set is $\mathbb{N}gs$-open but the invert may not be true.

Theorem 4.7. Every $\mathbb{N}g\alpha^*$-open set is $\mathbb{N}gp$-open but the invert may not be true.

Theorem 4.8. Every $\mathbb{N}g\alpha^*$-open set is $\mathbb{N}g\beta$-open but the invert may not be true.

Theorem 4.9. Every $\mathbb{N}g\alpha^*$-open set is $\mathbb{N}gr$-open but the invert may not be true.

Theorem 4.10. Every $\mathbb{N}g^*$-open set is $\mathbb{N}g\alpha^*$-open but the invert may not be true.

Theorem 4.11. Every $\mathbb{N}g\alpha^*$-open set is $\mathbb{N}g^*s$-open but the invert may not be true.

Theorem 4.12. Every $\mathbb{N}g\alpha^*$-open set is $\mathbb{N}g^*p$-open but the invert may not be true.

Theorem 4.13. Every $\mathbb{N}g^*r$-open set is $\mathbb{N}g\alpha^*$-open but the invert may not be true.

Theorem 4.14. If $\mathbb{G}$ and $\mathbb{H}$ are $\mathbb{N}g\alpha^*$-open sets in NTS, then $\mathbb{G} \cup \mathbb{H}$ is a $\mathbb{N}g\alpha^*$-open set.

Theorem 4.15. If $\mathbb{G}$ and $\mathbb{H}$ are $\mathbb{N}g\alpha^*$-open sets in NTS, then $\mathbb{G} \cap \mathbb{H}$ is a $\mathbb{N}g\alpha^*$-open set.

Corollary 4.16. If $\mathbb{G}$ is $\mathbb{N}g\alpha^*$-open and $\mathbb{H}$ is nano open in $\mathbb{U}$, then $\mathbb{G} \cap \mathbb{H}$ is $\mathbb{N}g\alpha^*$-open.

Corollary 4.17. If $\mathbb{G}$ is $\mathbb{N}g\alpha^*$-open and $\mathbb{H}$ is nano closed in $\mathbb{U}$, then $\mathbb{G}\backslash\mathbb{H}$ is $\mathbb{N}g\alpha^*$-open.

5. $\mathbb{N}g\alpha^*$-interior and $\mathbb{N}g\alpha^*$-closure

Definition 5.1. Let $\mathbb{U}$ be a NTS and let any point $a \in \mathbb{U}$. A subset $\mathbb{S}$ of $\mathbb{U}$ is called the $\mathbb{N}g\alpha^*$-nbhd of a if there exists a $\mathbb{N}g\alpha^*$-open set $\mathbb{K}$ such that $a \in \mathbb{K} \subseteq_{\mathbb{N}} \mathbb{S}$.

Definition 5.2. Let $\mathbb{S}$ be a subset of the NTS $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. A point $a \in \mathbb{S}$is called $\mathbb{N}g\alpha^*$-interior point of $\mathbb{S}$ if $\mathbb{S}$ is a$\mathbb{N}g\alpha^*$-nbhd of a. The set which contains all $\mathbb{N}g\alpha^*$-interior points of $\mathbb{S}$ is called $\mathbb{N}g\alpha^*$-interior of $\mathbb{S}$ and symbolized as $\mathbb{N}g\alpha^*$-int($\mathbb{S}$).

Definition 5.3. Let $\mathbb{S}$ be a subset of the NTS $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. Then the intersection of all $\mathbb{N}g\alpha^*$-closed sets containing $\mathbb{S}$ is called $\mathbb{N}g\alpha^*$-closure of $\mathbb{S}$. That is, $\mathbb{N}g\alpha^* - cl(\mathbb{S}) = \cap \{\mathbb{R}: \mathbb{R}\ is\ \mathbb{N}g\alpha^* - closed\ sets\ and\ \mathbb{S} \subseteq_{\mathbb{N}} \mathbb{R}\}$.

Theorem 5.4. If $\mathbb{S}$ be a subset of $\mathbb{U}$, then
$\mathbb{N}g\alpha^* - int(\mathbb{S}) = \cup \{\mathbb{R}: \mathbb{R}\ is\ \mathbb{N}g\alpha^* - open\ set\ and\ \mathbb{R} \subseteq_{\mathbb{N}} \mathbb{S}\}$.
Proof: Let $\mathbb{S}$ be a subset of $\mathbb{U}$.
$x \in \mathbb{N}g\alpha^* - int(\mathbb{S})$
$\Leftrightarrow$ x is a $\mathbb{N}g\alpha^*$-interior point of $\mathbb{S}$
$\Leftrightarrow$$\mathbb{S}$ is a $\mathbb{N}g\alpha^*$-nbhd of the point x
$\Leftrightarrow$There exists $\mathbb{N}g\alpha^*$-open set $\mathbb{R}$ such that $x \in \mathbb{R} \subseteq_{\mathbb{N}} \mathbb{S}$
$\Leftrightarrow$$x \in \cup \{\mathbb{R}: \mathbb{R}\ is\ \mathbb{N}g\alpha^* - open\ set\ and\ \mathbb{R} \subseteq_{\mathbb{N}} \mathbb{S}\}$
Hence $Ng\alpha^* - int(\mathbb{S}) = \cup \{\mathbb{R}: \mathbb{R}\ is\ \mathbb{N}g\alpha^* - open\ set\ and\ \mathbb{R} \subseteq_{\mathbb{N}} \mathbb{S}\}$.

Theorem 5.5. Let $\mathbb{R}$ and $\mathbb{S}$ be subsets of $\mathbb{U}$. Then
a) $\mathbb{N}g\alpha^* - int(\mathbb{U}) = \mathbb{U}$ and $\mathbb{N}g\alpha^* - int(\phi) = \phi$
b) $\mathbb{N}g\alpha^* - int(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{S}$
c) If $\mathbb{R}$ contains any $\mathbb{N}g\alpha^*$-open set $\mathbb{S}$, then $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{R})$
d) If $\mathbb{R} \subseteq_{\mathbb{N}} \mathbb{S}$, then $\mathbb{N}g\alpha^* - int(\mathbb{R}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{S})$
Proof: a) Since $\mathbb{U}$ and ϕ are $\mathbb{N}g\alpha^*$-open sets, by theorem 5.4, $\mathbb{N}g\alpha^* - int(\mathbb{U}) = \cup \{\mathbb{R}: \mathbb{R}\ is\ \mathbb{N}g\alpha^* - open\ set\ and\ \mathbb{R} \subseteq_{\mathbb{N}} \mathbb{U}\}$.
$\Rightarrow Ng\alpha^* - int(\mathbb{U}) = \cup \{\mathbb{S}: \mathbb{S}\ is\ a\ \mathbb{N}g\alpha^* - open\ set\}$
$\Rightarrow Ng\alpha^* - int(\mathbb{U}) = \mathbb{U}$
Since ϕ is the only $\mathbb{N}g\alpha^*$-open set contained in ϕ, $\mathbb{N}g\alpha^* - int(\phi) = \phi$.
b) Let $x \in \mathbb{N}g\alpha^* - int(\mathbb{S})$
$\Rightarrow x$ is a $\mathbb{N}g\alpha^*$-interior point of $\mathbb{S}$.
$\Rightarrow \mathbb{S}$ is a $\mathbb{N}g\alpha^*$-nbhd of x.
$\Rightarrow x \in \mathbb{S}$
Thus $\mathbb{N}g\alpha^* - int(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{S}$.
c) Let $\mathbb{S}$ be any $\mathbb{N}g\alpha^*$-open set such that $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{R}$ and let $x \in \mathbb{S}$.
Since $\mathbb{S}$ is a $\mathbb{N}g\alpha^*$-open set contained in $\mathbb{R}$, x is a $\mathbb{N}g\alpha^*$-interior point of $\mathbb{R}$.
That is, $x \in \mathbb{N}g\alpha^* - int(\mathbb{R})$
Hence $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{R})$.
d) Let $\mathbb{R}$ and $\mathbb{S}$ be subsets of $\mathbb{U}$ such that $\mathbb{R} \subseteq_{\mathbb{N}} \mathbb{S}$.
Let $x \in \mathbb{N}g\alpha^* - int(\mathbb{R})$.
Then x is a$\mathbb{N}g\alpha^*$-interior point of $\mathbb{R}$ and so $\mathbb{R}$ is a$\mathbb{N}g\alpha^*$-nbhd of x contained in $\mathbb{S}$.
Therefore x is a$\mathbb{N}g\alpha^*$-interior point of $\mathbb{S}$.
Thus $x \in \mathbb{N}g\alpha^* - int(\mathbb{S})$.
Hence $\mathbb{N}g\alpha^* - int(\mathbb{R}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{S})$.

Theorem 5.6. If $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-open then $\mathbb{N}g\alpha^* - int(\mathbb{S}) = \mathbb{S}$.
Proof: Let $\mathbb{S}$ be a $\mathbb{N}g\alpha^*$-open in $\mathbb{U}$.
We know that $\mathbb{N}g\alpha^* - int(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{S}$.
Also, $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-open set contained in $\mathbb{S}$.
By theorem 5.5 c), $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{S})$.
Hence $\mathbb{N}g\alpha^* - int(\mathbb{S}) = \mathbb{S}$.

Theorem 5.7. If $\mathbb{R}$ and $\mathbb{S}$ are subsets of $\mathbb{U}$, then
$\mathbb{N}g\alpha^* - int(\mathbb{R}) \cup \mathbb{N}g\alpha^* - int(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{R} \cup \mathbb{S})$.
Proof: We know that $\mathbb{R} \subseteq_{\mathbb{N}} \mathbb{R} \cup \mathbb{S}$ and $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{R} \cup \mathbb{S}$.
Then by theorem 5.5 d), $\mathbb{N}g\alpha^* - int(\mathbb{R}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{R} \cup \mathbb{S})$ and
$\mathbb{N}g\alpha^* - int(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{R} \cup \mathbb{S})$.
Thus $\mathbb{N}g\alpha^* - int(\mathbb{R}) \cup \mathbb{N}g\alpha^* - int(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{R} \cup \mathbb{S})$.

Theorem 5.8. If $\mathbb{R}$ and $\mathbb{S}$ subsets of $\mathbb{U}$, then
$\mathbb{N}g\alpha^* - int(\mathbb{R} \cap \mathbb{S}) = \mathbb{N}g\alpha^* - int(\mathbb{R}) \cap \mathbb{N}g\alpha^* - int(\mathbb{S})$.
Proof: We know that $\mathbb{R} \cap \mathbb{S} \subseteq_{\mathbb{N}} \mathbb{R}$ and $\mathbb{R} \cap \mathbb{S} \subseteq_{\mathbb{N}} \mathbb{S}$.
By theorem 5.5 d), $\mathbb{N}g\alpha^* - int(\mathbb{R} \cap \mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{R})$ and
$\mathbb{N}g\alpha^* - int(\mathbb{R} \cap \mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{S})$.

Then $\mathbb{N}g\alpha^* - int(\mathbb{R} \cap \mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{R}) \cap \mathbb{N}g\alpha^* - int(\mathbb{S})$(1)
Next, let $x \in \mathbb{N}g\alpha^* - int(\mathbb{R}) \cap \mathbb{N}g\alpha^* - int(\mathbb{S})$
Then $x \in \mathbb{N}g\alpha^* - int(\mathbb{R})$ and $x \in \mathbb{N}g\alpha^* - int(\mathbb{S})$
Hence x is a $\mathbb{N}g\alpha^*$-interior point of both sets $\mathbb{R}$ and $\mathbb{S}$.
It follows that $\mathbb{R}$ and $\mathbb{S}$ is a $\mathbb{N}g\alpha^*$-nbhd of x.
Thus $x \in \mathbb{N}g\alpha^* - int(\mathbb{R} \cap \mathbb{S})$
Hence $\mathbb{N}g\alpha^* - int(\mathbb{R}) \cap \mathbb{N}g\alpha^* - int(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - int(\mathbb{R} \cap \mathbb{S})$(2)
From (1) and (2), we get $\mathbb{N}g\alpha^* - int(\mathbb{R} \cap \mathbb{S}) = \mathbb{N}g\alpha^* - int(\mathbb{R}) \cap \mathbb{N}g\alpha^* - int(\mathbb{S})$.

Theorem 5.9. Let $\mathbb{S}$ be a subset of $\mathbb{U}$, then
a) $\mathbb{N}g\alpha^* - cl(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed in $\mathbb{U}$ and $\mathbb{N}g\alpha^* - cl(\mathbb{S})$ is the smallest $\mathbb{N}g\alpha^*$-closed set in $\mathbb{U}$ containing $\mathbb{S}$.
b) $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed if and only if $\mathbb{N}g\alpha^* - cl(\mathbb{S}) = \mathbb{S}$.
Proof: a) Since the intersection of all $\mathbb{N}g\alpha^*$-closed subsets of $\mathbb{U}$ containing $\mathbb{S}$ is $\mathbb{N}g\alpha^* - cl(\mathbb{S})$, $\mathbb{N}g\alpha^* - cl(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed. $\mathbb{N}g\alpha^* - cl(\mathbb{S})$ is contained in every $\mathbb{N}g\alpha^*$-closed set containing $\mathbb{S}$. Hence, the smallest $\mathbb{N}g\alpha^*$-closed set in $\mathbb{U}$ containing $\mathbb{S}$ is $\mathbb{N}g\alpha^* - cl(\mathbb{S})$.
b) Suppose $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed. By the definition of $\mathbb{N}g\alpha^*$-closure, $\mathbb{N}g\alpha^* - cl(\mathbb{S}) = \mathbb{S}$ Conversely, Suppose $\mathbb{N}g\alpha^* - cl(\mathbb{S}) = \mathbb{S}$. By theorem 3.42, $\mathbb{N}g\alpha^* - cl(\mathbb{S})$is the $\mathbb{N}g\alpha^*$-closed set. Therefore, $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed.

Theorem 5.10 Let $\mathbb{R}$ and $\mathbb{S}$ be subsets of $\mathbb{U}$, then
a) $\mathbb{N}g\alpha^* - cl(\phi) = \phi$
b) $\mathbb{N}g\alpha^* - cl(\mathbb{U}) = \mathbb{U}$
c) $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{S})$
d) If $\mathbb{R} \subseteq_{\mathbb{N}} \mathbb{S}$ then $\mathbb{N}g\alpha^* - cl(\mathbb{R}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{S})$
e) $\mathbb{N}g\alpha^* - cl(\mathbb{R} \cup \mathbb{S}) = \mathbb{N}g\alpha^* - cl(\mathbb{R}) \cup \mathbb{N}g\alpha^* - cl(\mathbb{S})$
f) $\mathbb{N}g\alpha^* - cl(\mathbb{R} \cap \mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{R}) \cap \mathbb{N}g\alpha^* - cl(\mathbb{S})$
Proof: a), b), c), d) follows from the definition of $\mathbb{N}g\alpha^*$-closure.
e) We know that $\mathbb{R} \subseteq_{\mathbb{N}} \mathbb{R} \cup \mathbb{S}$ and $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{R} \cup \mathbb{S}$.
By d), $\mathbb{N}g\alpha^* - cl(\mathbb{R}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{R} \cup \mathbb{S})$,$\mathbb{N}g\alpha^* - cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{R} \cup \mathbb{S})$.
Then $\mathbb{N}g\alpha^* - cl(\mathbb{R}) \cup \mathbb{N}g\alpha^* - cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{R} \cup \mathbb{S})$(1)
Next, we prove $\mathbb{N}g\alpha^* - cl(\mathbb{R} \cup \mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{R}) \cup \mathbb{N}g\alpha^* - cl(\mathbb{S})$
Let $x \notin \mathbb{N}g\alpha^* - cl(\mathbb{R}) \cup \mathbb{N}g\alpha^* - cl(\mathbb{S})$
$\Rightarrow x \notin \mathbb{N}g\alpha^* - cl(\mathbb{R})$ and $x \notin \mathbb{N}g\alpha^* - cl(\mathbb{S})$
By definition of $\mathbb{N}g\alpha^* - cl$, $\mathbb{N}g\alpha^* - cl(\mathbb{R}) = \cap \{F_i : \mathbb{R} \subseteq_{\mathbb{N}} F_i, F_i\ is\ \mathbb{N}g\alpha^* - closed\}$ and $\mathbb{N}g\alpha^* - cl(\mathbb{S}) = \cap \{F_i : \mathbb{S} \subseteq_{\mathbb{N}} F_i, F_i\ is\ \mathbb{N}g\alpha^* - closed\}$.
Then $x \notin F_i$ for some i.
Since $\mathbb{R} \subseteq_{\mathbb{N}} F_i$ and $\mathbb{S} \subseteq_{\mathbb{N}} F_i$, $\mathbb{R} \cup \mathbb{S} \subseteq F_i$.
Therefore $x \notin \mathbb{N}g\alpha^* - cl(\mathbb{R} \cup \mathbb{S})$
Hence $\mathbb{N}g\alpha^* - cl(\mathbb{R} \cup \mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{R}) \cup \mathbb{N}g\alpha^* - cl(\mathbb{S})$(2)
From (1) and (2) we have, $\mathbb{N}g\alpha^* - cl(\mathbb{R} \cup \mathbb{S}) = \mathbb{N}g\alpha^* - cl(\mathbb{R}) \cup \mathbb{N}g\alpha^* - cl(\mathbb{S})$.
f) We know that $\mathbb{R} \cap \mathbb{S} \subseteq_{\mathbb{N}} \mathbb{R}$ and $\mathbb{R} \cap \mathbb{S} \subseteq_{\mathbb{N}} \mathbb{S}$.

By d), $\mathbb{N}g\alpha^* - cl(\mathbb{R} \cap \mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{R})$ and $\mathbb{N}g\alpha^* - cl(\mathbb{R} \cap \mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{S})$
Then $\mathbb{N}g\alpha^* - cl(\mathbb{R} \cap \mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}g\alpha^* - cl(\mathbb{R}) \cap \mathbb{N}g\alpha^* - cl(\mathbb{S})$.

6. Conclusions

In this paper, we have introduced nano generalized α^*-closed sets and discussed some of its properties. Then we investigated its relation with many other nano closed sets. Further nano generalized α^*-open sets are defined and its properties and relations with other nano open sets are studied. Consequently, nano generalized α^*-interior and nano generalized α^*-closure are introduced and discussed.

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