Markov Chain Model and its Application Yearly Rainfall Data in Nagapattinam District

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Abstract

A stochastic model expresses a sequence of possible events in which the possible event of each event depends on the previous event and is called a Markov chain. This paper has analyzed yearly rainfall in the Nagapattinam district and formulated three-state models. The first-order Markov chain to determine the long-term probability of rainfall in the following years and the steady-state. It can be used to make a forecast of the annual rainfall pattern. This model can give some information about rainfall to farmers and the government to plan strategies for high crop production in the Nagapattinam district.

Keywords: Markov chain, Yearly Rainfall, Transition probability

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1. Introduction

Forecasting is a science of the future. Here based on the knowledge of the previous, predictions are made on the future. This involves a knowledge of all forecasting methods. The importance of forecasting is becoming necessary for prosperity. Preparation of a forecast needs mathematical formula and historical data into the future.

The most effective forecasting methods is to use mathematical techniques to routinely forecast demands. By combining mathematical techniques with informed judgement, they can serve checks on each other and tend to eliminate gross errors.

Agriculture largely depends on water resources. The variation and quantity of rainfall have two extreme impacts – bumper harvest or lean year. Rainfall modelling plays a prominent role for rainfall prediction. Apart from data generation, the application of rainfall modelling is vital for water resource management and in the field of hydrology and agriculture. Data relating to climate and rainfall needs a wide range of models in which time and spatial scales are involved.

Rainfall, its arrival, intensity, duration, was generally decided by the atmospheric factors which are available just before the onset. Hence, for forecasting the future rainfall it is difficult to estimate the probable environmental factors which are the causes of the rainfall. In view of this the researchers are left with the option of predicting it with the previous data set. Abubaker et al [10] have formulated a, four- state Markov model of annual rainfall in Minna with respect to crop production in the region. The present study emphasizes analyzing the annual rainfall data in a three-state model. In this paper the forecasting of annual rainfall pattern for the period of 20 years (2001 to 2020) in Nagapattinam District.

2. Methodology

Nagapattinam district is one of the 38 districts (a coastal district) of Tamilnadu state in southern India. The district lies between northern latitude 10.7906 degrees and 79.8428 degrees Eastern longitude. Since the study was forecasting of rainfall, the annual rainfall data was collected from the statistical department in Nagapattinam from the year 2001 to 2020.

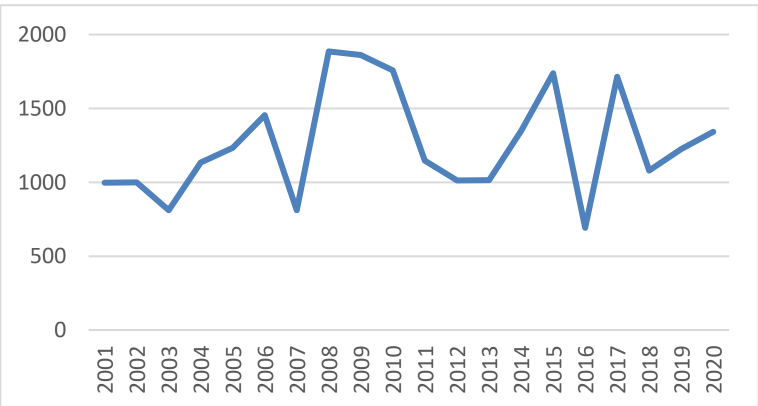

Figure: 1 Annual rainfall in Nagapattinam district from 2001-2020.

From Figure 1 time-series analysis of yearly rainfall is converted into rainfall states prepared by a suitable frequency distribution table. The frequency distribution of each class is specified states of rainfall and is denoted by S1, S2, and S3 in table 1.

SL. No	class interval	Frequency distribution	states
1.	below 1100	8	S_1
2	1100-1600	9	S_2
3	above 1600	3	S_3

Table: 1

3. Markov Chain Modeling

Definition: 3.1 A Markov chain is a random sequence (X_n, n Є N) for all X_o, X_1 ……. X_{n-1} Є I such that

$P(X_n = J_n / X_0 = J_0, X_1 = J_1, X_2 = J_2 \ldots\ldots X_{n-1} = J_{n-1}) = P(X_n = J_n / X_{n-1} = J_{n-1})$ (1)

Definition: 3.2 If a Markov Chain (J_n, n ≥ 0) is homogeneous.

We consider $P(X_n = j / X_{n-1} = i) = P_{ij}$ and we put the matrix P i.e., $P = [P_{ij}]$

The Markov process X has steady state transition probabilities if for any pair of states i, j: The first step transition probabilities P_{ij},

P_{ij}^n denote the n step transition probability.

That is, $P_{ij}^n = P\{X_{n+m} = J / X_m = i\}$ n ≥ 0 all i, j ≥ 0

We have $P_{ij}^1 = P_{ij}$.

$P_{ij}^{n+m} = \sum_{k=0}^{\infty} P_{ik}^n \; P_{kj}^m$ for all n, m ≥ 0 (2)

From (2)

$P^{n+m} = p^{(n)} \times p^{(m)}$

$P^{(2)} = P^{(1+1)} = P^2$

Hence by induction

$P^{(n)} = P^{(n-1+1)} = P^{(n-1)} P^1 = P^n$

We have $P^n = P^0 P^n$ (3)

Here P^0 denote the initial state vector of transition matrix

P^n denote limiting state probability.

Now, let $P^n = [P^n_1, P_2^n, P^n_3]$.

Also let $P^0 = [\, P^0_1 \;\; P^0_2 \;\; P^0_3]$.

Limiting state probabilities: 3.3

The probability distribution $\pi = [\pi_1\, \pi_2 \ldots\ldots \pi_n]$ is called the limiting distribution of the continuous time Markov chain X (t)

if $\pi = (\pi_1 \pi_2\, \pi_3)$ Howard [8]

let n→ ∞ in equation (3)

we get $\pi = \pi P$ (4)

also, $\pi = \sum_{i=1}^{3} \pi i = 1$ Abubakar et al [10]

These equations will be used to find the limiting state equilibrium probabilities.

4. Result and Discussion

Let the model for yearly rainfall is

S_1: less than 1100

S_2: in between 1100-1600

S_3: greater value of 1600.

Therefore, the transition probability matrix

$$P = \begin{bmatrix} P11 & P12 & P13 \\ P21 & P22 & 0 \\ P31 & 0 & 0 \end{bmatrix}$$

The probability of transition matrix

$$P = \begin{vmatrix} 4 & 3 & 1 \\ 4 & 5 & 0 \\ 3 & 0 & 0 \end{vmatrix} \quad (5)$$

i, e. $P_{ij} = \frac{fij}{\sum fij}$ i, j = 1,2,3. Tamil and Samuel (9)

Where $f_{ij} \rightarrow$ transition frequency from state i to state j, $0 \leq P_{ij} \leq 1$.

We get the probability matrix

$$P = \begin{vmatrix} 4/8 & 3/8 & 1/8 \\ 4/9 & 5/9 & 0 \\ 3/3 & 0 & 0 \end{vmatrix},$$

$$P = \begin{vmatrix} 0.5 & 0.375 & 0.125 \\ 0.44 & 0.56 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

n-step transition probability

we have

$$P^2 = \begin{bmatrix} 0.5415 & 0.396 & 0.0625 \\ 0.4689 & 0.4756 & 0.0555 \\ 0.5 & 0.375 & 0.0625 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.5091 & 0.4232 & 0.0442 \\ 0.5011 & 0.4402 & 0.0586 \\ 0.5415 & 0.396 & 0.0625 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0.5101 & 0.4262 & 0.0519 \\ 0.5046 & 0.4326 & 0.0521 \\ 0.5091 & 0.4232 & 0.0442 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 0.5079 & 0.4282 & 0.0509 \\ 0.5071 & 0.4297 & 0.0519 \\ 0.5101 & 0.4262 & 0.0519 \end{bmatrix}$$

$$P^6 = \begin{bmatrix} 0.5079 & 0.4284 & 0.0515 \\ 0.5074 & 0.4290 & 0.0514 \\ 0.5079 & 0.4282 & 0.0509 \end{bmatrix}$$

$$P^7 = \begin{bmatrix} 0.5078 & 0.4286 & 0.0515 \\ 0.5076 & 0.4287 & 0.0515 \\ 0.5079 & 0.4284 & 0.0515 \end{bmatrix}$$

$$P^8 = \begin{bmatrix} 0.5078 & 0.4286 & 0.0515 \\ 0.5077 & 0.4286 & 0.0515 \\ 0.5078 & 0.4286 & 0.0515 \end{bmatrix}$$

$$p^{10} = \begin{bmatrix} 0.5078 & 0.4286 & 0.0515 \\ 0.5078 & 0.4286 & 0.0515 \\ 0.5078 & 0.4286 & 0.0515 \end{bmatrix} \qquad (6)$$

$$p^{10} = \begin{bmatrix} 0.51 & 0.43 & 0.05 \\ 0.51 & 0.43 & 0.05 \\ 0.51 & 0.43 & 0.05 \end{bmatrix} \qquad \text{corrected to 2 decimal places}$$

Limiting state probabilities

After n steps p^n gets the fixed value (6) i, e. $n \geq 10$

Let us take $P^0 = (1 \quad 0 \quad 0)$

$$P^0 . P^n = (100)\begin{bmatrix} 0.507 & 0.4286 & 0.0515 \\ 0.5078 & 0.4286 & 0.0515 \\ 0.5078 & 0.4286 & 0.0515 \end{bmatrix} =$$

$$= (0.5078 \quad 0.4286 \quad 0.0515)$$

$$= (0.51 \quad 0.43 \quad 0.05)$$

From equation (4) and (6), n = nP = (0.51 0.43 0.05).

This result shows that the probabilities of yearly rainfall after ten years. In the first-year probability is (0.5 0.38 0.13) respectively. By the comparison of the probabilities, state 3 dropped slowly and the probability of state 1 and 2 increased in the above ten years. This shows that in the 51% annual rainfall in Nagapattinam will be state 1, 43%will be state 2 and 5% will be state 3.

5. Conclusion

This article show analyzes the yearly rainfall data used by the first-order Markov chain model. Yearly rainfall forecasting of the current year can be used to make a forecast for the following year and in the long run. A yearly rainfall forecasting pattern used to give details about the production of the crops in the Nagapattinam district.

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