Ratio Mathematica Volume 43, 2023

Application of homotopy to the ageing process of human bone

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Abstract

This article presents the ageing process of human bone which can play a major role in the structure of the human body from the concept of homotopy in algebraic topology. This article is about applications. Hence this title. In 2019, William Obeng-Denteh et al., Wrote the literature on the application of homotopy within the framework of algebraic topology on the ageing of the human body. We applied this application, which was generally created for the human body to the ageing of the human bone. The result was good. So we have solved this problem in the literature topologically. Also described by the Cartesian function. As a result of the literature, we have explained the assumption that bone age increases as human age increases with the use of homotopy. The structure of the human bone, which is precisely connected is considered here to be topologically equivalent to a cylinder [9]. The process of continuous ageing bone is considered to be a family of homotopy based on its functions. The study discusses the algebraic topology of homotopes through the homotopy of stable functions of the human bone from infancy to old age.

Keywords: Homomorphism, Homotopy, Homology, Chain complex, Topological Space.

2020 AMS subject classification: 51A10, 55P15, 55R40, 55U15, 18F60.

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§ Received on October 14, 2023. Accepted on February 3, 2023. Published on xxx xx, xxxx. DOI:10.23755/rm.v41i0.874. ISSN: xxxx-xxxx. eISSN: xxxx-xxxx.

1. Introduction

A lot of research articles have been published using the homotopy application. In particular, HPM also known as the homotopy perturbation method, plays in Engineering, Applied maths, Biology, Physics, Quantum Physics and real-world problems. It only began to expand in the 18th and 19th centuries. Homotopy analysis method for solving the biological population sample in 2011 used by Anas. A. M. Arafa et al., [2]. In 2012 F. Guerrero et al., as real-world applications of how smoking has evolved in Spain [4]. In 2019 William Obeng-Denteh et al., published a research paper application of homotopy to the ageing process of the human body [9]. Based on this, in this paper, we have described the ageing process of bone in the human body with homotopy application in algebraic topology.

The growth of human body organs or body growth occurs as a result of genetic, hormonal, dietary and other environmental factors. Bones play an important role in supporting the growing or developing organs. What is our condition if there are no bones? At birth, a baby has about 350 bones in its body and as the baby grows there is a large bone formed by the combination of tiny little bones found in many parts of the body. Physiological research reports that this results in a change in the number of bones reduced to 206. The development of human body organs from infancy to old age is considered a developmental change that does not only aggravate the appearance of the body. The internal organs of the body also continue to mature. The elegant but complex skeletal shape of the human bone is described by the Cartesian functions of $\alpha = S^1 \times I$ and $H, H': \alpha \to \alpha$ [9]. The bone shape of the human body at an early stage. That is the bone shape of the newborn is called $\alpha = S^1 \times I$ homotopy. We will explore the continuous changes between this childhood bone and the bone seen in old age using the properties of homotopy in mathematics. Especially in algebraic topology. For example, vector calculus is used to demonstrate the relationship between an integral line around a simple closure. Homotopy is used to define the surface area of an unaltered $a(\alpha) = 0$ human bone that can be calculated in algebraic topology where α is attached and closed. Thus, if the surface of a bone α is attached and closed $a(\alpha) = 0$. A few developed variations have been derived from algebraic topology. They reflect the connecting properties of spaces or objects.

More complex spatial biological systems are described by topological collection techniques. Spaces are objects connected in mathematics that have been considered for the introduction of topology. Homotopy theory is used to describe the ageing process of unchanged human bone. It offers numerous applications when Fredholm equations use homotopy analysis on integrated equations. The functions of the values of the given t and α parameters are given by selecting the appropriate value for the time parameters t

of the process functions of the ageing bones. In which $t \in [0,1]$. In this study the growth or age of human bone is compared with the age of the bone is the same as the age of the body. The homotopy $H_t(\alpha)$ from $H(\alpha)$ to $H'(\alpha)$ such that $H_0(\alpha) = H(\alpha)$ and $H_N(\alpha) = H'(\alpha)$ is currently the same age as bone. That is the age of humans is one. As the age of bone grows and it is functions change. This is what we call homotopy. This is because continuous transformations take place from one function $H(\alpha)$ to another function $H'(\alpha)$. Thus, we can take $t \in [a, n]$. where $[n > a]$. If we consider the first age as a function, we can refer to it as $H_0(\alpha) = H(\alpha)$ and the old or mature age as $H_N(\alpha) = H'(\alpha)$. The bone undergoes a series of changes from $H_a(\alpha)$ to $H_n(\alpha)$. In this shape, the bone develops as homotopy. That is $H_a(\alpha) = H(\alpha)$ to $H_n(\alpha) = H'(\alpha)$ as the bone age expanse to n. Hence the interval $t \in [a, n]$ the human bone α is closed and connected surface at each change in the variable the age of the human bone α growth from one level to another. For instance, if $a = H_0(\alpha) = H(\alpha) \in \alpha$ and $c = H_1(\alpha) = H'(\alpha) \in \alpha$ then a and c are connected by a line that is a homotopy path $H_t(\alpha)$ given by a continuous map $H: [0,1] \to \alpha$ in α.

2. Homotopy

Definition 2.1. [7] let A, B be topological spaces and H, H' be continuous maps and $H, H': A \to B$ homotopy from H to H' is continuous function $M: A \times [0,1] \to B$; satisfying, $M(\alpha, t) = H(\alpha)$ and $M(\alpha, 1) = H'(\alpha)$ for all $\alpha \in A$. if such a homotopy exists then H is homotopic to H' and it is denoted by $H \simeq H'$.

Definition 2.2. [7] Two paths H and H'. mapping the interval $I = [0,1]$ into A, are said to be path homotopy. If they have the same primary point α_0 and the same endpoint α_1 and if there is a continuous map $M: I \times I \to A$. Such that $M(u, 0) = H(u)$ and $M(u, 1) = H'(u)$. $M(0, t) = a_0$ and $M(1, t) = a_1$ for each $t, u \in I$. Then M is a path homotopy between H and H'. If H is pate homotopic to H' then notation $H \simeq_p H'$.

Definition 2.3. [5] Let A, B be topological spaces and the mapping $: A \to B$. A continuous real-valued function $H(\alpha)$, $\alpha \geq 0$ at any point α_0 in α. If for each neighbourhood $N(\alpha_0)$. Then there exists a neighbourhood $D(\alpha_0)$, such that $H(D) \subseteq N$.

Definition 2.4. [11] Let A, B be topological spaces. Let $H: B \to \mathbb{R}$. We say that H is continuous at β. If for every $\epsilon > 0$, there exists $\delta > 0$ such that $|\alpha - \beta| < \delta$ then $|H(\alpha) - H(\beta)| < \epsilon$ where α, $\beta \in B$. A function $H: A \to B$ is said to be continuous if for each open subset W of B, the set $H^{-1}(W)$ is an open subset of space A.

Definition 2.5. [8] Let A, B be topological spaces. Then the function $H: A \to B$ and H, $H'$are continuous functions from A to B. Then H is homotopic to H'. If there is a continuous family of functions, $H_t: A \to B$ for $0 \le t \le 1$. Then the following conditions are satisfied;

a. $H_0 = H$
b. $H_1 = H'$
c. $H_t(\alpha)$ is continuous both as a function $\alpha \in A$ and $t \epsilon [0, 1]$

Theorem 2.6. [7] Let $H: B \to \mathbb{R}$ and $H': A \to \mathbb{R}$ be two functions. Suppose H and H' are continuous at β, such that $H(B) \subset A$ then $(H' \circ H): B \to \mathbb{R}$ is continuous at β.

Theorem 2.7. [1] A topological space A is a path connected if any two points A could be connected by a line. For instance, if $a = H(0) \in A$ and $c = H(1) \in A$ then a, c are connected by a line H given by a continuous map $H: [0,1] \to A$ in a topological space A.

3. Homology

The fundamental group $\pi_1(x)$ is especially good for low-dimensional spaces. Because it's concerning loops. The definition of objects in the 2 dimensions expresses itself to the maximum. For example, when x is a CW complex then $\pi_1(x)$ depends only on the 2- skeleton of x. Homotopy is the best way to differentiate and construct all dimensions and spher**e**s. However, these high-dimensional homotopy groups have some drawbacks. These are a bit difficult to calculate. There is an alternative to this. It is homology groups. Yes. Homology is a commutative alternative to homotopy. The calculation or definition of homology groups is less explicit than the calculation or definition of homotopy groups. The chain complex is the algebraic structure of the abelian groups that form the image of each homomorphism and the sequence of homomorphisms between each group added to the next kernel homotopy is related to the chain complex.

Definition 3.1. [3] A chain complex $\mathcal{C}$ is a sequence of additive abelian groups and homomorphism's $\longrightarrow \mathcal{C}_{\wp+1} \xrightarrow{\mathfrak{d}_{\wp+1}} \mathcal{C}_{\wp} \xrightarrow{\mathfrak{d}_{\wp}} \mathcal{C}_{\wp-1} \longrightarrow \cdots \ldots \ldots \longrightarrow \mathcal{C}_0 \xrightarrow{\mathfrak{d}_0} 0$. The elements of $\mathcal{C}_{\wp}$ are called $\wp$-chains and these maps are boundary maps. Such that composition of 2 successive homomorphism's zero. That is, $\mathfrak{d}_{\wp} \circ \mathfrak{d}_{\wp+1} = 0$ for each $\wp$. Such a sequence is called a chain complex.

Definition 3.2. [3] The homology group of the chain complex is the quotient group $\mathcal{H}_{\wp} = Ker\, \mathfrak{d}_{\wp} \,/\, Im\ \mathfrak{d}_{\wp+1}$. That is, $\mathcal{H}_{\wp}(\mathcal{C}) = \mathfrak{Z}_{\wp}(\mathcal{C}) / \mathfrak{B}_{\wp}(\mathcal{C})$

Here, $\mathcal{H}_{\wp}$ is the $\wp^{th}$ homology group of $\mathcal{C}$.

Elements of $Ker\ \eth_{\wp}$ are called cycles.

Elements of $Im\ \eth_{\wp+1}$ are called boundaries.

Elements of $\mathcal{H}_{\wp}$ are cosets of $Im\ \eth_{\wp+1}$ called homology classes.

4. Description of ageing human bone function

The inherent properties of the human body can be explored topologically. That is various factors such as the functions, development and transformation of different body parts can be explored using the properties of algebraic topology. We can now compare the ageing functions of bone with the properties of homotopy in algebraic topology at intervals ranging from the age of human bone to the age of onset. The time interval is considered to be $t \in [0, 1]$ and the human bone A. The importance of algebraic invariants such as homotopy that reflect the connectivity of the bone. The structure of human bone $\alpha = S^1 \times I$ and let $\alpha \in A$ and $t \in T$ define the growth of the bone and the age of the bone respectively. Although we take the total duration of human bone to be from early age to final age. We cannot accurately determine the final bone age. That is, we must assume that the functions of the bone will last as long as man exists. So $t = N$ is the final age of the bone.

The topological shape of the bone at the beginning $H_t(\alpha)$ undergoes various changes every year. That is, $H_0(\alpha)$, $H_1(\alpha), H_2(\alpha), \dots \dots, H_N(\alpha)$. So, the full-scale development of this topological shape of the human bone can be referred to as $H_0(\alpha) = H(\alpha)$ and $H_t(\alpha) = H'(\alpha)$. From this, the age of the human which is the duration of the bone is determined as $t \in [0, 1]$. The total period time of a bone is divided into two functions $H,\ H'$ and the total bone life of a human being is assumed to be N. We know the definition of path homotopy. The paths specified in it are given the starting point $H(0, t) = \alpha_0$ and the endpoint $H(1, t) = \alpha_1$ which have the same starting point and ending point even after the paths have undergone various changes and transformations. And the relationship between the two is an equivalence relation. Here it's called path homotopy. Similarly, the homotopic and equivalence relationship between $H_0(\alpha)$ bone in childhood and $H_N(\alpha)$ bone in old age is similar. We can extend the N-period in which the closed attached human bones $\alpha = S^1 \times I$ divided into two functions H and H'. Defined by the equivalence functions of homotopy $H, I, J, K, \dots \dots \dots, H': \alpha \to \alpha$ on the whole interval $I = [0, N]$.

The interval $t = [0, 1]$ indicates the initial age and height or growth of the bone. Since it satisfies the initial condition $H_0(\alpha) = H(\alpha)$ and $H_1(\alpha) = I(\alpha)$ The intervals also change with increasing age.

That is $t = [1, 2]$ there is a continuous function $H_1(\alpha) = I(\alpha)$ and $H_2(\alpha) = J(\alpha)$,

Similarly, $t = [2, 3]$ there is a continuous function $H_2(\alpha) = J(\alpha)$ and $H_3(\alpha) = K(\alpha)$

$t = [3, 4]$ there is continuous function $H_3(\alpha) = L(\alpha)$ and $H_4(\alpha) = M(\alpha)$ etc., these functions are satisfied for the homotopy $H_t(\alpha)$ from $H(\alpha)$ to $H'(\alpha)$. The total lifespan of human bone is $(1 + 2 + 3 + \cdots \ldots N)$ for every function we have $H(\alpha)$, $I(\alpha)$, $J(\alpha)$, $K(\alpha), \ldots \ldots \ldots, H'(\alpha)$ are continuous functions from α to α. And these are homotopy to each other. Strictly speaking, $H(\alpha)$ is homotopic to $I(\alpha)$, $I(\alpha)$ is homotopic to $J(\alpha)$, $J(\alpha)$ is homotopic to $K(\alpha)$ etc. if there is homotopy $H_1(\alpha)$ where $t = [0, N]$ from $H(\alpha)$ to $H'(\alpha)$. Such that; $H_0(\alpha) = H(\alpha)$, $H_1(\alpha) = I(\alpha)$, $H_2(\alpha) = J(\alpha)$, $H_3(\alpha) = K(\alpha), \ldots, H_N(\alpha) = H'(\alpha)$

These are all continuous but undergo many changes at different intervals $I = [0, N]$. Here we see information about the age of the bone found in the body with homotopy. Homotopic functions have been shown to change the shape and age of the bone $t > 0$ each year. That is the function $H_{t=0}(\alpha)$. No matter how many changes occur in bone function or growth, they all occur within a period of $t \in [0, \mathrm{N}]$. To be clear, $t = [0, 1]$ in the first year, $t = [1, 2]$ in the second year, etc., if the homotopy $H_t(\alpha)$ is denoted by $H_0(\alpha)$ and $H_1(\alpha)$. That is, the age of human bone increased from $H_0(\alpha)$ to $H_1(\alpha)$ followed by $H_2(\alpha)$ etc., in this case, if we consider this series of functions available to us in homotopy $H: \alpha \to \alpha$ as a common element it will be available as $H_t(\alpha) = t\alpha$ where $t \in [0, N]$. That is, $H_0(\alpha) = (0.\alpha) = 0$ represents the initial age of the bone. $H_1(\alpha) = (1.\alpha) = \alpha$, $H_2(\alpha) = (2.\alpha) = 2\alpha$, $H_3(\alpha) = (3.\alpha) = 3\alpha$, etc.,where $t = 0, 1, 2, \ldots \ldots$ all are represents the age of the bone.

All of these continuous activities show different stages of bone age and development. The increasing sequence of the function $H_t(\alpha)$ of the bone $\alpha = S^1 \times I$ provides the chains of the bone. In which human bone X is a 2-cell complex and each subsequent growth of each bone forms 2 chains which can be referred to as $\Delta_2(\alpha)$. Thus with each age of the bone, an additional 2 chain complexes are formed. That as the bone ages many 2 chains are formed. These 2 chains are all referred to as $\Delta_2(\alpha) = \sigma$, where σ is a positive integer. Bone α is a closed structure. Hence the set of all closed bones is called the kernel $\xi_2(\alpha) = \sigma$. At this point, the boundary of cell 2 is zero. That is $\pi_2(\alpha) = 0$ Because any 2 successive homomorphisms then the composition to each other we get the zero map. So the homotopy groups $\eta_2(\alpha) = \frac{\xi_2(\alpha)}{\pi_2(\alpha)}$, where $\eta_2(\alpha)$ characterizes the connected 2-cell at each human age level. The homological value of the topologically unchanged $\eta_2(\alpha)$ is the closed and attached bone in the body it changes with age. So the homology theory gives the sequence of α abelian groups of bone H and the sequence of $H: \alpha \to \alpha$ homomorphisms $H_2: \eta_2(\alpha) \to \eta_2(\alpha)$ for the image that follows it. All the maps we have are continuous and homomorphism, the composition of two homotopic functions is homotopic. Therefore homotopic function is obtained by assembling two or more homotopic functions in this ageing process of human bone.

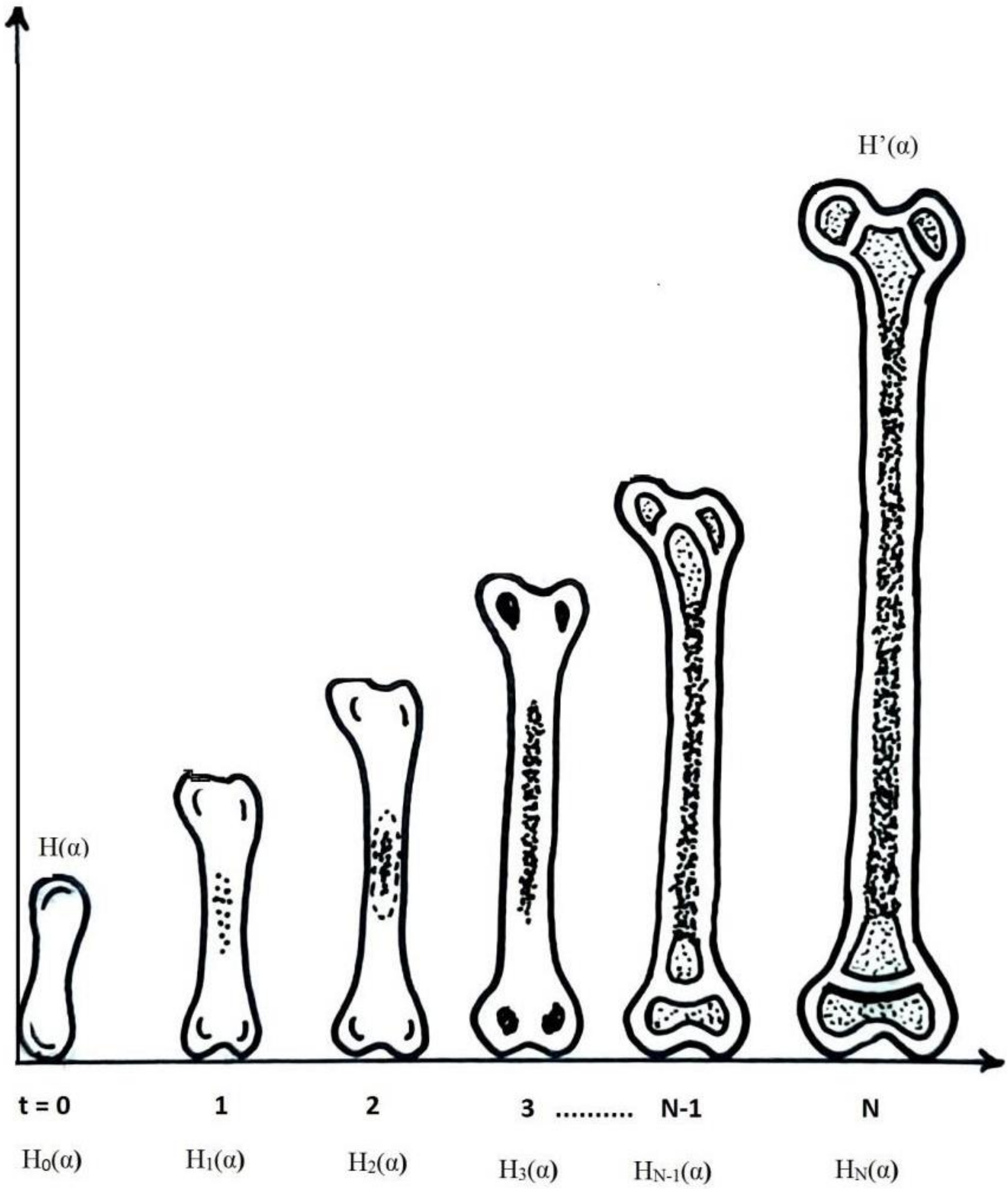

Figure 1. The shape of the bone N-years human from $H(\alpha)$ to $H'(\alpha)$

Remark 4.1. Let H, I, J be the maps and α be the space.

Proposition 4.2. If $H, I: \alpha \to \alpha$ and $I, J: \alpha \to \alpha$ are two continuous functions of the body. Show that $H \sim I$, $I \sim J$ then $(I \circ H) \sim (J \circ I)$.

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Theorem 4.3. Prove that $H_1, H_2 : \mathrm{P} \to \mathrm{T}$ and $H_3, H_4 : \mathrm{T} \to \Sigma$ be continuous and homotopic. Then $(H_3 \circ H_1), (H_4 \circ H_2): \mathrm{P} \to \Sigma$ be continuous and homotopic.

5. Conclusions

In this article, we talked about the ageing process of human bone. This is because the application of homotopy, which is generally accepted for the human body is to know whether it corresponds to a particular body organ. This paper is designed to stabilize human bone growth in the event of physical and to state its function and structure. The age of man increases as the time variable t increases. The age of the bone also increases with age. It also describes the continuous process of $H(\alpha)$ in the ageing process. The concepts of homotopy were very helpful in accessing the ageing function of the human bone. We add the basic definition of homotopy to the definition of homology in the literature to justify the publication of the paper. As a result of the literature, we have explained with the applications of homotopy that bone age increases as human age increases. Since we have included the basic definition of homotopy, the interval is t. So here the ageing process is considered by choosing suitable values for the time parameters t of the functions $H_t(\alpha)$. Hence $H_t(\alpha)$ is increases from $H_0(\alpha) = H(\alpha)$ to $H_n(\alpha) = H'(\alpha)$, $t \in [a, n]$. The homology theory is slightly incorporated here because the human bone $\alpha = S^1 \times I$ is denoted as a 2-cell complex and bone growths are also 2 chains. In the future, the development of homotopy will play an important role in the research of biologically interacting substances or elements that change over time.

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