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Mixed Picture Fuzzy Graph

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Abstract

A new form of picture fuzzy graph has been identified and extended here as Mixed Picture Fuzzy Graph (MPFG). The picture fuzzy set is formed from the fuzzy set and the intuitionistic fuzzy set. It is helpful when there are multiple options, such as yes, no, rejection and abstain. MPFG, which is dependent on the picture fuzzy relation, is defined in this paper. The properties of various types of degrees, order and size of MPFG are examined. Also some types of MPFG such as regular, strong, complete and complement of MPFG are introduced and their properties were analysed. As an application part, the concept of MPFG has been applied in instagram and the result has been discussed here.
Keywords: picture fuzzy graph, MPFG, degree, size & order of MPFG, regular, strong, complete MPFG and complement of MPFG
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1 Introduction

Many decision-making problems in unpredictable environments have been modelled using fuzzy graphs. A variety of generalisations of fuzzy graphs have really been implemented to deal with the uncertainty of complex real-life circumstances. Zadeh's(19) fuzzy set theory played a significant role in decision making in unpredictable environments. Rosenfeld(14), developed the basic conception of fuzzy graph 10 years after Zadeh's seminal article on fuzzy sets. As compared to the graph, the fuzzy graph seems to be a beneficial tool for modelling those problems because it is more efficient, flexible and compatible with any real-world problem. Mordeson & Nair(8) introduced the idea of a complement fuzzy graph, in which Sunitha & Kumar(17) expanded the concept.

The principle of Atanassov's(2) Intuitionistic Fuzzy Set (IFS) allocates a membership and non-membership degree individually, with the sum of the two degrees not exceeding the value one. Shannon and Atanassov proposed a description for intuitionistic fuzzy relations and intuitionistic fuzzy graphs, as well as a list of properties in (16). Different operations on intuitionistic fuzzy graphs were defined by Parvathi et al.(10; 11). Nagoor Gani and Shajitha Begum(9) has characterised about degree, order and size of intuitionistic fuzzy graphs.

The Picture Fuzzy Set (PFS) is a new idea that deals with uncertainties and is a direct continuation of the IFS. It can simulate uncertainty in circumstances including multiple types of answers: yes, abstention, rejection, and no. It is shown about one of the most fundamental concepts of degree of neutrality goes absent from the IFS principle. Cuong & Kreinovich(6) proposed PFS, a direct extension of fuzzy set and IFS that integrates the principle of positive, negative, and neutral membership degree of an element. Cuong(4) investigated some PFS properties and proposed distance measures between them. Phong and Co-authors(13) investigated some picture fuzzy relation compositions. Then, Cuong and Hai(5) extended some fuzzy logic operators for PFSs, including such conjunctions, complements, and disjunctions. Peng & Dai(12) proposed and implemented an algorithmic solution for PFS in a decision-making problem. New concepts of PFG with application was published by Cen Zuo et al.,(3). L. T. Koczy et al.(7) analyzed the study of social networks and Wi-Fi networks using the concept of picture fuzzy graphs. Wei Xiao, Arindam Dey, and Le Hoang Son(18) spoke about their research on regular picture fuzzy graphs and how they can be used in communication networks. Sankar Das and Ganesh Ghorai(15) investigated the creation of a road map based on a multigraph using picture fuzzy information. And Abdelkadir Muzey Mohammed(1) explained about mixed graph representation.

2 Basic Definitions

We stepped over some fundamental definitions in this section that are related to our main concept.

Definition 2.1. (3) Let $G^*_{pf} = (V, \mathcal{E})$ be a graph. A pair $G_{pf} = (A, B)$ is called a *picture fuzzy graph* on G^* where $A = (\mu_A, \eta_A, \nu_A)$ is a picture fuzzy set on V and $B = (\mu_B, \eta_B, \nu_B)$ is a picture fuzzy set on $\mathcal{E} \subseteq$ V $\times$ V such that for each arc $vu \in \mathcal{E}$.

$$\left.\begin{aligned} \mu_B(v,u) \leq \min(\mu_A(v), \mu_A(u)) \\ \eta_B(v,u) \leq \min(\eta_A(v), \eta_A(u)) \\ \nu_B(v,u) \geq \max(\nu_A(v), \nu_B(u)) \end{aligned}\right\} \tag{1}$$

denotes the degree of positive membership, neutral membership & membership membership of the edge $(v, u) \in \mathcal{E}$.

Definition 2.2. (1) A *mixed graph* $G_m = (V, \mathcal{E}, A)$ is a graph consists from set of vertices V, set of undirected edges $\mathcal{E}$ & set of directed edges(or arcs) A.

2.1 Notations

The following mathematical symbols were used throughout the paper:
G_{pf}–picture fuzzy graph
G_m–mixed graph
G_{mpf}–mixed picture fuzzy graph
$\mu_A(v), \eta_A(v), \nu_A(v)$–positive, neutral & negative membership of a vertex v in G_{mpf}
$\mu_B(v,u), \eta_B(v,u), \nu_B(v,u)$–positive, neutral & negative membership of an edge vu in G_{mpf}
$\mu_{\vec{B}}(v,u), \eta_{\vec{B}}(v,u), \nu_{\vec{B}}(v,u)$–positive, neutral & negative membership of an arc $\vec{vu}$ in G_{mpf}
$d(v_i)$–degree of a vertex v_i in G_{mpf}
$\delta(G_{mpf})$–minimum degree of a G_{mpf}
$\Delta(G_{mpf})$–maximum degree of a G_{mpf}
$(G_{mpf})^c$–complement of a G_{mpf}
$(G^c_{mpf})^c$–complement of complement G_{mpf}
$O(G_{mpf})$–order of a G_{mpf}
$S(G_{mpf})$–size of a G_{mpf}
$\mathbb{S}_P$–strength of a path P
H'–subgraph of G_{mpf}
$CD_{mpf}(v_i, v_j)$–circle-distance between v_i and v_j of G_{mpf}
$C(S)$–centrality of a squad

3 Mixed Picture Fuzzy Graph[MPFG]

The popularity of social media sites and networks are growing every day. Positive, neutral & negative membership of a vertex can be classified as good, neutral and bad activities in PFG. The situation now is why we should have to switch from PFG to MPFG? MPFG is the combination of both directed and undirected edges. Many real-life situations take the shape of MPFG. Further a real life problem has been identified and resolved using this MPFG.

Definition 3.1. Let $G^*_{mpf} = (V, \mathcal{E}, \vec{\mathcal{E}})$ be a graph. An ordered triple $G_{mpf} = (A, B, \vec{B})$ is called *mixed picture fuzzy graph* on G^*_{mpf}, where $A = (\mu_A, \eta_A, \nu_A)$ is a picture fuzzy set on V, $B = (\mu_B, \eta_B, \nu_B)$ is a picture fuzzy relation on the undirected edge $\mathcal{E} \subseteq V \times V$ and $\vec{B} = (\mu_{\vec{B}}, \eta_{\vec{B}}, \nu_{\vec{B}})$ is a picture fuzzy relation on the directed edge $\vec{\mathcal{E}} \subseteq V \times V$, which satisfies,

$$\left.\begin{aligned} \mu_B(v,u) &\le \min(\mu_A(v), \mu_A(u)) \\ \eta_B(v,u) &\le \min(\eta_A(v), \eta_A(u)) \\ \nu_B(v,u) &\ge \max(\nu_A(v), \nu_A(u)) \end{aligned}\right\} \forall (v,u) \in \mathcal{E} \quad \&$$

$$\left.\begin{aligned} \mu_{\vec{B}}(v,u) &\le \min(\mu_A(v), \mu_A(u)) \\ \eta_{\vec{B}}(v,u) &\le \min(\eta_A(v), \eta_A(u)) \\ \nu_{\vec{B}}(v,u) &\ge \max(\nu_A(v), \nu_A(u)) \end{aligned}\right\} \forall (v,u) \in \vec{\mathcal{E}} \tag{2}$$

Also $\vec{B}$ must not have a symmetric relation.

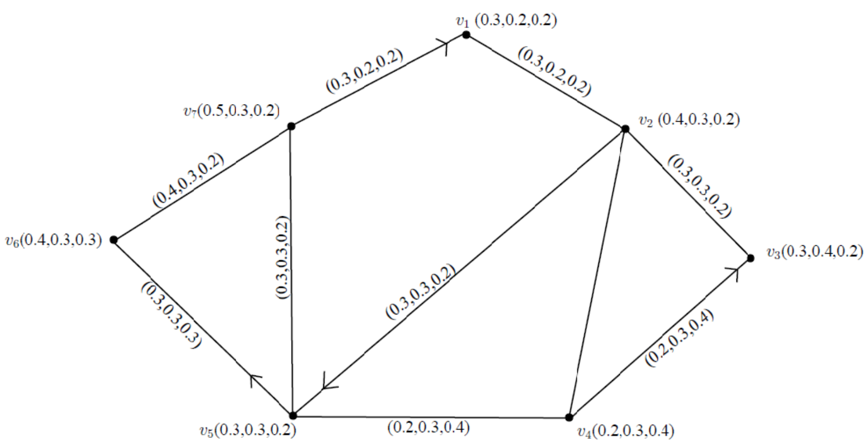

Figure 1: Mixed Picture Fuzzy Graph

Definition 3.2. Consider a graph $H^{'} = (V^{'}, \mathcal{E}^{'}, \overrightarrow{\mathcal{E}}^{'})$ is Mixed Picture Fuzzy Subgraph (MPFSG) of MPFG if $V^{'} \subseteq V, \mathcal{E}^{'} \subseteq \mathcal{E}$ and $\overrightarrow{\mathcal{E}}^{'} \subseteq \overrightarrow{\mathcal{E}}$ if, $\mu^{'}_A(v) \leq \mu_A(v), \eta^{'}_A(v) \leq \eta_A(v), \nu^{'}_A(v) \geq \nu_A(v), \mu^{'}_B(v,u) \leq \mu_B(v,u), \eta^{'}_B(v,u) \leq \eta_B(v,u), \nu^{'}_B(v,u) \geq \nu_B(v,u), \mu^{'}_{\overrightarrow{B}}(v,u) \leq \mu_{\overrightarrow{B}}(v,u), \eta^{'}_{\overrightarrow{B}}(v,u) \leq \eta_{\overrightarrow{B}}(v,u), \nu^{'}_{\overrightarrow{B}}(v,u) \geq \nu_{\overrightarrow{B}}(v,u)$.

Theorem 3.1. *A MPFG is a expandation of IFG.*

Proof. The statement becomes trivial by assuming the neutral membership/abstain is equal to zero. Hence MPFG can reduce to IFG.
Similarly, the statement "A MPFG is a generalization of PFG" is also true. □

Theorem 3.2. *If $V = \{v_1, v_2, ..., v_n\}$ is vertex set of MPFG, $G_{mpf} = (V, \mathcal{E}, \overrightarrow{\mathcal{E}})$. Then total number of edges denoted by $|\mathcal{E}_{mpf}|$ in MPFG G_{mpf} is given by,*

$$|\mathcal{E}_{mpf}| = 1/2\Big[\sum_{v\in V} deg(v) + \sum_{v\in V} deg_{in}(v)\Big] \quad \textit{or}$$
$$|\mathcal{E}_{mpf}| = 1/2\Big[\sum_{v\in V} deg(v) + \sum_{v\in V} deg_{out}(v)\Big]$$

Proof. Let $G_u = (V, \mathcal{E})$ be undirected subgraph of G_{mpf} and $G_d = (V, \overrightarrow{\mathcal{E}})$ with directed edges which are disjoint MPFSGs of MPFG $G_{mpf} = (V, \mathcal{E}, \overrightarrow{\mathcal{E}})$ such that

$$\mathcal{E}_{mpf} = \mathcal{E} \cup \overrightarrow{\mathcal{E}}$$

Handshaking theorem and Elementary counting principle, which states that

$$|\mathcal{E}| = \frac{1}{2}\sum deg(v) \quad \text{and} \quad |\overrightarrow{\mathcal{E}}| = \sum_{v\in V} deg_{in}(v) = \sum_{v\in V} deg_{out}(v) \tag{3}$$

$$|\mathcal{E}_{mpf}| = |\mathcal{E} \cup \overrightarrow{\mathcal{E}}| = |\overrightarrow{\mathcal{E}}| + |\mathcal{E}| - |\mathcal{E} \cap \overrightarrow{\mathcal{E}}| \tag{4}$$

since, $G_u = (V, \mathcal{E})$ and $G_d = (V, \overrightarrow{\mathcal{E}})$ are disjoint MPFSGs,

$$|\mathcal{E} \cap \overrightarrow{\mathcal{E}}| = 0$$

then (4) is reduced to

$$|\mathcal{E}_{mpf}| = |\overrightarrow{\mathcal{E}}| + |\mathcal{E}| \tag{5}$$

substituting (3) in (5), we get

$$|\mathcal{E}_{mpf}| = 1/2\Big[\sum_{v\in V} deg(v) + \sum_{v\in V} deg_{in}(v)\Big] \quad \text{or}$$
$$|\mathcal{E}_{mpf}| = 1/2\Big[\sum_{v\in V} deg(v) + \sum_{v\in V} deg_{out}(v)\Big]$$

$[\because \sum_{v\in V} deg_{in}(v)= \sum_{v\in V} deg_{out}(v)]$

Hence proved $\square$

Definition 3.3. The *degree of a vertex* in a MPFG denoted as,

$$d(v_i) = (d_\mu(v_i), d_\eta(v_i), d_\nu(v_i))$$

where,

$$\left.\begin{aligned} d_\mu(v_i) &= \sum_{v\neq u} \mu_B(v,u) + \frac{1}{2}\Big[\sum_{v\neq u} \mu_{\vec{B}_{in}}(v,u) + \sum_{v\neq u} \mu_{\vec{B}_{out}}(v,u)\Big] \\ d_\eta(v_i) &= \sum_{v\neq u} \eta_B(v,u) + \frac{1}{2}\Big[\sum_{v\neq u} \eta_{\vec{B}_{in}}(v,u) + \sum_{v\neq u} \eta_{\vec{B}_{out}}(v,u)\Big] \\ d_\nu(v_i) &= \sum_{v\neq u} \nu_B(v,u) + \frac{1}{2}\Big[\sum_{v\neq u} \nu_{\vec{B}_{in}}(v,u) + \sum_{v\neq u} \nu_{\vec{B}_{out}}(v,u)\Big] \end{aligned}\right\} \quad (6)$$

From figure 1, we get, $d(v_1)$=(0.45,0.3,0.3), $d(v_2)$=(0.95,0.95,0.9), $d(v_3)$=(0.4,0.45,0.4), $d(v_4)$=(0.5,0.75,1.0), $d(v_5)$=(0.8,0.9,0.85), $d(v_6)$=(0.55,0.45,0.35), $d(v_7)$=(0.45,0.3,0.3), $\delta(G_{mpf})$=(0.4,0.3,0.3) and $\Delta(G_{mpf})$=(0.95,0.95,1.0)

Definition 3.4. Consider $G_{mpf} = (V, \mathcal{E}, \vec{\mathcal{E}})$ be a MPFG. The *neighbourhood of a vertex* is represented as,

$$Nh(v) = (Nh_\mu(v), Nh_\eta(v), Nh_\nu(v))$$

where,

$$\left.\begin{aligned} Nh_\mu(v) &= \{u\in V/\mu_B(v,u) = \min(\mu_A(v),\mu_A(u)), \mu_{\vec{B}}(v,u) = \min(\mu_A(v),\mu_A(u))\} \\ Nh_\eta(v) &= \{u\in V/\eta_B(v,u) = \min(\eta_A(v),\eta_A(u)), \eta_{\vec{B}}(v,u) = \min(\eta_A(v),\eta_A(u))\} \\ Nh_\nu(v) &= \{u\in V/\nu_B(v,u) = \max(\nu_A(v),\nu_A(u)), \nu_{\vec{B}}(v,u) = \max(\nu_A(v),\nu_A(u))\} \end{aligned}\right\} \quad (7)$$

and $Nh[v] = Nh(v) \cup \{v\}$ represents closed neighbourhood of a vertex.

Definition 3.5. The *neighbourhood degree of a vertex* is represented as,

$$d_{Nh}(v) = (d_{Nh_\mu}(v), d_{Nh_\eta}(v), d_{Nh_\nu}(v))$$

where,

$$\left.\begin{aligned} d_{Nh_\mu}(v) &= \sum_{u\in Nh(v)} \mu_A(u) \\ d_{Nh_\eta}(v) &= \sum_{u\in Nh(v)} \eta_A(u) \\ d_{Nh_\nu}(v) &= \sum_{u\in Nh(v)} \nu_A(u) \end{aligned}\right\} \quad (8)$$

Note: If a vertex is an isolated vertex then $Nh(v) = \emptyset$

Definition 3.6. The *closed neighbourhood degree* of a vertex is denoted as,

$$d_{Nh}[v] = (d_{Nh_\mu}[v], d_{Nh_\eta}[v], d_{Nh_\nu}[v])$$

where,

$$\left.\begin{aligned} d_{Nh_\mu}[v] &= \sum_{u \in Nh(v)} \mu_A(u) + \mu_A(v), \\ d_{Nh_\eta}[v] &= \sum_{u \in Nh(v)} \eta_A(u) + \eta_A(v) \\ d_{Nh_\nu}[v] &= \sum_{u \in Nh(v)} \nu_A(u) + \nu_A(v) \end{aligned}\right\} \tag{9}$$

Definition 3.7. A *path* in $G_{mpf} = (A, B, \overrightarrow{B})$ is a distinct vertices sequence $v_0, v_1, v_2, ..., v_k$ one of the succeeding responses are satisfied with both directed & undirected edges,
$\mu_B(v_{i-1}, v_i), \eta_B(v_{i-1}, v_i) > 0$ and $\nu_B(v_{i-1}, v_i) = 0$
$\mu_B(v_{i-1}, v_i), \eta_B(v_{i-1}, v_i) = 0$ and $\nu_B(v_{i-1}, v_i) > 0$
$\mu_B(v_{i-1}, v_i), \eta_B(v_{i-1}, v_i), \nu_B(v_{i-1}, v_i) > 0$
$\mu_{\overrightarrow{B}}(v_{i-1}, v_i), \eta_{\overrightarrow{B}}(v_{i-1}, v_i) > 0$ and $\nu_{\overrightarrow{B}}(v_{i-1}, v_i) = 0$
$\mu_{\overrightarrow{B}}(v_{i-1}, v_i), \eta_{\overrightarrow{B}}(v_{i-1}, v_i) = 0$ and $\nu_{\overrightarrow{B}}(v_{i-1}, v_i) > 0$
$\mu_{\overrightarrow{B}}(v_{i-1}, v_i), \eta_{\overrightarrow{B}}(v_{i-1}, v_i), \nu_{\overrightarrow{B}}(v_{i-1}, v_i) > 0 \qquad i = 1, 2, ..., k.$
Where k denotes the *length of the path*.

Definition 3.8. A MPFG $G_{mpf} = (A, B, \overrightarrow{B})$ seems to be *connected*, if each set of vertices possesses atleast 1 mixed picture fuzzy path connecting them, else it is said to be disconnected.

Definition 3.9. If there is a path $P = v_n, v_1, ..., v_n$ for $n \geq 3$ then it's a *cycle.*

Definition 3.10. The *complement of a* $G_{mpf} = (A, B, \overrightarrow{B})$ is a $G^c_{mpf} = (A^c, B^c, \overrightarrow{B^c})$ iff it follows,
${\mu_A}^c = \mu_A$, ${\eta_A}^c = \eta_A$, ${\nu_A}^c = \nu_A$ and

$$\left.\begin{aligned} \mu^c_B(v,u) &= min(\mu_A(v), \mu_A(u)) - \mu_B(v,u) \\ \eta^c_B(v,u) &= min(\eta_A(v), \eta_A(u)) - \eta_B(v,u) \\ \nu^c_B(v,u) &= max(\nu_A(v), \nu_A(u)) - \nu_B(v,u) \\ {\mu_{\overrightarrow{B}}}^c(v,u) &= min(\mu_A(v), \mu_A(u)) - \mu_{\overrightarrow{B}}(v,u) \\ {\eta_{\overrightarrow{B}}}^c(v,u) &= min(\eta_A(v), \eta_A(u)) - \eta_{\overrightarrow{B}}(v,u) \\ {\nu_{\overrightarrow{B}}}^c(v,u) &= max(\nu_A(v), \nu_A(u)) - \nu_{\overrightarrow{B}}(v,u) \end{aligned}\right\} \tag{10}$$

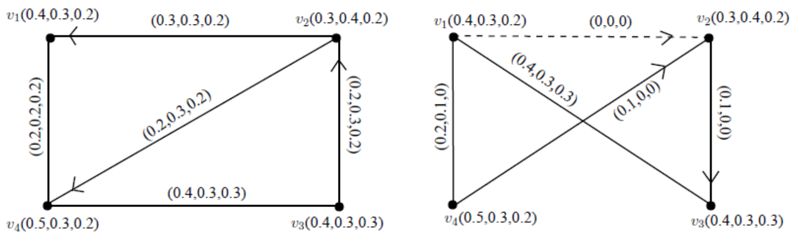

Figure 2: Complement of Mixed Picture Fuzzy Graph

Theorem 3.3. *If G^c_{mpf} be a complement of MPFG, then $(G^c_{mpf})^c = G$*

Note: A MPFG is self-complementary if $(G^c_{mpf})^c = G$

Definition 3.11. The *order of a G_{mpf}* is represented by,

$$O(G_{mpf}) = (O_\mu(G_{mpf}), O_\eta(G_{mpf}), O_\nu(G_{mpf}))$$

where,

$$\left.\begin{aligned} O_\mu(G_{mpf}) &= \sum_{u\in V} \mu_A(v) \\ O_\eta(G_{mpf}) &= \sum_{u\in V} \eta_A(v) \\ O_\nu(G_{mpf}) &= \sum_{u\in V} \nu_A(v) \end{aligned}\right\} \tag{11}$$

Here $O_\mu(G_{mpf})$, $O_\eta(G_{mpf})$ & $O_\eta(G_{mpf})$ are the order of positive, neutral & negative membership degree respectively.

Definition 3.12. Let $G_{mpf} = (A, B, \vec{B})$ is MPFG. The *size of a G_{mpf}* is represented by,

$$\mathcal{S}(G_{mpf}) = (\mathcal{S}_\mu(G_{mpf}), \mathcal{S}_\eta(G_{mpf}), \mathcal{S}_\nu(G_{mpf}))$$

where,

$$\left.\begin{aligned} \mathcal{S}_\mu(G_{mpf}) &= \sum_{v,u\in V} \mu_B(v,u) + \sum_{v,u\in V} \mu_{\vec{B}}(v,u) \\ \mathcal{S}_\eta(G_{mpf}) &= \sum_{v,u\in V} \eta_B(v,u) + \sum_{v,u\in V} \eta_{\vec{B}}(v,u) \\ \mathcal{S}_\nu(G_{mpf}) &= \sum_{v,u\in V} \nu_B(v,u) + \sum_{v,u\in V} \nu_{\vec{B}}(v,u), \forall j \neq i. \end{aligned}\right\} \tag{12}$$

Here $\mathcal{S}_\mu(G_{mpf})$, $\mathcal{S}_\eta(G_{mpf})$ and $\mathcal{S}_\nu(G_{mpf})$ are the size of positive, neutral & negative membership respectively.

Definition 3.13. For a path P,

$$\left.\begin{aligned}\mathbb{S}_\mu &= \min_{v,u\in V}\{\mu_B(v,u)\} + \min_{v,u\in V}\{\mu_{\vec{B}}(v,u)\}\\ \mathbb{S}_\eta &= \min_{v,u\in V}\{\eta_B(v,u)\} + \min_{v,u\in V}\{\eta_{\vec{B}}(v,u)\}\\ \mathbb{S}_\nu &= \max_{v,u\in V}\{\nu_B(v,u)\} + \max_{v,u\in V}\{\nu_{\vec{B}}(v,u)\}\end{aligned}\right\} \tag{13}$$

The *strength of a path* $\mathbb{S}_P = (\mathbb{S}_\mu, \mathbb{S}_\eta, \mathbb{S}_\nu)$.

Definition 3.14. A $G_{mpf} = (A, B, \vec{B})$ is said to be *strong MPFG* if,

$$\left.\begin{aligned}&\mu_B(v,u) = min(\mu_A(v), \mu_A(u))\\ &\eta_B(v,u) = min(\eta_A(v), \eta_A(u))\\ &\nu_B(v,u) = max(\nu_A(v), \nu_A(u)), \forall (v,u) \in \mathcal{E}\ \&\\ &\mu_{\vec{B}}(v,u) = min(\mu_A(v), \mu_A(u))\\ &\eta_{\vec{B}}(v,u) = min(\eta_A(v), \eta_A(u))\\ &\nu_{\vec{B}}(v,u) = max(\nu_A(v), \nu_A(u)), \forall (u,v) \in \vec{\mathcal{E}}.\end{aligned}\right\} \tag{14}$$

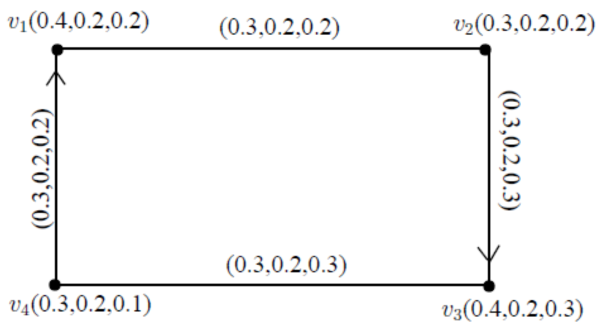

Figure 3: Strong Mixed Picture Fuzzy Graph

Note: $(G^c_{mpf})^c = G_{mpf}$ iff G is strong MPFG

Definition 3.15. *A MPFG $G_{mpf} = (A, B, \vec{B})$ is said to be complete MPFG if,*

$$\left.\begin{aligned}&\mu_B(v,u) = min(\mu_A(v), \mu_A(u))\\ &\eta_B(v,u) = min(\eta_A(v), \eta_A(u))\\ &\nu_B(v,u) = max(\nu_A(v), \nu_A(u))\ \textit{and}\\ &\mu_{\vec{B}}(v,u) = min(\mu_A(v), \mu_A(u))\\ &\eta_{\vec{B}}(v,u) = min(\eta_A(v), \eta_A(u))\\ &\nu_{\vec{B}}(v,u) = max(\nu_A(v), \nu_A(u)), \forall v,u \in V\end{aligned}\right\} \tag{15}$$

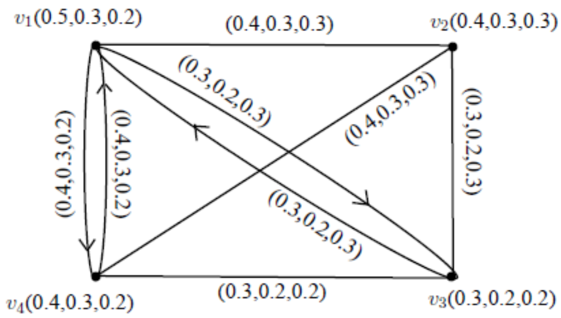

Figure 4: Complete Mixed Picture Fuzzy Graph

Note: Every complete MPFG becomes a strong MPFG. But the contrary, does not have to be true.

Theorem 3.4. *The order of a complete MPFG is equal to the closed neighbourhood degree of every vertex (i.e),* $O_\mu(G_{mpf}) = \{d_{N_\mu}[v]|v \in V\}, O_\eta(G_{mpf}) = \{d_{N_\eta}[v]|v \in V\}, O_\nu(G_{mpf}) = \{d_{N_\nu}[v]|v \in V\}$.

Proof. Consider $G_{mpf} = (V, \mathcal{E}, \vec{\mathcal{E}})$ be a complete MPFG. The μ, η and ν-order of G_{mpf}, is the sum of the positive, neutral and negative membership value of each vertex respectively.
We know that, if G_{mpf} is a complete MPFG, then the closed neighbourhood μ, η and ν-degree of every vertex is the sum of the positive membership, neutral membership & negative membership values of the vertices respectively. Therefore, $O_\mu(G_{mpf}) = \{d_{N_\mu}[v]|v \in V\}, O_\eta(G_{mpf}) = \{d_{N_\eta}[v]|v \in V\}, O_\nu(G_{mpf}) = \{d_{N_\nu}[v]|v \in V\}$. Hence the result. □

Definition 3.16. A MPFG $G_{mpf} = (A, B, \vec{B})$ is defined as *regular MPFG* if,

$$\left.\begin{aligned}
\mu_B(v,u) &= min(\mu_A(v),\mu_A(u)) \text{ and } \sum_{u\neq v}\mu_B(u,v) = constant,\\
\eta_B(v,u) &= min(\eta_A(v),\eta_A(u)) \text{ and } \sum_{u\neq v}\eta_B(u,v) = constant,\\
\nu_B(v,u) &= max(\nu_A(v),\nu_A(u)) \text{ and } \sum_{u\neq v}\nu_B(u,v) = constant,\\
\mu_{\vec{B}}(v,u) &= min(\mu_A(v),\mu_A(u)) \text{ and } \sum_{u\neq v}\mu_{\vec{B}}(u,v) = constant,\\
\eta_{\vec{B}}(v,u) &= min(\eta_A(v),\eta_A(u)) \text{ and } \sum_{u\neq v}\eta_{\vec{B}}(u,v) = constant,\\
\nu_{\vec{B}}(v,u) &= max(\nu_A(v),\nu_A(u)) \text{ and } \sum_{u\neq v}\nu_{\vec{B}}(u,v) = constant.
\end{aligned}\right\} \quad (16)$$

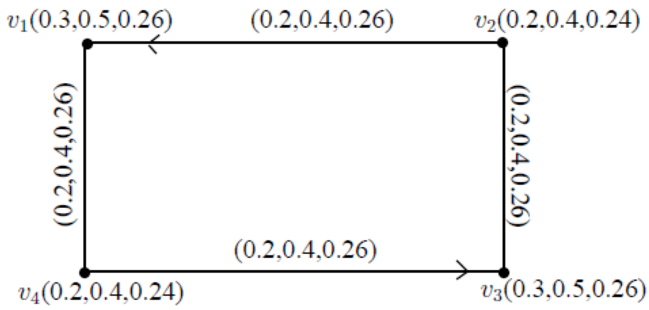

Figure 5: Regular Mixed Picture Fuzzy Graph

Theorem 3.5. *Every complete MPFG is a regular MPFG.*

Proof. Consider $G_{mpf} = (V, \mathcal{E}, \overrightarrow{\mathcal{E}})$ be a MPFG. From the definition of complete MPFG we have, $\mu_B(v,u) = min(\mu_A(v), \mu_A(u)), \eta_B(v,u) = min(\eta_A(v), \eta_A(u))$, $\nu_B(v,u) = max(\nu_A(v), \nu_A(u))$ and $\mu_{\overrightarrow{B}}(v,u) = min(\mu_A(v), \mu_A(u)), \eta_{\overrightarrow{B}}(v,u) = min(\eta_A(v), \eta_A(u))$,
$\nu_{\overrightarrow{B}}(v,u) = max(\nu_A(v), \nu_A(u))\ \forall v, u \in V$.
Then, the closed neighbourhood μ, η and ν-degree of every vertex is the sum of the positive membership, neutral membership & negative membership values of the vertices and itself respectively. As a result, the closed neighbourhood μ-degree, closed neighbourhood η-degree, & closed neighbourhood ν-degree were the same for all vertices. Therefore, min. closed neighbourhood degree is equal to max. closed neighbourhood degree. Hence G_{mpf} is a regular MPFG. □

Definition 3.17. Let $G_{mpf} = (A, B, \overrightarrow{B})$ be a MPFG. If two vertices v & u are linked by a length of a path k in G_{mpf} is $P : v_0, v_1, v_2, ..., v_{n-1}, v_n$ then $\mu_B(v,u), \eta_B(v,u), \nu_B(v,u)$ and $\mu_{\overrightarrow{B}}(v,u), \eta_{\overrightarrow{B}}(v,u), \nu_{\overrightarrow{B}}(v,u)$ are described as follows

$$\begin{aligned}
{\mu_B}^k(v,u) &= \min\{\mu_B(v,v_1), \mu_B(v_1,v_2), ..., \mu_B(v_{k-1},u)\}\\
{\eta_B}^k(v,u) &= \min\{\eta_B(v,v_1), \eta_B(v_1,v_2), ..., \eta_B(v_{k-1},u)\}\\
{\nu_B}^k(v,u) &= \max\{\nu_B(v,v_1), \nu_B(v_1,v_2), ..., \nu_B(v_{k-1},u)\}\\
{\mu_{\overrightarrow{B}}}^k(v,u) &= \min\{\mu_{\overrightarrow{B}}(v,v_1), \mu_{\overrightarrow{B}}(v_1,v_2), ..., \mu_{\overrightarrow{B}}(v_{k-1},u)\}\\
{\eta_{\overrightarrow{B}}}^k(v,u) &= \min\{\eta_{\overrightarrow{B}}(v,v_1), \eta_{\overrightarrow{B}}(v_1,v_2), ..., \eta_{\overrightarrow{B}}(v_{k-1},u)\}\\
{\nu_{\overrightarrow{B}}}^k(v,u) &= \max\{\nu_{\overrightarrow{B}}(v,v_1), \nu_{\overrightarrow{B}}(v_1,v_2), ..., \nu_{\overrightarrow{B}}(v_{k-1},u)\}
\end{aligned}$$

Let $\mu^\infty(v,u), \eta^\infty(v,u), \nu^\infty(v,u)$ is *Strength of connectedness* between the two nodes v & u of MPFG.

$$\mu_B^\infty(v,u) = sup\{\mu_B{}^k(v,u)/k = 1,2,...\}$$
$$\eta_B^\infty(v,u) = sup\{\eta_B{}^k(v,u)/k = 1,2,...\}$$
$$\nu_B^\infty(v,u) = inf\{\nu_B{}^k(v,u)/k = 1,2,...\}$$
$$\mu_{\vec{B}}^\infty(v,u) = sup\{\mu_{\vec{B}}{}^k(v,u)/k = 1,2,...\}$$
$$\eta_{\vec{B}}^\infty(v,u) = sup\{\eta_{\vec{B}}{}^k(v,u)/k = 1,2,...\}$$
$$\nu_{\vec{B}}^\infty(v,u) = inf\{\nu_{\vec{B}}{}^k(v,u)/k = 1,2,...\}$$

here inf has been used to determine the minimum membership value and sup is used to determine the maximum membership value.

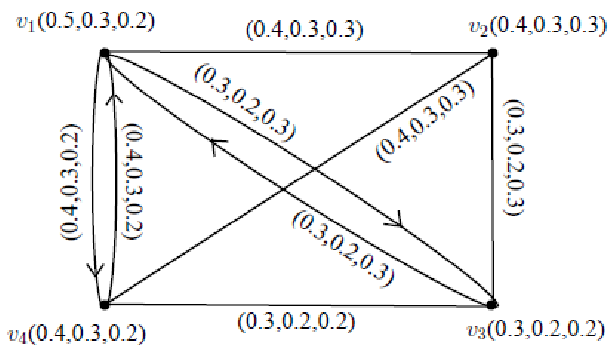

Figure 6: Strength of connectedness

Consider a conneted MPFG as shown in the figure 6
The possible paths between v_1 to v_4 are
$P_1 : v_1 - v_4$ along with the value of membership $(0.4, 0.3, 0.2)$
$P_2 : v_1 - v_2 - v_4$ along with the value of membership $(0.4, 0.3, 0.3)$
$P_3 : v_1 - v_3 - v_4$ along with the value of membership $(0.3, 0.2, 0.3)$
$P_4 : v_1 - v_2 - v_3 - v_4$ along with the value of membership $(0.3, 0.2, 0.3)$
$P_5 : v_1 - v_2 - v_3 - v_4$ along with the value of membership $(0.3, 0.2, 0.3)$
$P_6 : v_1 - v_3 - v_2 - v_4$ along with the value of membership $(0.3, 0.2, 0.3)$
We've arrived to this conclusion through routine calculations,
$\mu^\infty(v_1, v_4) = sup\{0.4, 0.4, 0.3, 0.3, 0.3, 0.3\} = 0.4$
$\eta^\infty(v_1, v_4) = sup\{0.3, 0.3, 0.2, 0.2, 0.2, 0.2\} = 0.3$
$\nu^\infty(v_1, v_4) = inf\{0.2, 0.3, 0.3, 0.3, 0.3, 0.3\} = 0.2$
The *strength of connectedness* between 2 vertices v_1 & v_4 of a MPFG is $(0.4, 0.3, 0.2)$

Definition 3.18. Consider $G_{mpf} = (V, \mathcal{E}, \vec{\mathcal{E}})$ be a MPFG & v, u be any two distinct vertices. In G_{mpf}, eliminating an edge or arc (v, u) decreases the strength between some pair of vertices and is described to as a bridge.

Definition 3.19. Let $G^{'}_{mpf} = (A_1, B_1, \vec{B_1})$ and $G^{''}_{mpf} = (A_2, B_2, \vec{B_2})$ be two MPFGs. A *homomorphism* $h : G^{'}_{mpf} \to G^{''}_{mpf}$ is a mapping function h from V_1 to V_2 if:

- $\mu_{A_1}(v_1) \leq \mu_{A_2}(h(v_1))$
  $\eta_{A_1}(v_1) \leq \eta_{A_2}(h(v_1))$
  $\nu_{A_1}(v_1) \geq \nu_{A_2}(h(v_1))$

- $\mu_{B_1}(v_1, u_1) \leq \mu_{B_2}(h(v_1), h(v_2))$
  $\eta_{B_1}(v_1, u_1) \leq \eta_{B_2}(h(v_1), h(v_2))$
  $\nu_{B_1}(v_1, u_1) \geq \nu_{B_2}(h(v_1), h(v_2)), \forall v_1 \in V_1 \quad \& \quad v_1, u_1 \in E_1$

- $\mu_{\vec{B_1}}(v_1, u_1) \leq \mu_{\vec{B_2}}(h(v_1), h(v_2))$
  $\eta_{\vec{B_1}}(v_1, u_1) \leq \eta_{\vec{B_2}}(h(v_1), h(v_2))$
  $\nu_{\vec{B_1}}(v_1, u_1) \geq \nu_{\vec{B_2}}(h(v_1), h(v_2)), \forall v_1 \in V_1 \quad \& \quad v_1, u_1 \in \vec{E_1}$

Definition 3.20. Let $G^{'}_{mpf} = (A_1, B_1, \vec{B_1})$ and $G^{''}_{mpf} = (A_2, B_2, \vec{B_2})$ be two MPFGs. An *isomorphism* $h : G^{'}_{mpf} \to G^{''}_{mpf}$ is a bijective mapping function h from V_1 to V_2 if:

- $\mu_{A_1}(v_1) = \mu_{A_2}(h(v_1))$
  $\eta_{A_1}(v_1) = \eta_{A_2}(h(v_1))$
  $\nu_{A_1}(v_1) = \nu_{A_2}(h(v_1))$

- $\mu_{B_1}(v_1, u_1) = \mu_{B_2}(h(v_1), h(v_2))$
  $\eta_{B_1}(v_1, u_1) = \eta_{B_2}(h(v_1), h(v_2))$
  $\nu_{B_1}(v_1, u_1) = \nu_{B_2}(h(v_1), h(v_2)), \forall v_1 \in V_1 \quad \& \quad v_1, u_1 \in E_1$

- $\mu_{\vec{B_1}}(v_1, u_1) = \mu_{\vec{B_2}}(h(v_1), h(v_2))$
  $\eta_{\vec{B_1}}(v_1, u_1) = \eta_{\vec{B_2}}(h(v_1), h(v_2))$
  $\nu_{\vec{B_1}}(v_1, u_1) = \nu_{\vec{B_2}}(h(v_1), h(v_2)), \forall v_1 \in V_1 \quad \& \quad v_1, u_1 \in \vec{E_1}$

Theorem 3.6. *Isomorphism of MPFG is an equivalence relation.*

Proof. For show that MPFG isomorphism is an equivalence relation, we must first prove that it is reflexive, symmetric, and transitive.
Reflexive: consider $\theta : G_{mpf} \to G_{mpf}$ is a mapping, therefore θ is an identity function. Hence it is reflexive.
Symmetric: In isomorphic MPFG G_{mpf} & H_{mpf}, there exist a 1-1 correspondence $\theta : G_{mpf} \to H_{mpf}$ which sustains adjacency. From θ is 1-1 correspondence from G_{mpf} to H_{mpf}, here 1-1 correspondence θ^{-1} from H_{mpf} to G_{mpf} which sustains adjacency. Hence isomporphism of MPFG is symmetric.
Transitive: If G_{mpf} is isomorpic to H_{mpf} and H_{mpf} is isomorphic to K_{mpf}, then there are 1-1 correspondences between θ & ϕ from G_{mpf} to H_{mpf} & H_{mpf} to K_{mpf} respectively, which sustains adjacency. It follows $\phi \circ \theta$ is a 1-1 correspondence between from G_{mpf} to K_{mpf} which sustains adjacency. Hence it is transitive.
Therefore, isomorphism of MPFG is an equivalene relation. □

Definition 3.21. Let $G_{mpf} = (A, B, \vec{B})$ be a MPFG. A vertex v of G_{mpf} is said to be *busy vertex* if $\mu_A(v) \leq d_\mu(v), \eta_A(v) \leq d_\eta(v), \nu_A(v) \geq d_\nu(v)$. otherwise, it is called *free vertex.*

Definition 3.22. Let $G_{mpf} = (A, B, \vec{B})$ be a MPFG. Then an edge (v, u) is defined as an *effective edge* iff $\mu_B(v, u) = \min(\mu_A(v), \mu_A(u))$,
$\eta_B(v, u) = \min(\eta_A(v), \eta_A(u)), \nu_B(v, u) = \max(\nu_A(v), \nu_A(u))$,
$\mu_{\vec{B}}(v, u) = \min(\mu_A(v), \mu_A(u)), \eta_{\vec{B}}(v, u) = \min(\eta_A(v), \eta_A(u))$,
$\nu_{\vec{B}}(v, u) = \max(\nu_A(v), \nu_A(u))$.

Note: When all edges in a graph are effective, the graph is complete.

4 Application of MPFG in instagram

Social media has grown gaining popularity in latest years of its user-friendliness. Social media services such as Whatsapp, Facebook, Twitter, and Instagram allow people to communicate across long distances. To put it another way, social media has made the entire globe available at the touch of a button. Social media sites are also valuable resources for public awareness creation, as they rapidly distribute information about natural disasters and terrorist/criminal attacks to a mass audience.

Social network is a collection of vertices and edges. Persons, groups, countries, associations, locations, business and other entities are represented by vertices, while edges define the relationship between vertices. We commonly use a classical graph to describe a social network, with vertices representing persons and edges representing relationships/flows between vertices. Several manuscripts have been shared on social media platforms. However, a classical graph cannot accurately model a social network. Since all vertices in a classical graph are extremely significant. As a result, in today's social networks, every social units (personal or organisational) are given equal weight. In fact, however, not all social units are equal in importance. In a classical graph, all edges (relationships) have the same weight. For example, a person may be well-versed in certain practises. On the other hand, they have no experience of certain activities, and he has a very little knowledge of others. We can easily represent these three kinds (positive, neutral and negative) of vertex and edge membership degrees with a picture fuzzy set, which has three membership values for each element.

In Instagram, we can classify three activities namely good, neutral and bad activities which is represented in PFG as positive, neutral & negative membership values of a vertex. Similarly, edge membership value can be used to describe the strength of relationship between two vertices. Since social media has such a vast number of clients, it also contains mutual and single-sided relationships; it is not restricted to directed or undirected relationships. As a result, we have introduced a mixed picture fuzzy graph which includes both directed and undirected edges. It provides a more accurate result than previous methods. For example, in Instagram an undirected edge exists when two friends have a mutual relationship. Similarly, if a friend-1 follows friend-2 but friend-2 doesn't then there occurs directed edge.

The vertex effect on a social media platform is identified via centrality, which is one of the most significant concepts in social networking. The degree of centrality determines how closely a social squad is linked to other social squads. It essentially provides the social squad's/person's participation in the social network. A vertex's centrality seems more central than that of other vertex's. The centre people are muchis closer to the others and has access to more information. It should be noticed that a person's information is shared by a friend of a friend. However, friends of friends communicate less information than direct friends. As a result, the importance of the relationship gradually decreases as it passes from one member to the next along a connected path.

In MPFG, suppose a friend-1 directly connected to a friend-2, then we say v_1 is circle distance-1(CD-1) friend of v_2. The set of all CD-1 friends of v represented as $cd_1(v)$. i.e., $cd_1(v) = \{v_i \in V; v_i \text{ is a CD-1 friends of } v\}$. Correspondingly, suppose there is a shortest path between v_1 & v_2 with m edges, then v_1 is a CD-m friend of v_2. That is, $cd_m(v) = \{v_i \in V; v_i \text{ is a CD-m friends of } v\}$. Now, consider $cd'_m(v) = cd_m(v) - cd'_{m-1}(v)$, where $m = 2, 3, ...$ and $cd'_m(v) = cd_m(v)$.

CD-1 friends are obviously more significant than CD-2 friends, and CD-2 friends are more significant than CD-3 friends, and so on. The linguistic term "more significant" could be denoted by weights(w_m). Let $0 \leq w_m \leq 1$ have been the weights that gradually decreases, when the CD between the friends increases. Then $w_1 \geq w_2 \geq ... \geq w_m \geq ....$

Let $u_1(=v_i), u_2, u_3, ...u_m(=v_j)$ are the vertices upon the path between v_i and v_j. We have to derive MPFCD $CD_{mpf}(v_i, v_j)$ between v_i and v_j with this path as

$$CD_{mpf}(v_i, v_j) = \sum_{n=1}^{m-1} \mu(u_n, u_{n+1}) + \sum_{n=1}^{m-1} \eta(u_n, u_{n+1}) + \sum_{n=1}^{m-1} \nu(u_n, u_{n+1})$$

There could be several paths connecting two vertices in a networks. Let us assume these paths of equal length whose MPFCD (CD_{mpf}) seems to be the maximum in G_{mpf}. Suppose these are n edgeds in this path of maximum MPFCD, we designate CD^n_{mpf} i.e, $CD^n_{mpf}(v_i, v_j)$ represents the MPFCD between the vertices v_i & v_j in MPFG with the particular path having accurately n edges. We stated that social squad S with atmost CD-p friends.
The centrality C(S) of a social squad is defined as follows:

$$C(S) = \sum_{u_1 \in cd^1_1(v)} w_1 CD^1_{mpf}(V, u_1) + \sum_{u_2 \in cd'_2(v)} w_2 CD^2_{mpf}(V, u_2) + ... + \sum_{u_p \in cd'_p(v)} w_p CD^p_{mpf}(V, u_p) \quad (17)$$

Close friends are valued more than the next closest friends, while the significance of the furthest friend gradually decreases. The significance is established by including the weight w_i, which stands for CD-i friend, I = 1,2,3,... For example, MPFG of 7 people after 7 days is shown in figure 7. Also the link membership values are shown in same figure. In the definition of centrality of a social unit, p can be taken as fixed for a social network. Here we assumed that p = 3 and measure the centrality of social squad. Here, we take $w_1 = 1$ and $w_{i+1} = 1/2w_i$, i = 1,2,...

4.1 Centrality of v_1

Here $cd_1(v_1) = \{v_2, v_4, v_3\} = cd'_1(v_1)$, $cd_2(v_1) = \{v_3, v_5, v_7, v_2\}$, $cd'_2(v_1) = cd_2(v_1) - cd'_1(v_1) = \{v_5, v_7\}$, $cd_3(v_1) = \{v_6, v_7, v_3\}$, $cd'_3(v_1) = cd_3(v_1) - cd'_2(v_1) = \{v_6, v_3\}$.

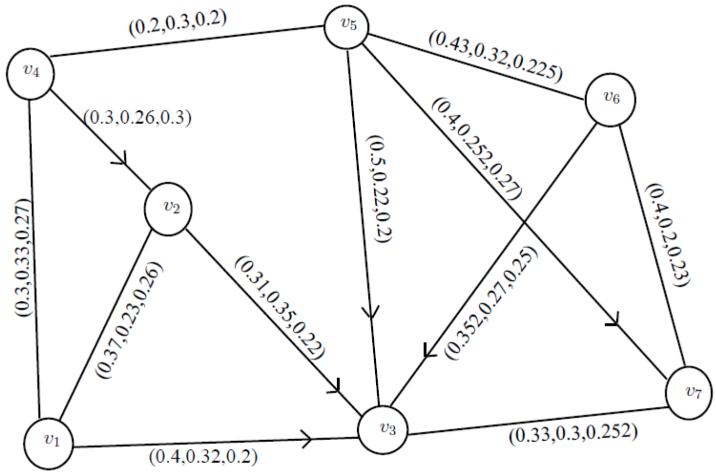

Figure 7: Mixed picture fuzzy network

Now,

$$\sum_{u_1 \in (cd_1)'(v_1)} (CD_{mpf})^1(v_1, u_1) = \{\text{membership values of}(v_1, v_2)\} + \{\text{membership values of}(v_1, v_4)\} + \{\text{membership values of } (v_1, v_3)\}$$
$$= (\mu(v_1, v_2), \eta(v_1, v_2), \nu(v_1, v_2)) + ((\mu(v_1, v_4), \eta(v_1, v_4), \nu(v_1, v_4)) + (\mu(v_1, v_3), \eta(v_1, v_3), \nu(v_1, v_3))$$

$$= (0.37, 0.23, 0.26) + (0.3, 0.33, 0.27) + (0.4, 0.32, 0.2)$$
$$= (1.07, 0.88, 0.73).$$

$$\sum_{u_2 \in (cd_2)'(v_1)} (CD_{mpf})^2(v_1, u_2) = (CD_{mpf})^2(v_1, v_5) + (CD_{mpf})^2(v_1, v_7)$$

$$= \{\text{membership values of}\{(v_1, v_5) + (v_1, v_5)\}\} + \{\text{membership values of}\{(v_1, v_3) + (v_3, v_7)\}\}$$

$$= \{(0.3, 0.33, 0.27) + (0.2, 0.3, 0.2)\} + \{(0.4, 0.32, 0.2) + (0.33, 0.3, 0.252)\}$$
$$= (1.23, 0.25, 0.922).$$

$$\sum_{u_3 \in (cd_3)'(v_1)} (CD_{mpf})^3(v_1, u_3) = (CD_{mpf})^3(v_1, v_6) + (CD_{mpf})^3(v_1, v_3)$$

$$= \{\text{membership values of}\{(v_1, v_3) + (v_3, v_7) + (v_7, v_6)\}\} + \{\text{membership values of}\{(v_1, v_4) + (v_4, v_5) + (v_5, v_3)\}\}$$

$$= \{(0.4, 0.32, 0.2) + (0.33, 0.3, 0.252) + (0.4, 0.2, 0.23)\} + \{(0.3, 0.33, 0.27) + (0.2, 0.3, 0.2) + (0.52, 0.27, 0.25)\}$$

$$= (2.15, 1.72, 1.402).$$

The centrality of v_1 is

$$C(v_1) = \sum_{u_1 \in cd_1'(v_1)} w_1 CD^1_{mpf}(v_1, u_1) + \sum_{u_2 \in cd_2'(v_1)} 0.5 \times CD^2_{mpf}(v_1, u_2) + \sum_{u_3 \in cd_3'(v_1)} 0.25 \times CD^3_{mpf}(v_1, u_3)$$

$$= (2.2225, 1.935, 1.5415)$$

Similarly, we can calculate centralities of other vertices.
$C(v_2)$=(1.8125,1.6125,1.363), $C(v_3)$=(0.985,0.755,0.66975),
$C(v_4)$=(1.715,1.781,1.4775), $C(v_5)$=(2.818,2.317,1.91225),
$C(v_6)$=(2.8005,2.1905,1.824), $C(v_7)$=(3.442,2.33,2.042).

4.1.1 Disscussion

Suppose there are more than one paths between two vertices, we have to choose the shortest distance path to calculate the centrality. From the results, we have centrality of v_3 is comparatively less than other vertices. Because v_3 has less number of mutual friends. So degree of centrality depends on mutual friends and friends of circle distance-i.
Social networks are built on the backs of millions of users and massive amounts of data. We used a simple numerical example of a MPFG to describe a small social network problem in this study. The smaller examples are really useful in understanding the benefits of our suggested model.

5 Conclusion

The prime goal of this paper is to just introduce the terms and concepts of a MPFG and examined the various types of MPFG. Initially, we present a definition of an MPFG built from a picture fuzzy graph in this paper. Few types of degrees were discussed with its properties. We discuss about regular, strong, complete and complement of MPFG are some of the different forms of MPFG. The isomorphic property has also been analysed in MPFG. When comparing to picture fuzzy graph models, the MPFG can boost effectiveness, reliability, flexibility and comparability in modelling complex real-world scenarios. A model has been developed to represent a social network problem using MPFG. The concept of a MPFG can be used to a database system, a computer network, a traffic signal system, a social network, a transportation network and image processing among other things.

6 Acknowledgements

Future work is to develop this concept in the field of transversals of MPFG.

References

[1] Abdelkadir Muzey Mohammed, *Mixed graph representation and mixed graph isomorphism,* Gazi University Journal of Science, 2017, 30(1), 303-310.

[2] K.T.Atanassov, *Intuitionistic fuzzy sets,* Fuzzy Sets and Systems, 1986, 20(1), 87-96.

[3] Cen Zuo, Anita Pal & Arindam Dey, *New Concepts of picture fuzzy graphs with application,* Mathematics, 2019, 7(5), 470, 1-18.

[4] B.C.Cuong, *Picture fuzzy sets,* Journal of Computer Science and Cybernetics, 2014, 30(4), 409-420.

[5] B.C.Cuong & P.V.Hai, *Some fuzzy logic operators for picture fuzzy sets,* Seventh International Conference on Knowledge and Systems Engineering, 2015 132-137.

[6] B.C.Cuong & V.Kreinovich, *Picture fuzzy sets-a new concept for computational intelligence problems,* Third World Congress on Information and Communication Technologies WIICT, 2013, 1-6.

[7] Laszlo T. Koczy, Naeem Jan, Tahir Mahmood & Kifayat Ullah, *Analysis of social networks and Wi-Fi networks by using the concept of picture fuzzy graphs,* Soft Computing - A Fusion of Foundations, Methodologies and Applications, 2020, 24(21), 16551–16563.

[8] J.N.Mordeson & P.R.Nair, *Fuzzy graphs and fuzzy hypergraphs,* 2012, Springer, Berlin, Germany.

[9] A.Nagoor Gani & S.Shajitha Begum, *Degree, order and size in intuitionistic fuzzy graphs,* International Journal of Algorithms Computing and Mathematics, 2010, 3(3), 11-16.

[10] R.Parvathi, M.G.Karunambigai & K.T.Atanassov *Operations on intuitionistic fuzzy graphs,* 2009 IEEE International Conference on Fuzzy Systems, 2009, 1396-1401.

[11] R.Parvathi & M.G.Karunambigai, *Intuitionistic fuzzy graphs,* In Computational Intelligence, Theory and Applications, Springer, Berlin, Germany, 2006, 139-150.

[12] Peng & X.Dai, *Algorithm for picture fuzzy multiple attribute decision-making based on new distance measure,* International Journal for Uncertainty Quantification, 2017, 7(2), 177-187.

[13] P.H.Phong, D.T.Hieu, R.H.Ngan & P.T.Them, *Some compositions of picture fuzzy relations,* Seventh National Conference on Fundamental and Applied Information Technology Research, FAIR'7, Thai Nguyen, 2014, 1-10.

[14] A.Rosenfeld, *Fuzzy graphs, fuzzy sets and their applications,* 1975, Springer, Physica, Heidelberg, New York, USA.

[15] Sankar Das & Ganesh Ghorai, *Analysis of road map design based on multigraph with picture fuzzy information,* International Journal of Applied and Computational Mathematics, 2020, 6(3), 1-17.

[16] A.Shannon & K.T.Atanassov, *On a generalization of intuitionistic fuzzy graphs,* Notes of Intuitionistic Fuzzy Sets, 2006, 12(1), 24-29 .

[17] Sunitha & Vijayakumar, *Complement of a fuzzy graph,* Indian Journal of Pure and Applied Mathematics, 2002, 33(9), 1451-1464.

[18] Wei Xiao, Arindam Dey & Le Hoang Son, *A study on regular picture fuzzy graph with applications in communication networks,* Journal of Intelligent & Fuzzy Systems, 2020, 39(3), 3633-3645.

[19] L.A.Zadeh, *Fuzzy sets,* Information and Control, 1965, 8(3), 338-356.