

Weaker Forms of Nano Irresolute and Its Contra Functions

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Abstract

In this paper the concept of some weaker forms of irresolute and contra irresolute functions in Nano Topological spaces are studied and its related characteristics are discussed. Also we introduced the notion called contra nano alpha irresolute function, contra nano semi irresolute function, contra nano pre irresolute function and its properties are examined. Finally, we have revealed some applications related to recent scenario of online teaching and COVID-19 which can be expressed as nano irresolute functions and contra irresolute functions respectively.

Keywords: Ns-irresolute function, Np-irresolute function, contra N_α -irresolute function, contra Ns-irresolute function, contra Np-irresolute function.

2020 AMS subject classifications: 54B05 ¹

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¹Received on April 18, 2022. Accepted on August 25, 2022. Published on September 25, 2022.

doi: 10.23755/rm.v43i0.764. ISSN: 1592-7415. eISSN: 2282-8214. ©The Authors.

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1 Introduction

In 1982 Pawlak [Pawlak, 1982] investigated about approximate operations, equality and inclusion on sets. In [Crossley and Hildebrand, 1972], irresolute functions was introduced and analysed by Crossley and Hildebrand in topological spaces. Weak and Strong forms of irresolute functions in topology were discussed by Maio and Noiri [Maio and Noiri, 1988]. The conception of nano-topology was initiated by Lellis Thivagar [Thivagar and Richard, 2013b],[M. Lellis Thivagar and Richard, 2013] and [M. Lellis Thivagar and Devi, 2017]. Also in [Thivagar and Richard, 2013a], nano continuous functions, nano interior and nano closure was look over by Lellis and Carmel Richard. Bhuvaneshwari and Ezhilarasi[Bhuvaneshwari and Ezhilarasi, 2016] introduced irresolute maps and semi-generalized irresolute maps in nano topological spaces. New functions called Ns-irresolute and Np-irresolute functions are originated and look into its behaviour in this article. Further the notions called contra $N\alpha$ -irresolute function, contra Ns-irresolute function, contra Np-irresolute function were introduced and examined their properties. Throughout this article we use the notation NTS, N-open, $N\alpha$ -open, Ns-open, Np-open, $N\alpha$ -continuous, Ns-continuous, Np-continuous for "Nano Topological spaces, Nano open, Nano α -open, Nano semi-open, Nano Pre-open sets, Nano α -continuous, Nano semi-continuous, Nano pre-continuous" respectively. Similar notation is used for their respective closed sets.

2 Nano Irresolute Functions

Definition 2.1. Let U_1 and U_2 be NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$. Then $h : U_1 \rightarrow U_2$ is called

1. Ns-irresolute if $h^{-1}(S)$ is Ns-open set in U_1 for each Ns-open set S in U_2 ,
2. Np-irresolute if $h^{-1}(S)$ is Np-open set in U_1 for each Np-open set S in U_2 .

Example 2.1. Take $U_1 = \{w,x,y,z\}$ with $U_1/R = \{\{x,z\},\{y,w\}\}$ and $X = \{x,z\}$. Then $\tau_R(X) = \{U_1, \phi, \{x,z\}\}$. Let $U_2 = \{q,r,s,t\}$ with $U_2/R' = \{\{q\},\{r,s\},\{t\}\}$ and $Y = \{q,t\}$. Then $\tau_{R'}(Y) = \{U_2, \phi, \{q,t\}\}$. We define $h : (U_1, \tau_R(X)) \rightarrow (U_2, \tau_{R'}(Y))$ as $h(x) = q, h(y) = r, h(z) = t, h(w) = s$. Then the inverse image of any Ns-open in U_2 is Ns-open in U_1 and the inverse image of any Np-open in U_2 is Np-open in U_1 . Therefore h is Ns-irresolute and Np-irresolute.

Theorem 2.1. Let U_1 and U_2 be the NTS with reference to $\tau_R(X)$ and $\tau_{R'}(Y)$ and $h : U_1 \rightarrow U_2$ be a mapping. Then the statements given below are equivalent.

1. h is $N\alpha$ -irresolute.

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2. $h^{-1}(S)$ is $N\alpha$ -closed in U_1 , for each $N\alpha$ -closed set S in U_2 .
3. $h(N\alpha cl(S)) \subseteq N\alpha cl(h(S))$ for each $S \subseteq U_1$.
4. $N\alpha cl(h^{-1}(S)) \subseteq h^{-1}(N\alpha cl(S))$ for each $S \subseteq U_2$.
5. $h^{-1}(N\alpha int(S)) \subseteq (N\alpha int(h^{-1}(S)))$ for each $S \subseteq U_2$.
6. h is $N\alpha$ -irresolute for each $x \in U_1$.

Proof. (i) \implies (ii). It is obvious.

(ii) \implies (iii). Let $S \subseteq U_1$. Then, $N\alpha cl(h(S))$ is a $N\alpha$ -closed set of U_2 . By (ii), $h^{-1}(N\alpha cl(h(S)))$ is a $N\alpha$ -closed set in U_1 and $N\alpha cl(S) \subseteq N\alpha cl(h^{-1}h(S)) \subseteq N\alpha cl(h^{-1}(N\alpha cl(h(S)))) = h^{-1}(N\alpha cl(h(S)))$. So $h(N\alpha cl(h(S))) \subseteq N\alpha cl(h(S))$.

(iii) \implies (iv). Let S be a subset of U_2 . By (iii) $h(N\alpha cl(h^{-1}(S))) \subseteq N\alpha cl(hh^{-1}(S)) \subseteq N\alpha cl(S)$. So $N\alpha cl(h^{-1}(S)) \subseteq h^{-1}h(N\alpha cl(h^{-1}(S))) \subseteq h^{-1}(N\alpha cl(S))$.

(iv) \implies (v). Let S be a subset of U_2 . By (iv), $h^{-1}(N\alpha cl(U_2-S)) \supseteq N\alpha cl(h^{-1}(U_2-S)) = N\alpha cl(U_1-h^{-1}(S))$. Since $U_1-N\alpha cl(U_1-S) = N\alpha int(S)$, then $h^{-1}(N\alpha int(S)) = h^{-1}(U_2-N\alpha cl(U_2-S)) = U_1-h^{-1}(N\alpha cl(U_2-S)) \subseteq U_1-N\alpha cl(U_1-h^{-1}(S)) = N\alpha int(h^{-1}(S))$.

(v) \implies (vi). Let S be a $N\alpha$ -open set of U_2 , then $S = N\alpha int(S)$. By (v), $h^{-1}(S) = h^{-1}(N\alpha int(S)) \subseteq N\alpha int(h^{-1}(S)) \subseteq h^{-1}(S)$. So, $h^{-1}(S) = N\alpha int(h^{-1}(S))$. Thus, $h^{-1}(S)$ is a $N\alpha$ -open set of U . Therefore h is $N\alpha$ -irresolute.

(i) \implies (vi). Let h be $N\alpha$ -irresolute, $x \in U_1$ and any $N\alpha$ -open set S of U_2 , such that $h(x) \in S$. Then $x \in h^{-1}(S) = N\alpha int(h^{-1}(S))$. Let $B = h^{-1}(S)$, then B is a $N\alpha$ -open set of U_1 and so $h(B) = hh^{-1}(S) \subseteq S$. Thus h is $N\alpha$ -irresolute for each $x \in U_1$.

(vi) \implies (i). Let S be a $N\alpha$ -open set of U_2 , $x \in h^{-1}(S)$. Then $h(x) \in S$. By hypothesis there exists a $N\alpha$ -open set B of U_1 such that $x \in B$ and $h(B) \subseteq S$. Hence $x \in B \subseteq h^{-1}(h(B)) \subseteq h^{-1}(S)$ and $x \in B = N\alpha int(B) \subseteq N\alpha int(h^{-1}(S))$. So, $h^{-1}(S) \subseteq N\alpha int(h^{-1}(S))$. Hence $h^{-1}(S) = N\alpha int(h^{-1}(S))$. Thus h is $N\alpha$ -irresolute. \square

Theorem 2.2. Let U_1 and U_2 be the NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$ and $h : U_1 \rightarrow U_2$ be a 1-1 and onto function. Then h is $N\alpha$ -irresolute iff $N\alpha int(h(S)) \subseteq h(N\alpha int(S))$ for each $S \subseteq U_1$.

Proof. Let S be any subset of U_1 . By Theorem 2.1 and since h is 1-1 and onto, $h^{-1}(N\alpha int(h(S))) \subseteq N\alpha int(h^{-1}(h(S))) = N\alpha int(S)$. So, $hh^{-1}(N\alpha int(h(S))) \subseteq h(N\alpha int(S))$. Thus $N\alpha int(h(S)) \subseteq h(N\alpha int(S))$.

Conversely, Let S be a $N\alpha$ -open set of U_2 . Then $S = N\alpha int(S)$. By hypothesis, $h(N\alpha int(h^{-1}(S))) \supseteq N\alpha int(h(h^{-1}(S))) = N\alpha int(S) = S$. Thus we get $h^{-1}h(N\alpha int(h^{-1}(S))) \supseteq h^{-1}(S)$. Since h is 1-1 and onto, $N\alpha int(h^{-1}(S)) = h^{-1}h(N\alpha int(h^{-1}(S))) \supseteq h^{-1}(S)$. Hence $h^{-1}(S) = N\alpha int(h^{-1}(S))$. So $h^{-1}(S)$ is $N\alpha$ -open set of U . Thus h is $N\alpha$ -irresolute. \square

Lemma 2.1. Let U_1 be a NTS with respect to $\tau_R(X)$ then

1. $N\alpha cl(S) \subseteq Ncl(S)$ for every subset S of U_1 ,
2. $Ncl(S) = N\alpha cl(S)$ for every α -open subset S of U_1 .

Theorem 2.3. Let U_1 and U_2 be the NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$ and $h : U_1 \rightarrow U_2$ be a $N\alpha$ -irresolute. Then $Ncl(h^{-1}(S)) \subseteq h^{-1}(Ncl(S))$ for every $S \subseteq U_2$.

Proof. Let S be any N-open subset of U_2 . Since h is $N\alpha$ -irresolute and $N\alpha cl(h^{-1}(S))$ is equal to $Ncl(h^{-1}(S))$. By Theorem 2.1, $N\alpha cl(h^{-1}(S)) \subseteq h^{-1}(N\alpha cl(S))$ and by Lemma 2.1 $h^{-1}(N\alpha cl(S)) \subseteq h^{-1}(Ncl(S))$. Then $N\alpha cl(h^{-1}(S)) \subseteq h^{-1}(Ncl(S))$. Therefore $Ncl(h^{-1}(S)) \subseteq h^{-1}(Ncl(S))$. \square

Theorem 2.4. Let U_1 and U_2 be the NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$. Then $h : U_1 \rightarrow U_2$ is a Ns-irresolute iff for each Ns-closed subset $h^{-1}(S)$ is Ns-closed in U_1 .

Proof. If h is Ns-irresolute, then $h^{-1}(B)$ is Ns-open in U_1 for each Ns-open set $B \subseteq U_2$. If S is any Ns-closed subset of U_2 , then $U_2 - S$ is Ns-open. Thus $h^{-1}(U_2 - S)$ is Ns-open in U_1 , but $h^{-1}(U_2 - S) = U_1 - h^{-1}(S)$ so that $h^{-1}(S)$ is Ns-closed in U_1 . Conversely, if for all Ns-closed set $S \subseteq U_2$, $h^{-1}(S)$ is Ns-closed in U_1 and if B is any Ns-open subset of U_2 , then $U_2 - B$ is Ns-closed. Also $h^{-1}(U_2 - B) = U_1 - h^{-1}(B)$ which is Ns-closed in U_1 . Therefore $h^{-1}(B)$ is Ns-open set in U_1 . Hence h is Ns-irresolute. \square

Theorem 2.5. If $h : U_1 \rightarrow U_2$ and $g : U_2 \rightarrow U_3$ is Ns-irresolute(Np-irresolute) then $g \circ h : U_1 \rightarrow U_3$ is Ns-irresolute(Np-irresolute).

Proof. (i) If $A \subseteq U_3$ is Ns-open(Np-open), then $g^{-1}(A)$ is Ns-open(Np-open) set in U_2 because g is Ns-irresolute(Np-irresolute). Consequently since h is Ns-irresolute(Np-irresolute), $h^{-1}(g^{-1}(A)) = (g \circ h)^{-1}(A)$ is Ns-open(Np-open) set in U_1 . Hence $g \circ h$ is Ns-irresolute(Np-irresolute).

Theorem 2.6. If $h : U_1 \rightarrow U_2$ is $N\alpha$ -irresolute(Ns-irresolute, Np-irresolute) and $g : U_2 \rightarrow U_3$ is $N\alpha$ -continuous(Ns-continuous, Np-continuous) then $g \circ h : U_1 \rightarrow U_3$ is $N\alpha$ -continuous(Ns-continuous, Np-continuous).

Proof. Let $S \subseteq U_3$ is N-open. Since g is $N\alpha$ -continuous(Ns-continuous, Np-continuous), $g^{-1}(S)$ is $N\alpha$ -open(Ns-open, Np-open) set in U_2 . Consequently since h is $N\alpha$ -irresolute(Ns-irresolute, Np-irresolute), $h^{-1}(g^{-1}(S)) = (g \circ h)^{-1}(S)$ is $N\alpha$ -open(Ns-open, Np-open) set in U_1 . Hence $g \circ h$ is $N\alpha$ -continuous(Ns-continuous, Np-continuous). \square

Theorem 2.7. Let U_1 and U_2 be the NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$. A function $h : U_1 \rightarrow U_2$ is

1. Ns -irresolute and Np -irresolute then h is $N\alpha$ -irresolute,
2. $N\alpha$ -continuous iff it is Ns -continuous and Np -continuous.

Proof. It is obvious.

3 Nano Contra Irresolute Functions

Here we introduce contra irresolute functions and its characteristics are discussed. The notations used are $NC\alpha$ -open, NCs -open, NCp -open for "Nano contra α -open, Nano contra semi-open, Nano contra pre-open functions" respectively.

Definition 3.1. Let U_1 and U_2 be the NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$. Then $h : U_1 \rightarrow U_2$ is said to be

1. $NC\alpha$ -open if $h(S)$ is $N\alpha$ -closed in U_2 for each N -open S in U_1 ,
2. NCs -open if $h(S)$ is Ns -closed in U_2 for each N -open S in U_1 ,
3. NCp -open if $h(S)$ is Np -closed in U_2 for each N -open S in U_1 .

Example 3.1. 1. Let $U_1 = \{j, k, l\}$ with $U_1/R = \{\{l\}, \{j, k\}\}$ and $X = \{k, l\}$. Then $\tau_R(X) = \{U_1, \phi, \{l\}, \{j, k\}\}$. Let $U_2 = \{x, y, z\}$, $U_2/R' = \{\{y\}, \{x, z\}\}$ and $Y = \{y, z\}$. Subsequently $\tau_{R'}(Y) = \{VU_2, \phi, \{y\}, \{x, z\}\}$. We label $h : (U_1, \tau_R(X)) \rightarrow (U_2, \tau_{R'}(Y))$ as $h(j) = x$, $h(k) = z$, $h(l) = y$. Subsequently $h(S)$ is $N\alpha$ -closed in U_2 for every N -open set S in U_1 . Hence h is $NC\alpha$ -open.

2. Let $U_1 = \{j, k, l, m\}$ with $U_1/R = \{\{j\}, \{l\}, \{k, m\}\}$ and $X = \{j, k\}$. Subsequently $\tau_R(X) = \{U_1, \phi, \{j\}, \{k, m\}, \{j, k, m\}\}$. Let $U_2 = \{p, q, r, s\}$ with $U_2/R' = \{\{p\}, \{s\}, \{q, r\}\}$ and $Y = \{p, r\}$. Subsequently $\tau_{R'}(Y) = \{U_2, \phi, \{p\}, \{q, r\}, \{p, q, r\}\}$. We label $h : (U_1, \tau_R(X)) \rightarrow (U_2, \tau_{R'}(Y))$ as $h(j) = s$, $h(k) = r$, $h(l) = p$, $h(m) = q$. Then $h(S)$ is Ns -closed in U_2 for every N -open set S in U_1 . Hence h is NCs -open.

3. Let $U_1 = \{j, k, l, m\}$ with $U_1/R = \{\{l\}, \{m\}, \{j, k\}\}$ and $X = \{j, l\}$. Subsequently $\tau_R(X) = \{U_1, \phi, \{l\}, \{j, k\}, \{j, k, l\}\}$. Let $U_2 = \{p, q, r, s\}$ with $U_2/R' = \{\{q\}, \{r\}, \{p, s\}\}$ and $Y = \{p, r\}$. Subsequently $\tau_{R'}(Y) = \{U_2, \phi, \{r\}, \{p, s\}, \{p, r, s\}\}$. We define $h : (U_1, \tau_R(X)) \rightarrow (U_2, \tau_{R'}(Y))$ as $h(j) = q$, $h(k) = s$, $h(l) = p$, $h(m) = r$. Then $h(S)$ is Np -closed in U_2 for every N -open set S in U_1 . Hence h is NCp -open.

Definition 3.2. Let U_1 and U_2 be the NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$. Then $h : U_1 \rightarrow U_2$ is said to be $CN\alpha$ -irresolute(CNs -irresolute, CNp -irresolute) if $h^{-1}(S)$ is $N\alpha$ -closed(Ns -closed, Np -closed) set in U_1 for every $N\alpha$ -open set(Ns -open, Np -open) in U_2 .

Example 3.2. 1. Let $U_1 = \{j, k, l, m\}$ with $U_1/R = \{\{j\}, \{k\}, \{l\}, \{m\}\}$ and $X = \{j\}$. Then $\tau_R(X) = \{U_1, \phi, \{j\}\}$. Let $U_2 = \{w, x, y, z\}$ with $U_2/R' = \{\{w\}, \{x\}, \{y\}, \{z\}\}$ and $Y = \{x, y, z\}$. Then $\tau_{R'}(Y) = \{V, \phi, \{x, y, z\}\}$. We label $h : (U_1, \tau_R(X)) \rightarrow (U_2, \tau_{R'}(Y))$ as $h(j) = w, h(k) = x, h(l) = y, h(m) = z$. Then $h^{-1}(S)$ is Ns -closed in U_1 for every Ns -open set S in U_2 . Therefore h is $CN\alpha$ -irresolute and CNs -irresolute.

2. Let $U_1 = \{p, q, r\}$ with $U_1/R = \{\{p\}, \{q, r\}\}$ and $X = \{q, r\}$. Then $\tau_R(X) = \{U_1, \phi, \{q, r\}\}$. Let $U_2 = \{j, k, l\}$ with $U_2/R' = \{\{j\}, \{k, l\}\}$ and $Y = \{j\}$. Then $\tau_{R'}(Y) = \{U_2, \phi, \{j\}\}$. We define $h : (U_1, \tau_R(X)) \rightarrow (U_2, \tau_{R'}(Y))$ as $h(p) = j, h(q) = k, h(r) = l$. Then $h^{-1}(S)$ is Np -closed in U_1 for every Np -open set S in U_2 . So h is CNp -irresolute.

Theorem 3.1. Consider U_1 and U_2 be the NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$. Then $h : U_1 \rightarrow U_2$ is $CN\alpha$ -irresolute iff for each $N\alpha$ -closed subset S of U_2 , $h^{-1}(S)$ is $N\alpha$ -open in U_1 .

Proof. If h is $CN\alpha$ -irresolute, then for each $N\alpha$ -open subset B in U_2 , $h^{-1}(B)$ is $N\alpha$ -closed in U_1 . If S is any $N\alpha$ -closed subset in U_2 , then $U_2 - S$ is $N\alpha$ -open. Thus $h^{-1}(U_2 - S)$ is $N\alpha$ -closed but $h^{-1}(U_2 - S) = U_1 - h^{-1}(S)$ so that $h^{-1}(S)$ is $N\alpha$ -open in U_1 .

Conversely, if, for all $N\alpha$ -closed subsets S of U_2 , $h^{-1}(S)$ is $N\alpha$ -open in U_1 and if B is any $N\alpha$ -open subset of U_2 , then $U_2 - B$ is $N\alpha$ -closed. Also $h^{-1}(U_2 - B) = U_1 - h^{-1}(B)$ is $N\alpha$ -open. Thus $h^{-1}(B)$ is $N\alpha$ -closed in U_1 . Hence h is $CN\alpha$ -irresolute. \square

Corolary 3.1. Let U_1 and U_2 be the NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$. Then $h : U_1 \rightarrow U_2$ is CNs -irresolute(CNp -irresolute) if and only if for each Ns -closed subset(Np -closed subset) S of U_2 , $h^{-1}(S)$ is Ns -open(Np -open) in U_1 .

Theorem 3.2. If the functions $h : U_1 \rightarrow U_2$ and $g : U_2 \rightarrow U_3$ are $CN\alpha$ -irresolute then $g \circ h : U_1 \rightarrow U_3$ is $N\alpha$ -irresolute.

Proof. If $S \subseteq U_3$ is $N\alpha$ -open, then $g^{-1}(S)$ is $N\alpha$ -closed in U_2 because g is $CN\alpha$ -irresolute. Consequently since h is $CN\alpha$ -irresolute, $h^{-1}(g^{-1}(S)) = (g \circ h)^{-1}(S)$ is $N\alpha$ -open set in U_1 , by corollary 4.6. Hence $g \circ h$ is $N\alpha$ -irresolute. \square

Corolary 3.2. If the functions $h : U_1 \rightarrow U_2$ and $g : U_2 \rightarrow U_3$ are CNs -irresolute (CNp -irresolute) then $g \circ h : U_1 \rightarrow U_3$ is Ns -irresolute(Np -irresolute).

Theorem 3.3. *If the function $h : U_1 \rightarrow U_2$ is $CN\alpha$ -irresolute and the function $g : U_2 \rightarrow U_3$ is $NC\alpha$ -continuous then $g \circ h : U_1 \rightarrow U_3$ is $N\alpha$ -continuous.*

Proof. Let $S \subseteq U_3$ is N -open. Since g is $NC\alpha$ -continuous, $g^{-1}(S)$ is $N\alpha$ -closed in U_2 . Consequently since h is $CN\alpha$ -irresolute, $h^{-1}(g^{-1}(S)) = (g \circ h)^{-1}(S)$ is $N\alpha$ -open set in U_1 , by theorem 4.5. Hence $g \circ h$ is $N\alpha$ -continuous. \square

Corollary 3.3. *If the function $h : U_1 \rightarrow U_2$ is CNs -irresolute(CNp -irresolute) and the function $g : U_2 \rightarrow U_3$ is NCs -continuous(NCp -continuous) then $g \circ h : U_1 \rightarrow U_3$ is Ns -continuous(Np -continuous).*

Theorem 3.4. *Let U_1 and U_2 be the NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$. Then $h : U_1 \rightarrow U_2$ is CNs -irresolute and CNp -irresolute then h is $CN\alpha$ -irresolute.*

Proof. It is obvious.

Theorem 3.5. *Let U_1 and U_2 be the NTS with respect to $\tau_R(X)$ and $\tau_{R'}(Y)$. Then $h : U_1 \rightarrow U_2$ is $CN\alpha$ -irresolute then it is $NC\alpha$ -continuous.*

Proof. Consider the N -open set $T \subseteq U_2$. Which implies T is a $N\alpha$ -open set in U_2 . But h is $CN\alpha$ -irresolute So $h^{-1}(T)$ is a $N\alpha$ -closed set in U_1 . It shows that h is $NC\alpha$ -continuous function. \square

4 Applications

Finally, we discuss the application of nano irresolute functions and its contra functions.

Example 4.1. *Advances in technology and some pandemic situations allow students to study entirely online. Consider the impact of e-learning on students characteristics, as a function of, the innovative strategies used in online teaching. Let us consider some of the strategies used in online teaching are powerpoint presentation (P), videos (V), mind map (M), Online Quiz (Q), Group discussion (G) and its impact on students characteristics are Intellectually curious (I), Good time management (T), Self-driven (S), Enhanced Communication skills (C). Let $U_1 = \{P, V, M, Q, G\}$ be the universe of the innovative strategies used in online teaching with $U_1/R = \{\{P, V\}, \{M, G\}, \{Q\}\}$ and $X_1 = \{P, Q\}$. Subsequently $\tau_R(X_1) = \{U_1, \phi, \{Q\}, \{P, V\}, \{P, V, Q\}\}$. Let $U_2 = \{I, T, S, C\}$ be the universe on students characteristics with $U_2/R' = \{\{I, S\}, \{T, C\}\}$ and $X_2 = \{T, C\}$. Then $\tau_{R'}(X_2) = \{U_2, \phi, \{T, C\}\}$. We define $h : (U_1, \tau_R(X_1)) \rightarrow (U_2, \tau_{R'}(X_2))$ as $h(P) = C$, $h(V) = C$, $h(M) = I$, $h(Q) = T$ and $h(G) = S$. Then for every $N\alpha$ -open set in U_2 , inverse image is $N\alpha$ -open set in U_1 and also for every Ns -open set in U_2 , inverse image is Ns -open set in U_1 . Hence h is $N\alpha$ -irresolute*

and N_s -irresolute. Thus, the impact of e-learning on students characteristics, as a function of the innovative strategies used in online teaching, are N_α -irresolute and N_s -irresolute function.

Example 4.2. The main cause of illness is the infectious diseases. However, some initial precautions may help to prevent infections. If not, it leads to serious medical conditions and sometimes to death. Consider the precautionary measures to be adopted to prevent affecting from COVID-19, as a function of, its symptoms. Let the symptoms of COVID-19 are Dry cough (K), Fever (F), Shortness of Breath (B), Loss of Taste/Smell (L) and the precautionary measures to be adopted are Sanitizing (S), Social distancing (D), Wearing mask (M), Boosting Immunity power (I). Let $U_1 = \{K, F, B, L\}$ be the universe of symptoms of COVID-19 with $U_1/R = \{\{K\}, \{F\}, \{B\}, \{L\}\}$ and $X_1 = \{K\}$. Then $\tau_R(X_1) = \{U_1, \phi, \{K\}\}$. Let $U_2 = \{S, D, M, I\}$ be the universe of the precautionary measures to be adopted with $U_2/R' = \{\{S\}, \{D\}, \{M\}, \{I\}\}$ and $X_2 = \{D, M, I\}$. Then $\tau_{R'}(X_2) = \{U_2, \phi, \{D, M, I\}\}$. We define $h : (U_1, \tau_R(X_1)) \rightarrow (U_2, \tau_{R'}(X_2))$ as $h(K) = S$, $h(F) = D$, $h(B) = M$ and $h(L) = I$. Then for every N_α -open set in U_2 , inverse image is N_α -closed set in U_1 and also for every N_s -open set in U_2 , inverse image is N_s -closed set in U_1 . Thus h is contra N_α -irresolute and contra N_s -irresolute. Thus, the precautionary measures to be adopted to prevent affecting from COVID-19, as a function of its symptoms, are contra N_α -irresolute and contra N_s -irresolute function.

5 Conclusions

Through the above discussions we have summarized the conceptualization of irresolute functions and contra irresolute functions in NTS along with examples. Further, We have revealed some applications related to current scenario of online teaching and COVID-19 which can be expressed as nano irresolute functions and contra irresolute functions respectively. Thus these notions can be applied in many real time situations.

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