Ratio Mathematica Volume 42, 2022

Some Edge Domination Parameters in Bipolar Hesitancy Fuzzy Graph

Jahir Hussain Rasheed*
Mujeeburahman Thacharakavil Chemmala †
Dhamodharan Durairaj‡

Abstract

In this article, we establish edge domination in Bipolar Hesitancy Fuzzy Graph(BHFG). Various domination parameters such as inverse edge domination and total edge domination in BHFG are determined. Some theorems related to edge domination and examples are also discussed.
Keywords: Bipolar fuzzy graph; Hesitant fuzzy graph; Edge domination number; Total edge domination; Inverse edge domination;
2020 AMS subject classifications: 05C72, 05C69, 94D05.[1]

*(PG and Research Department of Mathematics, Jamal Mohamed College(Autonomous), Affiliated to Bharathidasan University, Tiruchirapplli-620020, India); hssn_jhr@yahoo.com.
†(PG and Research Department of Mathematics, Jamal Mohamed College(Autonomous), Affiliated to Bharathidasan University, Tiruchirapplli-620020, India); mohdmujeebtc@gmail.com.
‡(PG and Research Department of Mathematics, Jamal Mohamed College(Autonomous), Affiliated to Bharathidasan University, Tiruchirapplli-620020, India); dharan_raj28@yahoo.co.in.
[1]Received on March 26th, 2022. Accepted on June 26th, 2022. Published on June 30th, 2022. doi: 10.23755/rm.v41i0.732. ISSN: 1592-7415. eISSN: 2282-8214.

1 Introduction

The concept of fuzzy sets was first originated by L.A. Zadeh [Zadeh [1965]]. In 1973, Kaufmann established fuzzy graph using Zadeh's fuzzy relation. The domination concept in fuzzy graph was first established by A. Somasundaram and S. Somasundaram [Somasundaram and Somasundaram [1998]]. The edge domination in fuzzy graphs was initiated by S. Velammal and K.Thiagarajan [Vellamal and Thiagarajan [2012]]. The notion of Bipolar Fuzzy Graph(BFG) was established by M.Akram [Akram [2002]].The approach of domination in bipolar fuzzy graphs was proposed by M.G. Karunambigai, Palanivel and Akram [Karunambigai et al. [2013]].The book by Akram, Sarwar and Dudek entitled *"Graphs for the Analysis of Bipolar Fuzzy Information"* [Akram et al. [2021]] is a great tool for understanding the concepts of domination in BFGs. S. Ramya and S. Lavanya developed edge domination in bipolar fuzzy graphs [Ramya and Lavanya [2017]].The notion of hesitant fuzzy sets was first introduced by V.Torra [Torra [2010]] in the year 2010. Hesitancy fuzzy graph, a new approach to fuzzy graph theory was first established by T. Pathinathan,et.al [Pathinathan et al. [2015]].The idea of domination in hesitancy fuzzy graph was investigated by R. Sakthivel et.al,[Sakthivel et al. [2019]]. In the year 2021, K. Anantha Kanaga Jothi and K. Balasangu [Anantha Kanaga Jothi and Balasangu [2021]]defined the idea of irregular and totally irregular bipolar hesitancy fuzzy graphs and some of its properties.

2 Preliminaries

Definition 2.1 (Akram [2002]). *Let $\mathcal{X}$ be a non empty set. A bipolar fuzzy set B in $\mathcal{X}$ is an object having the form $B = \{(x, \mu_B^P(x), \mu_B^N(x)) | x \in \mathcal{X}\}$ where, $\mu_B^P : \mathcal{X} \to [0,1]$ and $\mu_B^N : \mathcal{X} \to [-1,0]$ are mapppings.*

Definition 2.2 (Akram [2002]). *A Bipolar Fuzzy Graph (BFG) is of the form $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where*

1. $\mathcal{V} = \{v_1, v_2, ...v_n\}$ *such that* $\mu_1^P : \mathcal{V} \to [0,1]$ *and* $\mu_1^N : \mathcal{V} \to [-1,0]$
2. $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ *where* $\mu_2^P : \mathcal{V} \times \mathcal{V} \to [0,1]$ *and* $\mu_2^N : \mathcal{V} \times \mathcal{V} \to [-1,0]$ *such that*

$$\mu_2^P(v_i, v_j) \leq min(\mu_1^P(v_i), \mu_1^P(v_j))$$

and

$$\mu_2^N(v_i, v_j) \geq max(\mu_1^N(v_i), \mu_1^N(v_j))$$

for all $(v_i, v_j) \in \mathcal{E}$.

Definition 2.3 (Akram [2002]). *Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a BFG is said to be strong then $\mu_2^P = min(\mu_1^P(v_i), \mu_1^P(v_j))$ and $\mu_2^N = max(\mu_1^N(v_i), \mu_1^N(v_j))$ $\forall v_i, v_j \in \mathcal{V}$.*

Definition 2.4 (Akram [2002]). *Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a BFG is said to be complete then,*

$$\mu_2^P(v_i, v_j) = min(\mu_1^P(v_i), \mu_1^P(v_j))$$

$$\mu_2^N(v_i, v_j) = max(\mu_1^N(v_i), \mu_1^N(v_j))$$

for all $v_i, v_j \in \mathcal{V}$.

Definition 2.5 (Karunambigai et al. [2013]). *An arc (a, b) is said to be strong edge in a BFG, if*

$$\mu_2^P(a, b) \geq (\mu_2^P)^\infty(a, b) \text{ and } \mu_2^N(a, b) \geq (\mu_2^N)^\infty(a, b)$$

whereas $(\mu_2^P)^\infty(a, b) = max\{(\mu_2^P)^k(a, b)|k = 1, 2, ..., n\}$ and $(\mu_2^N)^\infty(a, b) = min\{(\mu_2^N)^k(a, b)|k = 1, 2, ..., n\}$.

Definition 2.6 (Karunambigai et al. [2013]). *Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a BFG, then cardinality of $\mathcal{G}$ is defined as*

$$|\mathcal{G}| = \sum_{v_i \in \mathcal{V}} \frac{(1 + \mu_1^P(v_i) + \mu_1^N(v_i))}{2} + \sum_{(v_i, v_j) \in \mathcal{E}} \frac{(1 + \mu_2^P(v_i, v_j) + \mu_2^N(v_i, v_j))}{2}$$

.

Definition 2.7 (Karunambigai et al. [2013]). *The cardinality of $\mathcal{V}$, i.e., amount of nodes is termed as the order of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and is signified by $|\mathcal{V}|$(or $O(\mathcal{G})$) and determined by*

$$O(\mathcal{G}) = |\mathcal{V}| = \sum_{v_i \in \mathcal{V}} \frac{(1 + \mu_1^P(v_i) + \mu_1^N(v_i))}{2}$$

The no. of elements in a set of S, i.e., amount of edges is termed as size of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and signified as $|S|$(or $S(\mathcal{G})$) and determined by

$$S(\mathcal{G}) = |S| = \sum_{(v_i, v_j) \in \mathcal{E}} \frac{(1 + \mu_2^P(v_i, v_j) + \mu_2^N(v_i, v_j))}{2}$$

for all $(v_i, v_j) \in \mathcal{E}$.

3 Bipolar hesitancy fuzzy graph

Definition 3.1 (Anantha Kanaga Jothi and Balasangu [2021]). *Let $\mathcal{X}$ be a non-empty set. A Bipolar hesitancy fuzzy set*

$$\mathtt{B} = \{x, \mu_1^P(x), \mu_1^N(x), \gamma_1^P(x), \gamma_1^N(x), \beta_1^P(x), \beta_1^N(x)/x \in \mathcal{X}\}$$

where $\mu_1^P, \gamma_1^P, \beta_1^P : \mathcal{X} \to [0,1]$ and $\mu_1^N, \gamma_1^N, \beta_1^N : \mathcal{X} \to [-1,0]$ are mappings such that,

$$0 \leq \mu_1^P(x) + \gamma_1^P(x) + \beta_1^P(x) \leq 1$$

and

$$-1 \leq \mu_1^N(x) + \gamma_1^N(x) + \beta_1^N(x) \leq 0$$

.

Definition 3.2 (Anantha Kanaga Jothi and Balasangu [2021]). *Let $\mathcal{X}$ be a non empty set.Then we call mappings $\mu_2^P, \gamma_2^P, \beta_2^P : \mathcal{X} \times \mathcal{X} \to [0,1]$, $\mu_2^N, \gamma_2^N, \beta_2^N : \mathcal{X} \times \mathcal{X} \to [-1,0]$ are bipolar hesitancy fuzzy relation on $\mathcal{X}$ such that,*
$\mu_2^P(x,y) \leq \mu_1^P(x) \wedge \mu_1^P(y); \mu_2^N(x,y) \geq \mu_1^N(x) \vee \mu_1^N(y)$; $\gamma_2^P(x,y) \leq \gamma_1^P(x) \wedge \gamma_1^P(y); \gamma_2^N(x,y) \geq \gamma_1^N(x) \vee \gamma_1^N(y)$; $\beta_2^P(x,y) \leq \beta_1^P(x) \wedge \beta_1^P(y); \beta_2^N(x,y) \geq \beta_1^N(x) \vee \beta_1^N(y)$.

Definition 3.3 (Anantha Kanaga Jothi and Balasangu [2021]). *A bipolar hesitancy fuzzy relation A on $\mathcal{X}$ is called symmetric relation if $\mu_2^P(x,y) = \mu_2^P(x,y)$, $\mu_2^N(x,y) = \mu_2^N(x,y)$,$\gamma_2^P(x,y) = \gamma_2^P(x,y)$, $\gamma_2^N(x,y) = \gamma_2^N(x,y)$, $\beta_2^P(x,y) = \beta_2^P(x,y)$, $\beta_2^N(x,y) = \beta_2^N(x,y)$ for all $(x,y) \in \mathcal{X}$*

Definition 3.4 (Pathinathan et al. [2015]). *A Hesitancy fuzzy graph is of the form $G = (V, E)$ where,*
$V = \{v_!, v_2, \dots v_n\}$ such that $\mu_1, \gamma_1, \beta_1 : V \to [0,1]$ denote the degree of membership, non-membership and hesitancy of the vertex $v_i \in V$ respectively and $\mu_1(v_i) + \gamma_1(v_i) + \beta_1(v_i) = 1$ for every $v_i \in V$ where $\beta_1(v_i) = 1 - [\mu_1(v_i) + \gamma_1(v_i)]$ and
$E \subseteq V \times V$ where $\mu_2, \gamma_2, \beta_2 : V \times V \to [0,1]$ denote the degree of membership, non-membership and hesitancy of the edge $(v_i, v_j) \in E$ respectively such that,
$\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$; $\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j)$; $\beta_2(v_i, v_j) \leq \beta_1(v_i) \wedge \beta_1(v_j)$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) + \beta_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.

Definition 3.5 (Anantha Kanaga Jothi and Balasangu [2021]). *A Bipolar Hesitancy Fuzzy Graph (BHFG) is of the form $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ where*

(i) $\mathtt{V} = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1^P, \gamma_1^P, \beta_1^P : \mathtt{V} \to [0,1]$ denote the degree of positive membership, positive non-membership and positive hesitancy of

the vertex $v_i \in \mathtt{V}$ *respectively,* $\mu_1^N, \gamma_1^N, \beta_1^N : \mathtt{V} \to [-1, 0]$ *denote the degree of negative membership,negative non-membership and negative hesitancy of the vertex* $v_i \in \mathtt{V}$*. For every* $v_i \in \mathtt{V}$*,*
$\mu_1^P(v_i) + \gamma_1^P(v_i) + \beta_1^P(v_i) = 1$ *and* $\mu_1^N(v_i) + \gamma_1^N(v_i) + \beta_1^N(v_i) = -1$
$\beta_1^P(v_i) = 1 - [\mu_1^P(v_i) + \gamma_1^P(v_I)]$ *and* $\beta_1^N(v_i) = -1 - [\mu_1^N(v_i) + \gamma_1^N(v_i)]$

(ii) $\mathtt{E} \subseteq \mathtt{V} \times \mathtt{V}$ *where,* $\mu_2^P, \gamma_2^P, \beta_2^P : \mathtt{V} \times \mathtt{V} \to [0, 1]$*;* $\mu_2^N, \gamma_2^N, \beta_2^N : \mathtt{V} \times \mathtt{V} \to [-1, 0]$ *are mappings such that*

$$\mu_2^P(v_i, v_j) \leq \mu_1^P(v_i) \wedge \mu_1^P(v_j)$$

$$\mu_2^N(v_i, v_j) \geq \mu_1^N(v_i) \vee \mu_1^N(v_j)$$

$$\gamma_2^P(v_i, v_j) \leq \gamma_1^P(v_i) \vee \gamma_1^P(v_j)$$

$$\gamma_2^N(v_i, v_j) \geq \gamma_1^N(v_i) \wedge \gamma_1^N(v_j)$$

$$\beta_2^P(v_i, v_j) \leq \beta_1^P(v_i) \wedge \beta_1^P(v_j)$$

$$\beta_2^N(v_i, v_j) \geq \beta_1^N(v_i) \vee \beta_1^N(v_j)$$

denote the degree of positive, negative membership, degree of positive, negative non membership and degree of positive, negative hesitancy of the edge $(v_i, v_j) \in \mathtt{E}$ *respectively and*

$$0 \leq \mu_2^P(v_i, v_j) + \gamma_2^P(v_i, v_j) + \beta_2^P(v_i, v_j) \leq 1$$

,

$$-1 \leq \mu_2^N(v_i, v_j) + \gamma_2^N(v_i, v_j) + \beta_2^N(v_i, v_j) \leq 0$$

for every $(v_i, v_j) \in \mathtt{E}$*.*

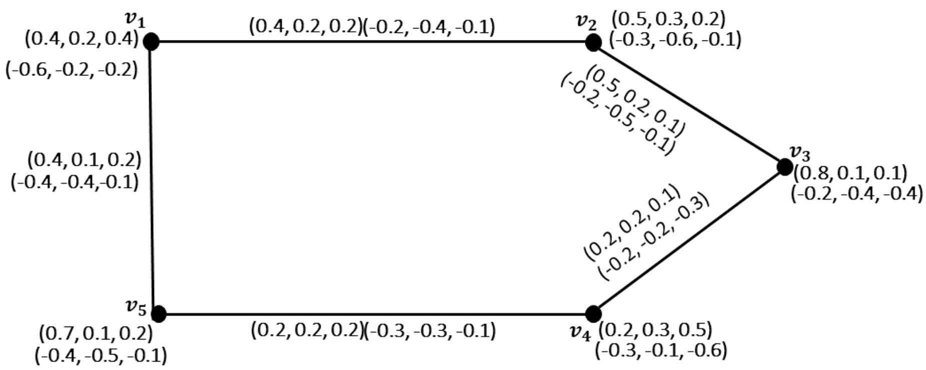

Figure 1: Bipolar Hesitancy Fuzzy Graph

Example 3.1. *From Fig 1, for vertex v_1,*
$\mu_1^P(v_1) + \gamma_1^P(v_1) + \beta_1^P(v_1) = 0.4 + 0.2 + 0.4 = 1$
$\mu_1^N(v_1) + \gamma_1^N(v_1) + \beta_1^N(v_1) = -0.6 - 0.2 - 0.2 = -1.$
For edge (v_1, v_2); $\mu_2^P(v_1, v_2) + \gamma_2^P(v_1, v_2) + \beta_2^P(v_1, v_2) = 0.8 \leq 1$
$\mu_2^N(v_1, v_2) + \gamma_2^N(v_1, v_2) + \beta_2^N(v_1, v_2) = -0.7 \geq -1.$

Definition 3.6. *A Bipolar Hesitancy Fuzzy Graph $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ is said to be complete when, $\mu_2^P(v_i, v_j) = \mu_1^P(v_i) \wedge \mu_1^P(v_j)$,$\mu_2^N(v_i, v_j) = \mu_1^N(v_i) \vee \mu_1^N(v_j)$, $\gamma_2^P(v_i, v_j) = \gamma_1^P(v_i) \vee \gamma_1^P(v_j)$, $\gamma_2^N(v_i, v_j) = \gamma_1^N(v_i) \wedge \gamma_1^N(v_j)$, $\beta_2^P(v_i, v_j) = \beta_1^P(v_i) \wedge \beta_1^P(v_j)$, $\beta_2^N(v_i, v_j) = \beta_1^N(v_i) \vee \beta_1^N(v_j)$ for every $v_i, v_j \in \mathtt{V}$.*

Definition 3.7. *A Bipolar Hesitancy Fuzzy Graph $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ is said to be strong when, $\mu_2^P(v_i, v_j) = \mu_1^P(v_i) \wedge \mu_1^P(v_j)$,$\mu_2^N(v_i, v_j) = \mu_1^N(v_i) \vee \mu_1^N(v_j)$ $\gamma_2^P(v_i, v_j) = \gamma_1^P(v_i) \vee \gamma_1^P(v_j)$, $\gamma_2^N(v_i, v_j) = \gamma_1^N(v_i) \wedge \gamma_1^N(v_j)$, $\beta_2^P(v_i, v_j) = \beta_1^P(v_i) \wedge \beta_1^P(v_j)$, $\beta_2^N(v_i, v_j) = \beta_1^N(v_i) \vee \beta_1^N(v_j)$ for every $(v_i, v_j) \in \mathtt{E}$.*

Definition 3.8. *Let $\mathtt{G}$ be a Bipolar hesitancy fuzzy graph. The neighbourhood of a vertex x in $\mathtt{G}$ is defined by*

$$N(x) = (N_\mu^P(x), N_\mu^N(x), N_\gamma^P(x), N_\gamma^N(x), N_\beta^P(x), N_\beta^N(x))$$

where
$N_\mu^P(x) = \{y \in \mathtt{V}/\mu_2^P(x, y) \leq \mu_1^P(x) \wedge \mu_1^P(x)\}$; $N_\mu^N(x) = \{y \in \mathtt{V}/\mu_2^N(x, y) \geq \mu_1^N(x) \vee \mu_1^N(x)\}$;$N_\gamma^P(x) = \{y \in \mathtt{V}/\gamma_2^P(x, y) \leq \gamma_1^P(x) \wedge \gamma_1^P(x)\}$; $N_\gamma^N(x) = \{y \in \mathtt{V}/\gamma_2^N(x, y) \geq \gamma_1^N(x) \vee \gamma_1^N(x)\}$;$N_\beta^P(x) = \{y \in \mathtt{V}/\beta_2^P(x, y) \leq \beta_1^P(x) \wedge \beta_1^P(x)\}$; $N_\beta^N(x) = \{y \in \mathtt{V}/\beta_2^N(x, y) \geq \beta_1^N(x) \vee \beta_1^N(x)\}$.

Definition 3.9. *Let $\mathtt{G}$ be a Bipolar Hesitancy Fuzzy Graph. The neighborhood degree of a vertex x in $\mathtt{G}$ is defined by*

$$deg(x) = [\deg \mu^P(x), \deg \mu^N(x), \deg \gamma^P(x), \deg \gamma^N(x), \deg \beta^P(x), \deg \beta^N(x)]$$

$y \in \mathtt{V}$, where

$$\deg \mu^P(x) = \sum_{y \in N(x)} \mu_1^P(y), \deg \mu^N(x) = \sum_{y \in N(x)} \mu_1^N(y), \deg \gamma^P(x) = \sum_{y \in N(x)} \gamma_1^P(y)$$

$$\deg \gamma^N(x) = \sum_{y \in N(x)} \gamma_1^N(y), \deg \beta^P(x) = \sum_{y \in N(x)} \beta_1^P(y), \deg \beta^N(x) = \sum_{y \in N(x)} \beta_1^N(y)$$

.

Definition 3.10. *Let $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ be a BHFG. The edge cardinality of $\mathtt{G}$ is given by,* $|\mathtt{E}| = r$

$$= \sum_{(u,v) \in \mathtt{E}} \frac{3 + \mu_2^P(u, v) + \mu_2^N(u, v) + \gamma_2^P(u, v) + \gamma_2^N(u, v) + \beta_2^P(u, v) + \beta_2^N(u, v)}{3}$$

.

Definition 3.11. *An Arc (u,v) is said to be strong edge in BHFG. then,*
$\mu_2^P(u,v) \geq (\mu_2^P)^\infty(u,v)$,$\mu_2^N(u,v) \geq (\mu_2^N)^\infty(u,v)$,$\gamma_2^P(u,v) \geq (\gamma_2^P)^\infty(u,v)$,
$\gamma_2^N(u,v) \geq (\gamma_2^N)^\infty(u,v)$,$\beta_2^P(u,v) \geq (\beta_2^P)^\infty(u,v)$, $\beta_2^N(u,v) \geq (\beta_2^N)^\infty(u,v)$
whereas $(\mu_2^P)^\infty(u,v) = max\{(\mu_2^P)^k(u,v)|k = 1,2,\ldots,n\}$;
$(\mu_2^N)^\infty(u,v) = min\{(\mu_2^N)^k(u,v)|k = 1,2,\ldots,n\}$;
$(\gamma_2^P)^\infty(u,v) = max\{(\gamma_2^P)^k(u,v)|k = 1,2,\ldots,n\}$;
$(\gamma_2^N)^\infty(u,v) = min\{(\gamma_2^N)^k(u,v)|k = 1,2,\ldots,n\}$;
$(\beta_2^P)^\infty(u,v) = max\{(\beta_2^P)^k(u,v)|k = 1,2,\ldots,n\}$;
$(\beta_2^N)^\infty(u,v) = min\{(\beta_2^P)^k(u,v)|k = 1,2,\ldots,n\}$.

4 Edge domination in bipolar hesitancy fuzzy graph

Definition 4.1. *Let* $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ *be a Bipolar Hesitancy Fuzzy Graph. A set* $\mathcal{S} \subseteq \mathtt{E}$ *is said to be an edge dominating set of* $\mathtt{G}$ *if every edge not in* $\mathcal{S}$ *is incident to some edge in* $\mathcal{S}$.

Definition 4.2. *An edge dominating set* $\mathcal{S} \subseteq \mathtt{E}$ *is said to be minimal if no proper subset of* $\mathcal{S}$ *is an edge dominating set.*

Definition 4.3. *The minimum cardinality out of all minimal dominating sets of BHFG* $\mathtt{G}$ *is said to be lower domination number of* $\mathtt{G}$ *and denoted as* $d_{bh}(\mathtt{G})$.

Definition 4.4. *The maximum cardinality out of all minimal dominating sets of BHFG* $\mathtt{G}$ *is said to be upper domination number of* $\mathtt{G}$ *and denoted as* $D_{bh}(\mathtt{G})$.

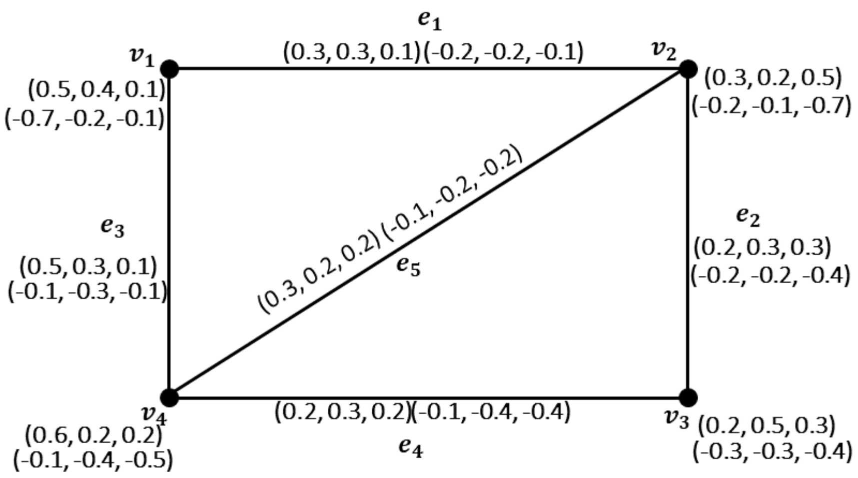

Figure 2: Edge domination in BHFG

Example 4.1. *In the above figure 2,* $\{e_1, e_2, e_4, e_5\}$,$\{e_2, e_3, e_5\}$,$\{e_1, e_3, e_4\}$ *are edge dominating sets of* $\mathtt{G}$.$\{e_1, e_4\}$ $\{e_3, e_2\}$,$\{e_5\}$ *are minimal edge dominating sets of* $\mathtt{G}$. *Among all the minimal dominating sets,* $\{e_5\}$ *has minimum cardinality and edge domination number* $\gamma_{bh}(\mathtt{G}) = 1.06$.

Theorem 4.1. *Let* $\mathcal{S}$ *be a minimal edge dominating set of a BHFG* $\mathtt{G} = (\mathtt{V}, \mathtt{E})$. *if for any edge* $\mathtt{e} \in \mathcal{S}$, *one of the following condition hold*
a) $N(\mathtt{e}) \cap \mathcal{S} = \phi$
b) $\exists \mathtt{e}' \in \mathtt{E} - \mathcal{S}$ *such that* $N(\mathtt{e}) \cap \mathcal{S} = \{\mathtt{e}\}$.

Proof. Given $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ is a BHFG and $\mathcal{S}$ is a minimal edge dominating set of $\mathtt{G}$, Then for every edge $\mathtt{e} \in \mathcal{S}$, $\mathcal{S} - \{\mathtt{e}\}$ is not an edge dominating set and hence there exists an edge $\mathtt{e}' \in \mathtt{E} - \mathcal{S}$ which is not adjacent to any element of $\mathcal{S} - \{\mathtt{e}\}$. Thus if $\mathtt{e}' = \mathtt{e}$ we get *(a)* and if $\mathtt{e}' \neq \mathtt{e}$ we get *(b)*. □

Definition 4.5. *An edge* $\mathtt{e}$ *of a BHFG* $\mathtt{G}$ *is called an an isolated edge if no effective edges is incident with the vertices of* $\mathtt{e}$ *and hence it doesn't dominate any other vertex in* $\mathtt{G}$.

Theorem 4.2. *If* $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ *is a BHFG without any isolated edges, then for every minimal edge dominating set* $\mathcal{S}$, *prove that* $\mathtt{E} - \mathcal{S}$ *is also an edge dominating set.*

Proof. Given $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ a BHFG without any isolated edges. Let $\mathcal{S}$ be minimal edge dominating set of $\mathtt{G}$, then there exists an edge $\mathtt{e}' \in N((\mathtt{e})$. From theorem 5.4 we get $\mathtt{e}' \in \mathtt{E} - \mathcal{S}$ which implies every edge in $\mathtt{E} - \mathcal{S}$ is adjacent to an edge in $\mathcal{S}$. Hence $\mathtt{E} - \mathcal{S}$ is also an edge dominating set. □

Corolary 4.1. *For any graph* $\mathtt{G}$ *without isolated edges* $\gamma_{bh}(\mathtt{G}) \leq \frac{r}{3}$.

Definition 4.6. *Let* $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ *be a BHFG. Let* $\mathcal{S}$ *be a minimum edge set of* $\mathtt{G}$. *If* $\mathtt{E} - \mathcal{S}$ *contains an edge dominating set* $\mathcal{S}'$ *of* $\mathtt{G}$, *then* $\mathcal{S}'$ *is said to be inverse edge dominating set of* $\mathtt{G}$. *The minimum cardinality out of all minmal inverse edge dominating sets is said to be inverse edge domination number and is denoted as* $\gamma_{bh}^{-1}(\mathtt{G})$.

Proposition 4.1. *For any graph* $\mathtt{G}$ *without isolated edges and vertices*

$$\gamma_{bh}(\mathtt{G}) \leq \gamma_{bh}^{-1}(\mathtt{G})$$

.

Proposition 4.2. *If* $\mathtt{G}$ *is a graph without isolated edges and vertices and if number of vertices are greater than or equal to 3, then*

$$\gamma_{bh}(\mathtt{G}) + \gamma_{bh}^{-1}(\mathtt{G}) \leq r$$

.

Definition 4.7. *Let $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ be BHFG without isolated edges. An edge dominating set $\mathcal{S}$ is called as total edge dominating set if $< \mathcal{S} >$ has no isolated edge.The minimum cardinality of all minimal total edge dominating sets is said to be total edge domination number of $\mathtt{G}$ and is denoted as γ_{tbh}.*
A set $\mathcal{F} \subseteq \mathtt{E}$ is said to be a total edge dominating set of $\mathtt{G}$ if for every edge in $\mathtt{E}$ is adjacent to at least one edge in $\mathcal{F}$.

Theorem 4.3. *For any bipolar hesitancy fuzzy graph $\mathtt{G}$, $\gamma_{bh}(\mathtt{G}) \leq \gamma_{tbh}(\mathtt{G})$.* □

Theorem 4.4. *For any bipolar fuzzy graph $\mathtt{G}$ with r edges then prove that $\gamma_{tbh} = r$ iff every edge of $\mathtt{G}$ has a unique neighbor.*

Proof. Given a BHFG $\mathtt{G}$ with r edges.Let us consider every edge of $\mathtt{G}$ has a unique neighbor, then $\mathcal{S}$ is the only total edge dominating set of $\mathtt{G}$ which implies $\gamma_{tbh} = r$. Conversely, suppose $\gamma_{tbh} = r$ and if there exists an edge with neighbors s and t then $\mathcal{S} - \{s\}$ gives a total edge dominating set of $\mathtt{G}$. Thus $\gamma_{tbh} < r$ which is a contradiction. □

5 Conclusions

We have established edge domination in Bipolar hesitancy fuzzy graph(BHFG). Along with various domination parameters such as inverse and total edge domination were also discussed. We have also given various examples and theorems supporting the main result. Our result can be extended to other domination parameters as well.

6 Acknowledgements

The authors thanks the management, Ratio Mathematica for their constant support towards the successful completion of this work. We wish to thank the anonymous reviewers for a careful reading of manuscript and for very useful comments and suggestions.

References

M. Akram. Bipolar fuzzy graphs. *Information Science*, 181:5548–5564, 2002.

M. Akram, M. Sarwar, and W. A. Dudek. *Graphs for the Analysis of Bipolar Fuzzy Information*. Springer, Singapore, 2021.

K. Anantha Kanaga Jothi and K. Balasangu. Irregular and totally irregular bipolar hesitancy fuzzy graphs and some of its properties. *Advances and Applications in Mathematical Sciences*, 20:1685–1696, 2021.

M. Karunambigai, M. Akram, and K. Palanivel. Domination in bipolar fuzzy graphs. *International conference on fuzzy systems*, 2013:7–10, 2013.

T. Pathinathan, Jonadoss Jon Arockiaraj, and Jesintha Rosline. Hesitancy fuzzy graphs. *Indian Journal of Science and Technology*, 8:1–5, 2015.

S. Ramya and S. Lavanya. Edge contraction in bipolar fuzzy graphs. *International Journal of Trend in Research and Development*, 4:475–477, 2017.

R. Sakthivel, R. Vikramaprasad, and N. Vinothkumar. Domination in hesitancy fuzzy graphs. *International Journal of Advanced Science and Technology*, 28: 1142 – 1156, 2019.

A. Somasundaram and S. Somasundaram. Domination in fuzzy graphs-i. *Pattern Recognition Letters*, 19:787–791, 1998.

V. Torra. Hesitant fuzzy sets. *International Journal of Intelligent Systems*, 25: 529–539, 2010.

S. Vellamal and K. Thiagarajan. Edge domination in fuzzy graphs. *International Journal of Theoretical and Applied Physics*, 2:33–40, 2012.

L. Zadeh. Fuzzy sets. *Information and Control*, 8:195–204, 1965.