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Analysis of Finite Population Stochastic modeling with State-Dependent Arrival and Service Facilities

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Abstract

This paper investigates a stock-dependent arrival process(SDAP) and queue-dependent service process(QDSP) in the stochastic queueing-inventory system(SQIS). The arriving units in the system generated from the finite source population. The arrival process holds the properties of quasi-random process and its intensity rate is defined based on the two-component demand rate(TCDR). The customers departure time is exponentially distributed. The concepts of non-SDAP and SDAP, non-QDSP and QDSP are to be generalized. The inventory system may have the perishable quality of the products. It adopts the (s, Q) reordering policy whenever the replenishment is required. Further, the join probability distribution of a Markov process is derived and necessary system performance measures are computed. The comparative discussion is presented to improve the quality of this model.

Keywords: Stock-dependent arrival process; Non-stock dependent arrival process; Two component demand rate; Finite population; queue-dependent service process

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1 Introduction

As the continuous increment in the population growth, the demand of the people such as foods, cloths and public service etc are also skyrocketed in this century. So the analysis of queueing-inventory management is inevitable in this current situation, especially in that food items related queueing-inventory systems. The results can be applied effectively in queue and inventory management systems and the optimum cost will increase the economy also. This model consists of a perishable inventory system with queue-dependent service rate and customers from a finite source. Due to the decay of the food items after some certain period, its must to reorder it at some fixed inventory level to avoid economical loss. The arrival rate of customer is dependent on the number of items available in the shops. Also service rate is dependent on the mood of server. The following illustration will give the exact model idea.

In the shopping malls and theaters, some stores are used to sell snack items for the customers. The possibility of customer occurrence is dependent on inventory level and also service rate can be dependent on the queue length. Here the population size is finite, since the only possible customers to purchase the snacks are from theater or the mall. Though there are many shops are available, most of the customers are interested to purchase from the shop where many items are available comparing to the shops where less items are available. Here the customer arrival depends on the inventory level. When a small queue is formed in the shop, server may serve slowly, speaking with someone or using mobile phone. Also server would provide a quick service if the queue length is big, in order to decrease the customer loss. Though the formed queue is bigger, customer won,t leave the queue as the service rate is high. This realistic situation motivates the author to develop the proposed model. In this model, stocks are replenished according to (s, Q) policy.

2 Literature review

As many researchers show their interest of research on the Stochastic Queueing Inventory modeling, this area has developed enormously in certain period. Queuing inventory systems with retrial is in-fusible in all walk of life. Reshmi and Jose studied the Queuing inventory model, considering perishable items and customer retrial on Reshmi and Jose [2019]. Periyasamy considered a finite population perishable inventory system where server is looking for the customers from the orbit to provide service after completing service to each primary customer on Periyasamy [2017]. Berman and Sapna Berman and Sapna [2002] discussed the rate of optimal service with perishable inventory in which instantaneous reordering policy was assumed. Considering the negative

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exponential rate for the life time of stocks, Kalpagam and Arivagam Kalpakam and Arivarignan [1988] analyzed the (s, S) inventory system in which stock one is evicted from the inventory whenever the demand or failure of item occurs. Sangeetha investigated the production optimal control of production time of perishable inventory system with finite source in order to get the minimal total cost on N. Sangeetha and Arivarignan [2015].

Alfres introduced the concept of occurring demand rate depends upon the stock level in the inventory system on Alfares [2007] and determined the total cost by variable holding cost assuming holding cost per unit item to be a monotonically increasing function of spending time in the storage. Diana Tom Varghese and Dhanya Shajin Varghese and Shajin [2018] studied the state dependent demand on the continuous review M/M/1/S inventory model. K. Venkata Subbaiah et al. K. Venkata Subbaiah and Satyanarayana [2004] developed the perishable inventory model with stock dependent demand rate. Rathod and Bhathawala Rathod and Bhathawala [2013] analyzed the inventory system with stock dependent demand having variable holding cost and shortages. The effect of demand rate depending on stock level was discussed through the proposed logistical growth model of Tsoularis Tsoularis [2014]. A shortage free inventory model with stock dependent demand was analyzed by Datta and Pal on Datta and Pal [1990]. Sudhir Kumar Sahu et al. Sudhir Kumar Sahu and Sahoo [2008] developed an inventory system with stock dependent demand rate and constant deterioration with the possibilities of partial or complete backlog and without it. Shib Sankar Sana Sana and Sankar [2010] proposed an EOQ model for the perishable inventory item with discount rate and the demand depending on stock level. Mandal Mandal and S. [1989] derived an inventory system with consumption rate depending on stock level.

For analyzing the local area management, Falin and Artalejo Falin and Artalejo [1998] proposed a retrial queue with finite source customer. Shophia Lawrence et al. A. Shophia Lawrence and Arivarignan [2013] discussed the perishable queueing-inventory system with demands from finite homogeneous source. Attahiru Sule Alfaa and Sapna Isotupa **?** discussed an M/PH/k retrial queue with the finite source. K. Jeganathan K. Jeganathan and Vigneshwaran [2015] analyzed the perishable inventory system with the possibility of server interruption and the multiple server vacation and customer is provided service only when customer level reaches to a particular N and no customer is left behind the system after service started. Jeganathan Jeganathan [2015] discussed finite source inventory system with an additional service for some customers which is called bonus service. Artalejo and Lopez-Herrero investigated retrial queue involving finite population with an BSDE approach on Artalejo and Lopez-Herrero [2012]. Sivakumar analyzed the perishable inventory system with retrial demand from finite source without service on Sivakumar [2009].

Shanthikumar and Yao Shanthikumar and Yao [1988] studied the upper and lower bounds on a closed queuing network with the queue dependent service rate. Menich ronald Ronald [1987] derived the optimal of shortest queue routing to the queue dependent service station considering a general Markovian system. Avhishek Chatterjee et al. Avhishek Chatterjee and varshney lav [2017] studied the information-theoretic limit of reliable information processing using queue dependent service facility. Jeganathan et al.[2021] proposed a finite inventory single server system and analyzed the queue dependent service rate.

Though the large number of researches have been done in this area, there is a research gap in analyzing stock dependent demand rate on the finite source queuing inventory system with the queue dependent service rate and retrial customer. As the demand rate depending upon stock level on the inventory and service rate depends on the queue length, this model simulates the realistic situation.

3 Model Developing

3.1 Mathematical Formulation of the model

This model deals the state dependent arrival and queue dependent service processes in a single server Markovian queueing-inventory system(SSMQIS) in a finite source environment. The system holds maximum of S units of inventory product in its storage place. It allows the customers to buy the product from a finite source, N only. It admits the arriving customers into the finite waiting hall of size N. There is only two possible choice of a customer such that they must be either free or in the waiting hall at any time.

The appearance of arrival process generates a output process called quasi-random process; that is, the probability that any particular customer generates a request for demand in any interval $(t, t+\mathrm{d}t)$ is $\theta_r \mathrm{d}t + o(\mathrm{~d}t)(r \in B_0^S)$ as d $t \rightarrow 0$ if the customer is free at time t and zero if the customer is in waiting hall at time t, independently of the behavior of any other customers. The arrival process of any individual customer is non-homogeneous, since the generation of arrivals must dependent upon the current stock level of the system. This non-homogeneous arrival streams come under the category of state dependent arrival stream. Next, the service pattern is processed following a first come and first serve(FCFS) service discipline. The service time of any customer at time t is non-homogeneous and exponentially distributed. That is, $\mu_s \ \ (s \in B_1^N)$ is the service rate of an individual at any time. This service process comes under the category of state dependent service processes. After each service completion, there will be one unit dropped in the storage place.

The stored products in the system does not have any guarantee about its life time till it will be sold. It may have the deteriorating quality. So this deterioration process follows exponential distribution and have the intensity rate $r\alpha_1$ where $r \in B_1^S$. The service and deterioration processes cause the depletion of an inventory product unit by unit. At one fine stage, the current number of product in the storage system will reach the predetermined value s. As and when the maximum inventory level reduced to s or less than s, then the replenishment process will be triggered immediately. Each time there are $Q = (S - s)$ items will be replaced whenever the reorder required. This policy is known as (s, Q) reordering policy and this processing time is exponentially distributed with an intensity rate α.

The defined arrival and service rates are ordered, $\theta_0 \leq \theta_1 \leq \theta_2 \leq \cdots \leq \theta_S$ and $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_N$ as an increasing manner. When the case $\theta_1 = \theta_2 = \cdots = \theta_S = \theta$, the considered model comes under the category of two component demand rate. That is the arrival rate is homogeneous in the positive stock period and during the stock out period, it is θ_0. When the case $\mu_1 = \mu_2 = \cdots = \mu_N = \mu$ means that the service rate become homogeneous.

Remark 3.1.

- *For a numerical computation θ_r can be defined by $\theta_r\ r^{\beta_1},\ \ 0 < \beta_1 \leq 1$ and $r \in B_1^S$.*
- *For a numerical computation μ_s can be defined by $\mu_s\ s^{\beta_2},\ \ 0 \leq \beta_2 \leq 1$ and $s \in B_1^N$.*
- *The case $\beta_1 = 0$ and $\beta_2 = 0$ explores the result of non-stock dependent arrival process and non- queue dependent service process of the proposed model.*

4 Analytical Discussion of the Model

Let $\{\ (R_1(t), R_2(t))\ ; t \geq 0\ \}$ be a stochastic process having state space $\{(r_1, r_2) :\ r_1 \in B_0^S$ and $r_2 \in B_0^N\}$ satisfies the Markov process, where $R_1(t)$ denotes the level of inventory at time t and $R_2(t)$ denotes the number of customers in the orbit at time t. The transition from any state (r_1, r_2) to other state (r_1', r_2') at any interval is denoted by $P\left((r_1, r_2), (r_1', r_2')\right)$. Any y items in the inventory perish alone at the rate of $r_1\gamma$ and the occurrence of primary demand is $(N - r_2)\,\theta_{r_1}$ from any one of the sources $(N - r_2)$. Hence, the probability of transition is

$P\left((r_1, r_2), (r_1 - 1, r_2)\right) = r_1\alpha_1\ r_1 \in B_1^S$ and $r_2 \in B_0^K$. Since the service rate is queue dependent service rate $P\left((r_1, r_2), (r_1 - 1, r_2 - 1)\right) = \mu_{r_2}$ where $r_1 \in B_1^S$ and $r_2 \in B_1^K$. If the arrival rate is dependent on inventory, arriving customers enter into the waiting hall. So the probability of the transition from the state (r_1, r_2) to the state $(r_1, r_2 + 1)$ is $P\left((r_1, r_2), (r_1, r_2 + 1)\right) = (K - r_2)\,\theta_{r_1}$ where $r_1 \in B_0^S,\ \ r_2 \in B_0^{K-1}$. When Q items are ordered, the probability of transition from the state (r_1, r_2) to state $(r_1 + Q, r_2)$ for all r_2 and $r_1 \in B_0^s$ is given by $P\left((r_1, r_2), (r_1 + Q, z)\right) = \alpha$. The rate of other transitions is zero. The sum of each row of this matrix should be zero. Hence, the diagonal entry is multiplied by a negative sign after summing all the entries from the row.

All the possible transitions are given below.

$$R\left(\left(r_1,r_2\right),\left(r_1',r_2'\right)\right)=\begin{cases} r_1\alpha_1, & r_1'=r_1-1, r_1\in B_1^S,\\ & r_2'=r_2, r_2\in B_0^K,\\ \mu_{r_2}, & r_1'=r_1-1, r_1\in B_1^S,\\ & r_2'=r_2-1, r_2\in B_1^K,\\ (K-r_2)\theta_{r_1}, & r_1'=r_1, r_1\in B_0^S,\\ & r_2'=r_2+1, r_1\in B_0^{K-1},\\ \alpha, & r_1'=r_1+Q, r_1\in B_0^s,\\ & r_2'=r_2, r_2\in B_0^K,\\ -(\bar{\delta}_{K,r_2}(K-r_2)\theta_{r_1}+\alpha), & r_1'=r_1, r_1\in B_0^0,\\ & r_2'=r_2,, r_2\in B_0^K,\\ -(\bar{\delta}_{K,r_2}(K-r_2)\theta_{r_1}+\alpha+\bar{\delta}_{0,r_2}\mu_{r_2}+r_1\alpha_1), & r_1'=r_1, r_1\in B_1^s,\\ & r_2'=r_2,, r_2\in B_0^K,\\ -(\bar{\delta}_{K,r_2}(K-r_2)\theta_{r_1}+\bar{\delta}_{0,r_2}\mu_{r_2}+r_1\alpha_1), & r_1'=r_1, r_1\in B_{s+1}^S,\\ & r_2'=r_2,, r_2\in B_0^K,\\ 0, & \text{Otherwise.} \end{cases}$$

The block partitioned matrices of the proposed model is structured as follows:

$$R=\begin{cases} L_y, & r_1'=r_1, \quad r_1\in B_0^S,\\ M_y, & r_1'=r_1-1, \quad r_1\in B_1^S,\\ N, & y'=Q+r_1 \quad r_1\in B_0^s,\\ 0, & \text{Otherwise.} \end{cases}$$

For $r_1\in B_1^S$,

$$M_{r_1}=\begin{cases} r_1\alpha_1, & r_2'=r_2, \quad r_2\in B_0^K,,\\ \mu_{r_2} & r_2'=r_2-1, \quad r_2\in B_1^K,\\ 0, & \text{Otherwise.} \end{cases}$$

For $r_1\in B_0^s$,

$$N=\begin{cases} \alpha, & r_2'=r_2, r_2\in B_0^K,\\ 0, & \text{Otherwise.} \end{cases}$$

For $r_1=0$,

$$L_{r_1}=\begin{cases} (K-r_2)\theta_0, & r_2'=r_2+1, \quad r_2\in B_0^{K-1},\\ -((K-r_2)\theta_0+\alpha), & r_2'=r_2, \quad r_2\in B_0^{K-1},\\ -\alpha, & r_2'=r_2, \quad r_2\in B_K^K,\\ 0, & \text{Otherwise.} \end{cases}$$

For $r_1 \in B_1^s$,

$$L_{r_1} = \begin{cases} (K - r_2)\theta_{r_1}, & r_2' = r_2 + 1, \quad r_2 \in B_0^{K-1}, \\ -(\bar{\delta}_{K,r_2}(K - r_2)\theta_{r_1} + \alpha + \bar{\delta}_{0,r_2}\mu_{r_2} + r_1\alpha_1), & r_2' = r_2, \quad r_2 \in B_0^K, \\ 0, & \text{Otherwise.} \end{cases}$$

For $r_1 \in B_{s+1}^S$,

$$L_{r_1} = \begin{cases} (K - r_2)\theta_{r_1}, & r_2' = r_2 + 1, \quad r_2 \in B_0^{K-1}, \\ -(\bar{\delta}_{K,r_2}(K - r_2)\theta_{r_1} + \bar{\delta}_{0,r_2}\mu_{r_2} + r_1\alpha_1), & r_2' = r_2, \quad r_2 \in B_0^K, \\ 0, & \text{Otherwise.} \end{cases}$$

4.1 Steady state analysis

The structure of the homogeneous Markov process $\{(R_1(t), R_2(t); t \geq 0\}$ with finite state space indicates that it is irreducible. Hence, the limiting distribution is

$$\xi^{(r_1,r_2)} = \lim_{t\to\infty} Pr\{(R_1(t) = r_1, R_2(t) = r_2)|(R_1(0), R_2(0))\}$$

$$\text{Let } \xi = (\xi^{(0)}, \xi^{(1)}, \dots, \xi^{(S)})$$

where each $\xi^{(r_1)} = (\xi^{(r_1,0)}, \xi^{(r_1,1)}, \dots, \xi^{(r_1,K)})$ for $r_1 \in B_0^S$ which satisfies

$$\xi P = \mathbf{0} \text{ and } \xi\mathbf{e} = 1 \tag{1}$$

From the above we get the following equation

$$\xi^{(r_1)}L_{r_1} + \xi^{(r_1+1)}M_{r_1+1} = \mathbf{0} \quad r_1 \in B_0^{Q-1}, \tag{2}$$

$$\xi^{(r_1)}L_{r_1} + \xi^{(r_1+1)}M_{r_1+1} + \xi^{(r_1-Q)}N = \mathbf{0} \quad r_1 = Q \tag{3}$$

$$\xi^{(r_1)}L_{r_1} + \xi^{(r_1+1)}M_{r_1+1} + \xi^{(r_1-Q)}N = \mathbf{0} \quad r_1 \in B_{Q+1}^{S-1}, \tag{4}$$

$$\xi^{(r_1)}L_{r_1} + \xi^{(r_1-Q)}N = \mathbf{0} \quad r_1 = S \tag{5}$$

Except the $r_1 = Q$ case, solving other equations recursively, we get,

$$\xi^{(r_1)} = \xi^{(Q)}\Delta_{r_1}, \quad r_1 \in B_0^S,$$

where

$$\Delta_i = \begin{cases} (-1)^{(Q-r_1)}(M_Q M_{Q-1} \dots M_{r_1+1})(L_{Q-1}^{-1}L_{Q-2}^{-1} \dots L_{r_1}^{-1}), & r_1 \in B_0^{Q-1}, \\ \qquad I & r_1 Q, \\ (-1)^{2Q+1-r_1} \sum_{j=0}^{C-r_1} (M_Q M_{Q-1} \dots M_{s+1-j})(L_{Q-1}^{-1}L_{Q-2}^{-1} \dots L_{s-j}^{-1})NL_{S-j}^{-1} & \\ (M_{S-j}M_{S-j-1} \dots M_{r_1+1})(L_{S-j-1}^{-1}L_{S-j-2}^{-1} \dots L_{r_1}^{-1}) & r_1 \in B_{Q+1}^S, \end{cases}$$

$\xi^{(Q)}$ can be yield solving

$$\xi^{(Q)}\Bigg[(-1)^{2Q+1-r_1}\sum_{j=0}^{S-r_1}\Bigg[(M_QL_{Q-1}^{-1}M_{Q-1}\dots M_{s+1-j}L_{s-j}^{-1})NL_{S-j}^{-1}(M_{S-j}L_{S-j-1}^{-1}M_{S-j-1}\dots \\ \dots M_{Q+2}L_{Q+1}^{-1})\Bigg]M_{Q+1}+L_Q+(-1)^QM_QL_{Q-1}^{-1}M_{Q-1}\dots M_1L_0^{-1}N\Bigg]=\mathbf{0}$$

$$\xi^{(Q)}\Bigg[\sum_{r_1=Q+1}^{S}\Bigg((-1)^{2Q-r_1+1}\sum_{j=0}^{S-r_1}\Bigg[(M_QL_{Q-1}^{-1}M_{Q-1}\dots M_{s+1-j}L_{s-j}^{-1})NL_{S-j}^{-1}(M_{S-j}L_{S-j-1}^{-1} \\ M_{S-j-1}\dots M_{r_1+1}L_{r_1}^{-1})\Bigg]M_{Q+1}\Bigg)+\sum_{r_1=0}^{Q-1}\big((-1)^{Q-r_1}M_QL_{Q-1}^{-1}\dots M_{r_1+1}L_{r_1}^{-1}\big)+I\Bigg]\mathbf{e}= \\ 1.$$

5 System Performance Measures

To make a detailed investigation of the proposed model, some significant system characteristics are to be computed as follows:

1. Expected present stock level $E[psl]=\sum_{r_1=1}^{S}\sum_{r_2=0}^{K}r_1\xi^{(r_1,r_2)}$.
2. Expected reorder level $E[reorder]=\sum_{r_2=1}^{K}\mu_{r_2}\xi^{(s+1,r_2)}+\sum_{r_2=0}^{K}(s+1)\alpha_1\xi^{((s+1),r_2)}$.
3. Expected perishable rate $E[perishable]=\sum_{r_1=1}^{S}\sum_{r_2=0}^{K}r_1\alpha_1\xi^{(r_1,r_2)}$.
4. Expected number of customers in the waiting hall $E[CWH]=\sum_{r_0=1}^{S}\sum_{r_2=1}^{K}r_2\xi^{(r_1,r_2)}$.
5. Expected number of customers enter into the waiting hall $E[CEWH]=\sum_{r_1=0}^{S}\sum_{r_2=0}^{K-1}(K-r_2)\theta_{r_1}\xi^{(r_1,r_2)}$.
6. Expected waiting time of a customer in the waiting hall $E[WT]=\dfrac{E[CWH]}{E[CEWH]}$
7. Probability that the server is busy $P(busy)=\sum_{r_0=1}^{S}\sum_{r_2=1}^{K}\xi^{(r_1,r_2)}$.
8. Probability that the server is idle $P(idle)=1-P(busy)$
9. The total expected cost value of the proposed model is defined as $TCV=c_aE[psl]+c_bE[reorder]+c_cE[perishable]+c_dE[WT]$

where
c_a- Holding cost per unit, c_b- Setup cost per unit, c_c- Perishable cost per unit and c_d- Waiting cost per customer.

Table 1: TCV for the case of SDAP and QDSP

s / S	7	8	9	10	11	12	13
58	2.805974	2.805623	2.806103	2.807504	2.809919	2.813445	2.818190
59	2.806534	2.805602	2.805445	2.806146	2.807788	2.810462	2.814264
60	2.807988	2.806522	2.805778	2.805835	2.806769	2.808661	2.811598
61	2.810287	2.808327	2.807044	2.806509	2.806792	2.807966	2.810111
62	2.813383	2.810970	2.809190	2.808110	2.807794	2.808309	2.809726
63	2.817234	2.814403	2.812165	2.810584	2.809717	2.809625	2.810374
64	2.821800	2.818583	2.815924	2.813880	2.812506	2.811856	2.811991

6 Simulation Analysis

In this section, the optimum cost analysis, monotonic behavior of some system characteristics are to be discussed by the numerical illustrations. This will be helpful to deliver a effective decision making polices for every inventory business tycoons. For knowing such curious results of our proposed model, we need to fix the value of the parameters and the cost values such that $\theta = 5, \theta_0 = 2, \mu = 9, \alpha = 0.9, \gamma = 0.07, \beta_1 = 0.5, \beta_2 = 0.5, S = 61, s = 10, N = 10, c_a = 0.05, c_b = 0.9, c_c = 0.1,$ and $c_d = 7$.

Example 6.1. ***Optimum cost analysis***

This example briefly investigate the minimum optimal TCV for the category of both arrival and service processes of homogeneous and non-homogeneous cases as shown in Table (1)-(2). In Table (1), $S \in B_{58}^{64}$ and $s \in B_{7}^{13}$ are used to find the minimal optimum TCV under the case of discussion between SDAP and QDSP. In this case, the $TCV^* = 2.805445$ and corresponding optimum $S^* = 59$ and $s^* = 9$ are obtained.

Next, the output values of the case non-SDAP and non-QDSP are given in Table (2). Here, $S \in B_{57}^{63}$ and $s \in B_{10}^{16}$ are varied to get an optimum TCV. In this case, the $TCV^* = 8.966159$ and corresponding optimum $S^* = 60$ and $s^* = 13$ are obtained.

As we expected due to the assumption of the proposed model, the case non-SDAP and non- QDSP have a higher TCV^* than the SDAP and QDSP case. That is, the minimal optimum TCV obtained in the case of SDAP and QDSP. Hence the arrival and service rates influence the cost value become a minimum one.

$k1 = 0, k2 = 0$

$k1 = 0, k2 = 0.6$

Example 6.2. ***The variation of TCV under the parameter variation***

In this example, we describe the path of TCV with each parameter considered in the model. In such a way, the major objective of this example is discussed with the

Table 2: TCV for the case of non-SDAP and non-QDSP

s / S	10	11	12	13	14	15	16
57	8.986034	8.976046	8.970983	8.970903	8.975952	8.986375	9.002534
58	8.985814	8.975121	8.969166	8.967979	8.971672	8.980449	8.994615
59	8.986685	8.975367	8.968613	8.966432	8.968907	8.976202	8.988575
60	8.988575	8.976700	8.969232	8.966159	8.967537	8.973497	8.984257
61	8.991414	8.979048	8.970942	8.967068	8.967457	8.972214	8.981518
62	8.995143	8.982342	8.973667	8.969073	8.968572	8.972242	8.980232
63	8.999705	8.986521	8.977338	8.972097	8.970793	8.973483	8.980284

scaling factors β_1 and β_2, because they are deciding factors whether the arrival and service processes are non-SDAP and non-QDSP or not respectively. In Table (3), the scaling factor β_1 increases the total cost if it is increasing. That is, β_1 increases means, the arriving customers in the system is increased. Subsequently, the sales of number of product in the inventory is raised. So the management is often ready to store or making reorder for their requirement. These jobs cause the increase of total cost. The same characteristics are holds the parameter θ. Simultaneously, when we are focusing the another scaling factor β_2, more interestingly it reduces the TCV. If β_2 increases means that the service time of an individual become reduced. So the number of customers leaves the system after a successful service completion of them is increasing. This helps to reduce the mean service time of a customer. So this is the reason for TCV is reduced if β_2 increases. If β_2 and μ are directly proportional to each other μ holds the same behavior as β_2. Then the perishable parameter α_1 affects the item life time. If α_1 raises, the number of current stock level starts falling down. If it happens, the management has to store more number of products which cause the extra expenditure to maintain the system. So this expenditure cause the increase of total cost. Finally, the reorder intensity rate α minimize the total cost when it is increasing. The successive mean reorder time reduced means the number of available product of the system become positive. Therefore, the service completion will be done as soon as possible. Hence, all the parameters involved in Table (3) and Table (4) are satisfies their own properties.

Example 6.3. ***Graphical Analysis***

• The scaling factors β_1 and β_2 shows the increasing/decreasing path due to its SDAP and non-SDAP, QDSP and non-QDSP. We observe that $0.2 \leq \beta_1 \leq 1$ the β_2 curves deviation is high and $0.5 \leq \beta_2 \leq 1.0$ the β_2 curves deviation is low for all $\beta_1 \in (0.2, 1)$.

• The graph of expected waiting time is shown in Figure (2) when β_1 and β_2 are varying together. Here, the deviation of β_2 curves coincides with the characteristics as we said in Figure (1).

Table 3: The variation of TCV under the parameter variation

β_2	θ	α_1	β_1	0			0.5			1		
			μ	7	9	11	7	9	11	7	9	11
			α									
0	4.5	0.05	0.70	11.08	8.92	7.55	44.64	35.40	29.67	136.60	104.43	85.33
			0.90	10.83	8.66	7.28	44.78	35.11	29.13	145.45	109.22	87.88
			1.10	10.71	8.52	7.14	45.25	35.24	29.04	154.70	114.88	91.46
		0.07	0.70	11.18	9.02	7.64	44.31	35.24	29.60	132.94	102.29	83.98
			0.90	10.92	8.74	7.36	44.40	34.91	29.03	141.43	106.85	86.36
			1.10	10.79	8.60	7.21	44.86	35.02	28.92	150.45	112.34	89.81
		0.09	0.70	11.29	9.12	7.73	44.01	35.10	29.54	129.67	100.36	82.73
			0.90	11.01	8.82	7.43	44.06	34.73	28.93	137.82	104.70	84.96
			1.10	10.87	8.68	7.28	44.50	34.82	28.80	146.61	110.02	88.30
	5	0.05	0.70	11.21	9.06	7.69	44.81	35.59	29.87	137.00	104.88	85.82
			0.90	10.95	8.79	7.41	44.93	35.28	29.31	145.74	109.58	88.28
			1.10	10.83	8.65	7.27	45.38	35.38	29.20	154.89	115.16	91.78
		0.07	0.70	11.31	9.15	7.78	44.48	35.43	29.80	133.34	102.75	84.46
			0.90	11.04	8.87	7.49	44.55	35.08	29.20	141.73	107.21	86.76
			1.10	10.91	8.73	7.34	44.98	35.16	29.08	150.65	112.62	90.14
		0.09	0.70	11.42	9.25	7.86	44.19	35.29	29.74	130.08	100.82	83.22
			0.90	11.13	8.95	7.57	44.21	34.90	29.11	138.12	105.06	85.36
			1.10	10.99	8.80	7.41	44.63	34.96	28.96	146.81	110.30	88.62
	5.5	0.05	0.70	11.31	9.17	7.80	44.95	35.74	30.03	137.33	105.26	86.22
			0.90	11.06	8.89	7.52	45.04	35.41	29.45	145.97	109.87	88.61
			1.10	10.93	8.76	7.38	45.48	35.50	29.33	155.04	115.37	92.04
		0.07	0.70	11.42	9.26	7.89	44.62	35.59	29.96	133.68	103.13	84.87
			0.90	11.14	8.98	7.60	44.67	35.21	29.35	141.96	107.50	87.09
			1.10	11.01	8.83	7.45	45.08	35.28	29.21	150.80	112.84	90.40
		0.09	0.70	11.52	9.36	7.97	44.33	35.45	29.91	130.41	101.20	83.63
			0.90	11.23	9.06	7.67	44.33	35.03	29.26	138.36	105.35	85.69
			1.10	11.09	8.91	7.52	44.73	35.08	29.10	146.97	110.52	88.89
0.5	4.5	0.05	0.70	2.02	1.83	1.70	5.58	4.87	4.41	18.73	16.13	14.48
			0.90	2.04	1.85	1.73	5.31	4.61	4.16	17.71	15.08	13.43
			1.10	2.06	1.88	1.76	5.16	4.48	4.03	17.18	14.51	12.85
		0.07	0.70	2.06	1.87	1.74	5.640	4.92	4.46	18.78	16.19	14.54
			0.90	2.08	1.89	1.76	5.36	4.66	4.21	17.75	15.13	13.48
			1.10	2.10	1.92	1.80	5.22	4.53	4.08	17.21	14.55	12.89
		0.09	0.70	2.10	1.90	1.77	5.69	4.98	4.50	18.83	16.25	14.59
			0.90	2.12	1.93	1.80	5.42	4.71	4.25	17.79	15.17	13.52
			1.10	2.15	1.96	1.84	5.27	4.58	4.12	17.23	14.59	12.93
	5	0.05	0.70	2.02	1.83	1.70	5.60	4.89	4.44	18.80	16.21	14.56
			0.90	2.04	1.85	1.73	5.33	4.63	4.18	17.773	15.15	13.50
			1.10	2.06	1.88	1.76	5.18	4.50	4.05	17.22	14.57	12.91
		0.07	0.70	2.06	1.86	1.73	5.66	4.95	4.48	18.85	16.27	14.62
			0.90	2.08	1.89	1.76	5.38	4.68	4.23	17.80	15.19	13.55
			1.10	2.11	1.92	1.80	5.24	4.55	4.10	17.25	14.61	12.95

Table 4: The variation of TCV under the parameter variation

β_2	θ	α_1	β_1	0			0.5			1		
			μ	7	9	11	7	9	11	7	9	11
			α									
		0.09	0.70	2.106	1.904	1.769	5.719	5.003	4.532	18.908	16.327	14.678
			0.90	2.125	1.931	1.801	5.441	4.737	4.276	17.845	15.242	13.596
			1.10	2.154	1.967	1.843	5.294	4.600	4.148	17.283	14.649	12.999
	5.5	0.05	0.70	2.029	1.832	1.699	5.618	4.918	4.460	18.860	16.278	14.637
			0.90	2.046	1.855	1.727	5.347	4.656	4.205	17.818	15.205	13.562
			1.10	2.071	1.888	1.765	5.202	4.519	4.076	17.266	14.616	12.965
		0.07	0.70	2.068	1.868	1.733	5.678	4.970	4.506	18.913	16.335	14.693
			0.90	2.087	1.893	1.763	5.403	4.706	4.250	17.854	15.250	13.609
			1.10	2.114	1.928	1.803	5.256	4.568	4.121	17.293	14.655	13.008
		0.09	0.70	2.107	1.903	1.766	5.737	5.022	4.552	18.966	16.392	14.749
			0.90	2.127	1.931	1.798	5.457	4.755	4.294	17.891	15.296	13.656
			1.10	2.156	1.967	1.840	5.309	4.617	4.165	17.321	14.695	13.051
1.0	4.5	0.05	0.70	1.021	0.981	0.957	1.290	1.190	1.124	3.460	3.200	3.026
			0.90	1.124	1.086	1.064	1.329	1.228	1.160	3.231	2.976	2.805
			1.10	1.207	1.173	1.153	1.382	1.282	1.215	3.129	2.882	2.716
		0.07	0.70	1.045	1.004	0.979	1.313	1.210	1.142	3.489	3.224	3.047
			0.90	1.152	1.113	1.090	1.355	1.251	1.181	3.261	3.001	2.827
			1.10	1.239	1.203	1.183	1.410	1.307	1.239	3.160	2.908	2.739
		0.09	0.70	1.069	1.027	1.002	1.335	1.230	1.161	3.518	3.248	3.068
			0.90	1.179	1.140	1.116	1.380	1.274	1.202	3.290	3.026	2.849
			1.10	1.270	1.233	1.212	1.438	1.333	1.262	3.191	2.935	2.763
	5	0.05	0.70	0.998	0.955	0.929	1.283	1.182	1.114	3.469	3.209	3.036
			0.90	1.101	1.060	1.035	1.322	1.219	1.150	3.239	2.985	2.814
			1.10	1.185	1.147	1.125	1.374	1.272	1.204	3.136	2.890	2.725
		0.07	0.70	1.022	0.978	0.951	1.306	1.202	1.132	3.498	3.234	3.057
			0.90	1.129	1.086	1.061	1.347	1.241	1.170	3.269	3.010	2.836
			1.10	1.216	1.177	1.154	1.402	1.297	1.227	3.167	2.916	2.748
		0.09	0.70	1.045	1.000	0.973	1.328	1.221	1.150	3.527	3.258	3.078
			0.90	1.156	1.113	1.087	1.372	1.264	1.191	3.298	3.035	2.858
			1.10	1.247	1.207	1.183	1.429	1.322	1.250	3.198	2.942	2.771
	5.5	0.05	0.70	0.978	0.932	0.904	1.277	1.175	1.106	3.477	3.218	3.045
			0.90	1.081	1.037	1.010	1.315	1.211	1.141	3.246	2.992	2.822
			1.10	1.166	1.125	1.100	1.367	1.264	1.194	3.143	2.897	2.732
		0.07	0.70	1.001	0.955	0.926	1.300	1.194	1.124	3.506	3.242	3.066
			0.90	1.108	1.063	1.036	1.340	1.233	1.161	3.276	3.017	2.844
			1.10	1.197	1.154	1.129	1.395	1.289	1.217	3.174	2.923	2.755
		0.09	0.70	1.024	0.977	0.947	1.322	1.214	1.142	3.534	3.266	3.086
			0.90	1.135	1.089	1.060	1.365	1.256	1.181	3.305	3.042	2.866
			1.10	1.227	1.183	1.157	1.423	1.314	1.240	3.204	2.949	2.778

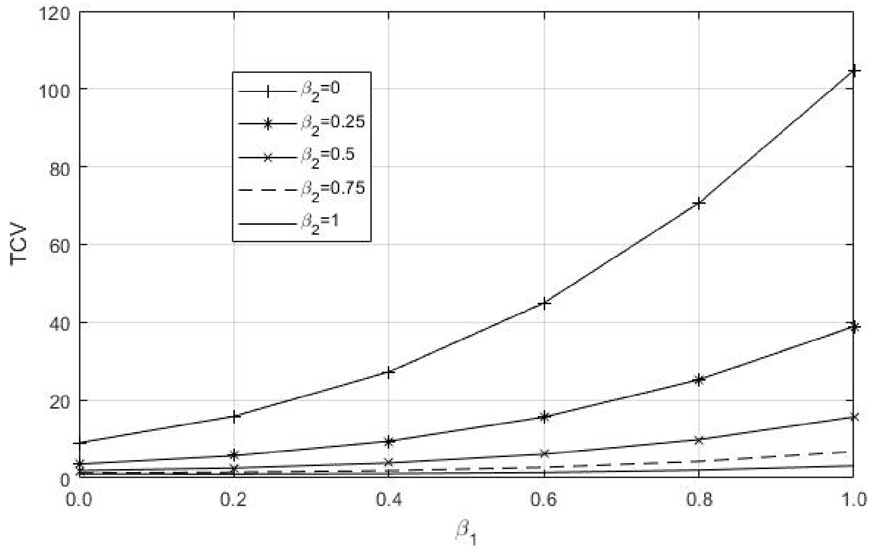

Figure 1: Impact of TCV on β_1 vs β_2

• Figure (3) explores expected present stock level of the system on the combination of β_1 vs β_2. Both β_1 vs β_2 reduce the $E[psl]$ when they are increasing. Here, the deviation of β_2 curves is high when $0.2 \leq \beta_1 \leq 1$.

• The parameters α and α_1 are affects the $E[WT]$ as shown in Figure 4. In this graph, the beta curve has the high deviation with themselves and low deviation with α_1.

• The $E[WT]$ is shown in Figure 5 if θ and μ are increasing together. The θ curves are decreasing when μ is increasing and it means that the increased service rate cause less mean service time of an individual.Therefore, the $E[WT]$ is decreased. If θ and μ are inversely proportional each other, θ reacts against μ.

• The average waiting time of a customer is discussed for the case of θ vs K in Figure 6 and μ vs K in Figure 7. As we have enough discussion about θ and μ on the $E[WT]$, we shall move to analyses the impact of K.When the number of finite source population is increases, for the $E[WT]$, the θ curve will be straight line.

Example 6.4. ***Impact of*** $E[psl], E[reorder]$ ***and*** $E[perishable]$ ***with the parameter variation***

This example describes the important system performance measures, $E[psl], E[reorder]$ and $E[perishable]$ are to be discussed with the parameter analysis of α, α_1, θ, μ and β_2 as shown in Table 5-7. As per the scaling factor, $\beta_2 = 0, \beta_2 = 0.5$ and $\beta_2 = 1$ are to be explored in Table 5, 6 and 7 respectively. If we increase the reorder rate, the expected present stock level increases. For every replenishment, there are Q items replaced as it reaches the system. So it makes the E[psl] is increasing when it is increase. When α

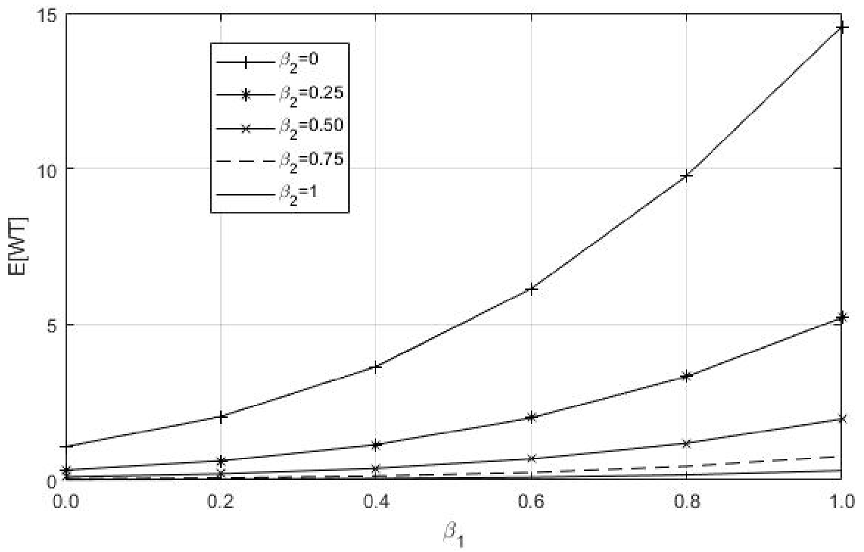

Figure 2: Impact of $E[WT]$ on β_1 vs β_2

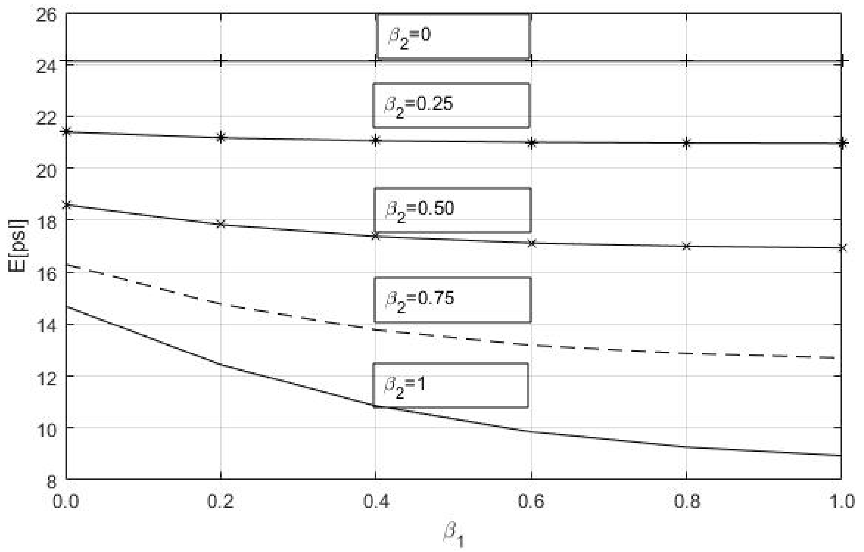

Figure 3: Impact of $E[psl]$ onβ_1 vs β_2

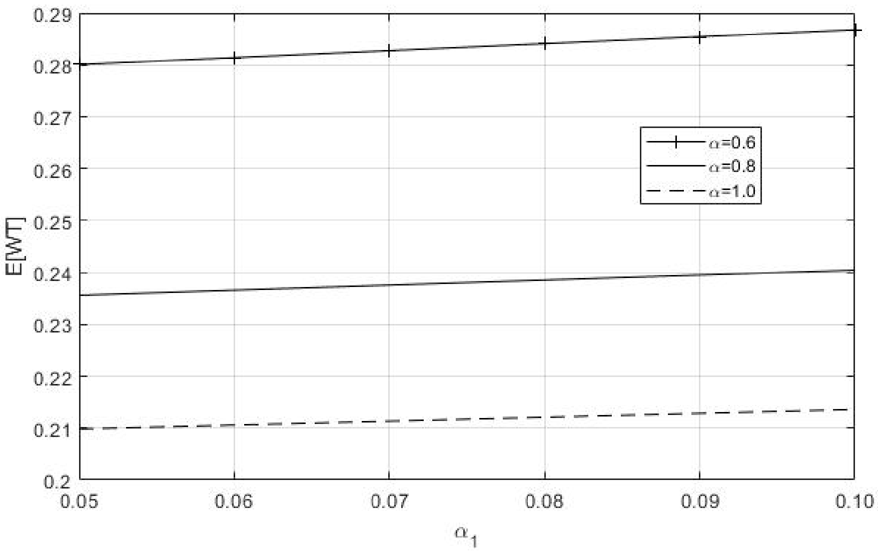

Figure 4: Impact of $E[WT]$ on α vs α_1

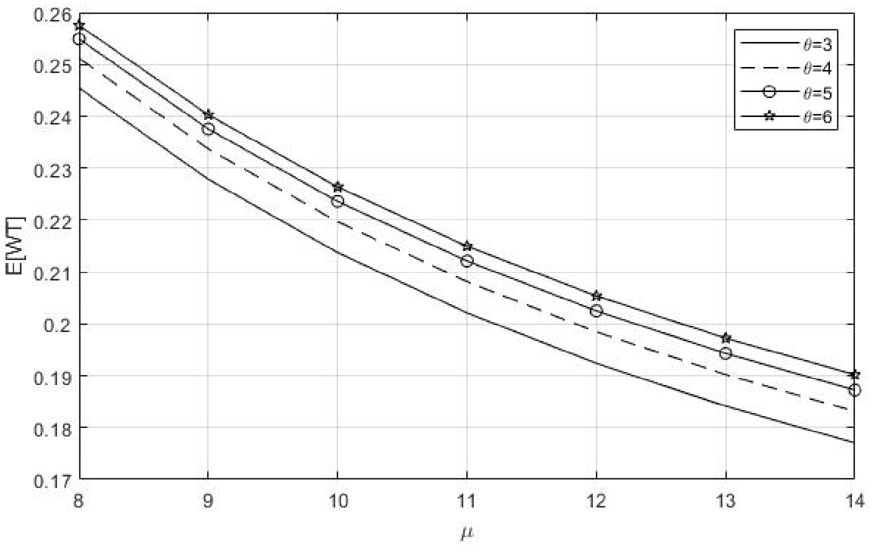

Figure 5: Impact of $E[WT]$ on μ vs θ

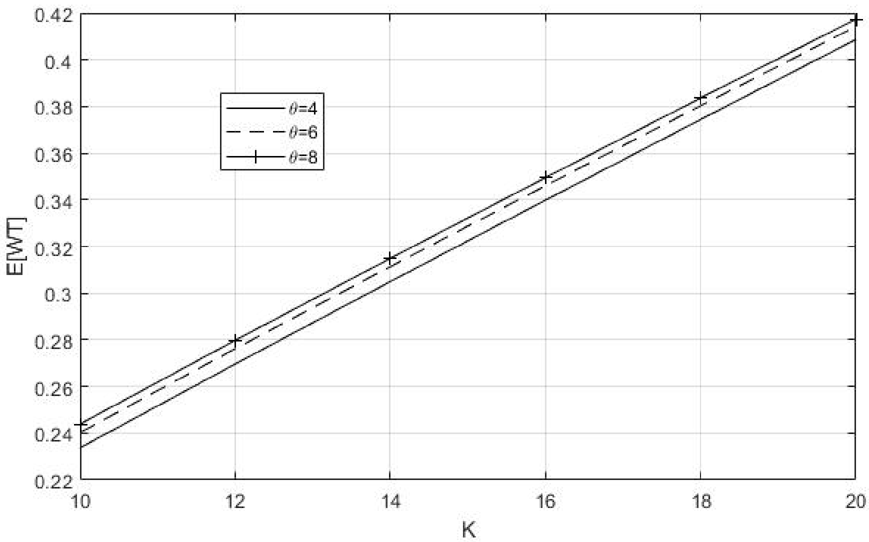

Figure 6: Impact of Impact of $E[WT]$ on K vs θ

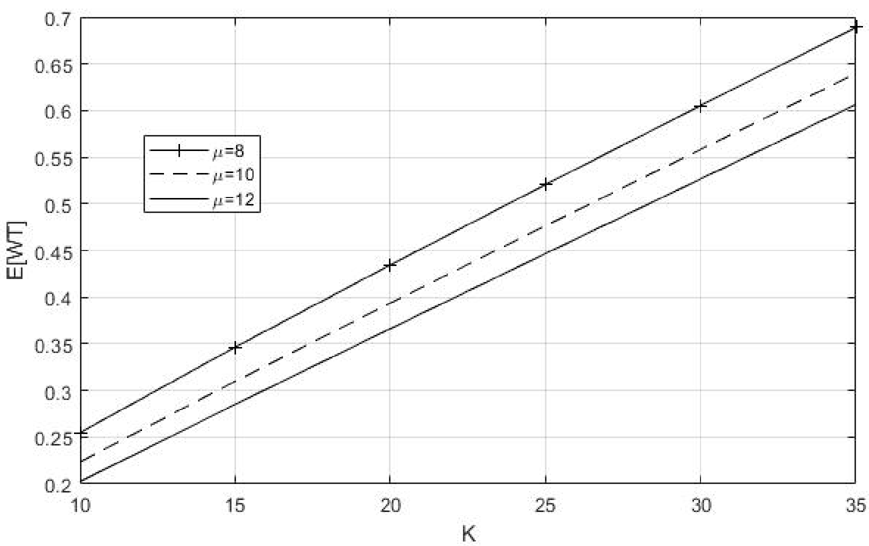

Figure 7: Impact of $E[WT]$ on μ vs$K$

is increasing, for the value of $\mu = 7$ E[reorder] behaves first increasing and then decreasing but at $\mu = 11$ it is increasing only. Further, for both μ, the E[perishable] will increase. Here, the perishable quality of the products depends on the number of present stock level of the system.

The parameter α_1 affects the E[psl] to fall down if it is increasing. Perishable products starts deterioration process depending the existing current stock level. So it is decreased. Since the items in the inventory storage system are perished, the system requires more number of products to provides the sales service. Hence the expected reorder level is increased. Here the raise of a perishable rate obviously influence the increase of E[perishable] . Then the parameter θ changes the E[psl] and E[reorder] by direct variation where as with E[perishable] it varies by indirect variation. For every arrival, there will be an unit in the system getting down when they leave the system. To fulfill such required number of items, there must be a reorder needed. Since the inventory reduces by the more sales, there must be less number of items remaining in the inventory storage place. This cause the E[perishable] become less.

More interestingly, as we predicted earlier, the intensity rate μ is inversely proportional to each E[psl], E[perishable]. If we increase μ, each of them starts falling down. If mean service time of individual customer too short, number of inventory falls down fast and less inventory requires more reorder and less number of perishable items.

Table 5: Impact of E[psl], E[reorder and E[perishable] with the parameter variation

β_2	θ	α_1	μ / α	E[stock] 7	E[stock] 11	E[reorder] 7	E[reorder] 11	E[perishable] 7	E[perishable] 11
0	4.5	0.05	0.70	24.9645	22.6523	0.3136	0.4386	1.2482	1.1326
			0.90	26.3157	24.3173	0.3138	0.4543	1.3158	1.2159
			1.10	27.2531	25.5090	0.3089	0.4602	1.3627	1.2755
		0.07	0.70	24.4147	22.2220	0.3277	0.4503	1.7090	1.5555
			0.90	25.8047	23.9103	0.3293	0.4679	1.8063	1.6737
			1.10	26.7722	25.1230	0.3252	0.4751	1.8741	1.7586
		0.09	0.70	23.9075	21.8168	0.3408	0.4614	2.1517	1.9635
			0.90	25.3337	23.5268	0.3439	0.4809	2.2800	2.1174
			1.10	26.3299	24.7591	0.3408	0.4893	2.3697	2.2283
	5	0.05	0.70	24.9645	22.6523	0.3188	0.4503	1.2482	1.1326
			0.90	26.3157	24.3173	0.3191	0.4665	1.3158	1.2159
			1.10	27.2531	25.5090	0.3142	0.4726	1.3627	1.2755
		0.07	0.70	24.4147	22.2220	0.3330	0.4621	1.7090	1.5555
			0.90	25.8047	23.9103	0.3347	0.4803	1.8063	1.6737
			1.10	26.7722	25.1230	0.3307	0.4877	1.8741	1.7586
		0.09	0.70	23.9075	21.8168	0.3463	0.4733	2.1517	1.9635
			0.90	25.3337	23.5268	0.3495	0.4934	2.2800	2.1174
			1.10	26.3299	24.7591	0.3464	0.5022	2.3697	2.2283
	5.5	0.05	0.70	24.9645	22.6523	0.3211	0.4553	1.2482	1.1326
			0.90	26.3157	24.3173	0.3214	0.4718	1.3158	1.2159
			1.10	27.2531	25.5090	0.3165	0.4780	1.3627	1.2755
		0.07	0.70	24.4147	22.2220	0.3353	0.4672	1.7090	1.5555
			0.90	25.8047	23.9103	0.3371	0.4856	1.8063	1.6737
			1.10	26.7722	25.1230	0.3330	0.4932	1.8741	1.7586
		0.09	0.70	23.9075	21.8168	0.3486	0.4785	2.1517	1.9635
			0.90	25.3337	23.5268	0.3519	0.4988	2.2800	2.1174
			1.10	26.3299	24.7591	0.3488	0.5078	2.3697	2.2283

Table 6: Impact of E[psl], E[reorder and E[perishable] with the parameter variation

β_2	θ	α_1	μ / α	E[stock] 7	E[stock] 11	E[reorder] 7	E[reorder] 11	E[perishable] 7	E[perishable] 11
0.5	4.5	0.05	0.70	18.5347	15.4582	0.2337	0.2938	0.9267	0.7729
			0.90	20.5232	17.5264	0.2533	0.3280	1.0262	0.8763
			1.10	22.0246	19.1546	0.2662	0.3532	1.1012	0.9577
		0.07	0.70	18.2647	15.2755	0.2384	0.2973	1.2785	1.0693
			0.90	20.2574	17.3398	0.2589	0.3323	1.4180	1.2138
			1.10	21.7661	18.9683	0.2725	0.3582	1.5236	1.3278
		0.09	0.70	18.0051	15.0982	0.2430	0.3007	1.6205	1.3588
			0.90	20.0014	17.1583	0.2644	0.3366	1.8001	1.5442
			1.10	21.5169	18.7869	0.2787	0.3631	1.9365	1.6908
	5	0.05	0.70	18.2833	15.0507	0.2366	0.2977	0.9142	0.7525
			0.90	20.2883	17.1217	0.2573	0.3339	1.0144	0.8561
			1.10	21.8085	18.7631	0.2712	0.3609	1.0904	0.9382
		0.07	0.70	18.0201	14.8771	0.2412	0.3009	1.2614	1.0414
			0.90	20.0281	16.9432	0.2628	0.3380	1.4020	1.1860
			1.10	21.5549	18.5840	0.2773	0.3657	1.5088	1.3009
		0.09	0.70	17.7668	14.7084	0.2456	0.3041	1.5990	1.3238
			0.90	19.7774	16.7695	0.2681	0.3420	1.7800	1.5093
			1.10	21.3101	18.4095	0.2834	0.3705	1.9179	1.6569
	5.5	0.05	0.70	18.1713	14.8689	0.2379	0.2993	0.9086	0.7434
			0.90	20.1830	16.9399	0.2591	0.3364	1.0092	0.8470
			1.10	21.7112	18.5861	0.2733	0.3642	1.0856	0.9293
		0.07	0.70	17.9110	14.6992	0.2424	0.3025	1.2538	1.0289
			0.90	19.9254	16.7649	0.2644	0.3404	1.3948	1.1735
			1.10	21.4596	18.4102	0.2794	0.3689	1.5022	1.2887
		0.09	0.70	17.6605	14.5342	0.2468	0.3056	1.5894	1.3081
			0.90	19.6770	16.5946	0.2697	0.3443	1.7709	1.4935
			1.10	21.2168	18.2387	0.2854	0.3736	1.9095	1.6415

Table 7: Impact of E[psl], E[reorder and E[perishable] with the parameter variation

<table>
<tr><th rowspan="3">β_2</th><th rowspan="3">θ</th><th rowspan="3">α_1</th><th></th><th colspan="2">E[stock]</th><th colspan="2">E[reorder]</th><th colspan="2">E[perishable]</th></tr>
<tr><th>μ</th><th rowspan="2">7</th><th rowspan="2">11</th><th rowspan="2">7</th><th rowspan="2">11</th><th rowspan="2">7</th><th rowspan="2">11</th></tr>
<tr><th>α</th></tr>
<tr><td rowspan="27">1</td><td rowspan="9">4.5</td><td rowspan="3">0.05</td><td>0.70</td><td>12.2895</td><td>10.2694</td><td>0.1562</td><td>0.1954</td><td>0.6145</td><td>0.5135</td></tr>
<tr><td>0.90</td><td>14.2804</td><td>12.1335</td><td>0.1792</td><td>0.2281</td><td>0.7140</td><td>0.6067</td></tr>
<tr><td>1.10</td><td>15.9157</td><td>13.7102</td><td>0.1973</td><td>0.2549</td><td>0.7958</td><td>0.6855</td></tr>
<tr><td rowspan="3">0.07</td><td>0.70</td><td>12.1776</td><td>10.1919</td><td>0.1586</td><td>0.1974</td><td>0.8524</td><td>0.7134</td></tr>
<tr><td>0.90</td><td>14.1615</td><td>12.0488</td><td>0.1822</td><td>0.2306</td><td>0.9913</td><td>0.8434</td></tr>
<tr><td>1.10</td><td>15.7933</td><td>13.6209</td><td>0.2007</td><td>0.2578</td><td>1.1055</td><td>0.9535</td></tr>
<tr><td rowspan="3">0.09</td><td>0.70</td><td>12.0681</td><td>10.1157</td><td>0.1610</td><td>0.1994</td><td>1.0861</td><td>0.9104</td></tr>
<tr><td>0.90</td><td>14.0450</td><td>11.9654</td><td>0.1851</td><td>0.2330</td><td>1.2640</td><td>1.0769</td></tr>
<tr><td>1.10</td><td>15.6733</td><td>13.5331</td><td>0.2041</td><td>0.2606</td><td>1.4106</td><td>1.2180</td></tr>
<tr><td rowspan="9">5</td><td rowspan="3">0.05</td><td>0.70</td><td>11.2674</td><td>9.0422</td><td>0.1473</td><td>0.1796</td><td>0.5634</td><td>0.4521</td></tr>
<tr><td>0.90</td><td>13.2040</td><td>10.7934</td><td>0.1709</td><td>0.2123</td><td>0.6602</td><td>0.5397</td></tr>
<tr><td>1.10</td><td>14.8221</td><td>12.3059</td><td>0.1899</td><td>0.2400</td><td>0.7411</td><td>0.6153</td></tr>
<tr><td rowspan="3">0.07</td><td>0.70</td><td>11.1733</td><td>8.9823</td><td>0.1494</td><td>0.1812</td><td>0.7821</td><td>0.6288</td></tr>
<tr><td>0.90</td><td>13.1024</td><td>10.7265</td><td>0.1734</td><td>0.2144</td><td>0.9172</td><td>0.7509</td></tr>
<tr><td>1.10</td><td>14.7161</td><td>12.2343</td><td>0.1928</td><td>0.2424</td><td>1.0301</td><td>0.8564</td></tr>
<tr><td rowspan="3">0.09</td><td>0.70</td><td>11.0811</td><td>8.9232</td><td>0.1514</td><td>0.1828</td><td>0.9973</td><td>0.8031</td></tr>
<tr><td>0.90</td><td>13.0026</td><td>10.6606</td><td>0.1759</td><td>0.2164</td><td>1.1702</td><td>0.9594</td></tr>
<tr><td>1.10</td><td>14.6120</td><td>12.1636</td><td>0.1958</td><td>0.2448</td><td>1.3151</td><td>1.0947</td></tr>
<tr><td rowspan="9">5.5</td><td rowspan="3">0.05</td><td>0.70</td><td>10.7714</td><td>8.4426</td><td>0.1426</td><td>0.1709</td><td>0.5386</td><td>0.4221</td></tr>
<tr><td>0.90</td><td>12.6732</td><td>10.1258</td><td>0.1662</td><td>0.2033</td><td>0.6337</td><td>0.5063</td></tr>
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<tr><td>1.10</td><td>14.1766</td><td>11.5305</td><td>0.1883</td><td>0.2332</td><td>0.9924</td><td>0.8071</td></tr>
<tr><td rowspan="3">0.09</td><td>0.70</td><td>10.6007</td><td>8.3388</td><td>0.1464</td><td>0.1738</td><td>0.9541</td><td>0.7505</td></tr>
<tr><td>0.90</td><td>12.4872</td><td>10.0088</td><td>0.1709</td><td>0.2069</td><td>1.1239</td><td>0.9008</td></tr>
<tr><td>1.10</td><td>14.0798</td><td>11.4676</td><td>0.1910</td><td>0.2354</td><td>1.2672</td><td>1.0321</td></tr>
</table>

7 Conclusion

The finite source population is considered to explore the non-SDAP and non-QDSP in the SQIS. The generalization of homogeneous and non-homogeneous arrival and service processes are given in the steady state of the model. Also, the comparative discussion is made in the numerical investigations.The illustrations given in the examples enhance minimum optimal total cost for the QDSP category. The SDAP will increase the number of units arriving to the inventory system. This increased units of arrival will produce the more sales of the inventory. When we are focusing the development of the inventory business, the first step has to be initialized to attract the customers towards the system. For such process, displayed stock level will assure the increase of customers in the inventory system. And Maintaining the sufficient current stock level will play the crucial role for the development of an inventory system. Simultaneously, the QDSR contribute the reduce of waiting time of a customer in the system. If a management provides some polices to reduce the waiting time of an individual in the servicing system, the customers are impressed and they will come to the same system often. In such a way that, the proposed model will applicable in a economically.

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