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Study of feedback retrial queueing system with working vacation, setup time and perfect repair

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Abstract

This manuscript analyses a retrial queueing system with working vacation, interruption, feedback, and setup time with the perfect repair. In the proposed model, the server takes vacation whenever the system gets empty but it still serves the customers at a relatively lower rate. To save power, the concept of setup time is included in the model. At vacation completion instant, the server is immediately turned off as soon as the system gets empty. The customer, who arrives during the closed-down state, activates the server and waits for his turn till the server is turned on. The unreliable server may sometimes fail to activate during setup. The failed server will resume service on being repaired. In the paper, explicit expressions for system size, sojourn times, and probabilities of various states of the server are obtained and results are analyzed graphically using MATLAB software.

Keywords: Retrial queue; working vacation; feedback; setup time.

2010 AMS subject classification[2]: 60K25, 60K30

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1. Introduction

Retrial queueing systems with vacations and feedback of customers have attracted many researchers due to their widespread applications in real-life situations such as cellular networks, call centers, computer systems, inventory systems, and production management. In such systems, the customer, who finds the server busy, joins the orbit (free pool) and after a random period, it retries for the service. The retrials may follow a constant or classical retrial policy. In a constant retrial policy, the customer at the head in retrial orbit can reattempt for the service, but in a classical one, all the customers can retry for their turn independent of all others in the orbit. Many researchers analyzed the applications of retrial queueing systems. Falin and Templeton [6] did pioneering work on retrial queues. A good survey on the retrial queueing system was done by Artalejo et al. [2] and Yang et al. [20].

Queueing systems with vacations play a vital role in many real-life systems. The vacations may be due to many reasons. In classical vacation policy, no service is provided to the customers during vacation. Servi and Finn [18] introduced a new vacation class i.e., working vacation, in which service is provided to customers but at a comparatively lower rate. Readers may refer Do [5], Arivudainambi et al. [1] and Chandrasekaran et al. [3] for reference. Furthermore, the concept of vacation interruption has been widely used in queueing systems. In this policy, at service completion instant, the server interrupts the ongoing vacation and returns to a normal working state on finding waiting customers. Keeping in view the importance of the concept, many researchers have analyzed the queueing systems with vacation interruption. Li and Tian [12] analyzed M/M/1 queueing system with working vacation and interruption using the matrix geometric method. A pioneer work on M/G/1 queueing model with vacation interruption was done by Zhang and Hou [21]. Later, Gupta and Kumar [7, 8, 9] studied retrial queues with different vacation policies, impatient behaviour of customers and obtain closed-form expressions for important performance measures.

The retrial queueing system with Bernoulli feedback of customers is characterized by the feature that the unsatisfied customers may rejoin the system with some probability until they receive satisfactory service. Choi et al. [4] analyzed a retrial queueing model with geometric loss and feedback. Kumar et al. [10,11] studied M/G/1 retrial queueing model with feedback and starting failure. Mokaddis [15] analyzed feedback queueing systems with vacations and system failure. Varalakshmi et al. [19] discussed a single server queue with immediate feedback and server breakdown. The concept of power saving is very important in today's scenario. To save power, the system should be turned off when not in use. Realizing the need for power saving, many researchers studied queueing model with setup time. Phung- Duc [16, 17] incorporated the concept of setup time to retrial queueing system. We may refer the reader to [13, 14] for the related works.

In this manuscript, a single server retrial queueing system with feedback, setup time, working vacation, and interruption under perfect repair is analyzed. If setup time is taken as zero, the model reduces to M/M/1 feedback retrial queueing system with working vacation and interruption. Further, the model changes to a retrial queueing system with feedback and setup time with perfect repair if vacation time tends to zero. Thus, our model generalizes some of the retrial queueing models existing in queueing literature.

2. Practical application of the model

Consider a manufacturing system consisting of an iron re-rolling mill, a Foreman(server), and a worker(assistant) to operate the mill. The foreman will operate the mill if the raw material is available and produce the products i.e., iron angles, iron rods, etc. If the raw material is not available due to transport issues, an increase in the price of raw material, etc., then the foreman may go on vacation(rest). During the vacation period of the foreman, if raw material becomes available then the worker will operate the mill, but the production will be relatively at a slow speed. When a batch of the product is completed, then the worker will call the foreman to resume the production at a higher speed by interrupting the vacation of the foreman.

In another situation, if the foreman’s vacation period completes, he will return to the production to operate the mill. If the raw material is available then he will manage the production at a higher speed otherwise, if the raw material is not available, to save power, he may turn off the mill. Again, the availability of new raw material will initiate the setup of the machine (re-rolling mill) and production starts again if setup occurs successfully otherwise the machine will be sent for repair, and during this period there will be no production.

3. Model description and assumptions

We considered a Markovian retrial queueing system with working vacation, interruption, feedback, and setup time under perfect repair. The following assumptions are taken for the proposed model.

1. The arrival of customers is by the Poisson process with rate λ. The customer who finds the server busy; joins the free pool (orbit) and waits for his turn. The customers in the orbit are assumed to follow classical retrial policy. The retrial time is exponentially distributed with parameter ξ.
2. The service time in the normal state of the server is assumed to be exponentially distributed with rate μ. When all the customers are served, the server goes to a working vacation state in which it still provides service to customers with a slow rate θ. The vacation time and service time in vacations again follow an exponential distribution with vacation rate ϕ.
3. The unsatisfied customers may rejoin the orbit as feedback customers with probability 'f' or may leave the system with complementary probability $\bar{f}$= (1-f).
4. On completion of vacation, if customers are found still waiting for their turn, the normal service period resumes otherwise, the server is turned off immediately to save power.
5. The customer who arrives in the off-state of the server; waits for his turn in front of the server till it is turned on. The setup time is required to restart the server. The setup time is assumed to follow an exponential distribution with a mean of 1/s. The customers arriving in the setup state, have to join the orbit.
6. The server is assumed to be unreliable i.e., during the set-up state, activation of the server may fail with probability $\bar{p}$ = (1-p). The failed server is sent for repair and repair time is exponentially distributed with parameter r.
7. The inter-arrival time, service time, vacation time, retrial time, and setup time are all mutually independent.

   Taking N(t) as the number of customers in the orbit at time t and J(t) as the state of the server.

   where,

$$J(t) = \begin{cases} 0, & the\ system\ is\ busy\ in\ a\ normal\ state\ at\ time\ t \\ 1, & the\ server\ is\ free\ in\ a\ normal\ state\ at\ time\ t \\ 2, & the\ server\ is\ busy\ in\ working\ vacation\ state \\ 3, & the\ server\ is\ free\ in\ working\ vacation\ state \\ 4, & the\ server\ is\ in\ setup\ or\ close-down\ state \\ 5, & the\ server\ is\ under\ repair\ state \end{cases}$$

{N(t), J(t)} represents a Markov process with following state-space
{(n, j), n ≥ 0, j=0,1,2,4,5} U {(0,3)}.

Here (n, 0), n ≥ 0 represents that server is busy in a regular service period with n customers waiting in the orbit. The state (0, 1) represents that system is in a close-down period. States (n, 1), n ≥1 shows that system is free in the regular service period. States (n, 2), n ≥0 represents the state that the system is busy in the working vacation period. The state (0, 3) represents that the server is free during the vacation period. States (n, 3), n≥1 do not exist due to the inclusion of the concept of vacation interruption. States (n, 4), n≥0 represent that system is in a setup state with n customers in the orbit and one customer waiting in the service area for successful set up of the system. States (n, 5), n≥0, show that the system is under repair due to sudden breakdowns with n customers waiting in the orbit for their turn.

4. Steady-state equations and stationary probabilities

Denoting by $p_{n\,j}$, the probability of n customers waiting in the orbit when the system is in state j and using Markov process for the quasi-birth death model, the stationary state equations governing the model are

$$(\lambda+\mu)p_{0\,0} = \phi p_{0\,2} + \xi p_{1\,1} + sp p_{0\,4} + r p_{0\,5} \tag{1}$$

$$(\lambda+\mu)p_{n\,0} = \lambda p_{n-1\,0} + \phi p_{n\,2} + (n+1)\xi p_{n+1\,1} + \lambda p_{n\,1} + sp p_{n\,4} + r p_{n\,5}, \quad n \geq 1 \tag{2}$$

$$\lambda p_{0\,1} = \phi p_{0\,3} \tag{3}$$

$$(\lambda+\xi)p_{n\,1} = \bar{f}\theta p_{n\,2} + f\theta p_{n-1\,2} + f\mu p_{n-1\,0} + \bar{f}\mu p_{n\,0}\ , \quad n \geq 1 \tag{4}$$

$$(\lambda+\theta+\phi)p_{0\,2} = \lambda p_{0\,3} \tag{5}$$

$$(\lambda+\theta+\phi)p_{n\,2} = \lambda p_{n-1\,2}, \qquad n \geq 1 \tag{6}$$

$$(\lambda+\phi)p_{0\,3}=\mu\bar{f}p_{0\,0}+\bar{f}\theta p_{0\,2} \quad (7)$$

$$(\lambda+s)p_{0\,4}=\lambda p_{0\,1} \quad (8)$$

$$(\lambda+s)p_{n\,4}=\lambda p_{n-1\,4}, \quad n\geq 1 \quad (9)$$

$$(\lambda+r)p_{0\,5}=s\bar{p}p_{0\,4} \quad (10)$$

$$(\lambda+r)p_{n\,5}=s\bar{p}p_{n\,4}+\lambda p_{n-1\,5}\;, \quad n\geq\ 1 \quad (11)$$

Defining probability generating functions

$$G_i(z)=\sum_{n=0}^{\infty}p_{n\,i}z^n\;, \quad i=0,1,2,4,5 \quad (12)$$

Multiplying equations (1) and (2) with appropriate power of z and taking summation for all possible values of n and using above-defined generating functions,

$$(\lambda+\mu-\lambda z)G_0(z)$$
$$=\phi G_2(z)+\lambda G_1(z)+\xi G_1'(z)+spG_4(z)+rG_5(z)-\lambda p_{0\,1} \quad (13)$$

Multiplying equations (3) and (4) with appropriate power of z and taking summation for all possible values of n and using generating functions we get,

$$(\lambda+\xi)G_1(z)=(fz+\bar{f}\,)\mu G_0(z)+\left(\bar{f}+fz\right)\theta G_2(z)-A\,p_{0\,1} \quad (14)$$

$$where\ A=\frac{\bar{f}\theta p_{0\,2}+\mu\bar{f}p_{0\,0}-\xi p_{0\,1}-\phi p_{0\,3}}{p_{0\,1}} \quad (15)$$

$$p_{0\,2}=\frac{\lambda^2}{\phi(\lambda+\theta+\phi)}p_{0\,1} \quad (16)$$

$$p_{0\,3}=\frac{\lambda}{\phi}p_{0\,1} \quad (17)$$

$$p_{0\,0}=\left(\frac{\lambda(\lambda+\phi)}{\mu\phi\bar{f}}-\frac{\lambda^2\theta}{\mu\phi(\lambda+\theta+\phi)}\right)p_{0\,1} \quad (18)$$

Again using probability generating functions along with equations (5) and (6)

$$(\lambda+\theta+\phi-\lambda z)G_2(z)=\lambda p_{0\,3}$$
$$=\frac{\lambda^2}{\phi}p_{0\,1}$$

$$G_2(z) = \frac{\lambda^2}{\phi(\lambda+\theta+\phi-\lambda z)} p_{0\,1} \tag{19}$$

Using equations (8), (9), and (12) together

$$(\lambda+s)G_4(z) = \lambda z G_4(z) + \lambda p_{0\,1}$$

$$G_4(z) = \frac{\lambda}{(\lambda+s-\lambda z)} p_{0\,1} \tag{20}$$

Similar calculations in equations (10) and (11) yield

$$(\lambda+r-\lambda z)G_5(z) = s(1-p)G_4(z)$$

Makin use of equation (19) in the above equation, we obtain

$$G_5(z) = \frac{\lambda s(1-p)}{(\lambda+r-\lambda z)(\lambda+s-\lambda z)} p_{0\,1} \qquad (21$$

Using the value of $G_0(z)$ from equation (14) in equation (13), and rearranging the terms we get the following differential equations

$$G_1'(z) + \frac{1}{\xi}\left(\lambda - \frac{(\lambda+\xi)(\lambda+\mu-\lambda z)}{\mu(\bar{f}+fz)}\right)G_1(z) = B(z) \tag{22}$$

$$\begin{aligned} where\ B(z) = \frac{1}{\xi}\Bigg[&\left(\lambda + \frac{A(\lambda+\mu-\lambda z)}{\mu(\bar{f}+fz)}\right)p_{0\,1} - \left(\phi + \frac{\theta(\lambda+\mu-\lambda z)}{\mu}\right)G_2(z) \\ &- (spG_4(z) + rG_5(z))\Bigg] \end{aligned} \tag{23}$$

To solve the differential equation we first find integrating factor,

$$I.F = e^{\frac{\lambda z}{\xi}\left(1+\frac{\lambda+\xi}{\mu f}\right)}\left((\bar{f}+fz)^{-\frac{(\lambda+\xi)(\lambda+\mu f)}{\mu\xi f^2}}\right) \tag{24}$$

The solution of the differential equation (23) is

$$\begin{aligned} G_1(z) = e^{\frac{-\lambda z}{\xi}\left(1+\frac{\lambda+\xi}{\mu f}\right)}&\left((\bar{f}+fz)^{\frac{(\lambda+\xi)(\lambda+\mu f)}{\mu\xi f^2}}\right) \\ &\int_0^z e^{\frac{\lambda z}{\xi}\left(1+\frac{\lambda+\xi}{\mu f}\right)}\left((\bar{f}+fz)^{-\frac{(\lambda+\xi)(\lambda+\mu f)}{\mu\xi f^2}}\right)B(z)dz \end{aligned} \tag{25}$$

$G_0(z)$ is obtained from equation (14) as follows

$$G_0(z) = \frac{(\lambda+\xi)G_1(z) - (\bar{f}+fz)\theta G_2(z) + Ap_{0\,1}}{(fz+\bar{f}\,)\mu} \tag{26}$$

Taking limit $z \to 1$ in equations (19), (20), (21), (25), and (26) we obtain the expressions

$$G_2(1) = \frac{\lambda^2}{\phi(\theta+\phi)}p_{0\,1} \tag{27}$$

$$G_4(1) = \frac{\lambda}{s}p_{0\,1} \tag{28}$$

$$G_5(1) = \frac{\lambda(1-p)}{r}p_{0\,1} \tag{29}$$

$$G_1(1) = e^{\frac{-\lambda}{\xi}\left(1+\frac{\lambda+\xi}{\mu f}\right)}\int_0^1 e^{\frac{\lambda z}{\xi}\left(1+\frac{\lambda+\xi}{\mu f}\right)}\left(\left(\bar{f}+fz\right)^{-\frac{(\lambda+\xi)(\lambda+\mu f)}{\mu\xi f^2}}\right)B(z)dz \tag{30}$$

$$G_0(1) = \frac{(\lambda+\xi)G_1(1) - \theta G_2(1) + Ap_{0\,1}}{\mu} \tag{31}$$

Differentiating equations (19), (20), (21) and taking limit $z \to 1$we get

$$G_2'(1) = \frac{\lambda^3}{\phi(\theta+\phi)^2}p_{0\,1} \tag{32}$$

$$G_4'(1) = \frac{\lambda^2}{s^2}p_{0\,1} \tag{33}$$

$$G_5'(1) = \frac{\lambda^2(1-p)(r+s)}{sr^2}p_{0\,1} \tag{34}$$

Equation (23) on taking limit $z \to 1$ implies

$$G_1'(1) = \frac{1}{\xi}[\xi G_1(1) - (\theta+\phi)G_2(1) - spG_4(1) - rG_5(1) + (\lambda+A)p_{0\,1}] \tag{35}$$

Similarly differentiating equation (26) and taking limits we obtain

$$G_0'(1) = \frac{1}{\mu}[(\lambda+\xi)G_1'(1) - \theta G_2'(1) - f\theta G_2(1) - \mu f G_0(1)] \tag{36}$$

We observe that all the closed-form expressions for $G_i(z)$ and their derivatives for i=0, 1, 2, 4, 5 are implicitly expressed in terms of $p_{0\,1}$.

$p_{0\,1}$ may be obtained by using the normalization condition

$$G_0(1) + G_1(1) + G_2(1) + G_4(1) + G_5(1) = 1 \quad (37)$$

5. Performance measures

In the present section, we obtain some important system performance measures of our proposed model as follows.

Expected orbit length $E[L_O] = G_0'(1) + G_1'(1) + G_2'(1)$

$$+G_4'(1) + G_5'(1) \quad (38)$$

Expected sojourn time in orbit $E[W_O]$

$$= E[L_O]/\lambda$$

$$= \frac{G_0'(1) + G_1'(1) + G_2'(1) + G_4'(1) + G_5'(1)}{\lambda} \quad (39)$$

Expected system length $E[L_S] = E[L_O] + G_0(1) + G_2(1)$ (40)

Expected sojourn time in system $E[W_S] = E[L_S]/\lambda$

$$= \frac{E[L_O] + G_0(1) + G_2(1)}{\lambda} \quad (41)$$

Probability of server being in off state = $p_{0\,1}$

Probability of server being in working vacation state (Pr_{wv})

$$= G_2(1) + \frac{\lambda}{\phi} p_{0\,1}$$

$$= \frac{\lambda^2}{\phi(\theta + \phi)} p_{0\,1} + \frac{\lambda}{\phi} p_{0\,1} \quad (42)$$

Probability of server being in setup state (Pr_S) = $G_4(1)$

$$= \frac{\lambda}{s} p_{0\,1} \quad (43)$$

Probability of server in repair state $(Pr_R) = G_5(1)$

$$= \frac{\lambda(1-p)}{r} p_{0\,1} \qquad (44)$$

6. Numerical and graphical analysis

In the present section, the numerical and graphical interpretation of derived closed-form expressions of various system performance measures, for the proposed mathematical model is performed. For this purpose, some of the system parameters are assumed to be fixed as λ=3, μ=7, ξ=1.8, ϕ=2, θ=3, f=0.7, p=0.7, r=0.8, s=0.6, unless otherwise mentioned. The behaviours of important performance measures, for a different set of values of one or more of the parameters is analyzed in the below-plotted graphs.

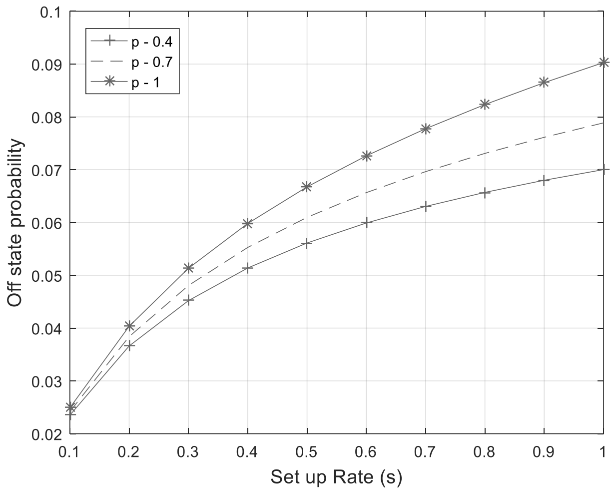

Figure1: Off-state probability versus setup rate for different values of p

From figure 1 we see that with an increase in setup rate, the probability of the server being in off-state increases. This is due to the reason that with an increase in the setup rate, the setup time decreases which causes early return in a normal state of the server hence increasing the probability of the server being in an off state. This probability of off-state increases with an increase in p, for a fixed value of set up rate; this is again due to an increase in chances of successful set up of server that further increases the off state probability.

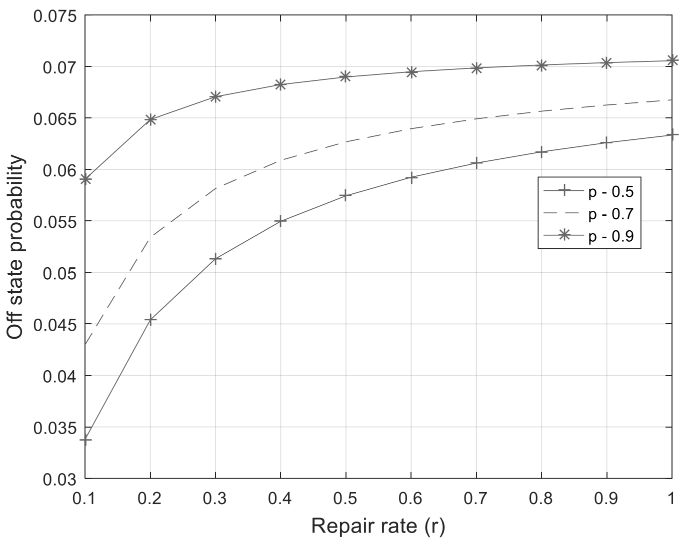

Figure2: Off-state probability versus repair rate for different values of p

We observe from figure 2 that the off-state probability of server increases with repair rate r, for a fixed value of p. This is due to a reduction in repair time with an increase in repair rate which leads to quick repair hence faster return to normal service thereby increasing the off-state probability.

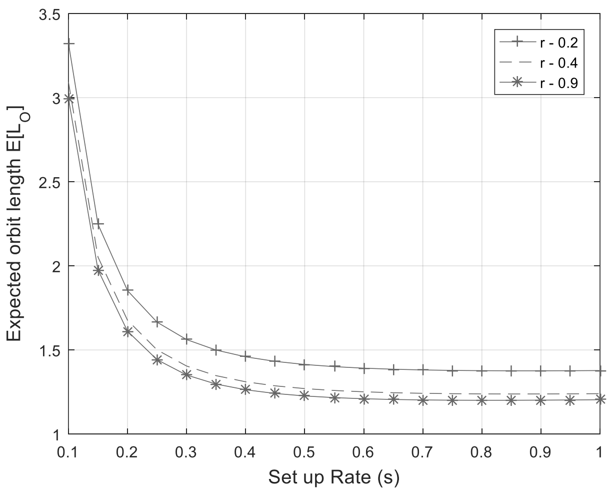

Figure3: Effect of setup rate on mean orbit length for different repair rates

Figure 3 reveals that the expected orbit length decreases with an increase in setup rate. This is because with an increase in setup rate, the time required for set up of server decreases which results in a quick return to normal service period of server thereby reducing the orbit length. For the same reason, mean orbit length decreases with a decrease in repair time.

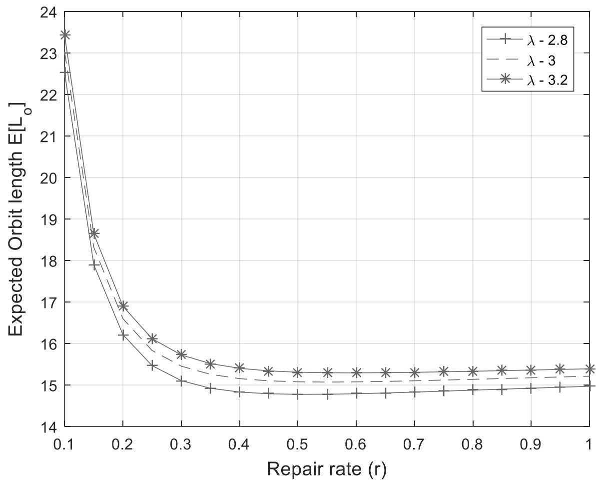

Figure 4: Effect of repair rate on mean orbit length for different arrival rates

We see from figure 4, the expected orbit length decreases with an increase in repair rate. As expected the mean orbit length increases with an increase in arrival rate, for a fixed repair rate. This is due to a reduction in inter-arrival time which increases mean orbit length.

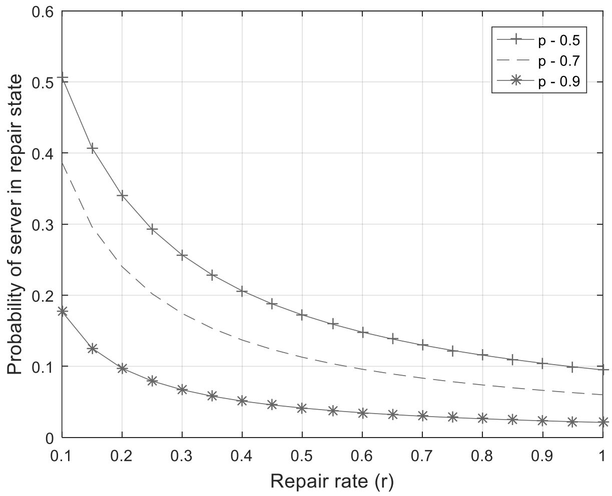

Figure 5: Effect of repair rate on repair state probability of server for different values of p

Figure 5 depicts that with an increase in repair rate, the probability of the server being in repair state decreases. This agrees with our expectations. As the repair rate increases, the repair is done in a lesser time that makes a faster return to the normal state from the repair state hence the probability of the server being in the repair state decreases. Again with an increase in p, for a fixed repair rate, the chances of successful activation (set up) of server raise hence the probability of server being in repair state decreases.

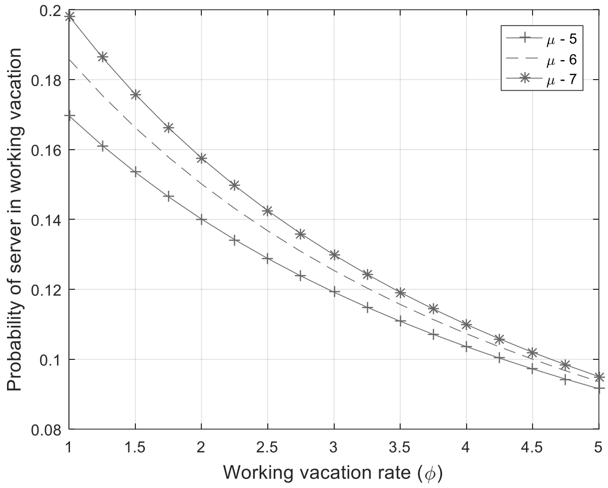

Figure 6: Probability of server in vacation versus vacation rate for different service rates

We observe from figure 6 that the probability of the server being in vacation state decreases with an increase in the rate of working vacation. The reason behind the observation is a decrease in the duration of vacation with an increase in the vacation rate. Further, the probability of the server in vacation state increases with service rate μ; this is due to faster service which promotes server vacations.

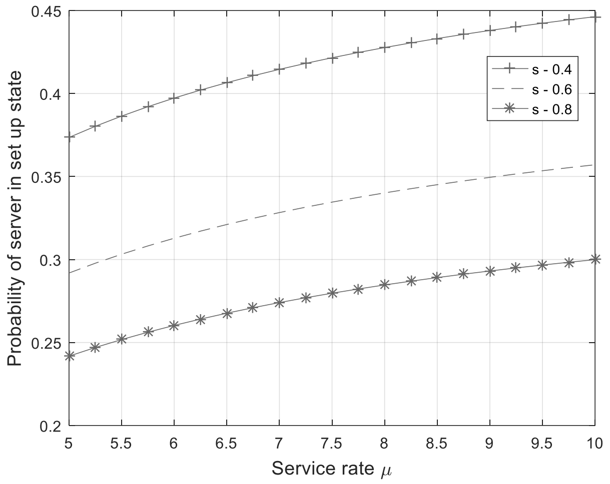

Figure 7: Probability of server in set up versus service rate for different setup rates

Figure 7 depicts that with an increase in service rate, the probability of the server being in setup state increases, this is as expected intuitively. For fixed service rate, as set up rate s increases, the probability decreases; this is due to faster activation of server with reduced setup time.

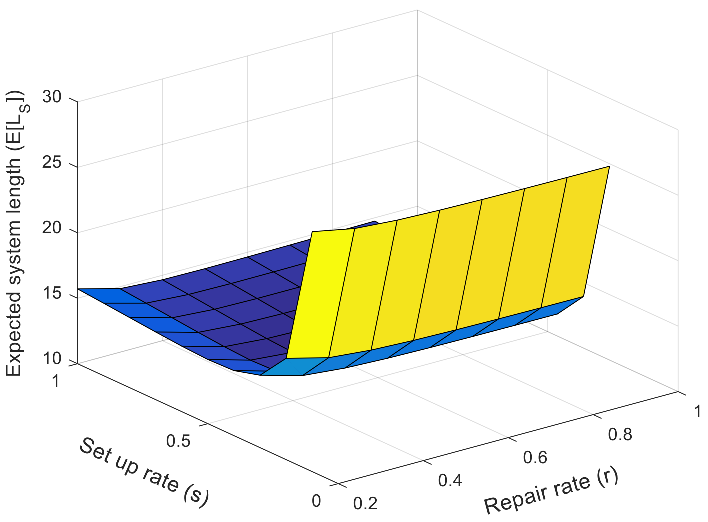

Figure 8: Variation in Expected system length with setup and repair rate

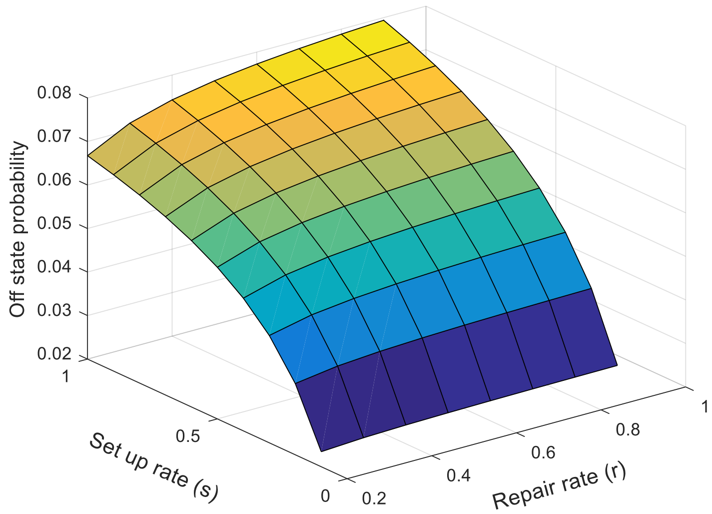

Figure 9: Variation in off state probability of server with setup and repair rate

Figures 8 and 9 represent the graphical behaviour of mean system length and off-state probability with setup and repair rate respectively. As expected, the mean system length decreases whereas off-state probability increases with an increase in the setup rate, for a fixed value of repair rate.

7. Conclusion and future scope

This paper analyses a single server retrial queueing system with working vacation, vacation interruption, Bernoulli feedback and setup time under perfect repair. The closed-form expressions for expected system size along with the probability of various system state probabilities, closed-down state have been obtained via the probability generating functions approach. The variation of the derived expressions against some of the system parameters is graphically studied by using MATLAB software. The observed graphical results are analyzed and are found to agree with the theoretically expected behaviour. The retrial queueing model with imperfect repair and multiple waiting servers can be considered for future investigations.

Conflict of interests

The authors declare that there is no conflict of interest.

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