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Contra Nα-I-Continuity over Nano Ideals

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Abstract

The conceptualization of Nα-I-open sets and Nα-I-continuous functions in nano ideal topology are used to study contra Nα-I-continuity. Also the characteristics and behaviours of contra Nα-I-continuity based on Nano Urysohn Space and Nano Ultra Hausdorff Space are discussed. **Keywords**: CNα-Cts function, CNα-I-Cts function, Nα-I-T_2 space, Nα-I-connected.
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1 Introduction

The ideal concept in topology was developed by Kuratowski [Kuratowski, 1966].The notion of α-I-continuity was introduced in 2004 [A. Acikgoz and Yuksel, 2004].The conception of nano topology was initated by L.Thivagar [Thivagar and Richard, 2013a].In addition to that the concept of continuity, α-continuity, kernal and clopen in Nano topology was introduced by [Karthiksankari and Subbulakshmi, 2019] [Thivagar and Richard, 2013b] and [M. Lellis Thivagar and SuthaDevi, 2017].Parimala and Jafari [Parimala and Jafari, 2018] had worked on Nano ideals.This work aims the introduction of contra Nα-I-continuous functions by applying the concept of Nα-I-open and Nα-I-continuity in nano ideal topology.Also this contra Nα-I-continuity are compared with some existing functions.Moreover, new class of functions are obtained. At every places the new notions have been substantiated with suitable examples.Throughout this article we use the notation NTS, NITS, N-regular, N-open, Nα-open, N-clopen, Nα-Cts for Nano Topological Spaces, Nano Ideal Topological Spaces, nano regular space, nano open, nano α-open, nano clopen, nano α-continuous respectively.Similar notations are used for their respective closed sets.

2 Preliminaries

Definition 2.1. *[M. Lellis Thivagar and SuthaDevi, 2017] Let (U, $\tau_R(X)$) be a NTS and S is a subset of U.The nano kernel of S is defined as NKer(S)=$\cap$ $\{U : S$ is a subset of U, $U \in \tau_R(X)\}$.*

Theorem 2.1. *[M. Lellis Thivagar and SuthaDevi, 2017] Let (U, $\tau_R(X)$) be a NTS and A_1, $A_2 \subseteq$ U.We have*

1. *$x \in$ NKer(A_1) iff for any N-closed set F containing x, A_1 and F are disjoint,*
2. *If $A_1 \subseteq$ NKer(A_1) and then A_1= Ker(A_1) if A_1 is N-open in U,*
3. *If $A_1 \subseteq A_2$, then NKer(A_1) $\subseteq$ NKer(A_2).*

Definition 2.2. *[Thivagar and SuthaDevi, 2016] A NTS (U, $\tau_R(X)$) along with an ideal I defined on U is called as a NITS and is denoted by (U, $\tau_R(X)$,I).Throughout this paper U represents a NTS (U, $\tau_R(X)$) and U_I represents a NITS (U, $\tau_R(X)$,I).*

Definition 2.3. *[Rajasekaran and Nethaji, 2018] Let (U, $\tau_R(X)$,I) be a nano ideal topological space and A $\subseteq$ U.Then A is said to be Nα-I-open if A $\subseteq$ Nint(Ncl*(Nint (A))).The complements of Nα-I-open is Nα-I-closed set.*

Theorem 2.2. *[V. Inthumathi and Krishnaprakash, 2020] Let (U_1, $\tau_R(X_1)$,I) be a NITS and (U_2, $\tau_R(X_1)$) be a NTS.Then $h : U_1 \rightarrow U_2$ is called Nα-I-Cts on U_1 if h^{-1}(S) is Nα-I-open in U_1 for any N-open set S in U_2.*

Definition 2.4. *Bhuvaneswari and Nagaveni [2018] A NTS (U, $\tau_R(X)$) is called N-regular Space, if for each N-closed set T and each point $x \notin T$, ∃ disjoint N-open sets G and H such that $x \in G$ and $T \subset G$.*

3 Contra Nα-I-Continuity

The notations used are CNα-open, CN-Cts function, CNα-Cts function, CNα-I-Cts function for contra nano α-open, contra nano continuous, contra Nα- continuous, contra Nα-I-continuous function resp.

Definition 3.1. *Let (U_1, $\tau_R(X_1)$) and (U_2, $\tau_{R'}(X_2)$) be NTS.Then $h : U_1 \rightarrow U_2$ is CNα-Cts if h^{-1}(S) is Nα-closed in U_1 whenever S is N-open set in U_2.*

Definition 3.2. *Let h : (U_1, $\tau_R(X_1)$,I) → (U_2, $\tau_{R'}(X_2)$) is CNα-I-Cts if h^{-1}(S) is Nα-I-closed in U_1 whenever S is N-open set in U_2.*

Example 3.1. *Let U_1={i,j,k,l}, U_1/R={{i},{j},{k},{l}} and X_1={i}.Then $\tau_R(X_1)$ ={U_1,ϕ,{i}}.Let I={ϕ}.Here the N α-I-open sets are{U_1,ϕ,{i},{i,j},{i,k},{i,l},{i, j,k},{i,j,l},{i,k,l}}. Let U_2={m,n,o,p} with U_2/R'={{m},{n},{o,p}} and X_2={n, o}.Then $\tau_{R'}(X_2)$={ U_2,ϕ,{n},{o,p},{n,o,p}}.We define h:(U_1,$\tau_R(X_1)$,I)→(U_2, $\tau_{R'}$ (X_2)) as f(i)=m, f(j)=n, f(k)=o and f(l)=p.Then h^{-1}(S) is Nα-I-closed in U_1 whenever S is N-open in U_2.Therefore h is CNα-I-Cts.*

Proposition 3.1. *1. Any CNα-I-Cts function is CNα-Cts.*

2. Any CN-Cts function is CNα-I-Cts.

Proof. *(i) Let h : (U_1, $\tau_R(X_1)$,I) → (U_2, $\tau_{R'}(X_2)$) be a CNα-I-Cts function.Let S be a N-open in U_2.Since h is CNα-I-Cts, h^{-1}(S) is Nα-I-closed in U_1.We know that each Nα-I-closed set is Nα-closed.Hence h^{-1}(S) is Nα-closed in U_1.Hence h is CNα-Cts.*
(ii) Let h : (U_1, $\tau_R(X_1)$,I) → (U_2, $\tau_{R'}(X_2)$) be a CN-Cts function.Let S be a N-open set in U_2.Since h is CN-Cts, h^{-1}(S) is N-closed in U_1.It is obvious that every N-closed set is Nα-I-closed.Thus h^{-1}(S) is Nα-I-closed in U_1.Which implies h is CNα-I-Cts function.□

Example 3.2. *CNα-Cts ⇏ CNα-I-Cts*
Let U_1={i,j,k,l} with U_1/R={{i},{j,k},{l}} and X_1={l}.Then $\tau_R(X_1)$={U_1,ϕ,{l}}. Let I={ϕ,{l}}.Here the Nα-open sets are {U_1,ϕ,{l},{i,l},{j,l},{k,l},{i,j,l},{i,k,l},

$\{j,k,l\}\}$ and $N\alpha$-I-open sets are $\{U_1,\phi,\{l\}\}$.Let $U_2=\{m,n,o,p\}$ with $U_2/R'=\{\{m\},\{n,o\},\{p\}\}$ and $X_2=\{m,n\}$.Then $\tau_{R'}(X_2) = \{U_2,\phi,\{m\},\{n,o\},\{m,n,o\}\}$. We define $h : (U_1, \tau_R(X_1),I) \to (U_2, \tau_{R'}(X_2))$ as h(i)=m, h(j)=n, h(k)=o and h(l)=p.Then $h^{-1}(S)$ is $N\alpha$-closed in U_1 but not $N\alpha$-I-closed whenever S is N-open set in U_2. Hence h is $CN\alpha$-Cts but not $CN\alpha$-I-Cts function.

Example 3.3. *$CN\alpha$-I-Cts $\nRightarrow$ CN-Cts*
Let $U_1=\{i,j,k,l\}$ with $U_1/R= \{\{i\},\{j\},\{k\},\{l\}\}$ and $X_1=\{i\}$.Then $\tau_R(X_1)=\{U_1,\phi,\{i\}\}$.Let $I = \{\phi\}$.Here the $N\alpha$-I-open sets are $\{U_1,\phi,\{i\},\{i,j\},\{i,k\},\{i,l\},\{i,j,k\},\{i,j,l\},\{i,k,l\}\}$.Let $U_2=\{m,n,o,p\}$ with $U_2/R'=\{\{m\},\{o,p\},\{n\}\}$ and $X_2=\{n,o\}$.Then $\tau_{R'}(X_2) = \{U_2,\phi,\{n\},\{o,p\},\{n,o,p\}\}$.We define $h : (U_1, \tau_R(X_1),I) \to (U_2, \tau_{R'}(X_2))$ as h(i) = m, h(j) = n, h(k) = o and h(l) = p.Then $h^{-1}(S)$ is $N\alpha$-I-closed in U_1 but not N-closed whenever S is N-open set in U_2. Hence h is $CN\alpha$-I-Cts but not CN-Cts function.

Theorem 3.1. *Let $h : (U_1, \tau_R(X_1),I) \to (U_2, \tau_{R'}(X_2))$, then the following statements are equivalent:*

1. *h is $CN\alpha$-I-Cts,*
2. *for each N-closed subset T of U_2, $h^{-1}(T) \in N\alpha IO(U_1)$,*
3. *for each $x \in U_1$ and each N-closed set T of U_2 containing h(x), $\exists\ U \in N\alpha IO(U_1)$ such that $h(U) \subset T$,*
4. *$h(N\alpha I\text{-}cl(V)) \subset NKer(h(V))$ for each $V \subseteq U_1$,*
5. *$N\alpha I\text{-}cl(h^{-1}(W)) \subset h^{-1}(NKer(W))$ for each $W \subseteq U_2$.*

Proof. *(i) $\Rightarrow$ (ii) and (ii) $\Rightarrow$ (iii) are obvious.*
(iii) $\Rightarrow$ (ii) Let T be any N-closed set of U_2 and $x \in h^{-1}(T)$.Then $h(x) \in T$ and $\exists\ U_x \in N\alpha IO(U_1)$ such that $h(U_x) \subset T$.Therefore, we obtain $h^{-1}(T) = \cup \{ U_x : x \in h^{-1}(T)\}$ and hence $h^{-1}(T) \in N\alpha IO(U_1)$.
(ii) $\Rightarrow$ (iv)Let $V \subseteq U_1$.If $y \notin NKer(h(V))$, then by Thm 2.1, $\exists$ a N-closed set T of U_2 containing y such that $h(V) \cap T=\phi$.Therefore $V \cap h^{-1}(T) = \phi$ and $N\alpha I\text{-}cl(V) \cap h^{-1}(T)=\phi$.Hence $h(N\alpha I\text{-}cl(V)) \cap T=\phi$ and $y \notin h(N\alpha I\text{-}cl(V))$.Thus $h(N\alpha I\text{-}cl(V)) \subset NKer(h(V))$.
(iv) $\Rightarrow$ (v) Let $W \subseteq U_2$.By the hypothesis and Thm 2.1, $h(N\alpha I\text{-}cl(h^{-1}(W))) \subset NKer(h(h^{-1}(W))) \subset NKer(W)$ and $N\alpha I\text{-}cl(h^{-1}(W)) \subset h^{-1}(NKer(W))$.
(v) $\Rightarrow$ (i) Let W be a N-open set of U_2.By Thm 2.1, $N\alpha I\text{-}cl(h^{-1}(W)) \subset h^{-1}(NKer(W)) = h^{-1}(W)$ and $N\alpha I\text{-}cl(h^{-1}(W)) = h^{-1}(W)$.Therefore $h^{-1}(W)$ is $N\alpha$-I-closed in $(U_1, \tau_R(X),I)$. □

Theorem 3.2. *If a function h : (U_1, $\tau_R(X_1)$,I) $\rightarrow$ (U_2, $\tau_{R'}(X_2)$) is CNα-I-Cts and V is N-regular, then h is Nα-I-Cts.*
Proof. *Let x $\in U_1$ and Y a N-open set of U_2 containing h(x).Since U_2 is N-regular, $\exists$ a N-open set Z in U_2 containing h(x) such that Ncl(Z) $\subset$ Y.Since h is CNα-I-Cts, by the above theorem, $\exists$ X $\in$ NαIO(U_1) such that h(X) $\subset$ Ncl(Z).Therefore h(X) $\subset$ Ncl(Z) $\subset$ Y.Hence h is Nα-I-Cts.*□

Definition 3.3. *A function h : (U_1, $\tau_R(X_1)$,I) $\rightarrow$ (U_2, $\tau_{R'}(X_2)$) satisfy the Nα-I-interiority rule if NαI-int(h^{-1}(Ncl(W))) $\subset$ h^{-1}(W) Whenever W is N-open set of (U_2, $\tau_{R'}(X_2)$).*

Theorem 3.3. *If a function h : (U_1, $\tau_R(X_1)$,I) and (U_2, $\tau_{R'}(X_2)$) is CNα-I-Cts and satisfies Nα-I-interiority rule, then h is Nα-I-Cts.*
Proof. *Let Y be any N-open set of U_2. Since h is CNα-I-Cts and Ncl(Y) is N-closed, by Thm 3.1, h^{-1}(Ncl(Y)) is Nα-I-open in (U_1, $\tau_R(X)$,I).By hypothesis of h, h^{-1}(Y) $\subset$ h^{-1}(Ncl(Y)) $\subset$ NαI-int(h^{-1}(Ncl(Y))) $\subset$ NαI-int(h^{-1}(Y)) $\subset$ h^{-1}(Y).Thus, we obtain h^{-1}(Y)=NαI-int(h^{-1}(Y)) and consequently h^{-1}(Y) $\in$ NαIO(U).Therefore h is Nα-I-Cts.*□

Theorem 3.4. *Let (U_1, $\tau_R(X_1)$,I) be any NITS and h : (U_1, $\tau_R(X_1)$,I) $\rightarrow$ (U_2, $\tau_{R'}(X_2)$) be a function and g : $U_1 \rightarrow U_1 \times U_2$ be the graph function, given by g(x) = (x, h(x)) for every x $\in U_1$.Then f is CNα-I-Cts if and only if g is Nα-I-Cts.*
Proof. *Let x $\in U_1$ and let T be a N-closed subset of $U_1 \times U_2$ containing g(x).Then T $\cap$ ($\{x\} \times U_2$) is N-closed in $\{x\} \times U_2$ containing g(x).Also $\{x\} \times U_2$ is homeomorphic to U_2. Hence $\{y \in U_2$: (x, y) $\in T\}$ is a N-closed subset of U_2.Since h is CNα-I-Cts, $\cup$ $\{$ h^{-1}(Y) $\in U_2$: (x, y) $\in$ T $\}$ is a Nα-I-open subset of (U_1, $\tau_R(X_1)$,I).Further, x $\in$ $\cup$ $\{$ h^{-1}(Y) $\in U_2$: (x, y) $\in$ T $\}$ $\subset$ g^{-1}(T).Hence g^{-1}(T) is Nα-I-open.Then g is CNα-I-Cts. Conversely, let F be a N-closed subset of U_2.Then $U_1 \times$ F is a N-closed subset of $U_1 \times U_2$.Since g is CNα-I-Cts, g^{-1}($U_1 \times$ F) is a Nα-I-open subset of U_1. Also, g^{-1}($U_1 \times$ F)=h^{-1}(F).Hence h is CNα-I-Cts.*□

Definition 3.4. *A NITS (U_1, $\tau_R(X_1)$,I) is called Nα-I-T_2 if for any distinct two points x, y $\in U_1$, $\exists$ X, Y $\in$ NαIO(U_1) containing x and y, resp., such that X $\cap$ Y=ϕ.*

Definition 3.5.

1. *A NTS (U_1, $\tau_R(X_1)$) is termed as a N-Urysohn Space if for any two distinct points x, y $\in U_1$, $\exists$ disjoint N-open subsets x $\in$ A, y $\in$ B such that the N-closures $\overline{A}$ and $\overline{B}$ are disjoint N-closed subsets of U_1.*
2. *A NTS (U_1, $\tau_R(X_1)$) is called N-Ultra Hausdorff if any two distinct points of U_1 can be separated by disjoint N-clopen sets.*

Theorem 3.5. *If (U_1, $\tau_R(X_1)$,I) is an NITS and for any two distinct points x_1 , x_2 $\in U_1$, $\exists$ a function h into a N-Urysohn Space (U_2, $\tau_{R'}(X_2)$) such that h(x_1) $\neq$ h(x_2) and h is CNα-I-Cts at x_1 , x_2, then (U_1, $\tau_R(X_1)$,I) is Nα-I-T_2.*

Proof. Let x_1 , x_2 be any two distinct points of U_1.Then by hypothesis there is a N-Urysohn Space (U_2, $\tau_{R'}(X_2)$) and a function h : (U_1, $\tau_R(X_1)$,I) and (U_2, $\tau_{R'}(X_2)$), which satisfies the required condition.Let y_i= h(x_i) for i=1,2.Then $y_1 \neq y_2$. Since (U_2, $\tau_{R'}(X_2)$) is N-Urysohn, $\exists$ N-open neighbourhoods X_{y_1} and X_{y_2} of y_1, y_2 respectively in U_2 such that Ncl(X_{y_1}) $\cap$ Ncl(X_{y_2})=ϕ.Since h is CNα-I-Cts at x_i, $\exists$ Nα-I-open neighbourhoods W_{x_i} of x_i in U_1 such that h(W_{x_i}) $\subset$ Ncl(X_{y_i}) for i=1,2. Hence we get $W_{x_1} \cap W_{x_2}$=ϕ because Ncl(X_{y_1}) $\cap$ Ncl(X_{y_2})=ϕ.Therefore (U_1, $\tau_R(X_1)$,I) is Nα-I-T_2.□

Corolary 3.1. *If h is a CNα-I-Cts injective function of a NITS (U_1, $\tau_R(X_1)$,I) into a N-Urysohn space (U_2, $\tau_{R'}(X_2)$), then (U_1, $\tau_R(X_1)$,I) is a Nα-I-T_2 space.*
Proof. *For any to two distinct points x_1 , x_2 in U_1, h is CNα-I-Cts function of U_1 into a N-Urysohn space (U_2, $\tau_{R'}(X_2)$) such that h(x_1) $\neq$ h(x_2) because h is injective.By Thm 3.5, the space (U_1, $\tau_R(X_1)$,I) is Nα-I-T_2.*□

Theorem 3.6. *If h is a CNα-I-Cts injective function of a NTS (U_1, $\tau_R(X_1)$,I) into N-Ultra Hausdorff space (U_2, $\tau_{R'}(X_2)$), then (U_1, $\tau_R(X_1)$,I) is a Nα-I-T_2 space.*
Proof. *Let the pair of distinct points of U_1 be x_1 , x_2.Since f is injective, U_2 is N-Ultra Hausdorff h(x_1) $\neq$ h(x_2) $\exists$ N-clopen sets Z_1, Z_2 such that h(x_1) $\in Z_1$, h(x_2) $\in Z_2$ and $Z_1 \cap Z_2$=ϕ.Then $x_i \in h^{-1}(Z_i) \in$ NαIO(U_1) for i=1,2 and $h^{-1}(Z_1) \cap h^{-1}(Z_2)$=$\phi$.Therefore ($U_1$, τ_R(X),I) is a Nα-I-T_2 space.*□

Definition 3.6. *Let h : (U_1, $\tau_R(X_1)$,I) $\rightarrow$ (U_2, $\tau_{R'}(X_2)$).The graph G(h) of the function h is called be CNα-I-closed in $U_1 \times U_2$ if for any (x_1, x_2) $\in$ ($U_1 \times U_2$)$\backslash$G(h), $\exists$ A $\in$ NαIO(U_1) and a N-closed set T of U_2 containing x_2 such that ($U_1 \times U_2$) $\cap$ G(h)=ϕ.*

Lemma 3.1. *Let h : (U_1, $\tau_R(X_1)$,I) $\rightarrow$ (U_2, $\tau_{R'}(X_2)$).The graph G(h) of the function h is CNα-I-closed in $U_1 \times U_2$ if and only if for each (x_1, x_2) $\in$ ($U_1 \times U_2$)$\backslash$G(h), $\exists$ A $\in$ NαIO(U_1, x_1) such that h(A) $\cap$ Ncl(T)=ϕ where T is a N-closed subset of $U_1 \times U_2$ containing g(x_1).*

Theorem 3.7. *If h : (U_1, $\tau_R(X_1)$,I) $\rightarrow$ (U_2, $\tau_{R'}(X_2)$) is a CNα-I-Cts function and U_2 is a N-Urysohn space, then G(h) is CNα-I-closed in $U_1 \times U_2$.*
Proof. *Let (x_1, x_2) $\in$ ($U_1 \times U_2$)$\backslash$G(h).Then $x_2 \neq$ h(x_1) and $\exists$ N-open set A, B of U_2 such that h(x_1) $\in$ A, $x_2 \in$ B and Ncl(A) $\cap$ Ncl(B)=ϕ.Since h is CNα-I-continuous, $\exists$ U $\in$ NαIO(U_1,x_1) such that h(U) $\subset$ Ncl(A).Therefore h(U) $\cap$ Ncl(B) =ϕ.Hence G(h) is CNα-I-closed.*□

Theorem 3.8. *If h : (U_1, $\tau_R(X_1)$,I) $\rightarrow$ (U_2, $\tau_{R'}(X_2)$) is a CNα-I-Cts function and (U_2, $\tau_{R'}(X_2)$) is T_2, then G(h) is CNα-I-closed.*
Proof. *Let (x_1, x_2) $\in$ ($U_1 \times U_2$)$\backslash$G(h).Then $x_2 \neq$ h(x_1) and $\exists$ N-open set B of U_2 such that h(x_1) $\in$ B, $x_2 \neq$ B. Since h is CNα-I-Cts, $\exists$ U $\in$ NαIO(U_1,x_1) such that*

h(U) ⊂ Ncl(B).Therefore h(U) ∩ (U_2 - B)=ϕ and U_2-B is a N-closed set of U_2 containing x_2.Hence G(h) is CNα-I-closed.□

Definition 3.7. *A NITS (U, τ_R(X),I) is called Nα-I-connected if there exists Nα-I-open sets A and B which form a separation of X.*

Proposition 3.2. *A CNα-I-Cts image of a Nα-I-connected space is connected.*

Definition 3.8. *A NITS (U, τ_R(X),I) is called Nα-I-normal if given any non-empty disjoint N-closed sets T and F such that ∃ Nα-I-open sets A of T and B of F such that A ∩ B=ϕ.*

Definition 3.9. *A NTS (U, τ_R(X)) is called N-Ultra normal if given any non-empty disjoint N-closed sets T and F such that ∃ N-clopen sets A of T and B of F such that A ∩ B=ϕ.*

Theorem 3.9. *If h : (U_1, τ_R(X_1),I) → (U_2, $\tau_{R'}$(X_2)) is a CNα-I-Cts closed injective function and (U_2, $\tau_{R'}$(X_2)) is N-Ultra-normal space, then (U_1, τ_R(X_1),I) is a Nα-I-normal space.*
Proof. *Let the two disjoint N-closed subsets of U_1 be F_1 and F_2. Since h is N-closed and injective, h(F_1) ∩ h(F_2)=ϕ where h(F_1) and h(F_2) are N-closed subsets of U_2.Since U_2 is N-Ultra normal, ∃ N-clopen sets Y_1 of h(F_1) and Y_2 of h(F_2) in U_2 such that Y_1 ∩ Y_2=ϕ.Hence $F_i \subset f^{-1}(Y_i)$, $f^{-1}(Y_i)$ ∈ NαIO(U) for i=1,2 and $f^{-1}(Y_1) \cap f^{-1}(Y_2)$=ϕ.Therefore ($U_1$, τ_R(X),I) is a Nα-I-normal.*□

Theorem 3.10. *For the functions h : (U_1, τ_R(X_1),I) → (U_2, $\tau_{R'}$(X_2)) and g : (U_2, $\tau_{R'}$(X_2),I') → (U_3, $\tau_{R''}$(X_3)), We have*

1. *g ∘ h is Nα-I-Cts, if h is CNα-I-Cts and g is CN-Cts.*

2. *g ∘ h is CNα-I-Cts, if h is CNα-I-Cts and g is N-Cts.*

Remark 3.1. *In general, g ∘ h is not CNα-I-Cts functions if g and f are CNα-I-Cts functions.The below example illustrate this result.*

Example 3.4. *Let U_1={i,j,k,l} with U_1/R={{i,k},{j},{l}}, and X_1={i,l}.Then τ_R(X_1)={U_1,ϕ,{l},{i,k},{i,k,l}}.Let I_1={ϕ,j}.Let U_2={m,n,o,p} with U_2/R'={{m, n},{o,p}} and Y={o,p}.Then $\tau_{R'}$(X_2)={U_2,ϕ,{p},{m,o},{m,o,p}}.Let I_2={ϕ,m}. Let W={t,u,v,w} with W/R''={{t},{u,v},{w}} and Z={w}.Then $\tau_{R''}$(Z)={W,ϕ,{w} }.Define h : (U_1, τ_R(X),I_1) → (U_2, $\tau_{R'}$(X_2)) by h(i)=n, h(j)=p, h(k)=m, h(l)=o and g : (U_2, $\tau_{R'}$(Y),I_2) → (U_3, $\tau_{R''}$(Z)) by g(m)=w, g(n)=t, g(o)=u, g(p)=v.Then h and g are CNα-I-Cts functions but (g ∘ h)$^{-1}$(w)=k which does not belongs to Nα-I-closed in (U_1, τ_R(X_1),I).*

4 Conclusion

Through the above discussions we have summarized the conceptulation of contra Nα-I-continuity and its characteristics based on Nano Urysohn Space and Nano Ultra Hausdorff Space.Also, We compared contra Nα-I-continuity with some existing functions using suitable examples.Further, this concept may be extended to Frechet Urysohn Space and Completely Hausdorff space in Nano Ideal Topology.

References

T. Noiri A. Acikgoz and S. Yuksel. On α-i-continuous and α-i-open functions. *Acta Mathematica Hungarica*, 105:27–37, 2004.

M. Bhuvaneswari and N. Nagaveni. On nwg-normal and nwg-regular spaces. *International Journal of Mathematics Trends and Technology*, 2018.

P. Karthiksankari and P. Subbulakshmi. On nano totally semi continuous functions in nano topological space. *International Journal of Mathematical Archive*, 10: 14–18, 2019.

K. Kuratowski. Topology. *Academic Press, New York*, I, 1966.

Saeid Jafari M. Lellis Thivagar and V. SuthaDevi. On new class of contra continuity in nano topology. *Italian Journal of Pure and Applied Mathematics*, 43: 1–11, 2017.

M. Parimala and S. Jafari. On some new notions in nano ideal topological space. *Eurasian Bulletin of Mathematics*, 1:51–64, 2018.

I. Rajasekaran and O. Nethaji. Simple forms of nano open sets in an ideal nano topological spaces. *Journal of New Theory*, 24:35–43, 2018.

M. Lellis Thivagar and Carmel Richard. On nano forms of weakly open sets. *International Journal of Mathematics and Statistics Invention*, I:31–37, 2013a.

M. Lellis Thivagar and Carmel Richard. On nano continuity. *Mathematical Theory and Modelling*, 3:32–37, 2013b.

M. Lellis Thivagar and V. SuthaDevi. New sort of operators in nano ideal topology. *Journal of Ultra Scientist of Physical Sciences*, 28:51–64, 2016.

S. Narmatha V. Inthumathi and S. Krishnaprakash. Decompositions of nano-r-i-continuity. *International Journal of Advanced Science and Technology*, 29: 2578–2582, 2020.