Ratio Mathematica Volume 38, 2020, pp. 139-222

Studies on A. Einstein, B. Podolsky and N. Rosen argument that "quantum mechanics is not a complete theory," III: Illustrative examples and applications

Ruggero Maria Santilli*

Abstract

In the preceding Papers I and II of this series, we have presented a review and upgrade of novel mathematical, physical and chemical methods, and shown their use for a confirmation of the apparent proof of the EPR argument that extended particles within physical media admit classical counterparts, while Einstein's determinism appears to be progressively verified with the increase of the density of the medium. In this third paper, we have additionally shown, apparently for the first time, the validity of the EPR final statement to the effect that *the wavefunction [of quantum mechanics] does not provide a complete description of the physical reality.*" In fact, we have studied the axiom-preserving "completion" of the quantum mechanical wavefunction due to deep wave-overlapping when represented via isomathematics, and shown that it permits an otherwise impossible representation of the attractive force between identical electrons pairs in valence coupling, as well as the representation of *all* characteristics of various physical and chemical systems existing in nature.

Keywords: EPR argument, isomathematics, isomechanics.
2010 AMS subject classifications: 05C15, 05C60. [1]

*Institute for Basic Research, Palm Harbor, FL 34683 U.S.A., research@i-b-r.org

[1]Received 26th of April, 2020, accepted 19th of June, 2020, published 30th of June, 2020, doi 10.23755/rm.v38i0.517, ISSN 1592-7415; e-ISSN 2282-8214.

TABLE OF CONTENTS

1. INTRODUCTION.

1.1. The EPR argument.

As it is well known, Albert Einstein did not accept quantum mechanical uncertainties as being final, for which reason he made his famous quote "God does not play dice with the universe."

Einstein communicated his views to B. Podolsky and N. Rosen and they jointly published in 1935 the historical paper [1] that became known as the *EPR argument.*

Objections against the EPR argument have been voiced by numerous scholars, including by N. Bohr [2], J. S. Bell [3] [4], J. von Neumann [5] and others (see Ref. [6] for a review and comprehensive literature).

The field became known as *local realism* and included the dismissal of the EPR argument based on claims that quantum axioms do not admit *hidden variables* λ [7] [8].

1.2. Outline of preceding works.

Following various preparatory works, in Ref. [9] of 1998, R. M. Santilli:

1) Assumed the validity of quantum mechanics, with consequential validity of the objections against the EPR argument [2] - [6], for point-like particles in empty space under linear, local and potential interactions (*exterior dynamical problems*);

2) Proved the inapplicability (and not their violation) of said objections for the broader class of extended, deformable and hyperdense particles within physical media under the most general known linear and non-linear, local and non-local and potential as well as non-potential interactions (*interior dynamical problems*); and

3) Provided the apparent proof that *interior dynamical systems admit classical counterparts in full accordance with the EPR argument* via the representation of interior systems with of *isomathematics* also called *isotopoic branch of hadronic mathematics,* and *isomechanics,* also called *isotopic branch of hadronic mechanics*.

In the 2019 paper [10], Santilli provided the apparent proof that *Einstein's determinism is progressively approached in the interior of hadrons, nuclei and stars and it is fully achieved in the interior of gravitational collapse.*

In Ref. [11], herein referred to as Paper I, we provided a review of isomathematics and isomechanics.

In the subsequent Ref. [12], hereinafter referred as Paper II, we provided a review of isosymmetries, with particular reference to the isotopies of the spin and rotational symmetries, and provided an apparent confirmation of the proofs [9] and [10] of the EPR argument.

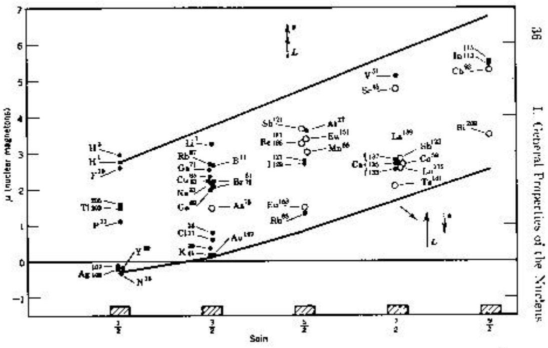

Figure 1: *In this figure, we present the so-called Schmidt limits essentially representing the deviations of experimental data on nuclear magnetic moments from the predictions of quantum mechanics. In Santilli's view [11], this occurrence is clear evidence on the need for a "completion" of quantum mechanics along Einstein's legacy [1], beginning with a "completion" of applied mathematics into a form suitable to represent extended and deformable nucleons according to the founders of nuclear physics, such as E. Fermi [13] and V. F. Weisskopf who states in page 31 of his treatise in nuclear physics with J. M. Blatt [14]: " it is possible that the intrinsic magnetism of a nucleon is different when it is in close proximity to another nucleon. " For penetrating appraisals of the insufficiencies of 20th century science, one should also see K. R. Popper [15]. J. Dunning-Davies [16], J. Horgan [17] and others. In this paper, we outline and update the achievement by hadronic mathematics and mechanics of exact representations of nuclear experimental data along the indicated historical legacies, and point out important implications for much needed new, clean energies and fuels (Sections 2.6.5, 2.7.3, 2.8.3).*

In this third and final paper of this series: we present apparent experimental verifications of the EPR argument in physics and chemistry; we outline expected industrial applications; and identify the most salient implications of the EPR argument.

1.3. Basic notions.

The basic assumptions underlying the apparent proofs of the EPR argument are the following::

1) The axiom-preserving lifting of the conventional associative product $ab = a \times b$ between *all* possible quantum mechanical quantities (numbers, functions, matrices, etc.) into the *isoproduct,* first introduced in the 1978 Harvard university paper [18] (see also Refs. [19] to [23])

$$a \star b = a \, \hat{T} \, b, \tag{1}$$

where $\hat{T}$, called the *isotopic element,* is restricted to be positive-definite,

$\hat{T} > 0$, but possesses otherwise an unrestricted functional dependence on all needed local variables, including wavefunctions and their derivatives.

20th century applied mathematics and quantum mechanics are reformulated in an axiom-preserving, thus isotopic form, via isoproduct (1) (Paper I and Section II-2).

3) The axiom-preserving isotopy of the various branches of Lie's theory, first achieved by Santilli in Refs. [18] [23] and are today known as the *Lie-Santilli isotheory* [24] (Section I-3.7) with Lie-Santilli algebras of the type [9]

$$[X_i \hat{,} X_j] = X_i \star X_j - X_j \star X_i = C_{ij}^k X_k. \;\; i, j = 1, 2, ..., N. \tag{2}$$

and ensuing systematic isotopy of space-time and internal symmetries (Section II-2).

3) Immediate explicit and concrete realizations of "hidden variables" [7] [8] of the type

$$\hat{T} = Diag.(1/\lambda, \lambda), \;\; Det\hat{T} = 1. \tag{3}$$

Ref. [9], therefore establishing that, contrary to objections [2] to [6], *the abstract axioms of quantum mechanics do indeed admit explicit and concrete realizations of hidden variables.*

4) Representation of extended particles in conditions of mutual penetration via realizations of the isotopic element $\hat{T}$ of isoproduct (1) of the type [25]

$$\begin{gathered}\hat{T} = \Pi_{k=1,...,N} Diag.(\tfrac{1}{n_{1k}^2}, \tfrac{1}{n_{2k}^2}, \tfrac{1}{n_{3k}^2}, \tfrac{1}{n_{4k}^2}) e^{-\Gamma}, \\ k = 1, 2, ..., N, \quad \mu = 1, 2, 3, 4,\end{gathered} \tag{4}$$

where n_1^2, n_2^2, n_3^2, (called *characteristic quantities*) represent the deformable semi-axes of the particle normalized to the values $n_k^2 = 1, `k = 1, 2, 3$ for the sphere; n_4^2 represents the *density* of the particle considered normalized to the value $n_4 = 1$ for the vacuum; and Γ represents non-linear, non-local and non-Hamiltonian interactions caused by mutual penetrations/entanglement of particles.

In particular, the isotopic element $\hat{T}$ has resulted to have numeric values *smaller* than 1 in all known applications [23]-

$$|\, \hat{T} \,| \;\leq\; 1, \tag{5}$$

which property permitted Ref. [10] to show that *the standard deviations Δr and Δp progressively tend to zero with the increase of the density of the medium of interior problems.*

5) Isosymmetries do not preserve over time the basic unit 1 of conventional numeric fields $F(n, \times, 1)$ with consequential lack of experimental

verifications. This occurrence mandated Santilli to formulate isomathematics on *isofields* $\hat{F}(\hat{n}, \star, \hat{I})$ [26] [27](Section I-3.3) with *isonumbers* $\hat{n} = n\hat{I}$ equipped with isoproduct (1) and basic *isounit*

$$\hat{I} = 1/\hat{T} > 0, \tag{6}$$

which remains numerically invariant under isosymmetries as necessary for consistency (Sections I-3.8, I-3.9 and II-2).

5) Recall that the Newton-Leibnitz differential calculus provides the ultimate characterization of the point-like character of particles admitted by quantum mechanics due to the known feature that said calculus can be solely defined as a finite number of isolated points.

This occurrence mandated Santilli to construct the covering of the Newton-Leibnitz differential calculus into the *isodifferential isocalculus* (Section I-3.6) with basic *isodifferential* [28] [29]

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(r, ...)] = dr + r\hat{T}d\hat{I}(r, ...), \tag{7}$$

and corresponding *isoderivative*

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\frac{\partial\hat{f}(\hat{r})}{\partial\hat{r}}, \tag{8}$$

which are defined on *volumes* represented by the isotopic element $\hat{T}$, rather than points.

Recall that Bell's inequality [3] D^{qm}_{Bell} does not admit a classical counterpart D^{class} because always smaller than the classical counterpart,

$$D^{qm}_{Bell} < D^{class}. \tag{9}$$

In Ref. [9], Eqs. (5.8), page 189, proved that, thanks to the existence in hadronic mechanics of hidden variables of type (3), the corresponding inequality in hadronic mechanics (hm), D^{hm} is bigger than Bell's inequality, and always admits a classical counterpart

$$D^{hm} \equiv D^{class}, \tag{10}$$

resulting in the following:

LEMMA II-3.6 (EXISTENCE OF CLASSICAL COUNTERPART). Extended particles within physical media that are invariant under the Lie-Santilli isosymmetry $\hat{SU}(2)$ *admit identical classical counterparts.*

The main result of Ref. [10], Eqs. (35), page 14,

$$\Delta r \Delta p \approx \frac{1}{2} | < \hat{\psi}(\hat{r}) | \star [\hat{r}, \hat{p}] \star | \hat{\psi}(\hat{r}) >=$$

$$= \frac{1}{2} | < \hat{\psi}(\hat{r}) | \hat{T} \, [\hat{r}, \hat{p}] \, \hat{T} | \hat{\psi}(\hat{r}) >= \tag{11}$$

$$\int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^{\dagger} \hat{T} \hat{\psi}(\hat{r}) d\hat{r} = T \ll 1,$$

verifying Einstein's determinism, was expressed with the following:

LEMMA II-3.7 (EINSTEIN DETERMINISM): The isostandard isodeviations for isocoordinates $\Delta\hat{r}$ and isomomenta $\Delta\hat{p}$, as well as their product, progressively approach classical determinism for extended particles in the interior of hadrons, nuclei and stars, and achieve classical determinism at the extreme densities in the interior of gravitational collapse.

By recalling that isoperturbative series of hadronic mechanics has based on isoproduct (1) (Section I-4),

$$A(0) = \hat{I} + (A\hat{T}H - H\hat{T}A)/1! + ... \tag{12}$$

and that $\hat{T} \ll 1$ from Lemma II-3.7, the following consequential property was introduced, apparently for the first time in Paper II:

COROLLARY 3.7.1 (LACK OF DIVERGENCIES) Einstein's determinism according to Lemma II-3.7 implies the lack of quantum mechanical divergencies in hadronic mechanics.

It should be indicated that the technical understanding of the verifications and applications presented in this paper require a knowledge of isomathematics and isomechanics.

2. VERIFICATIONS AND APPLICATIONS.

2.1. Foreword.

Being dimensionless, *Newtonian massive points cannot experience resistive or contact force of any type.* By recalling that Newton's equation have been the foundations of physics for the past four centuries, 20th century mainstream particle physics has been developed without the notion of resistive force, with ensuing lack of treatment of the *pressure* exercised by a medium on a particle in its interior, contrary to clear evidence that a proton in the core of a star is exposed to extremely big pressures (Figure 1).

In this section, we shall illustrate the fact that, once admitted, the pressure exercised on extended particles characterizes in their interior characterizes standard deviations $\Delta\hat{r}$ and $\Delta\hat{p}$ that, being *constrained* by said pressure, verify the isodeterministic principle of Lemma II-3.7 as well as the rapid convergence of isoperturbative series according to Corollary II-3.7.1, by progressively approaching classical determinism with the increase of the pressure, up to the apparent achievement of classical determinism for the interior of gravitational collapse as predicted by Einstein, Podolsky and Rosen [1].

A physically important notion emerging from the examples provided below is that *the EPR argument appears to be verified by strong interactions* because, as indicated by Santilli in the 1978 paper [19], contact non-Hamiltonian interactions responsible for the synthesis of the neutron and other hadrons are short range, strongly attractive, charge independent and non-Hamiltonian (technically identified as *variationally non-selfadjoint interactions* [22]), thus providing a conceivable, first known, explicit and concrete representation of strong interactions.

The models outlined in this section were first proposed by Santilli in Ref. [19] in their time irreversible form, as requested for decaying bound states, thus being elaborated with Lie-admissible genomathematics, in which case, the need for a "completion" of quantum mechanics is beyond scientific doubt (Section I-1.3).

However, the objections against the EPR argument [2] - [6] have been formulated for conventional quantum axioms, thus implying the sole consideration of time-reversal invariant states. In this section, we illustrate the need for a "completion" of quantum mechanics also for time-reversal invariant systems of extended particles in interior conditions.

Therefore, unstable strongly interacting particles (hadrons) are hereon studied for such a small period of time to allow their time-reversal invariant approximation.

2.2. Particles under pressure.

One of the simplest illustrations of Lemma II-3.7 is given by a particle in the center of a star, thus being under extreme pressures π from the surrounding hadronic medium in all radial directions (Figure 1).

By ignoring particle reactions in first approximation, the conditions here considered can be rudimentarily represented for very short periods of time by assuming that the function $\Gamma > 0$ in the exponent of the isotopic element (4) is linearly dependent on the pressure π, resulting in a

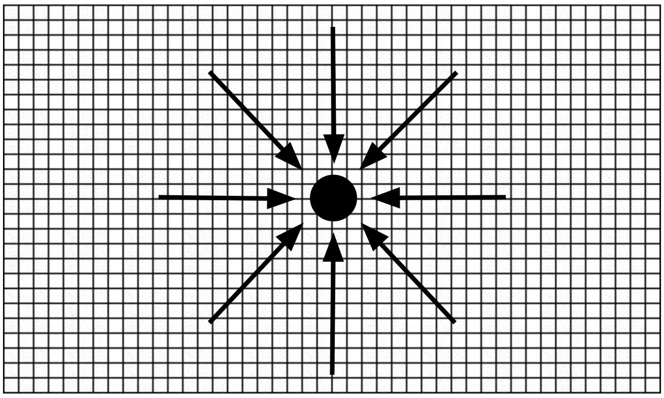

Figure 2: *In this figure, we present a conceptual rendering of the central notion used for the verification of the EPR argument, namely, the pressure experienced by extended particles immersed in hyperdense media, such as a proton in the core of a star, which pressure evidently restricts uncertainties in favor of Einstein's determinism (Lemma II-3.7). Note that the notion of pressure does not exist in 20th century physics due to the approximation of particles as being point-like.*

realization of the isotopic element of the simple type

$$\hat{T} = e^{-w\pi} \ll 1, \quad \hat{I} = e^{+w\pi} \gg 1, \tag{13}$$

where w is a positive constant.

Isodeterministic principle (11) for the considered particle is then given by

$$\Delta r \Delta p \approx \frac{1}{2} e^{-w\pi} \ \ll \ 1, \tag{14}$$

and tends to null values for diverging pressures.

The above example illustrates the consistency of isorenormalization (II-171) because a constant isotopic element verifies the isonormalization

$$\begin{gathered} \hat{<}\hat{\psi}(\hat{r})|\hat{T}|\hat{\psi}(\hat{r}) > \hat{I} = \\ = T < \hat{\psi}(\hat{r})| \, |\hat{\psi}(\hat{r}) > \hat{I} = \\ =< \hat{\psi}(\hat{r})| \, |\hat{\psi}(\hat{r}) >, \end{gathered} \tag{15}$$

but not necessarily other isorenormalizations.

Note that we have considered an individual extended particle immersed in a hadronic medium, rather than the bound state of extended particles in condition of mutual penetration that are studied in the next sections.

Consequently, isotopic element (14) represents a *subsidiary constraint on standard deviations* caused, as indicated, by the pressure of the surrounding hadronic medium on the particle considered.

It is easy to see that, since $\Gamma(\pi) > 0$, more complex functional dependence on the pressure π continue to verify Lemma II-3.7 as well as Corollary II-3.7.1.

2.3. Non-relativistic hadronic bound states.

As recalled in Paper I, the dynamical equations of quantum mechanics, such as the Schrödinger and the Dirac equation, are characterized by the conventional differential calculus which can solely be defined at a finite set of isolated points on a given representation space. Consequently, quantum mechanical bound states are solely possible for *point-like constituents* under linear, local and potential interactions (technically identified as *variationally self-adjoint interactions* [22]). This it is the case for the familiar Schrödinger equation for the bound state of two point-like particles of mass m with Coulomb potential $V(r)$ in a Euclidean space $E(r, \delta, I)$ formulated on a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$

$$
\begin{aligned}
i\tfrac{\partial}{\partial t}\psi(t,r) = H\psi(t,r) = \left[\tfrac{\hbar^2}{m}\Sigma_k p_k p_k - V(r)\right]\psi(t,r) = \\
= \left[\tfrac{1}{m}\Sigma_k(-i\hbar\partial_k)(-i\hbar\partial_k) - V(r)\right]\psi(t,r) = \\
= \left[-\tfrac{\hbar^2}{m}\Delta_r - V(r)\right]\psi(t,r) = E\psi(t,r).
\end{aligned}
\tag{16}
$$

By contrast, *hadronic bound states* are bound states of *extended particles* at mutual distances smaller or equal to the *hadronic horizon* (Figure I-13)

$$
R = \frac{1}{b} \approx 10^{-13}\, cm. \tag{17}
$$

In such a region, bound states verify hadronic mechanics, can be represented with isomathematics (Section I-3) and isomechanics (Section I-4) for the case of time-reversal invariant bound states (such as the deuteron), or genomathematics and genomechanics for time irreversible bound states (such as all hadrons produced in particle physics laboratories).

By using the methods outlined in Paper I and in the preceding sections, when assumed to be stable in first approximation (thus being time reversal invariant), hadronic bound states are characterized by the following main features:

1) The bound states occur between *isoparticles,* namely, isoirreducible, isounitary isorepresentations of the isospinorial covering of the Galileo-Santilli isosymmetry $\hat{\mathcal{G}}$ for non-relativistic treatments (Sections II-2.5.1 and II-3.9) or of the Lorentz-Poincaré-Santilli isosymmetry $\hat{\mathcal{P}}$ for relativistic treatments (Sections II-2.5.11 and II-3.9) represented by hadronic mechanics including the verification of Einstein's determinism (Lemma II-3.7) and the absence of divergencies (Corollary II-3.7.1).

2) The representation of the extended character of the isoparticles is done with isoproduct (1) and isotopic element (4), resulting in iso-Schrödinger equations of type (I-80), while the deep mutual penetration of the wavepackets and/or charge distributions of isoparticles generates novel non-linear, non-local and non-potential interactions represented by the exponent of the the isotopic element (4) and other means. Note that the latter interactions are short range, strongly attractive, charge independent, and non-Hamiltonian according to all studies conducted to date in the field, thus allowing an initial yet explicit and concrete realization of strong interactions [19].

3) By recalling that isosymmetries $\hat{\mathcal{G}}$ and $\hat{\mathcal{P}}$ are all *irregular* realizations of the Lie-Santilli isotheory (Sections I-2.7 and II-2.5.4), a necessary condition for the invariance of hadronic dynamical equations under isosymmetries is that contact interactions *cannot* be derived via non-unitary transforms of quantum mechanical potentials, thus being *basically new interactions.* The physically equivalent property is that, as it is well known, *strong interaction cannot be derived via non-unitary or other known transformations of electromagnetic interactions,* thus confirming the necessary use of the irregular Lie-Santilli isotheory and hadronic dynamical equations.

The notion of hadronic bound states was proposed, apparently for the first time, by Santilli in the 1978 Ref. [19] and was extensively studied hereafter in various works by various authors (see the 2001 monograph [30], the 2011 review [31], and papers quoted therein).

It is important to review the derivation of the basic non-relativistic and relativistic isoequations for hadronic bound states to show their apparent verification of the isodeterministic principle of Lemma 3.7.

The fundamental *non-relativistic, irregular isoequation of a time-reversal invariant hadronic bound state* of two isoparticles of mass m at mutual distances of the order of the hadronic horizon $R = 10^{-13}\,cm$ in an iso-Euclidean isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ formulated on a Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ over the isofield of isocomplex isonumbers $\hat{\mathcal{C}}$ can be written

$$i\frac{\hat{\partial}}{\hat{\partial}\hat{t}}\hat{\psi}(\hat{t},\hat{r}) = \Big[(\hat{1}/\hat{m})\star\Sigma_k\hat{p}_k\star\hat{p}_k \pm \hat{V}(\hat{r}) - \hat{S}(\hat{\psi})\Big]\star\hat{\psi}(\hat{t},\hat{r}) =$$

$$= \Big[(1/m)\Sigma_k\hat{p}_k\hat{T}\hat{p}_k\hat{T} \pm V(\hat{r}) - S(\hat{\psi})\Big]\hat{\psi}(\hat{t},\hat{r}) = \tag{18}$$

$$\hat{E}\star\hat{\psi}(\hat{t},\hat{r}) = E\hat{\psi}(\hat{t},\hat{r}),$$

where $\hat{V}(\hat{r}) = V(\hat{r})\hat{I}$; $\hat{S}(\hat{\psi}) = S(\hat{\psi})\hat{I}$ represents the novel short range, strongly attractive force; the value $-\hat{V}(\hat{r})$ occurs for bound states with opposite charge (as it is the case for the synthesis of hadrons reviewed below); the value $+\hat{V}(\hat{r})$ occurs for isoparticles with the same charge (as occurring for valence electron bonds reviewed below); and one should note that isoeigenvalues can always be reduced to conventional eigenvalues, thus allowing experimental verifications.

Due to the large representational capabilities of isoequations (182), we use the following simplifying assumptions:

1) The isotime is equal to the conventional time, $\hat{t} = t\hat{I}_t = t,\ \hat{I}_t = 1$;

2) Being extremely small, the orbits of the isoparticles are assumed to be nearly constant circles, thus implying that the n_k characteristic quantities of then isotopic element (4) can be normalized to the sphere, $n_k = 1,\ k = 1, 2, 3$;

3) The isotopic element is assumed to be given by the exponential term of Eq. (4) with realizations of the non-linear, non-local and non-potential interactions of the type (Eq. (4.7), page 170 Ref. [30])

$$\hat{T} = e^{-\Gamma} = e^{-N\psi/\hat{\psi}} \approx 1 - N\psi/\hat{\psi}, \tag{19}$$

where ψ behaves like the solution of quantum equation (180),

$$\psi(r) \approx W_1 e^{-br}, \tag{20}$$

and $\hat{\psi}$ behaves like the solution of the hadronic equation expected to be of the type

$$\hat{\psi} \approx W_2(1 - e^{\frac{(1--br)}{r}}, \tag{21}$$

where W_1 and W_2 are positive normalization constants.

We therefore have the following explicit form of the isotopic element

$$\hat{T} = e^{-W_2\frac{e^{-br}}{(1-e^{-br})/r}} \approx 1 - W\frac{e^{-br}}{(1-e^{-br})/r}, \tag{22}$$

exhibiting the *Hulten potential*

$$V_{Hult} = W_2 \frac{e^{-br}}{1 - e^{-br}}, \tag{23}$$

directly in the exponent of the isotopic element, where W is normalization constants.

It should be recalled that Santilli suggested the use of the Hulten potential in the 1978 paper [19], Eq. (5.1.6), page 833, as an *initial yet explicit and concrete representation of strong interactions.*

Under the above assumptions, isotopic element (186) verifies the central condition for the validity of the isodeterministic principle as well as the rapid convergence of isoperturbative series inside the hadronic horizon (Lemma II-3.7 and Corollary II-3.7.1), in a way fully compatible with the validity of conventional uncertainties as well as divergence of perturbative series outside said horizon

$$\begin{aligned} |\hat{T}| &\ll 1, \\ Lim_{r \gg R} \hat{T} &= 1. \end{aligned} \tag{24}$$

The use of the isolinear isomomentum (I-79), the projection of isodynamical equation (II-182) into our Euclidean space can be written in the form first derived in Eq. (5.1.9), page 833, Ref [19]

$$\begin{aligned} i\tfrac{\partial}{\partial t}\hat{\psi}(t,r) &= \left[\tfrac{1}{m}\Sigma_k \hat{p}_k \star \hat{p}_k \star \pm V \tfrac{e^2}{r} - W \tfrac{e^{-br}}{1-e^{-br}}\right]\hat{\psi}(t,r) = \\ &= \left[\tfrac{1}{m}\Sigma_k(-i\hat{I}\partial_k)(-i\hat{I}\partial_k) \pm W_1 \tfrac{e^2}{r} - W_2 \tfrac{e^{-br}}{1-e^{-br}}\right]\hat{\psi}(t,r) = \\ &= \left[-\tfrac{1}{\bar{m}}\Delta_r \pm V \tfrac{e^2}{r} - W \tfrac{e^{-br}}{1-e^{-br}}\right]\hat{\psi}(t,r) = E\hat{\psi}(t,r), \end{aligned} \tag{25}$$

where: W_1' and W_2 are renormalization constants; E is the binding energy of the hadronic bound state; the total energy E_{tot} is given by

$$E_{tot} = E_1 + E_2 + -E, \tag{26}$$

where $E_k, k = 1,2$ are the total energies of particles 1 and 2, respectively, and

$$\bar{m} = \frac{m}{\hat{I}^2}. \tag{27}$$

The nonrelativistic expression of the mean-life of the hadronic bound state can be derived via an isotopy of the quantum mechanical form, yielding the expression (Ref. [19], Eq. (5.1.13), page 835)

$$\tau^{-1} = 2\pi\lambda^2|\hat{\psi}(0)|^2\frac{\alpha^2 E_k}{\hbar},\ k = 1, 2. \tag{28}$$

The angular component of Eqs. (85) has been studied in detail in Ref. [33] via the isospherical isoharmonics.

The radial component of the non-relativistic, irregular, hadronic isoequation for the characterization of the total energy E_{tot}, mean life τ and charge radius R of a time-reversal invariant hadronic bound state can be written (Ref. [19], Eqs. (5.1.40, page 835)

$$\begin{gathered}\left[\tfrac{1}{r^2}(\tfrac{d}{dr}r^2\tfrac{d}{dr}) + \bar{m}(E \pm W_1 e^{\frac{2}{r}} + W_2\tfrac{e^{-br}}{1-e^{-br}})\right] = 0,\\ E_{tot} = E_1 + E_2 - E,\ \ \bar{m} = \tfrac{m}{I^2}\\ \tau^{-1} = 2\pi\lambda^2|\hat{\psi}(0)|^2\tfrac{\alpha^2 E_k}{\hbar},\ k = 1, 2,\\ R = b^{-1},\end{gathered} \tag{29}$$

where the last two equations are subsidiary constraints on the first two.

The analytic solution of the above equations has been studied in detail in Section 5, Ref. [19], including boundary conditions requested by the subsidiary constraints, and we cannot review it here for brevity. We merely limit ourselves to the indication that said analytic solution was reduced to the solution of the following two algebraic equations on the parameters k_1 and k_2 (Ref. [19], Eq. (5.1.32), page 840)

$$\begin{gathered}k_1[k_1 - (k_2 - 1)^3] = \tfrac{1}{2c}E_{tot}R,\\ \tfrac{(k_2-1)^3}{k_1} = \tfrac{9\times 10^6}{3\pi c}\tfrac{R}{\tau}.\end{gathered} \tag{30}$$

It is now important to evaluate the numeric value of the binding energy E in Eq. (198). For this purpose, we recall that the Hulten potential behaves like the Coulomb potential at short distances

$$V_{hp} \approx K\frac{1}{r}, \tag{31}$$

where K is a positive constant. Consequently, *the Hulten potential can absorb the Coulomb potential resulting in a short range strongly attractive force irrespective of whether the Coulomb force is attractive or repulsive.*

The radial equation can then be reduced in first approximation to the expression (Eq. (5.1.14a), page 836, Ref. [19])

$$\left[\frac{1}{r^2}(\frac{d}{dr}r^2\frac{d}{dr}) + \bar{m}(E + W_2\frac{e^{-br}}{1-e^{-br}})\right] = 0, \quad (32)$$

and its energy spectrum results to be the typical *finite* spectrum of the Hulten potential

$$|E_{hp}| = \frac{1}{4R^2\bar{m}}(k_2\frac{1}{n} - n)^2, \;\; n = 1, 2, 3, ... \quad (33)$$

An important feature of hadronic bound states is that, since we can ignore Coulomb interactions and solely assume *contact* interactions represented with the exponent of the isotopic element (4), *the value of the binding energy E is expected to be small or null,*

$$|E| = \frac{1}{4R^2\bar{m}}(k_2\frac{1}{n} - n)^2 = 0. \quad (34)$$

This is due to the fact that *contact interactions do not carry potential energy* (this is classically the case for a balloon moved by winds in our atmosphere).

Therefore, *the Hulten potential for consistent hadronic bound states is expected to admit one single energy level, the ground state* since all possible excited states imply radial distances bigger than the hadronic horizon R, with consequential recovering of quantum mechanics.

In particular, property (34) is solely possible for the following values of the k-[arameters

$$k_1 \; > \; 0, \;\; k_2 \; \geq \; 1. \quad (35)$$

The absence of a spectrum of energies was called in Section 5 of Ref. [19] the *hadronic suppression of quantum mechanical energy spectra* in order to differentiate the *quantum mechanical classification of hadrons into families* (which is characterized by energy spectra) from the *structure of individual hadrons of a given classification family* (which is expected to require different constituents for different particles due to the general difference of spontaneous decays with the lowest mode).

2.4. Relativistic hadronic bound states.

The relativistic counterpart of Eqs. (25) was identified, apparently for the first time, in Refs. [35] [36] and was formulated in the isoproduct of a real-valued iso-Minkowski isospace for orbital motions and a complex valued iso-Euclidean isospace for the hadronic spin

$$\hat{S}_{tot} = \hat{M}(\hat{x}, \hat{\eta}, \hat{I}_{orb}) \star \hat{R}(\hat{z}\hat{\delta}, \hat{I}_{spin}), \quad (36)$$

resulting in the following *irregular extension of the Dirac-Santilli isoequation (I-88)* ,

$$[\hat{\Omega}^{\mu\nu} \star \hat{\Gamma}_\mu \star \hat{\partial}_\nu + \hat{M} \star \hat{C} - \hat{V}_{hp}]\hat{\psi}(\hat{x}) = \\ = (-i\hat{I}\hat{\eta}^{\mu\nu}\hat{\gamma}_\mu\partial_\nu + mC - V_{hp})\hat{\psi}(\hat{x}) = 0, \tag{37}$$

where $\hat{S} = S\hat{I}_{orb}$ represents strong interactions, and the *Dirac-Santilli isogamma isomatrices* $\hat{\Gamma} = \hat{\gamma}\hat{I}$ are given by

$$\hat{\gamma}_k = \frac{1}{n_k}\begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \\ \hat{\gamma}_4 = \frac{i}{n_4}\begin{pmatrix} I_{2\times2} & 0 \\ 0 & -I_{2\times2} \end{pmatrix}; \tag{38}$$

where $\hat{\sigma}_k$ are the *irregular Pauli-Santilli isomatrices* studied in Section II-3.4 with the following *anti-isocommutation rules*

$$\{\hat{\gamma}_\mu \hat{,} \hat{\gamma}_\nu\} = \hat{\gamma}_\mu\hat{T}\hat{\gamma}_\nu + \hat{\gamma}_\nu\hat{T}\hat{\gamma}_\mu = \\ = 2\hat{\eta}_{\mu\nu}. \tag{39}$$

where $\hat{\eta}$ is the isometric of the orbital iso-Minkowskian isospace.

2.5. Einstein's determinism in the structure of mesons.

2.5.1. Insufficiencies of quark conjectures. While the classification of hadrons into families has received a rather large consensus since its initiation by M. Gell-Mann in the 1960's [75], the conjecture that the hypothetical quarks are the actual physical constituents of hadrons has been controversial since its inception. These problematic aspects were reviewed in detail in the 1979 paper [20] (written at the Department of Mathematics of Harvard University under DOE support), and can be summarized as follows:

1) The quantum mechanical classification of point-like particles permitted by the $SU(3)$ model and, more recently, by the standard model, has indeed achieved a satisfactory *classification* of hadrons into families.

2) Quarks are purely mathematical representations of a purely mathematical unitary symmetry defined on a purely mathematical complex-valued internal space and, as such, quarks cannot be the actual physical constituents of hadrons for numerous insufficiencies or sheer inconsistencies, such as:

2A) By recalling that quarks have to be point-like as a necessary condition to maintain the validity of quantum mechanics in the interior of hadrons, the ensuing conception of the hyperdense hadrons as ideal spheres with point-particles in their interior is not realistic;

2B) Quarks cannot be physical particles in our spacetime, and the same holds for their masses, because they cannot be defined as unitary irreducible representations of the Lorentz-Poincaré symmetry (Section 3.9);

2C) Quarks cannot be rigorously confined inside hadrons (i.e,. confined with a rigorously proved, identically null probability of tunnel effect) due to the uncertainty principle;

2D) Quarks have not been directly detected under collisions at the extremely high energies achieved at CERN and at other particle physics laboratories;

2E) The wavepackets of all particles are of the same order of magnitude of the size of all hadrons. Hence, the hyperdense character of hadrons is due to the total mutual penetration of the wavepackets of their constituents, resulting in non-linear, non-local and non-Hamiltonian internal interactions under which the SU(3) and other symmetries cannot be consistently defined (Sections 3, 4).

3) History has thought that the study of atoms (as well as of other natural systems) required *two* different yet compatible models, one for the *classification* of atoms into family, and a different model for the *structure* of each atom of a given classification family. Particularly significant is the fact that the classification of atoms could be achieved via the use of *pre-existing mathematics,* while the structure of atoms required *new mathematics,* such as the Hilbert spaces, that are unnecessary for the classification of atoms.

In order to resolve the insufficiencies of the conjecture that quarks are physical particles, Santilli [20] suggested to follow the teaching of the history of science, and study hadrons via *two* different models, the standard model for the classification of hadrons and a different, yet compatible, model for the structure of individual hadrons of a given classification multiplet.

In particular, the classification of hadrons can be effectively done via quantum mechanics because individual hadrons can be well approximated as being point-like particles in vacuum. By contrast, the structure of hadrons requires a necessary "completion" of quantum mechanics into a covering theory suggested beginning with the title of Ref. [20] in view of the unavoidable, internal, non-linear, non-local, and non-potential interactions.

2.5.2. Hadronic structure model of mesons. A primary aim of papers [18] [19] [20] and monographs [22] [23] of 1978-1979 was the "completion" of quantum mechanics (qm) into the covering hadronic mechanics (hm) for the specific intent of achieving a representation of *all* characteristics of mesons as hadronic bound states of actual, massive, physical particles

The Possible Invalidation of Quark Conjectures 429

Table I. The Technical Origin of Some of the Controversies in Hadron Physics[a]

$\{\pi^0 : 1^-, 0^-, +, 135, 0.8 \times 10^{-16}, \sim 1\ F\}$, $\{\pi^{\pm} : 1^-, 0^-, \ldots, 139, 2.6 \times 10^{-8}, \sim 1\ F\}$
$\{K^{\pm} : \frac{1}{2}, 0^-, \ldots, 439, 1.2 \times 10^{-8}, \sim 1\ F\}$, $\{K_S^0 : \frac{1}{2}, 0^-, \ldots, 498, 0.9 \times 10^{-10}, \sim 1\ F\}$
$\{K_L^0 : \frac{1}{2}, 0^-, \ldots, 498, 5.1 \times 10^{-8}, \sim 1\ F\}$, $\{\eta : 0^+, 0^-, +, 549, \Gamma = 0.8\ \text{keV}, \sim 1\ F\}$

$\pi^0 \to \gamma\gamma$, 98.8 %	$K^{\pm} \to e\pi^0\nu\gamma$, 3.7×10^{-4}
$\pi^0 \to \gamma e^+e^-$, 1.15 %	$K^{\pm} \to \pi e^+e^-$, 2.6×10^{-7}
$\pi^0 \to \gamma\gamma\gamma$, $<5 \times 10^{-6}$	$K^{\pm} \to \pi^{\mp}e^{\pm}e^{\pm}$, $<1.5 \times 10^{-8}$
$\pi^0 \to e^+e^-e^+e^-$, 3.3×10^{-5}	$K^{\pm} \to \pi\mu^+\mu^-$, $<2.4 \times 10^{-6}$
$\pi^0 \to \gamma\gamma\gamma\gamma$, $<6 \times 10^{-5}$	$K^{\pm} \to \pi\gamma\gamma$, $<3.5 \times 10^{-5}$
$\pi^0 \to e^+e^-$, $<2 \times 10^{-6}$	$K^{\pm} \to \pi\gamma\gamma\gamma$, $<3.0 \times 10^{-4}$
$\pi^{\pm} \to \mu\nu$, 100 %	$K^{\pm} \to \pi\nu\nu$, $<0.6 \times 10^{-6}$
$\pi^{\pm} \to e\nu$, 1.2×10^{-4}	$K^{\pm} \to \pi\gamma$, $<4 \times 10^{-6}$
$\pi^{\pm} \to \mu\nu\gamma$, 1.2×10^{-4}	$K^{\pm} \to e\pi^{\mp}\mu^{\pm}$, $<2.8 \times 10^{-8}$
$\pi^{\pm} \to \pi^0 e\nu$, 1.02×10^{-8}	$K^{\pm} \to e\pi^{\pm}\mu^{\mp}$, $<1.4 \times 10^{-8}$
$\pi^{\pm} \to e\nu\gamma$, 3×10^{-8}	$K^{\pm} \to \mu\nu\nu\nu$, $<6 \times 10^{-6}$
$\pi^{\pm} \to e^+e^-\nu$, $<3.4 \times 10^{-8}$	$K_S^0 \to \pi^+\pi^-$, 68.7 %
$K^{\pm} \to \mu\nu$, 63.6 %	$K_S^0 \to \pi^0\pi^0$, 31.3 %
$K^{\pm} \to \pi\pi^0$, 21.05 %	$K_S^0 \to \mu^+\mu^-$, $<3.2 \times 10^{-7}$
$K^{\pm} \to \pi\pi^-\pi^+$, 5.6 %	$K_S^0 \to e^+e^-$, $<3.4 \times 10^{-4}$
$K^{\pm} \to \pi\pi^0\pi^0$, 1.7 %	$K_S^0 \to \pi^+\pi^-\gamma$, 2.0×10^{-3}
$K^{\pm} \to \mu\pi^0\nu$, 3.2 %	$K_S^0 \to \gamma\gamma$, $<0.4 \times 10^{-3}$
$K^{\pm} \to e\pi^0\nu$, 4.8 %	$K_L^0 \to \pi^0\pi^0\pi^0$, 21.4 %
$K^{\pm} \to \mu\nu\gamma$, 5.8×10^{-3}	$K_L^0 \to \pi^+\pi^-\pi^0$, 12.2 %
$K^{\pm} \to e\pi^0\pi^0\nu$, 1.8×10^{-5}	$K_L^0 \to \pi\mu\nu$, 27.1 %
$K^{\pm} \to \pi\pi^{\pm}e^{\mp}\nu$, 3.7×10^{-5}	$K_L^0 \to \pi e\nu$, 39.0 %
$K^{\pm} \to \pi\pi^{\pm}e^{\pm}\nu$, $<5 \times 10^{-7}$	$K_L^0 \to \pi e\nu\gamma$, 1.3 %
$K^{\pm} \to \pi\pi^{\pm}\mu^{\mp}\nu$, 0.9×10^{-5}	$K_L^0 \to \pi^+\pi^-$, 0.2 %
$K^{\pm} \to \pi\pi^{\pm}\mu^{\pm}\nu$, $<3.0 \times 10^{-6}$	$K_L^0 \to \pi^0\pi^0$, 0.09 %
$K^{\pm} \to e\nu$, 1.5×10^{-5}	$K_L^0 \to \pi^+\pi^-\gamma$, 6.0×10^{-5}
$K^{\pm} \to e\nu\gamma$, 1.6×10^{-5}	$\eta \to \gamma\gamma$, 38.0 %
$K^{\pm} \to \pi\pi^0\gamma$, 2.7×10^{-4}	$\eta \to \pi^0\gamma\gamma$, 3.1 %
$K^{\pm} \to \pi\pi^+\pi^-\gamma$, 1×10^{-4}	$\eta \to 3\pi^0$, 29.9 %
$K^{\pm} \to \mu\pi^0\nu\gamma$, $<6 \times 10^{-5}$	$\eta \to \pi^+\pi^-\pi^0$, 23.6 %
$K_L^0 \to \pi^0\gamma\gamma$, $<2.4 \times 10^{-4}$	$\eta \to \pi^+\pi^-\gamma$, 4.9 %
$K_L^0 \to \gamma\gamma$, 4.9×10^{-4}	$\eta \to e^+e^-\gamma$, 0.5 %
$K_L^0 \to e\mu$, $<2.0 \times 10^{-9}$	$\eta \to \pi^0e^+e^-$, <0.04 %
$K_L^0 \to \mu^+\mu^-$, 1.0×10^{-8}	$\eta \to \pi^+\pi^-$, <0.15 %
$K_L^0 \to \mu^+\mu^-\gamma$, $<7.8 \times 10^{-6}$	$\eta \to \pi^+\pi^-e^+e^-$, 0.1 %
$K_L^0 \to \mu^+\mu^-\pi^0$, $<5.7 \times 10^{-6}$	$\eta \to \pi^+\pi^-\pi^0\gamma$, $<6 \times 10^{-4}$
$K_L^0 \to e^+e^-$, $<2.0 \times 10^{-9}$	$\eta \to \pi^+\pi^-\gamma\gamma$, <0.2 %
$K_L^0 \to e^+e^-$, $<2.8 \times 10^{-6}$	$\eta \to \mu^+\mu^-$, 2.2×10^{-5}
$K_L^0 \to \pi^0\pi^{\pm}e^{\mp}\nu$, $<2.2 \times 10^{-3}$	

[a] In this table we have listed most of the experimental data on the octet of light mesons from the particle data[100] with only one addition, the inclusion of the (approximate) charge radius of the particles. The data are divided into two groups: those for total

Figure 3: *A reproduction of Table 1, page 429, Ref. [20] used to identify the physical constituents of mesons in the massive particles produced free in the spontaneous decays, generally those with the lowest mode. The "completion" of quantum mechanics into hadronic mechanics then becomes recommendable for any quantitative treatment of the indicated structure model.*

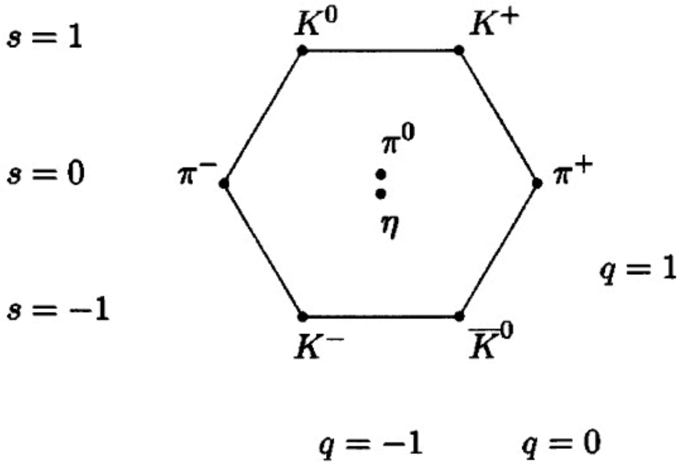

Figure 4: *An illustration of the compatibility between the hadronic structure model of mesons, Eqs. (40)-(45), with physical constituents and the conventional $SU(3)$ classification. This compatibility is achieved via the identification of the isounits or isotopic elements per each meson, Eq. (46), and their use as the hyperunit of the $\tilde{SU}(3)$ hypersymmetry in view of its local isomorphism to the conventional symmetry.*

produced free in the spontaneous decays , generally those with the lowest mode illustrated in Table 1, page 429, Ref. [20] (reproduced in Figure 2).

The above view suggested the following new structure models for the octet of mesons:

$$\pi^0 = (\tilde{e}^-, \tilde{e}^+)_{hm}, \tag{40}$$

$$\pi^\pm = (\tilde{\pi}^0, \tilde{e}^\pm)_{hm}, \tag{41}$$

$$K^0 = (\tilde{\pi}^0, \tilde{\pi}^0)_{hm}, \tag{42}$$

$$K^\pm = (\tilde{\pi}^0, \tilde{\pi}^\pm)_{hm}, \tag{43}$$

$$K_S = (\tilde{\pi}^-, \tilde{\pi}^+)_{hm}, \tag{44}$$

$$K_L = (\tilde{K}^-, \tilde{\pi}^+)_{hm}. \tag{45}$$

In the above models, the role of positrons as physical constituents of mesons provides the first known numerical representation of their very short meanlives, while all other characteristics are numerically represented via the hadronic bound states of Sections 2.3 and 2.4.

Structure models (40) - (45) are incompatible with quantum mechanics because the rest energy of all particles are *bigger* than the sum of the rest energies of the constituents (thus requiring *positive binding energies,* with ensuing *mass excesses* that are anathema for quantum mechanics as discussed in Section I.3), and for other reasons.

Necessary conditions to prevent misrepresentations is that models (40) - (45) are treated with hadronic mechanics and their constituents are *isoparticles and anti-isoparticles* (Section II-3.9) hereon denoted with an upper tilde.

Therefore, the elementary constituents of the π^0 in Eq.(40) are *one isoelectron* $\tilde{e}^-$ *and one isopositron* $\tilde{e}^+$ (called *eletons* in Ref. [19]); the constituents of the $\pi^\pm$, Eq. (51) are *one iso-meson* $\tilde{\pi}^0$ and *one isoelectron or isopositron;* etc.

Following their emission in the spontaneous via isotunnel effects (i.e., tunnel effects in the iso-Hilbert isospace), the isoconstituents assume conventional quantum mechanical characteristics plus possible secondary effects with the emissions of massless particles.

Note that *all models (40)-(45) either are or can be reduced to two-body hadronic bound states,* thus admitting analytic solutions in their representation via Eqs. (29). Note also that models (40)-(45) have a kind of bootstrap structures, since a given meson appears in the structure of heavier mesons.

Note additionally that *models (40)-(45) imply the increase of the number of "elementary" constituents with the increase of the rest energy.* In fact, the π^0 has only two elementary constituents while K_L has eight elementary constituents.

Note finally that the role of isopositrons as actual physical constituents of mesons provides the *only* known mechanism via particle-antiparticle annihilation for the *quantitative representation* of the very small mean-lives of mesons, by keeping in mind that the rest energy of electron-positron bound states is positive in our world and negative in the antimatter world (Section 2.5.3).

Since different structure models are characterized by numerically different isounits, the compatibility of the above hadronic structure model of mesons with their classification is readily achieved at the higher level of the *hyperstructural branch of hadronic mechanics* [33] [37], with the following total multi-valued *hyperunit*

$$\hat{I}_{tot} = \{\hat{I}_{\pi^0}, \hat{I}_{\pi^\pm}, \hat{I}_{K^0}, \hat{I}_{K^\pm}, \hat{I}_{K_S}, \hat{I}_{K_L}\}. \tag{46}$$

2.5.3. Positive energy of particle-antiparticle bound states. The *positive* total energy of particle-antiparticle hadronic bound states has been studied in detail in monograph [38] (see, e.g., Section 2.3.14, page 131), and in various additional works, including all historical references.

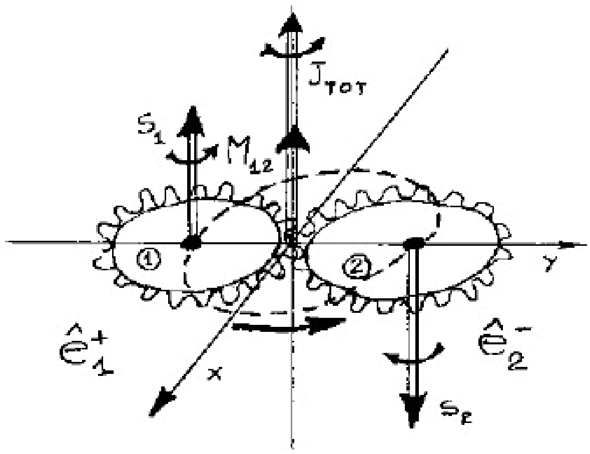

Figure 5: *A reproduction of "gear model" used in Section 5, page 852, Ref. [19] to illustrate the strongly attractive character of contact non-Hamiltonian forces when the constituents are in singlet coupling and their strongly repulsive character when in triplet couplings.*

We cannot possibly review here the underlying *isodual theory of antimatter,* but an indication of the following main notions appears recommendable to avoid basic misconceptions.

Recall that *negative energies violate causality,* for which reason P. A. M. Dirac was forced to work out his "hole theory."

Santilli resolved the problem of causality by referring all physical quantities, thus including energies, to *isodual units defined on isodual fields with a 'negative' unit.*

Hence, the terms "negative energy of antiparticles" have no meaning in the field here considered, because the correct statement is "negative energies of antiparticles measured to negative units of energy."

Within the above setting, Ref. [38], Eqs. (2.3.77), page 131, shows that *the total energy of a particle-antiparticle bound state is positive when measured in our world (with positive units) and negative when measured in the antimatter word (with negative units).*

The above property is truly fundamental for a consistent quantitative representation of all characteristics of unstable particles.

In fact, the presence of antiparticles in the structure of mesons and (unstable) baryons, first proposed by Santilli in 1978 [19], appears to be the most plausible origin of the extreme instability of mesons and (unstable) baryons with mean lives such as $\tau = 10^{-16}\ s$ or less, with no equally effective or plausible alternative known to this day.

2.5.4. Hadronic structure model of the π^0 meson. The characteristics of the π^0 meson are:

1. Rest energy $E = 134.96 MeV$,
2) Mean-life $\tau = 0.828 \times 10^{-16}\ s$,
3) Charge radius $R = 10^{-13}\ cm$,
4) Null charge and spin.
5) Null electric and magnetic moments,
6) Negative parity; and
7) Primary decay

$$\pi^0 \rightarrow \gamma + \gamma, \ \ 98.85\ \%. \tag{47}$$

The above characteristics are *all* numerically represented by hadronic bound state (29)) as a "compressed" form of the positronium (Pos), resulting in hadronic bound state (40) of one isoelectron and one isopositron (Ref. [19], Section 5, page 828 on)

$$Pos = (e^-, e^+)_{qm} \ \ \rightarrow \ \ \pi^0 = (e^-, e^+)_{hm}, \tag{48}$$

with hadronic structure equations

$$\begin{gathered}
\left[\tfrac{1}{r^2}(\tfrac{d}{dr}r^2\tfrac{d}{dr}) + \bar{m}(E \pm W_1 e^{\frac{2}{r}} + W_2\tfrac{e^{-br}}{1-e^{-br}})\right] = 0, \\
E_{tot} = E_{\tilde{e}^-} + E_{\tilde{e}^+} - E = 135\ MeV \ \ \bar{m} = \tfrac{m}{I^2} \\
\tau^{-1} = 2\pi\lambda^2|\hat{\psi}(0)|^2\tfrac{\alpha^2 E_{\tilde{e}}}{\hbar} = 10^{-16}s \\
R = b^{-1} = 10^{-13}\ cm = 1\ fm,
\end{gathered} \tag{49}$$

and numeric solution

$$k_1 = 0.34, \ \ k_2 = 1 + 4.27 \times 10^{-2}, \tag{50}$$

verifying conditions (35) as expected.

In appraising representation (48), the reader should keep in mind the assumption of Section 2.3 of '*constrained, nearly constant and circular orbits of the constituents*' under which the isounit can be considered independent from local coordinates.

The above results confirm all expectations indicated in preceding sections, namely,

1) Hadronic spectrum (33) admits one and only one energy level, the π^0, since all excited states are those of the positronium;

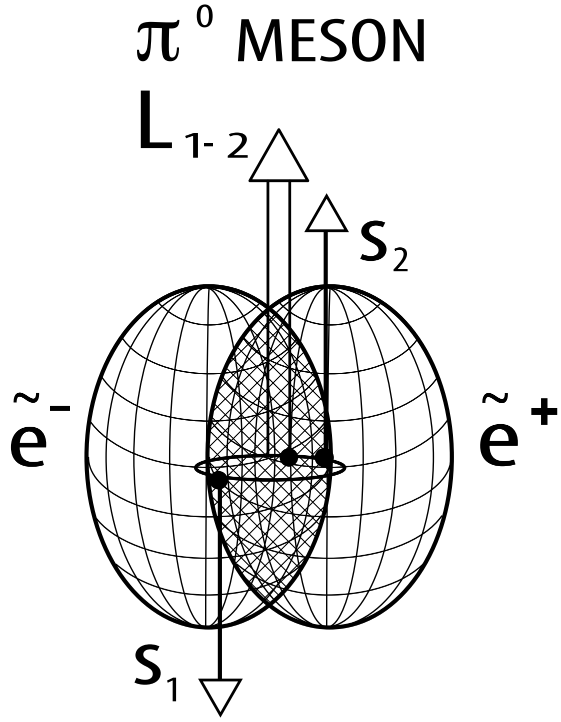

Figure 6: *A conceptual rendering of the structure model of the π^0 meson as a hadronic bound state of one isoelectron $\tilde{e}^-$ and one isopositron $\tilde{e}^+$ in singlet coupling with unmutated spins $s_1 = -s_2 = 1/2$ and orbital hadronic momentum in the ground state $L_{1-2} = 0$. The dashed area represents the contact, non-linear, non-local and non-potential interactions responsible for the bound state that, being not representable with a Hamiltonian H, are representable via the isotopic element $\hat{T}$ in the iso-Schrödinger equation $H \star \hat{\psi} = H\hat{T}\hat{\psi} = E\hat{\psi}$.*

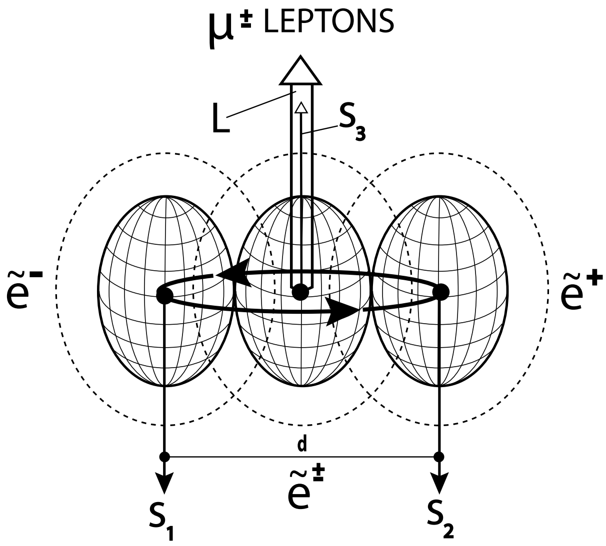

Figure 7: *To achieve compatibility with the standard model, it is generally believed that muons are elementary particles in contrast with the experimental evidence that they are naturally unstable with various decay modes (51) that strongly suggest a composite structure. In this figure, we present a conceptual rendering of the structure of the* $\mu^{\pm}$ *leptons predicted by hadronic mechanics as a three-body of elementary isoconstituents identified in the muon decay with the lowest mode. The weak interaction character of the particles is derived from a weak hadronic bound state essentially given by the overlapping of the classical radius of the constituents indicated by dashed ellipses. By recalling that all three constituents have a point-like charge, the dimension* d *of the charge radius of the two peripheral constituents is purely nominal.*

2) The presence of the antiparticle $\tilde{e}^+$ in the π^0 structure (40) explains its very short mean-life as well as its main decay (47);

3) Isodeterministic conditions (11) are verified by model (40), as a result of which standard deviations Δr and Δp have individual values smaller than one (Lemma II-3.7) and the isoperturbation series have no divergences (Corollary II-3.7.1).

2.5.5. Hadronic structure model of the remaining mesons. Recall the main characteristics of the muons $\mu^\pm$ are:

1) Rest energy $105,658\ MeV$;
2) Charge radius $R = 10^{-13}\ cm$;
3) Mean-life $\tau = 2.19703 \times 10^{-6}\ s$;
4) Spin $1/2$ and elementary charge;
5) Spontaneous decay

$$\begin{gathered} \mu^\pm \rightarrow e^- + \nu + \bar{\nu}, \\ \mu^\pm \rightarrow e^- + \gamma, \\ \mu^\pm \rightarrow e^- + 2\gamma, \\ \mu^\pm \rightarrow e^- + e^\pm + e^+. \end{gathered} \tag{51}$$

For the intent of achieving compatibility with the standard model, the $\mu^\pm$ leptons are considered to be "elementary particles." Santilli [19] cannot accept such a view because in contrast with the experimental evidence that *the muons are naturally unstable particles with decays (51).*

Therefore, Ref. [19], Section 5, proposed the structure model of the muons with the physical constituents identified by the decay mode (51) with the smallest mode $< 10 \times 10^{-12}$ (Figure 5)

$$\mu^\pm = (e^-, e^\pm, e^+)_{hm}. \tag{52}$$

The birth of weak interactions was suggested as being due to a weak form of wave-overlapping essentially that of the classical size of the constituents

$$R_{e^\pm} = 2.28 \times 10^{-13}\ cm, \tag{53}$$

which said weak contact interactions can be quantitatively represented by hadronic mechanics via an isotopic element $\hat{T}$ with values close to 1.

A main aspect is that there is the birth of strong interactions in model (52) because we do not have a wave-overlapping inside the hadronic horizon, as it is the case for the π^0.

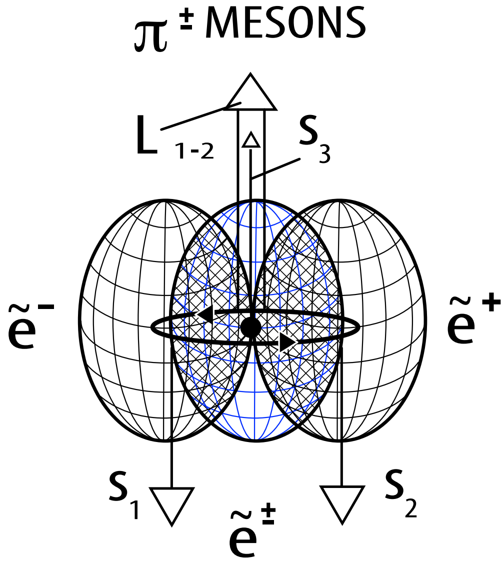

Figure 8: *A conceptual rendering of the structure of the* $\pi^{\pm}$ *mesons as a three-body hadronic bound states (57) of elementary isoparticles which can be interpreted as a "compressed* $\mu^{\pm}$ *lepton" (Figure 6). Note that individual pairs of isoconstituents are coupled in singlet as necessary for consistency. Note also that the isoconstituents have a point-like charge structure. Therefore, the charge diameter of the structure is given by the distance between the centers of the two peripheral isoconstituents.*

It is easy to see that model (52) can represent the muon spin $1/2$ and decay (51) with the lowest mode as a tunnel effect of the constituents.

The remaining characteristics can be represented via a direct analytic solution of model (52) as a restricted three-body problem.

Ref. [19] suggested the approximation of the model into a two-body structure with weakly bounded constituents and the structure equations

$$\begin{gathered}\left[\frac{1}{r^2}(\frac{d}{dr}r^2\frac{d}{dr}) + \bar{m}(E \pm W_1 e^{\frac{2}{r}} + W_2 \frac{e^{-br}}{1-e^{-br}})\right] = 0,\\ E_{tot} = E_{100\ Mev} + E_{e^\pm} - E, = 105\ MeV\ \ \bar{m} = \frac{m}{I^2}\\ \tau^{-1} = 2\pi\lambda^2|\hat{\psi}(0)|^2\frac{\alpha^2 E_e}{\hbar} = 10^{-6}\ s\\ R = b^{-1} = 10^{-13\ cm},\end{gathered} \tag{54}$$

admitting the following values for the the k-parameters

$$k_1 = 0.93,\ \ k_2 = 1 + 8.47 \times 10^{-2}, \tag{55}$$

that also verify conditions (35).

The main characteristics of the $\pi^\pm$ mesons are:

1) Rest energy $139.570\ MeV$;

2) Charge radius $R = 10^{-15}\ cm$;

3) Mean-life $\tau = 2.603 \times 10^{-8}\ s$;

4) Spin $J = 0$ and elementary charge;

5) Decay with lowest mode $< 3.2 \times 10^{-9}$)

$$\pi^\pm \rightarrow e^- + e^\pm + e^+ + \nu\pi^\pm \tag{56}$$

.

The above characteristics are all represented by the hadronic structure model of the $\pi^\pm$ mesons first proposed in Section 5, Ref. [19]

$$\pi^\pm\ = (\tilde{e}^-, \tilde{e}^\pm, \tilde{e}^+)_{hm} \approx (\tilde{\pi}^0, \tilde{e}^\pm)_{hm}, \tag{57}$$

for which, unlike the case of the muons, the constituents are elementary isoparticles in condition of deep mutual penetration inside the hadronic horizon (Figure 7).

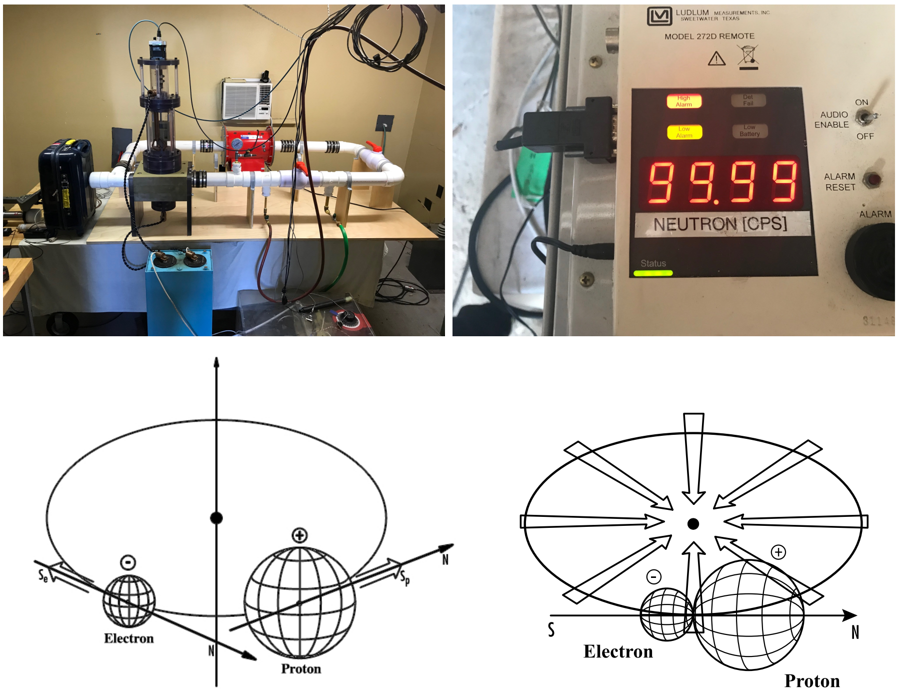

Figure 9: *In this figure we show: 1) At the top left a picture of the Directional Neutron Source (DNS) developed by Santilli at Thunder Energies Corporation, now Hadronic Technologies Corporation [39] which synthesizes on demand neutral and negatively charged hadrons from a hydrogen gas with preferred directionality, energy and flux; 2) At the top right a typical case of neutron CPS; 3) At the bottom left a conceptual rendering of the ionization of the hydrogen gas by the electric arc during its activation and the proper axial alignment of the proton and electron with opposite charge and magnetic polarities; and 4) At the bottom right a conceptual rendering of the "compression" of the electron inside the proton by the electric arc during its disconnection.*

The rest energy, mean-life and charge radius of the $\pi^{\pm}$ are readily represented by model (57) with the hadronic structure equations

$$\begin{aligned}
&\left[\tfrac{1}{r^2}(\tfrac{d}{dr}r^2\tfrac{d}{dr}) + \bar{m}(E \pm W_1 e^{\frac{2}{r}} + W_2 \tfrac{e^{-br}}{1-e^{-br}})\right] = 0,\\
&E_{tot} = E_{\tilde{\pi}^{\pm}} + E_{\tilde{e}^{\pm}} - E = 139\ MeV,\ \ \bar{m} = \tfrac{m}{I^2}\\
&\tau^{-1} = 2\pi\lambda^2|\hat{\psi}(0)|^2\tfrac{\alpha^2 E_{\tilde{e}^{\pm}}}{\hbar} = 2.603 \times 10^{-8}\ s,\\
&R = b^{-1} = 10^{-13}\ cm,
\end{aligned} \tag{58}$$

admitting solutions for the k-parameters

$$k_1 = 0.34,\ \ k_2 = 1 + 3.67 \times 10^{-3}, \tag{59}$$

that again verify conditions (35).

The main recent advance since the above 1978 proposal [19] is the achievement of a consistent representation of the total angular momentum $J = 0$ of the $\pi^{\pm}$ meson thanks to the irregular $SU(2)$ isosymmetry (Section 3.5). In essence, when the three isoparticles are all compressed inside the hadronic horizon, the orbital motion of the two peripheral isoconstituents L_{1-2} is constrained, for stability, to be equal to the spin $S_{\tilde{e}^{\pm}}$ of the central isoparticle, thus having value $L_{1-2} = S_{tildee^{\pm}} = 1/2$ with total angular momentum (Sections II-3.4 and II-3.5, Section II-3.5.3 in particular).

$$J_{tot} = s_1 + s_2 + s_3 + L_{1-2} = -1/2 + 1/2 - 1/2 - 1/2 = 0. \tag{60}$$

Note that model (57) can be interpreted as a form of "compressed muons" (52) (see Figures 6 and 7). The orbital motion of the peripheral electrons is unrestricted in model (52), yielding a total angular momentum $1/2$. By contrast, the peripheral isoelectrons and isopositron in model (57) are *constrained to orbit inside the central isoelectron or isopositron*, thus being forced to have the orbital value equal to the spin of the isoparticle. In fact, orbital values different than $1/2$ would imply extreme resistive forces with ensuing excessive instabilities.

In Santilli's view, an important aspect of model (57) is its apparent verification of the isodeterminism of Lemma II-3.7 as well as of the lack of divergencies in the isoperturbation seres (Corollary II-3.7.1) due to the small value of the isotopic element assured by values (59).

Interested readers can verify that values (35) remain verified under the use of more accurate experimental values for the characteristics of muons and mesons.

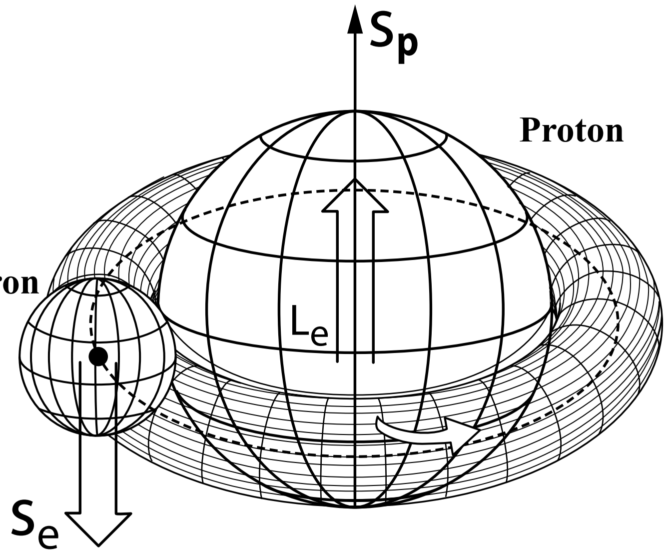

Figure 10: *A conceptual rendering of the structure of a particle called the neutroid $\tilde{n} = (e^-, p^+)_{hm}$, Eq. (61), with spin $J = 1$, charge radius $R = 10^{-13}$ cm, mass essentially that of the proton and mean-life of the order of* 5 s *which has been undetected in all experiments to date [I-90]-[I-95] by neutron detectors, yet has caused nuclear transmutations typical of neutron irradiation. The bound state is created by the extremely strong,* $r - p$ *Coulomb attraction which is of the order of* 10^{24} N *plus weak contact interactions implying ignorable mutations of the electron and the proton. The* $e - p$ *singlet coupling and the orbital motion in the ground state imply the spin* $J = 1$ *explaining the lack of detection of neutroids by neutron detectors. Note that the neutroid is impossible for quantum mechanics.*

It is an instructive exercise for the interested reader to work out the hadronic structure models of the remaining mesons, Eqs. (42)-(45) showing the increase of the k_1 value and the decrease of the k_2 value with the increase of the rest energy (see Section 6.2 of Ref. [31] for an independent review).

2.6. Einstein's determinism in the structure of baryons.

2.6.1. Structure of the neutroid. Stars initiate their lives as being composed of hydrogen; they grow in time via the accretion of interstellar-intergalactic hydrogen; and eventually reach such pressures and temperatures in their core to synthesize the neutron as a "compressed" hydrogen atom according to H. Rutherford [40].

A main aspect of the studies herein considered is that, in Santilli's view [20], the synthesis of the neutron from the hydrogen in the core of stars is one of the best illustrations of the validity of the EPR argument [1] because said synthesis cannot be represented with quantum mechanics for various technical reasons reviewed in Section I-1, in denial of the experimental evidence that, *having opposite charges, the proton and the electron at mutual distances of* 10^{-13} cm *are attracted to each other with a Coulomb force of about* 230 N *(See Eq, (107) for its calculation). Such an attractive force is so big for particle standards to dismiss as inapplicable any theory unable of producing a bound state under such an anormous attractive force.*

Consequently, Santilli dedicated decades of mathematical, theoretical, experimental and industrial research on the neutron synthesis due to its truly fundamental character for all quantitative sciences (see Refs. (I-85] to [I-95] and independent reviews [31] [41]).

Besides a number of preceding attempts without neutron detection, the first indirect experimental detection of the synthesis of the neutron from the hydrogen was initiated by in the 1960's by Don Carlo Borghi [76], confirmed in subsequent tests [I-90] - [I-95], and routinely verifiable with the Directional Neutron Source developed by Hadronic Technologies Corporation [39] (Figure 8), indicate the existence of an unstable, neutral, intermediate state called *neutroid* (denoted with the symbol $\tilde{n}$) which is unidentified by neutron detectors, while causing nuclear transmutations typically triggered by neutron irradiation.

Hadronic mechanics suggests the following representation of the structure of the neutroid (Figure 9)

$$\tilde{n} = (\tilde{e}^-_\downarrow, p^+_\uparrow)_{hm}, \tag{61}$$

consisting of an isoelectron $\tilde{e}^-$ with spin $s_1 = 1/2$ in singlet contact coupling with a standard proton p^+ with spin $s_2 = 1/2$ and orbital motion $L_{1-2} = 0$ in the ground state.

These assumptions suggest the following predicted features of the neutroid:

1) Rest energy estimated to be about $940\ MeV$,

2) Mean-life estimated to be of at least $\tau = 5\ s$,

3) Charge radius estimated to be of about $R \approx 10^{-13}\ cm$,

4) Spin predicted to be $J = 0$, and

5) Spontaneous decay

$$\tilde{n} \rightarrow e^- + p^{\cdot} \tag{62}$$

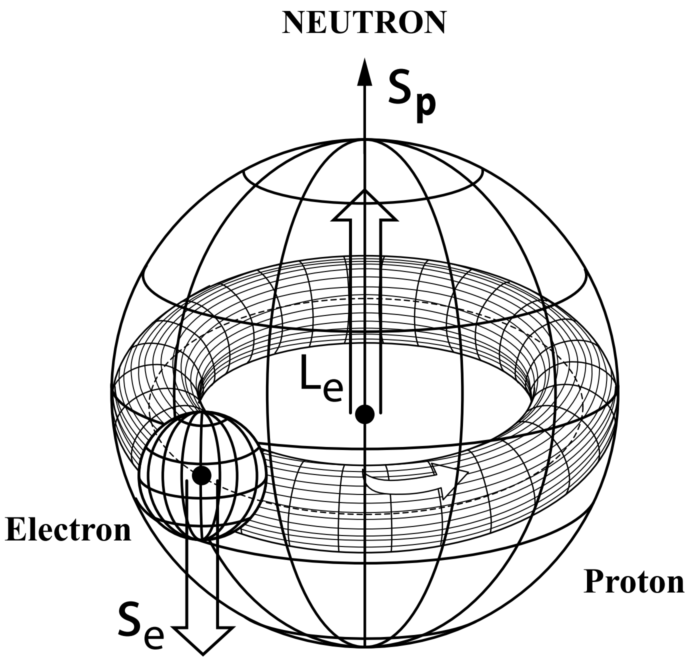

Figure 11: *A conceptual rendering of the structure of the neutron as "compressed" hydrogen atoms in the core of stars according to H. Rutherford [40]. When the proton is represented as an extended particle, there is the emergence of a constrained orbital motion of the electron inside the proton verifying the isodeterminism of Lemma 3.7, which allows the first known exact representation of all the characteristics of the neutron at the nonrelativistic [42] and relativistic [36] levels without need for the neutrino hypothesis.*

The above features can be represented with hadronic bound state (29) specialized to the indicated values

$$\left[\frac{1}{r^2}\left(\frac{d}{dr}r^2\frac{d}{dr}\right)+\bar{m}\left(E\pm W_1\frac{e^2}{r}+W_2\frac{e^{-br}}{1-e^{-br}}\right)\right]=0,$$
$$E_{tot}=E_{\tilde{e}}+E_p-E=940\ MeV$$
$$\tau^{-1}=2\pi\lambda^2|\hat{\psi}(0)|^2\frac{\alpha^2 E_1}{\hbar}=5\ s \tag{63}$$
$$R=b^{-1}=10^{-13}\ cm,$$

with values of the k-parameters

$$k_1\approx 1,\ \ k_2\approx 1+10^{-6}, \tag{64}$$

verifying the crucial condition (35).

Once absorbed by a stable nucleus $N(A,Z,J)$, the neutroid is transformed by strong interactions into a neutron plus secondary emissions, resulting in a generally untabulated unstable nucleus $\tilde{N}(A+1,Z,J+1/2)$ that decays into stable nuclei plus radiations.

The spin 0 of the neutroid explains the impossibility of its detection via conventional neutron detectors.

The decay of the nucleus $\tilde{N}(A+1,Z,J+1/2)$ explains the triggering by undetectable particles of conventional nuclear transmutations normally triggered by neutron irradiations.

Hadronic bound state (61) is clearly impossible for quantum mechanics, but readily possible for hadronic mechanics via structure model (29) due to the combination of strongly attractive Coulomb and contact forces at mutual distances of the order of $10^{-13}\ cm$ with isotopic elements of the simple type (19) verifying isodeterministic Lemma II-3.7. and the rapid convergence of isoperturbation series of Corollary II-3.7.1.

2.6.2. Nonrelativistic representation of the neutron synthesis. We consider now the synthesis of the neutron on demand from a hydrogen gas in the needed directionality, (low) energy and CPS.

Such a synthesis has been first achieved by the Directional Neutron Source (DNS) developed by Santilli at Thunder Energies Corporation, now Hadronic Technologies Corporation [39].

Recall that, when seen in an oscilloscope set for milliseconds, electric arcs between carbon electrodes, are continuously connected and disconnected to seek the shortest distance for the discharge.

With reference to Figure 9, the electric arc ionizes the hydrogen gas during its connection and creates an axial alignment along a magnetic line of electrons and protons with opposite charge and magnetic polarities.

By contrast, during their disconnection electric arcs “compress” the electron inside the proton by synthesizing in this way the neutron in a way crucially dependent on the shape of the arc, its power and other factors.

With reference to Figure 11, when compressed inside the hyperdense proton, the much lighter electron is “constrained” to have an orbital motion equal to the proton spin so as to avoid extreme resistive forces with ensuing high instabilities.

Note that the neutroid (figure 10) appears to be an unavoidable intermediate step prior to the full synthesis of the neutron (Figure 11).

The well known main characteristics of the neutron are the following:

1) Rest energy $939.565\ MeV$;

2) Charge radius $R = 1.73 \times 10^{-13}\ cm$;

3) Mean-life $\tau = 881\ s(about\ 15\ m)$;

4) Spin $1/2$;

5) Charge ;0

6) Anomalous magnetic moment $\mu_n = -1.9\frac{e}{2m_pc}$ and null electric dipole moment;

7) Decay

$$n \ \rightarrow \ p^+ + e^- + \bar{\nu}. \tag{65}$$

The nonrelativistic representation of *all*— characteristics of the neutron in its synthesis from the hydrogen was first achieved by Santilli in the 1990 paper [43] via hadronic bound state

$$n \ = \ (\tilde{e}^-_\downarrow, \tilde{p}^+_\uparrow)_{hm}, \tag{66}$$

where one should note that, unlike the case of the neutroid (61), both the electron and the proton are mutated into isoparticles.

Note also that model (66) represents a *compressed neutroid* in a way much similar to the structure of the $\pi^\pm$ mesons as compressed muons , $\mu^\pm$ (Section 2.5.4).

The representation of the rest energy, charge radius, mean-life, charge, parity and tunnel effect decay of the neutron have been first achieved in

Ref. [43] Eqs. (2.19), page 521, via hadronic two-body bound state

$$\left[\frac{1}{r^2}(\frac{d}{dr}r^2\frac{d}{dr}) + \bar{m}(E \pm W_1\frac{e^2}{r} + W_2\frac{e^{-br}}{1-e^{-br}})\right] = 0,$$

$$E_{tot} = E_{\tilde{e}} + E_{\tilde{p}} - E = 939\ MeV$$

$$\tau^{-1} = 2\pi\lambda^2|\hat{\psi}(0)|^2\frac{\alpha^2 E_{\tilde{e}}}{\hbar} = 881\ s\ (about 15m) \tag{67}$$

$$R = b^{-1} = 10^{-13}cm,$$

with values of the k-parameters

$$k_1 = 2.6,\ \ k_2 = 1 + 0.81 \times 10^{-8}, \tag{68}$$

verifying conditions (35) for the validity of isodeterminism inside the neutron and the validity of conventional uncertainties in its outside.

The representation of the spin $1/2$ of the neutron was also achieved for the first time in Ref. [43], Eqs. (2.22)-(2.37), thanks to the appearance of a constrained *orbital* motion of the isoelectron when totally "compressed" inside the proton (which orbital motion is completely non-existent for quantum mechanics).

The representation of the spin $1/2$ is additionally permitted by the *isotopies of spin-orbit couplings* (see Chapter 6, page 209 on, Ref. [33], for a detailed treatment), which the hadronic angular momentum of the isoelectron is constrained to be equal to the spin of the isoproton, as a necessary condition to avoid big instabilities according to Eqs. (iI-118) under constraint (II-119). Hence, $J^n_{tot} = s_p + s_e + L_e = 1/2$, namely, *the spin of the neutron coincides with that of the proton.*

Recall the following experimental values of magnetic moments in nuclear units $\mu_N = \frac{e}{2m_p c}$

$$\mu_n^{exp} = -1.91\ \mu_N$$

$$\mu_p^{exp} = 2.7\ \mu_N \tag{69}$$

$$\mu_e^{exp} = 1\ \mu_B = 5 \times 10^{-4}\ \mu_N,$$

where one can see that the magnetic moment of the neutron is "anomalous" because outside the prediction of quantum mechanics both in its direction and numeric value.

The first known exact representation of the anomalous magnetic moment of the neutron was achieved in Ref. [43], Eqs. (2.39)-(2.41), page 526, via the novel contribution of the internal orbital motion of the isoelectron.

By noting from values (233) that the magnetic moment of the electron is very small for nuclear standard, thus being ignorable in first approximation, the main assumption of Ref. [43] is that *the magnetic moment of the neutron of model (66) is given by the magnetic moment of the proton plus the orbital magnetic moment ot the isoelectron inside the proton.*

Note that said contribution is completely absent in quantum mechanics due to its point-like approximation of particles as being point-like when at mutual distances *smaller* than their charge diameter.

Note that the electron is negatively charged and, consequently, the contribution to the magnetic moment of the neutron from its rotation inside the proton in the direction of the proton spin is *negative,* thus providing the first known representation of the *anomalous direction* of the neutron magnetic moment.

Calculations done in Ref. [43] for the magnetic moment of the isoelectron orbiting inside the proton yield the value $\mu_{\tilde{e}}^{orb} \approx -4.6\,\mu_N$, by therefore reaching the first known representation of the *anomalous value* of the neutron magnetic moment (Eqs. (2.40), page 526, Ref. [43], see also Ref. [31], Section 6.3.D and paper [41], Section 4, page 41)

$$\mu_n = \mu_{\tilde{p}} + \mu_{\tilde{e}}^{orb} + \mu_{\tilde{e}}^{intr} \approx$$
$$\approx \mu_{\tilde{p}} + \mu_{\tilde{e}}^{orb} = \tag{70}$$
$$= 2.7\,\mu_N - 4.6\,\mu_N = -1.91\,\mu_N = \mu_n^{exp}.$$

Note that *the negative sign of the neutron magnetic moment can be considered as direct evidence of the presence of an electron in singlet internal coupling in the proton,* since no other known particle besides the electron can provide the internal, rather large contribution $\mu_{\tilde{e}}^{orb} = -4.6\,\mu_N$.

In Santilli's view, *the inability by quantum mechanics to represent both the anomalous direction and value of the neutron magnetic moment constitutes an additional evidence beyond scientific doubt supporting Einstein's vision on the lack of "completion" of quantum mechanics.*

Furthermore, *the impossibility for quantum mechanics to characterize the motion of the electron compressed inside the proton, constitutes evidence that the most important "completion" of quantum mechanics needed to represent experimental values is the characterization of particles with their actual shape and density.*

The parity and null value of the electric dipole moment were represented via a isotopy of conventional lines.

For historical comments on the birth of the neutrino hypothesis in quantum mechanics [13] and its possible replacement with the etherino hypothesis [42] in hadronic mechanics, one may consult Section II-3.4.1 on Irregular Pauli-Santilli isomatrices.

The reader should keep in mind that the neutron is naturally unstable (when isolated). Consequently, the synthesis of the neutron is a time-irreversible process. It then follows that the Lie-isotopic isomechanical treatment presented in this section is an approximation of the broader treatment via the covering Lie-admissible genomechanics outlined in Section 1-2 (see Refs. [32] [33] for extensive treatments).

With reference to the geno-Schrödinger and geno-Heisenberg equations (I-10) and (I-11), respectively, the genomechanical treatment can be studied via the extension of isotopic element (19) into a time-dependent form characterizing the genotopic elements for motions forward and backward in time.

2.6.3. Relativistic representation of the neutron synthesis. The relativistic representation of *all* characteristics of the neutron in synthesis (66) was first achieved by Santilli in Refs. [35] [36] (see independent review [31], Section 6.3, page 342 on) via the isosymmetry of the irregular Dirac-Santilli isoequation (37), namely, the isotopy $\hat{\mathcal{P}}(3.1)$ of the spinorial covering of the Poincaré symmetry (Section 2.5.11) and can be outlined as follows.

Recall that the non-potential hadronic representation of strong interactions via the isotopic element (1) (Section 2.3) implies the "absorption" of the Coulomb potential by the Hulten potential, as essentially implied by the charge independence of strong interactions.

This feature permits to ignore Coulomb binding energies in first approximation, resulting in a *weakly bounded* relativistic hadronic structure of the neutron, namely, a state with a small binding energy typical of all non-potential interactions.

Consider first synthesis (61) of the neutroid, and assume its representation in the iso-Minkowski isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ with isospacetime (II-19). Assume in first approximation that the proton is perfectly spherical for which $n_1 = n_2 = n_3 = 1$ and assume that the density of the region of neutroid-proton overlapping is close to that of the vacuum, thus implying the value $n_4 = 1$. These assumptions imply the simplified isometric

$$\hat{\eta} = Diag.(1, 1, 1, -1)e^{-K}, \;\; K > 0. \tag{71}$$

The relativistic version of the synthesis (61) with the representation of all characteristics of the neutroid then follows via a simply isotopy of the relativistic treatment of the hydrogen atom.

In the transition to the relativistic treatment of the structure of the neutron, Eq. (66), the isoproton cannot any longer be assumed to be perfectly spherical and the density of the overlapping region becomes dominant, resulting in values of the characteristic quantities $n_\mu \neq 1,\ \mu = 1, 2, 3, 4$.

In Refs. [35] [36], Santilli assumes that the value of the characteristic quantity n_4 representing the density of the neutron is equal to the density of the proton-antiproton fireball of the Bose-Einstein correlation [46] [47], resulting in the values

$$n_4 = 0.62,\ \ b_4 = \frac{1}{n_4} = 1.62, \tag{72}$$

where $b_4 = 1/n_4$ is the notation used in Ref. [36].

The Lorentz-Santilli isotransforms (II-42) then imply the following *isorenormalization of the rest energy of the electron,* namely a renormalization caused by non-potential interactions (Ref. [36], Eqs. (7.1), page 191)

$$E_e = m_e c^2 = 0.511\ MeV \ \ \rightarrow \ \ E_{\tilde{e}} = m_e \frac{c^2}{n_4^2} = 1.341\ MeV. \tag{73}$$

As one can see, the above isorenormalization removes the problem of the missing $0.782\ MeV$ energy in the neutron synthesis when represented on the iso-Minkowskian isospace over an isofield, thus rendering consistent the needed isorelativistic isoequations.

It should be stressed that the above isorenormalization continues to be based on the etherino mechanism for the delivery of the missing energy to the neutron [42].

The relativistic representation of the spin of the neutron in synthesis (66) was first achieved in Refs. [35] [36]. Recall from Figure 10 that the spin S_1 of isoelectron is opposite to the spin S_2 of the isoproton. The relativistic spin-orbit coupling implies the *constraint* that the orbital angular momentum L_1 of the isoelectron inside the isoproton be equal to the isoproton spin,

$$L_1 = S_2. \tag{74}$$

The above identity is manifestly impossible for the spinorial covering of the Poincaré symmetry $\mathcal{P}(3.1)$ and relativistic quantum mechanics, but it is indeed possible for the covering isosymmetry $\hat{\mathcal{P}}(3.1)$. In fact, with reference to Section II-3.5.3, Eqs. (II-122), identity (II- 241) implies the con-

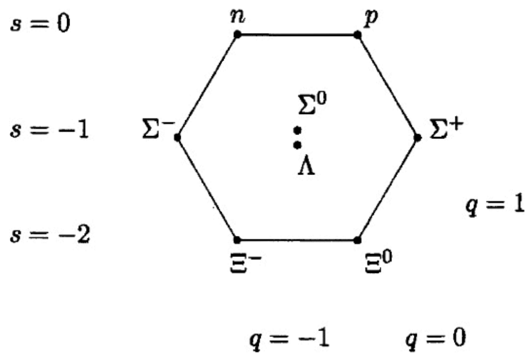

Figure 12: *An illustration of the compatibility of the quantum mechanical $SU(3)$ and subsequent classifications of baryons unto families, and the structure of individual baryons as a hadronic bound state with physical constituents generally emitted free in the spontaneous decays with the lowest mode, Eqs. (241). Said compatibility is achieved via the multi-valued hyperunit characterized by the isounits or isotopic elements of the individual baryons, Eqs. (242), and its use to build the hyperstructural image of the applicable Lie symmetry that turns out to be locally isomorphic to the original symmetry due to the positive-definite character of the isounit.*

ditions (Ref. [36], Eqs. (7.2), page 192)

$$\hat{L}_3 \star |\hat{\psi}>= \pm n_1 n_2 |\hat{\psi}>=$$
$$= \hat{S}_3 \star |\hat{\psi}>= \frac{1}{2}\frac{1}{n_1 n_2}|\hat{\psi}>,$$
$$\hat{L}^{\hat{2}} \star |\hat{\psi}>= (n_1^2 n_2^2 + n_2^2 n_3^2 + n_3^2 n_1^2)|\hat{\psi}>=$$
$$= \hat{S}^{\hat{2}} \star |\hat{\psi}>= \frac{1}{4}(n_1^{-2} n_2^{-2} + n_2^{-2} n_3^{-2} + n_3^{-2} n_1^{-2})|\hat{\psi}>, \quad (75)$$

admitting the simple solution

$$n_k^2 = \frac{1}{\sqrt{2}} = 0.706,$$
$$b_k^2 = \frac{1}{n_k^2} = \sqrt{2} = 1.415, \quad k = 1, 2, 3, \quad (76)$$

where $b_k^2 = 1/n_k^2$ in the notation of Ref. [36].

The relativistic representation of the anomalous magnetic moment of the neutron was also achieved for the first time in Ref. [36], Eqs. (7.4), page 192. The representation is again permitted by the contribution from the orbital motion of the isoelectron inside the isoproton, and it is given by non-relativistic expression (234) with a more accurate representation of the orbital contribution.

It should be indicated that the non-relativistic and relativistic structure models of the neutron outlined in this section are mere approximation of a much more complex reality in which all characteristics of the constituents are mutated, thus including the charge.

2.6.4. Remaining baryons. A central requirement for the consistency of model (66) is that *the excited states of the neutron are the conventional states of the hydrogen atom.* This illustrates again the hadronic suppression of quantum mechanical energy spectra, since the latter are typical for the classification, rather than the structure of hadrons.

By recalling the condition that the number of elementary constituents of hadrons increases with the increase of the rest energy, the hadronic structure of the remaining baryons is reducible to two isoparticle structures derived from the spontaneous decays generally those with the lowest mode, much along the structure of mesons [21] (see [31], Section 6.3.J, page 366 for an independent review)

$$\begin{aligned}
&p^+(938\ MeV) = stable,\\
&n(940\ MeV) = (\tilde{p}^+, \tilde{e}^-)_{hm},\\
&\Lambda(1115\ MeV) = (\tilde{p}^+, \tilde{\pi}^-)_{hm},\\
&\Sigma^+(1189\ MeV) = (\tilde{p}^+, \tilde{\pi}^0)_{hm},\\
&\Sigma^0(1192\ MeV) = (\tilde{n}, \tilde{\pi}^0)_{hm},\\
&\Sigma^-(1197\ MeV) = (\tilde{n}, \tilde{\pi}^-)_{hm},\\
&\Sigma^0(1314\ MeV) = (\tilde{\Lambda}, \tilde{\pi}^0,\\
&\Xi^-(1321\ MeV) = (\tilde{\Lambda}, \tilde{\pi}^-)_{hm},
\end{aligned} \tag{77}$$

by keeping in mind that numerous alternative internal exchanges of isoparticles do indeed occur while keeping constant the total characteristics.

It is an instructive exercise for the interested reader to see that all the above models verify condition (II-199) on the lack of hadronic excited states, as well as conditions (II-188) for the validity of isodeterministic Lemma II-3.7 and related rapid convergence of isoseries.

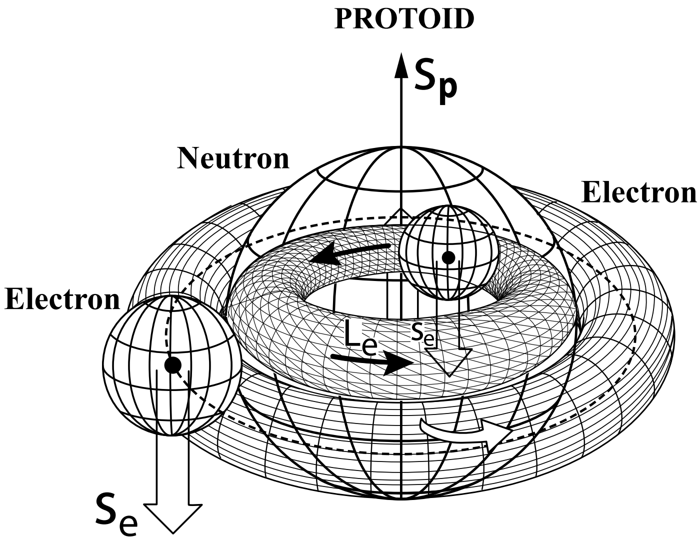

Figure 13: *A conceptual rendering of the protoid as weakly bounded electron and neutron in singlet contact coupling, resulting in a negatively charged hadron with spin $S = 0$ with intriguing applications.*

The compatibility of hadronic structure models (77) with the $SU(3)$-color or more recent classifications is achieved at the hyperstructural level [33] [37] with the following ordered hyperunit [21] (Figure 11)

$$\hat{I}_{tot} = \{\hat{I}_p, \hat{I}_n, \hat{I}_\Lambda, \hat{I}_{\Sigma^+}, \hat{I}_{\Sigma^0}, \hat{I}_{\sigma^-}, \hat{I}_{\Xi^0}, \hat{I}_{\Xi^-}\} \tag{78}$$

Said compatibility is ultimately due to the positive-definiteness of the individual isounits that, in turn, implies the local isomorphism between the hypersymmetry $\tilde{SU}(3)$ and the conventional $SU(3)$ symmetry.

2.6.5. Industrial applications. There is no doubt that the "completion" of quantum mechanics is, by far, Einstein's most important legacy because of its *basic* implications for mathematics, physics, chemistry and other quantitative sciences, with expected industrial applications generally beyond our expectation at this writing.

As an illustration, we outline in this section the industrial applications expected from the technology underlying the synthesis of the neutron from the hydrogen which technology has been possible thanks to the "completion" of quantum into hadronic mechanics.

Besides the synthesis of neutroids (Figure 10) and neutrons (11), the

Directional Neutron Source (DNS) [39] (Figure 9) permits the study of the synthesis of a *negatively charged, strongly interacting particles* [48] (patent pending) with rest energy of about $940\ MeV$, spin 0, charge radius $R = 1\ fm$, and mean-life of about $3\ s$ which is called the *protoid*, and denoted $\hat{p}_1$ (Figure 13).

Recall that contact non-Hamiltonian interactions solely depend on wave-overlapping, thus being charge independent. The protoid is predicted to be given by a singlet coupling of an electron essentially at contact with a neutron, thus yielding total angular momentum 0, with the following structure model:

$$\hat{p}_1 = (e^-_\downarrow, n_\uparrow)_{hm}. \tag{79}$$

Hadronic mechanics predicts a second negatively charged proton called *pseudo-proton*, and denoted $\hat{p}_2$, with essentially the same rest energy, charge radius and mean-life of the neutroid but with spin 1 (Figure 14).

Recall that an electric arc submerged within a hydrogen gas creates a plasma in its surrounding comprising: protons, electrons, neutrons and valence electron pairs in singlet couplings known as *isoelectronia* with rest energy of about $1\ MeV$, null spin and magnetic moment, and a mean-life expected to be of the order of $1\ s$ or fractions thereof (Section 2.8.3).

Following the synthesis of the neutron via the "compression" of an electron inside the proton (Section 2.6.2), the pseudo-proton is predicted to be generated by a "compression" of an electron, this time, inside the neutron, or equivalently, by the "compression" of an isoelectronium inside the proton, with structure model

$$\hat{p}_2 = (\tilde{e}, \tilde{n})_{hm} \approx [\tilde{p}, (\tilde{e}^-_\uparrow, \tilde{e}^-_\downarrow)]_{hm}. \tag{80}$$

The spin $\S_{\hat{p}2} = 1$ is due to the sum of the proton spin $S_p = 1/2$ plus the constrained orbital motion of the electron pair along the proton spin $L_{1,2} = 1/2$.

Recall that protons are *repelled* by nuclei. Hence, the industrially significant feature of negatively charged hadrons is that they are *attracted* by nuclei, thus initiating a basically new approach toward the controlled nuclear fusions that resolves the extremely large Coulomb repulsion that has opposed nuclear fusions to date.

In view of the above features, the technologies that can be developed with the DNS are the following:

1. The detection of fissionable nuclear material that can be concealed in suitcases, containers or underground, since the neutron irradiation of fissionable material under controlled directionality, energy and flux, triggers the decay of some of the nuclei with a shower of clearly detectable

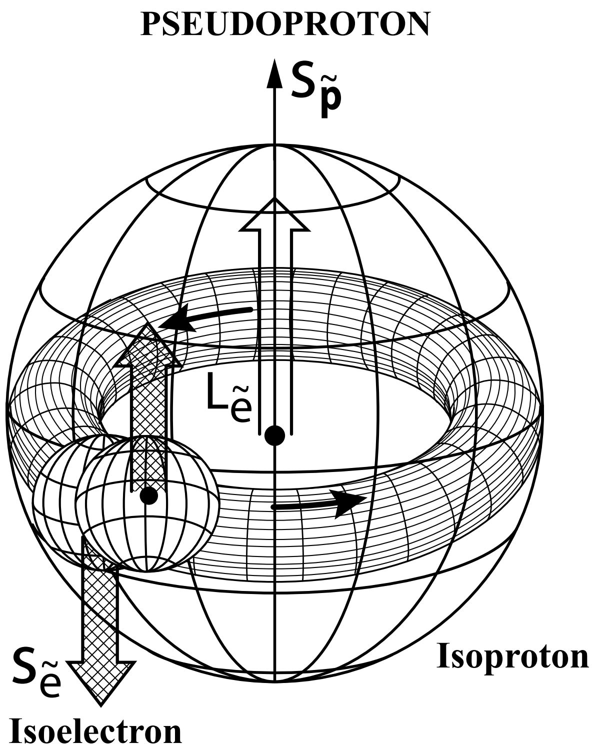

Figure 14: *A conceptual rendering of the pseudo-proton created by the "compression" of an electron, this time, inside a neutron, or equivalently, of a valence electron pair in singlet coupling, resulting in a negatively charged hadron with spin* 1.

radiations. By comparison, most nuclear material are detected as ordinary metals under currently available X-ray, microwave and other scans.

2. The detection of the presence and concentration of precious metals in mining operations, since the neutron irradiation, for instance, of the walls of mining tunnel triggers known nuclear transmutations with the emission of sharp, clearly detectable photons, while statistical data identify the concentration.

3. The control of large metal welds in civilian and military naval construction, which control is rendered particularly effective by the easy mobility of the DNS.

4. Cancer treatments via pseudo-proton irradiation which is expected to be less invasive and more localized than currently available proton irradiation since, in the former case, pseudo-protons are *attracted* by cancerous nuclei, while, in the latter case, protons are *repelled*.

5. Initiation of the much overdue research and development toward the recycling of nuclear waste via its stimulated decay while delivering energy, which recycling is possible in a number of ways, including the irradiation of nuclear waste pellets with a sufficiently intense pseudo-proton flux resulting in a deficiency of the nuclear charge Z for a given atomic number A and ensuing decay.

Note that the above applications are possible thanks to the *mean-life of DNS synthesized particle of the order of seconds or fractions thereof,* which are quite large for particle standard.

By comparison, negatively charged strongly interacting particles synthesized in contemporary physics laboratory with mean-lives of the order of 10^{-30} s or so have no known industrial or medical applications.

2.7. Einstein's determinism in nuclear structures.

2.7.1. Historical notes. As it is well known, nuclei were first assumed to be bound states of protons and electrons under their mutual Coulomb attraction which force, being inversely proportional to the square of the distance, is extremely big at nuclear distances.

This assumption was soon dismissed because it was considered inconsistent with quantum mechanics and unable to represent experimental data.

Consequently, nuclei were assumed to be bound states of protons and neutrons under a strong nuclear force that remains unknown to this day.

In reality, nuclei provide additional, rather clear evidence on the "lack of completeness" of quantum mechanics beyond scientific doubt, due to the well known inability by quantum mechanics to achieve an exact representation of nuclear experimental data.

As an illustration, in about one century of efforts and the use of large public funds, quantum mechanics has been unable to achieve a representation of the characteristics of the simplest nucleus, the *deuteron,* with embarrassing deviations between the prediction of the theory and the experimental data for heavier nuclei such as the Zirconium [25] (see also K. R. Popper [15], J. Dunning-Davies [16], J. Horgan [17] and quoted literature):

1. Quantum mechanics has been unable to represent the spin $J = 1$ of the deuteron in its ground state $L = 0$. This is due to the fact that the sole stable quantum mechanical bound state of two particles with spin $1/2$ is the singlet state, in which case the spin of the deuteron should be $J = 0$. For the intent of preserving the validity of quantum mechanics in nuclear physics, the spin $J = 1$ of the deuteron is represented via the assumption of a *combination of excited orbital spaces,* $L = 0, 1, 2$ contrary to the evidence that isolated deuterons are in their ground state, with the consequential inability to represent the deuteron positive parity $P = (-1)^L - 0$, and other insufficiencies.

2. Quantum mechanics has been unable to achieve an exact representation of the deuteron magnetic moment, because the representation of about 1% of the experimental value is still missing despite all possible relativistic (as well as quark-inspired) corrections, with large deviations occurring for heavier nuclei.

3. Quantum mechanics has been unable to represent the stability of the deuteron, since neutrons are well know to be naturally unstable, thus mandating a quantitative representation of the mechanism turning the neutron into a permanently stable particle when bonded to a proton.

4. Quantum mechanics has been unable to represent the proton-neutron exchange in the deuteron structure.

5. Quantum mechanics has been unable to achieve an explicit and concrete representation of the attractive strong nuclear force bonding together the proton and the neutron. Due to the intent of preserving the exact validity of quantum mechanics in nuclear physics, with the consequential sole representation of strong interactions via a potential in a Hamiltonian, the representation of strong nuclear forces has been attempted by adding potentials in the Hamiltonian, up to fifty additive potentials (sic) without the achievement of the needed representation of experimental data.

In a variety of works outlined in these papers (see monograph [25] and

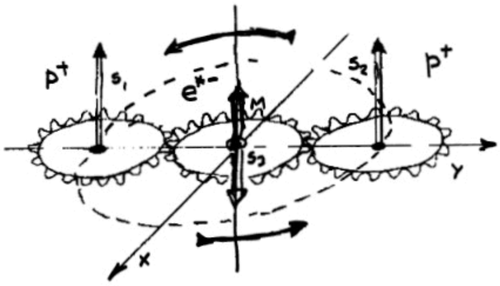

Figure 15: *The "gear model" used by Santilli [25], page 180, to illustrate the prediction from the spin $J_D = 1$ that the deuteron is a three-body hadronic bound state of one isoelectron and two isoprotons, with the individual couplings $p - e$ and $e - p$ being in singlet as necessary for stability. The spin $J_D = 1$ is then achieved from the "constrained angular momentum" of the bound state studied in Sections II-3.5.3 and II-3.5.4.*

papers quoted therein), Santilli has expressed the view that the "completion" of quantum into hadronic mechanics permits the apparent resolution of the above insufficiencies or sheer inconsistencies via a return to the original conception of nuclei as bound state of electrons and protons, although under non-linear, non-local and non-potential interactions caused by deep mutual overlapping.

2.7.2. Deuteron three-body model. Following the achievement of mathematical, theoretical, experimental and industrial advances in structure model (66) of the neutron as a hadronic bound state of one isoelectron and one isoproton, Santilli proposed in Section IV of Ref. [25], page 143 on (see also independent reviews [31] and [49]), the structure model of the deuteron D as a *restricted three-body hadronic bound state of two isoprotons and one isoelectron* (Figures 13, 14) which can be reduced in first approximation to a two-body hadronic bound state of one isoproton and one isoneutron

$$D = (\tilde{p}^+_\uparrow, \tilde{e}_\downarrow, \tilde{p}^+_\uparrow)_{hm} \approx (\tilde{p}^\uparrow, \tilde{n}^\uparrow)_{hm}. \quad (81)$$

Ref. [25] then presented various arguments showing that model (81) apparently resolves quantum mechanical insufficiencies (1 - 5) of the preceding section.

The analysis of model (81) was conducted in the 1999 monograph [25] following the publication in 1998 of the first proof of the EPR argument [9], as well as its detailed study in the 1995 monograph [33], Chapter 4, particularly Appendix 4C, page 166. Therefore, Santilli was aware that, unlike the case for atomic structures, deuteron model (81) does admit a classical counterpart (Section II-3.7) due to the inapplicability of Bell's inequality (9) [3] in favor of isoidentity (10).

However, at the time of writing the 1999 monograph [25], Santilli was unaware of the additional apparent proof of the progressive validity of Einstein's determinism in the interior of hadrons, nuclei and stars [10], and its apparent full achievement in the interior of gravitational collapse.

It is, therefore, important to review the main aspects of model (81) to indicate, apparently for the first time, its verification of Einstein's determinism we have called isodeterministic according to Lemma II-3.7 and Corollary II-3.7.1, and verify the apparent resolution of historical problems (1 - 5) of the preceding section.

1. NONRELATIVISTIC REPRESENTATION OF THE SPIN $J_D = 1$ IN THE GROUND STATE $L_D = 0$. With reference to Figure 24, model (81) includes *five* angular momenta: 1) The parallel spins S_1 and S_2 of the isoprotons; 2) The angular momentum L_{1-2} of the two isoprotons; 3) The spin of the isoelectron S_3; and 4) The angular momentum L_3 of the isoelectron inside one of the two isoprotons.

Recall that, according to experimental data on nuclear dimensions, the isoproton and the isoneutron are expected to have a diameter of $1.73\ fm$, while the diameter of the deuteron is $4,26\ fm$.

Consequently, the two isoprotons are separated by about $0.886\ fm$. In model (81), this space is occupied by the isoelectron acting like a "gluon" of the two isoprotons, while permitting their triplet alignment necessary for the spin $J_D = 1$.

In fact, the isoelectron is in the singlet coupling with each of the two isoprotons necessary for stability according to hadronic mechanics (Figure 14).

Recall also that, in semiclassical approximation, the wavepacket of the electron is of the order of $2.2\ fm$.

The above data provide not only a considerable synchronicity in model (81), but also a rather strong bond caused by the nuclear forces, the Coulomb attraction between the isoelectron and the isoprotons, as well as the strong attractive force originating from deep wave-overlapping in singlet coupling.

Recall finally that the diameter of the horizon for the full applicability of hadronic mechanics has been selected in Paper I to be $d = 2 \times 10^{-13}\ cm = 2 fm$, while the deuteron is about double that size.

The above data on dimensions suggest the use of the following isosymmetries for the characterization of the angular momentum and spin of the deuteron:

Use the regular isosymmetries $\hat{O}(3)$ and $\hat{SU}(2)$ for the characterization of the angular momentum and spin outside the hadronic horizon. We are here referring to isosymmetries which can be constructed via a non-unitary transform of the corresponding Lie symmetries, thus preserving conventional eigenvalues for angular momentum and spin (Section I-3.8 and I-3.9).

Use the irrreegular isosymmetries $\hat{O}(3)$ and $\hat{SU}(2)$ for the characterization of the angular momentum and spin inside the hadronic horizon. We are here referring to isosymmetries which cannot be constructed via non-unitary transforms o the corresponding Lie symmetries, by therefore having anomalous values of the angular momentum and spin (Section II-3.4).

It is then easy to see that the diameter $D = 2.59\ fm$ of the orbital motion of the two isoprotons is close to, but bigger than the hadronic horizon $d = 2\ fm$. We can therefore see the regular isorotational symmetry $\hat{O}3)$ admitting conventional angular momentum eigenvalues with ground state $L_{1-2} = 0$.

By contrast we have to use the irregular $\hat{O}(3)$ isosymmetry for the rotation of the isoelectron in the interior of the isoproton, resulting in the constrained value $L_3 = 1/2$ for the neutron (Sections 2.6.2 and 2.6.3).

In this way, Ref. [25] achieved the first known representation of the spin of the deuteron in its true ground state according to values:

$$\begin{aligned} J_D = S_1 + S_2 + S_3 + L_{1-2} + L_3 = \\ = \tfrac{1}{2} + \tfrac{1}{2} - \tfrac{1}{2} + 0 + \tfrac{1}{2} = 1. \end{aligned} \tag{82}$$

By comparing the hadronic structure model of the neutron, Figure 10, and that of the deuteron, Figure 15, it is evident that the isoelectron is entirely compressed inside the isoproton in the former case, but only partially compressed in the latter case.

This occurrence is expected due to the presence in model (81) of the second isoproton with ensuing strongly attractive Coulomb force responsible for the partial extraction of the isoelectron from the isoproton, although this possibility has not been investigated to date.

Similarly, structure (81) appears to be particularly suited for the representation of the proton-neutron exchange in the deuteron structure, although this possibility has also not been subjected to a quantitative study to our best knowledge.

2. RELATIVISTIC REPRESENTATION OF THE SPIN $J_D = 1$ IN THE GROUND STATE $L_D = 0$. Ref. [25] was written following the 1995 publication of *Elements* *of Hadronic Mechanics,* Vols. I [32] and II [33]. Therefore, Ref. [25] studied model (81) with the entire sue of said Volumes whose knowledge, particularly that reviewed in Section II-3, is herein tacitly assumed.

The study of model (81) was done in Ref. [25], Section IV-2.3.4, page 161, via the use of the irregular iso-Dirac equation (II-201) defined on iso-semi-product (II-200) of an iso-Minkowskian space $\hat{M}(\hat{x}, \hat{\eta}, \hat{I}_{orb})$ for the relativistic description of the hadronic orbital motion, multiplied by a three-dimensional complex-valued isounitary isospace for the characterization of the hadronic spin $\hat{E}(\hat{z}, \hat{\eta}, \hat{I}_{spin}$.

It should be recalled that, in principle, *all* conventional intrinsic characteristics of the proton and the electron, including their charges, are expected to be mutated when said particles are in conditions of total mutual penetration. These mutations are quantitatively represented by the Lorentz-Poincaré-Santilli isosymmetry of the irregular iso-Dirac equation (Section II-2.5.11) and its irregular Pauli-Santilli isomatrices (Section II-3.4).

However, Ref. [25], Section 2.3.5, page 162, has shown that, for the case of the deuteron (but not necessarily so for heavier nuclei), the spins and changes of the isoprotons and of the isoelectron can be assumed in first approximation to have conventional values. In this case, the sole mutations achieving the representation of experimental data of the deuteron are those for shapes and constrained angular momenta.

The first known relativistic representation of the spin $J_D = 1$ of the deuteron in its ground state $L_D - 0$ was achieved in Ref. [25], Section 3.5.6, thanks to constrained spin-orbit coupling inside the isoneutron of model (81).

The latter coupling has been reviewed at the non-relativistic level in Section II-3.5.3 and at the relativistic level in Section II-3.5.4, see in particular Eqs. (II-118), (II-119), with a detailed description available in Ref. [33], Chapter 6.

The resulting relativistic characterization of hadronic angular momenta and spins is given by

$$
\begin{gathered}
S_{k\alpha} = \epsilon_{kij} \hat{\gamma}_{i\alpha} \star \hat{\gamma}_{j\alpha}, \quad \alpha = \tilde{p}1, \tilde{p}2, \tilde{e}, \\
L_{k,p1-p2} = \epsilon_{kij} r_{i,p1-p2} \star p_{j,p1-p2},
\end{gathered}
\tag{83}
$$

where $\hat{\gamma}_k$ are the irregular Dirac-Santilli isomatrices.

We then have the irregular isocommutation rules Ref. [36], Eqs. (6.4),

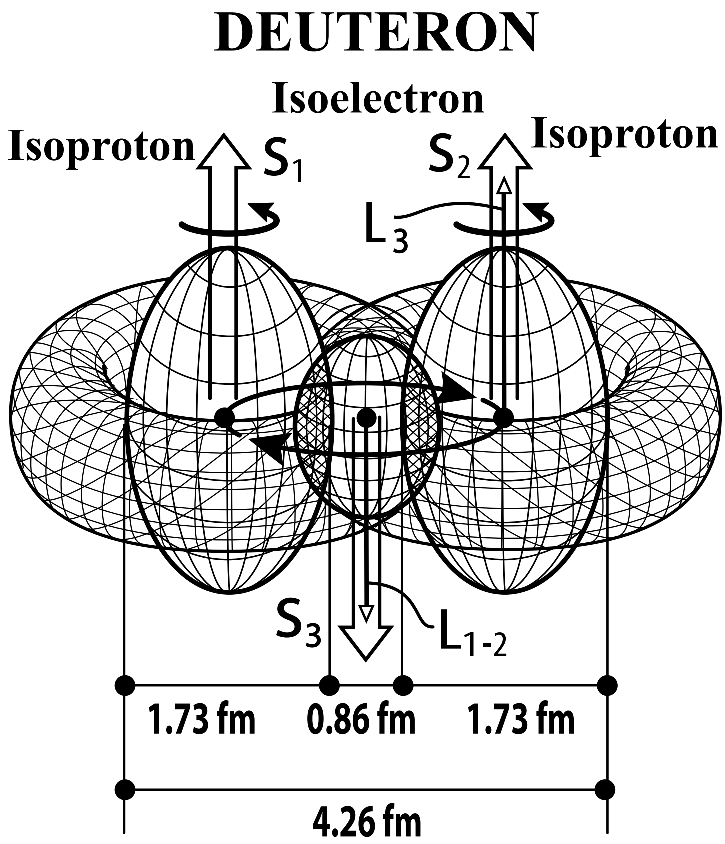

Figure 16: *A conceptual rendering of the deuteron model (91) illustrating the representation for the first time of spin $J_D = 1$ with angular momentum $L_D = 0$, where: the two isoprotons have the charge diameter $D_{\tilde{p}} = 1.73\ fm$; the deuteron has the charge diameter $D_D = 4.26\ fm$; the interspace between the isoprotons is then of $0.86\ fm$; the isoelectron has a point-like charge, the hadronic diameter $D^{hm}_{\tilde{e}} = 1\ fm$, and a semi-classical wavepacket with $D^{cl}_{\tilde{e}} = 2.2\ fm$. The above data illustrate the stability of model (81) with the central isoelectron allowing the isoprotons in the triplet coupling needed to achieve the spin $J_D = 1$ in the true ground state (Figure 14). The doughnuts around the two isoprotons are used to represent the proton-neutron exchange forces.*

page 190,

$$[S_{i,\alpha}\hat{;}S_{j\alpha}] = 2\epsilon_{kij} m_{k\alpha}^2 \hat{S}_{k\alpha},$$
$$[L_{i,1-2}\hat{;}L_{j,1-2}] = \epsilon_{ijk} m_{k,1-2}^2 L_{k,1-2}, \tag{84}$$

and isoeigenvalues, Eqs, [36], Eqs. (6.4d), page 190,

$$\hat{S}_{3\alpha} \star |\hat{\psi} >= \pm \tfrac{1}{m_{1\alpha} m_{2\alpha}} |\hat{\psi} >,$$

$$\hat{S}_{\alpha}^{\hat{2}} \star |\hat{\psi} >= (m_{1\alpha}^{-2} m_{2\alpha}^{-2} + m_{2\alpha}^{-2} m_{3\alpha}^{-2} + m_{3\alpha}^{-2} m_{1\alpha}^{-2}) |\hat{\psi} >,$$

$$\hat{L}_{3,1-2} \star |\hat{\psi} >= \pm m_{1,1-2} m_{2,1-2} |\hat{\psi} >,$$

$$\hat{L}_{\beta}^{\hat{2}} 1-2 \star |\hat{\psi} >= (m_{1,1-2}^2 m_{2,1-2}^2 + m_{2,1-2}^2 m_{3,1-2}^2 + m_{3,1-2}^2 m_{1,1-2}^2) |\hat{\psi} > . \tag{85}$$

Under the assumption that the hadronic medium in the interior of the proton is homogeneous, we can assume the values [36]

$$m_{1\alpha} = m_{1,1-2} = m_{2\alpha} = m_{2,1-2} = m_{3\alpha} = m_{3,1-2} = \frac{1}{\sqrt{2}} = 0.842, \tag{86}$$

under which the isoeigenvalues become

$$\hat{S}_{3\alpha} \star |\hat{\psi} >= \pm \tfrac{1}{2} |\hat{\psi} >,$$

$$\hat{S}_{\alpha}^{\hat{2}} \star |\hat{\psi} >= \tfrac{3}{4} |\hat{\psi} >,$$

$$\hat{L}_{3,1-2} \star |\hat{\psi} >= \pm \tfrac{1}{2} |\hat{\psi} >, \tag{87}$$

$$\hat{L}_{1-2}^{\hat{2}} \star |\hat{\psi} >= \tfrac{3}{4} |\hat{\psi} > .$$

It then follows that the total angular momentum of the isoneutron is given by

$$J_{\tilde{n}} = S_2 + S_{\tilde{e}}^{spin} + L_{\tilde{e}}^{orb} = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2}. \tag{88}$$

The total value of the deuteron spin $J_D = 1$ in its ground state $L_D = 0$ then follows from the fact indicated above that the orbital diameter is bigger than the hadronic horizon.

3. REPRESENTATION OF THE DEUTERON REST ENERGY, STABILITY AND SIZE. The hadronic representation of the rest energy, stability and charge radius of the deuteron were first achieved in Ref. [25], Eqs. (5.2.16), page 179, via the use of structural equations (29) (see also independent reviews [31], [49])

$$\left[\frac{1}{r^2}\left(\frac{d}{dr}r^2\frac{d}{dr}\right) + \bar{m}\left(E \pm W_1 e^{\frac{2}{r}} + W_2 \frac{e^{-br}}{1-e^{-br}}\right)\right] = 0,$$

$$E_{tot} = E_1 + E_2 - E = 1875.7\ MeV, \quad \bar{m} = \frac{m}{I^2} \tag{89}$$

$$\tau^{-1} = 2\pi\lambda^2 |\hat{\psi}(0)|^2 \frac{\alpha^2 E_1}{\hbar} = \infty,$$

$$R = b^{-1} = 2.13 \times 10^{-13}\ cm = 2.13\ fm,$$

with solutions for the k-parameters

$$k_1 = 1 \ \ k_2 = 2.5. \tag{90}$$

The total mass/rest energy of the deuteron is then given by

$$M_D = M_p + M_n - E = 1875.7\ MeV, \tag{91}$$

where the binding energy is given by $E = 2.2\ MeV$.

Value (90) should be compared to the corresponding values for the neutron, Eqs. (69). As one can see, values (90) verify conditions (35) on the validity of the isodeterminism of Lemma II-3.7 as well as with the increase of the rapid convergence of isopperturbative series of Corollary II-3.7.1.

Note that, according to our assumption, values (81) verify the crucial condition (35) for the lack of existence of excited states, because excited states would imply the existing of the deuteron constituents out of the hadronic horizon with its consequential disintegration.

Note also that the stability of the deuteron is intrinsic in model (81) since the deuteron is reduced to the only known, massive, permanently stable particles existing in the universe, the proton and the electron.

Needless to say, the above representation should be considered as a first approximation, with a number of improvements being possible such as the inclusion of Coulomb interactions with possible excited states within the hadronic horizon.

4. REPRESENTATION OF THE DEUTERON MAGNETIC MOMENT. As it is well known, the experimental value of the deuteron magnetic moment is given by [13] [14]

$$\mu_D^{exp} = 0.857\ \mu_N. \tag{92}$$

For the quantum mechanical ground state with $L_D = 0$, in case consistent, one would obtains

$$\mu_D^{qm} = \frac{1}{2}(g_p^S + g_n^S) = 2.792 - 1.913 = 0.879\ \mu_N, \tag{93}$$

(where the g's are tabulated g-factors), by therefore missing about 3% of experimental value (256), which deviation cannot be entirely resolved via relativistic or other corrections.

The first, exact, non-relativistic representation of the deuteron magnetic moment was achieved by Santilli for the approximate two-body version of model (81) in paper [50] of 1994 (written at the Joint Institute for Nuclear Research, Dubna, Russia) under the sole assumption that the charge distributions of protons and neutrons are deformed under strong nuclear forces, resulting in a corresponding mutation of their intrinsic magnetic moment as predicted by E. Fermi [13], J.M. Blatt and V. F. Weisskopf [14], and other founders of nuclear physics (see their recollection in Ref. [25], page 158-159, Section IV-2.3.1, under the title *The Historical Hypothesis*).

Santilli achieved the first, exact, relativistic representation of magnetic moment of the two-body deuteron model in paper [51] of 1996 via the use of the iso-Dirac equation.

The 1998 Ref. [25], Section IV-2.3.6, page 163, presents the first known exact representation of the deuteron magnetic moment for the full three-body model (81).

It is important to review and upgrade the latter representation to show its compatibility with Einstein's determinism according to Lemma II-3.7 and rapid convergence according to Corollary II-3.7.1.

Assume, in first approximation, that the isoparticles of model (81) have the same spheroidal shape with semiaxes $n_1^2,\ n_2^2,\ n_3^2$ under the condition of preserving the volume of the original particles assumed for simplicity to be normalized to 1

$$n_1^2 \times n_2^2 \times n_3^2 = 1. \tag{94}$$

Represents with n_4^2 the density of the two isoprotons (denoted with the subindeces 1 and 2), resulting in the isotopic element for the two isoprotons

$$\hat{T}_{1,2} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2). \tag{95}$$

Then, the Lorentz-Santilli isotransforms (II-42) imply the following mutation of the intrinsic magnetic moment of the proton (see Ref. [36], page 190 for a detailed derivation with the notation $b_\mu = 1/n_\mu,\ \mu = 1, 2, 3, 4$)

$$\begin{gathered} \mu_{1,2}^{qm} \ \rightarrow \ \tilde{\mu}_{1,2}^{hm} = \tfrac{n_4}{n_3}\mu_{1,2}^{qm} = \tilde{g}^S \mu_N, \\ \tilde{g} = \tfrac{n_4}{n_3} g. \end{gathered} \tag{96}$$

Recall that the isoprotons have Spin $S_{1,2} = 1/2$ and $L_D = 0$.

In order to compute the magnetic moment of the three-body model (81), we first compute *the intrinsic and orbital contributions of the isoprotons* via a simple isotopy of the corresponding quantum mechanical expression [45]

$$\tilde{\mu}_k^{hm} =$$

$$\tfrac{1}{J+1} < \hat{Y}_{L,S} | \tfrac{1}{2}\tilde{g}_k^S S + \tfrac{1}{2}\tilde{g}_k^L L) | \hat{Y}(L,S) >= \tag{97}$$

$$= \tfrac{1}{2(J+1)}\tilde{g}_k^S \left[J(J+1) - L(L+1) + S(S+1), \right], \ \ k = \tilde{p}1, \tilde{p}2,$$

where $\hat{Y}_{L,S}$ are the isospherical isoharmonics (Ref. [33], Section 6.4, page 240) and $J_k = S_k + L_k, \ k = 1,2$ is the total angular momentum.

Simple calculations show that, for $L_k = 0$ and $\mu_p^{exp} = 2.79 \ \mu_N$, expression (261) yields the value

$$\tilde{\mu}_k^{hm} =$$

$$= \tfrac{1}{2}\tilde{g}_k^s = \tfrac{n_4}{2n_3} g_k^S = \tfrac{n_4}{n_3} 2.79 \ \mu_N. \tag{98}$$

To compute *the intrinsic and orbital contribution from the isoelectron,* we recall from Eq. (69) that its intrinsic value is ignorable in first approximation, the sole expected contribution being that from its constrained orbital motion inside the proton.

By recalling that the electron has a point-like charge which cannot be mutated by isotopies, then the density of the electron remains the conventional one for the vacuum with $n_{4,\tilde{e}} = 1$.

Under the approximation that the deformation of the isoelectron is the same as those of the two isoprotons, we have the mutation of the value (70) for the neutron,

$$\mu_{\tilde{e},orb}^{hm} = -\frac{1}{n_3}\mu_{\tilde{e}}^{orb} = -\frac{1}{n_3} 4.6 \ \mu_N. \tag{99}$$

The quantum mechanical magnetic moment of the three-body model (81) of the deuteron, in case consistent, would be given by

$$\mu_D^{qm} = 2\mu_p + \mu_e = 2 \times 2.79 - 4.6 \ \mu_N = 0.98 \mu_N, \tag{100}$$

thus lacking an exact representation of value (92) because in *excess* for about 8%.

By comparison, the magnetic moment for model (81) according to hadronic mechanics is given by

$$\mu_D^{hm} = 2\frac{n_4}{n_3}\mu_{\tilde{p}} + \frac{1}{n_3}\mu_{\tilde{e}} = \frac{n_4}{n_3} 5.78 - \frac{1}{n_3} 4.6 \ \mu_N = 0.857 \ \mu_N. \tag{101}$$

By assuming the value $n_4 = 0.62$ from the density of the proton-antiproton fireball in the Bose-Einstein correlation [46] [47], we can write the *hadronic representation of the deuteron magnetic moment*

$$\mu_D = \frac{0.62 \times 5.78}{n_3} - \frac{4.6}{n_3} = 0.857\, \mu_N, \tag{102}$$

which is verified for

$$n_3 = 1.20. \tag{103}$$

The use of the volume-preserving condition (94) then yields the remaining ellipsoidal values

$$n_1 = n_2 = 0.91. \tag{104}$$

The above values imply that the shape deformation of the deuteron constituents caused by the internal strong nuclear force turns charge distributions and/or wavepackets from a spherical to a *prolate* spheroidal ellipsoid with semiaxes

$$n_1^2 = n_2^2 = 0.83, \;\; n_3^2 = 1.44, \tag{105}$$

which prolate deformation was predicted by all preceding works by Santilli in the field.

Note that *the magnetic moment of the deuteron is represented exactly not only in its value, but also in its sign,* that of being parallel to the spins of the isoprotons.

It then follows that *representation (101) can be construed as evidence on the presence of two protons with parallel spins inside the deuteron,* which presence mandates three-body model (81) with a central exchanged electron for stability (Figures 14, 15).

Note also that the *prolate* deformation of shape is necessary because the quantum mechanical expression (93) is in *excess* of experimental value (92). By comparison, an *oblate* deformation would *increase* said excess value due to the increase of the rotating charge distribution at the equator.

Almost needless to say, representation (101) is intended to illustrate the *capability* by hadronic mechanics to achieve an exact representation of the deuteron magnetic moment.

A number of improvement of representation (101) are possible, among which we mention a more accurate value of the density n_4 of the proton, a more accurate value of the orbital contribution of the isoelectron due to its geometric difference with that for the neutron indicated above, and other improvements.

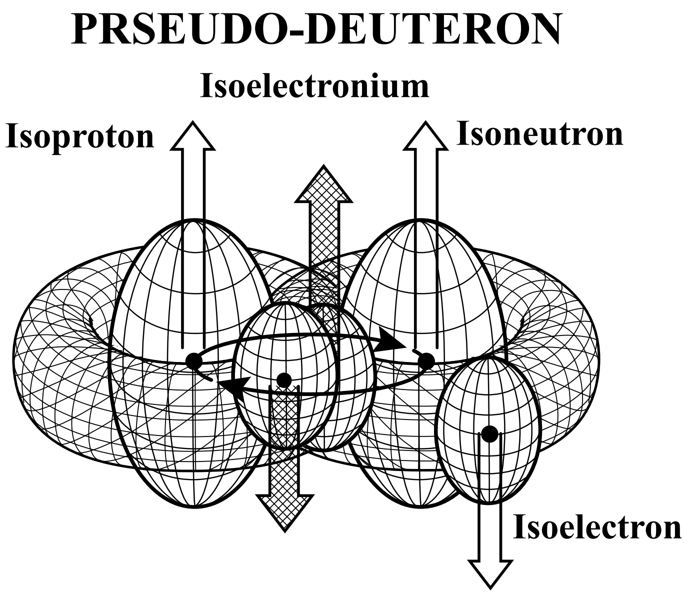

Figure 17: *An illustration of the negatively charged pseudo-deuteron predicted by hadronic mechanics via the use of a deuterium gas in the Directional Neutron Source of Figure 8. Due to its negative charge, the pseudo-deuteron can potentially initiate a basically new type of nuclear fusions, here called hyperfusions, based on the natural attraction of the pseudo-deuteron by natural deuteron resulting in the helium without production of harmful radiations or release of radioactive waste.*

5. REPRESENTATION OF NUCLEAR FORCES. Hadronic structure model (81) achieves the first known explicit concrete representation of strong nuclear forces thanks to the structural lifting of the mathematics underlying quantum mechanics into the covering isomathematics.

Such a generalization resulted to be necessary due to the non-Hamiltonian character of strong interactions, with particular reference to the generalization of the associative product into isoproduct (1) and the representation of strong nuclear forces via isotopic element (4), with ensuing lifting of the Schrödinger equation, from its historical form $H\psi = E\psi$, into the isotopic form

$$H \star \psi = H(r,p)\hat{T}(\psi, ...)\hat{\psi} = E\hat{\psi}. \tag{106}$$

6. REDUCTION OF STABLE NUCLEI TO ISOELECTRONS AND ISOPROTONS. The reduction of neutrons to isoprotons and isoelectrons clearly implies the possible reduction of all stable matter in the universe to the only known, massive and stable particles, protons and electrons [52].

This important possibility has been confirmed by the reduction of the deuteron to isoprotons and isoelectrons [25], and it is under study by A. A. Bhalekar and R. M. Santilli [54].

2.7.3. Industrial applications. In Section I-1.4 we have indicated the societal duty of seeking basically *new* forms of clean energies, while continuing research along conventional lines.

This is due to the inability of identifying industrially viable nuclear fusions in about three quarters of a century and the investment of billions of dollars of public funds.

Recall that the primary obstacle opposing the achievement of the controlled nuclear fusion of two deuterons $D(1,2,1)$ into the Helium $He(2.4.0)$ is the *extreme repulsive Coulomb force between nuclei* that, the distance of $1\ fm = 10^{-15}\ m$ needed to activate strong nuclear forces is given by

$$\begin{aligned} F &= k\tfrac{e^2}{r^2} = \\ &= (8.99 \times 10^9)\tfrac{(1.60\times 10^{-19})^2}{(10^{-15})^2} = 230\ N, \end{aligned} \tag{107}$$

which force is extremely large for nuclear standards.

As it is well known, the efforts done to date for overcoming such a large repulsive force have been the use of high energies resulting, as expected, in uncontrollable instabilities at the initiation of the fusion itself.

Another problem that has prevented the achievement of the controlled

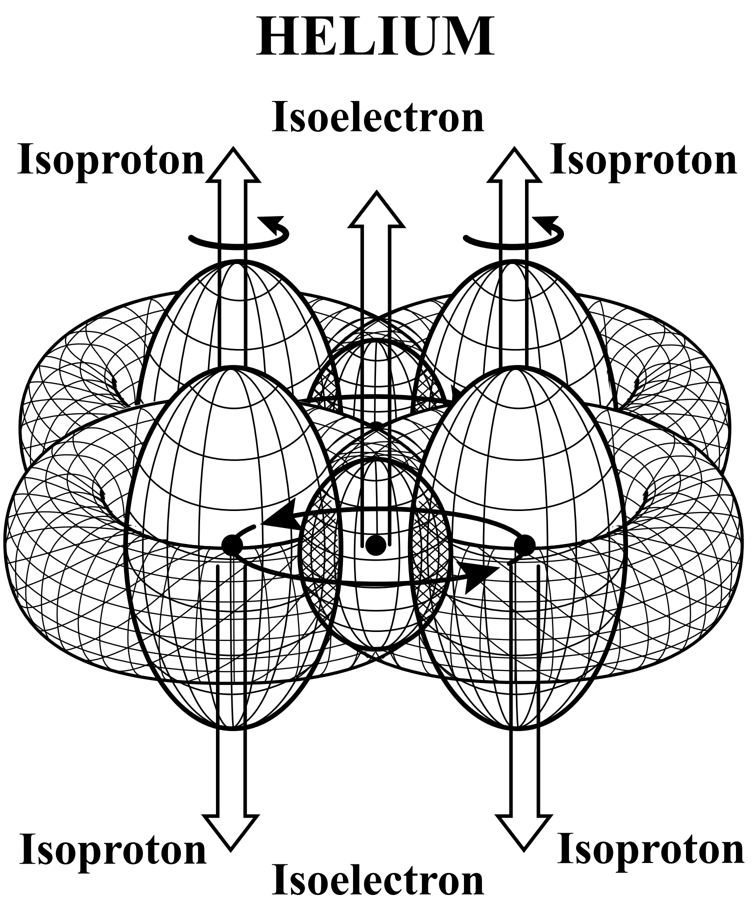

Figure 18: *An illustration of the helium nucleus as predicted by hadronic mechanics and consisting of four isoprotons and two isoelectrons. Note that the illustration depicts the conventional conception of the Helium as being composed by two protons and two neutrons, with the sole replacement of the neutrons with their physical constituents permitted by hadronic mechanics, the proton and the electron. Note finally that the uncertainties of the constituents of the helium follow Einstein's determinism per Lemma II-3.7 and Corollary II-3.7.1.*

fusion of deuterons into helium is the need for their *singlet coupling*

$$D(1,2,1_{\downarrow}) + D(1,2,1_{\uparrow}) \rightarrow He(2,4,0), \tag{108}$$

which is of extremely difficult engineering realization at low energies, and virtually impossible at high energies, resulting in nuclear fusions that can at best be at random.

For additional technical requirements needed to achieve the fusion of deuterons into helium, interested readers may consult the *hadronic laws for nuclear fusions,* which have been systematically studied in Ref. [25], Chapter 4 and Section 4.2, page 188, in particular.

The Directional Neutron Source (DNS, Figure 8) is produced and sold by Thunder Energies Corporation, now called Hadronic Technologies Corporation, for the synthesis of neutrons (Section 2.6.3) and pseudo-protons (Section 2.6.5) from a commercially available hydrogen gas.

However, the same DNS is produced for the use, without any modification, of a commercially available *deuterium gas* as a basic feedstock, in which case, hadronic mechanics predicts the synthesis of a *negatively charged deuteron* called *pseudo-deuteron* [48] (patent pending) which is predicted to have a structure of the type (Figure 17)

$$\hat{D} = \left[\tilde{p}_{\uparrow}, (\tilde{e}^{-}_{\uparrow}, \tilde{e}^{-}_{\downarrow}), \tilde{n}_{\uparrow})\right]_{hm}. \tag{109}$$

The mechanism for the synthesis of the pseudo-deuteron (which we write with nuclear symbols $D(-1,2,1)$) from a deuterium gas stems from the main feature of the DNS, that of actually "compressing" electrons in the interior hadrons (Figure 9) thanks to a specially conceived electric arc and other engineering features. Consequently, pseudo-deuteria are expected to be synthesized in the DNF operating on deuterium gas, following molecular separation with the ensuing presence of valence electron pairs called *isoelectronia* (Section 2.8.2 and Figure 20), of course, in a way dependent on power, pressure, flow, and other factors.

Preliminary calculations via hadronic mechanics indicate that the pseudo-deuteron should have a mean-life of the order of a second, thus being sufficient for industrial applications.

The importance of the research under consideration is that, contrary to current trends, *pseudo-deuterons are attracted, rather than being repelled by natural deuterons.*

This basic feature provides plausible means to search for basically *new* nuclear fusions called *hyper-fusions,* here referred to fusions of natural, positively charged nuclei and synthesized negatively charged nuclei, such as the hyper-fusion of a pseudo-deuteron and a deuteron into the helium

$$\hat{D}(-1,2,1_{\uparrow}) + D(1,2,1_{\downarrow}) \ \rightarrow \ He(2,4,0) + 2e^{-}, \tag{110}$$

where the singlet coupling is naturally achieved because the pseudo-deuteron has not only the charge, but also the magnetic moment opposite that of the natural deuteron.

The energy expected to be released by each hyperfusion (110) is given by

$$\Delta E = E_{He} - (E_{\hat{D}} + E_D) == 23.8\ MeV = 3.81 \times 10^{-12}\ J, \tag{111}$$

and occurs *without any emission of harmful radiation (since electrons can be stopped with a metal shield), and without the release of radioactive waste.*

As an illustration, the possible achievement of the rather realistic number of 10^{18} controlled fusions (110) per hour would yield the significant release of about $10^6\ J$ of clean energy per hour without harmful radiations or waste.

In regard to the possible achievement of controlled hyper-fusions, we should mention that the DNS is considered in this section for the mere intent of achieving experimental measurements on the sole *existence* of pseudo-deuterons. Their production in the needed number and energy, and the hadronic reactor needed for the utilization of the produced heat, evidently require specialized equipment under proper funding.

By recalling that conventional nuclear fusions have requested billions of dollars of investments of public funds without any industrially viable result to date, readers should be warned against any expectation prior to the investment of equally large funds

2.8. Einstein's determinism in molecular structures.

2.8.1. Insufficiencies of quantum chemistry. There is no doubt that quantum chemistry has permitted truly historical advances in the 20th century. However, as it is the case for nuclear physics, serious scholars are expected to admit that quantum chemistry cannot be *exactly valid* for chemical structures and processes because of a number of insufficiencies, such as [30] (for dissident views, see also Refs. [15] [16] [17]):

1. Even though possessing excellent practical values, *the quantum chemical notion of valence is a 'nomenclature' without quantitative treatment because, due to their equal charge, the Schrödinger equation predicts that valence electron pairs 'repel' (rather than attract) each other,* due to the necessary sign + of the Coulomb potential in the equation for the electron pair

$$i\frac{\partial}{\partial t}\psi(t,r) = \left[-\frac{\hbar^2}{m}\Delta_r + \frac{e^2}{r}\right]\psi(t,r). \tag{112}$$

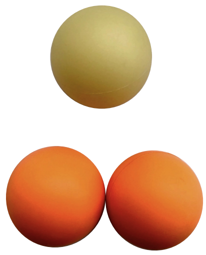

Figure 19: *Lectures in hadronic chemistry are generally initiated by showing a ball in representation of a hydrogen atom with the recollection that quantum mechanics achieved an exact representation of all its experimental data. The speaker then shows two joined balls representing a hydrogen molecule, with the recollection that, this time, quantum mechanics and chemistry have not achieved an exact representation of molecular data. The reason indicated for this dichotomy is that mutual distances in the atomic structure, that are of the order of* 10^{-8} cm*, are such to allow an effective point-like approximation of the constituents. By contrast, mutual distances for valence electron pairs in molecular structures, which are of the order of* 10^{-13} cm*, are smaller than the size of the electron wavepackets, which is of about* 2.2×10^{-13} cm*, thus prohibiting an effective point-like approximation of particles in favor of the representation of their actual size and as well as of the ensuing contact non-potential interactions due to deep wave overlapping.*

In particular, the *repulsive* force between the two identical valence electrons has the extremely big value of $230\ N$ at mutual distances of $1\ fm$, Eq. (107), with repulsive values remaining big at atomic distances. Consequently, *quantum chemistry misses a quantitative model of molecular structures.*

2. According to quantum chemistry (see, e.g., Ref. [55]), valence electron bonds between two atoms are created by the overlap of atomic orbitals. Even though approximately valid, such a model is "incomplete" because, due to the point-like approximation of the electrons, said orbitals remain mostly independent, thus allowing their polarizations under a sufficiently strong electric or magnetic field, with the consequential prediction that substances are generally ferromagnetic.

3. Quantum mechanics has achieved an exact representation of all experimental data of the *hydrogen atom* H, but in the transition to the *hydrogen molecule* $H_2 = H - H$ (where $-$ represents valence bond), quantum mechanics and chemistry still miss $1\ \%$ of the H_2 binding energy which corresponds to the rather significant value of $950\ BTU$.

4. Quantum chemistry still misses to this day the quantitative identi-

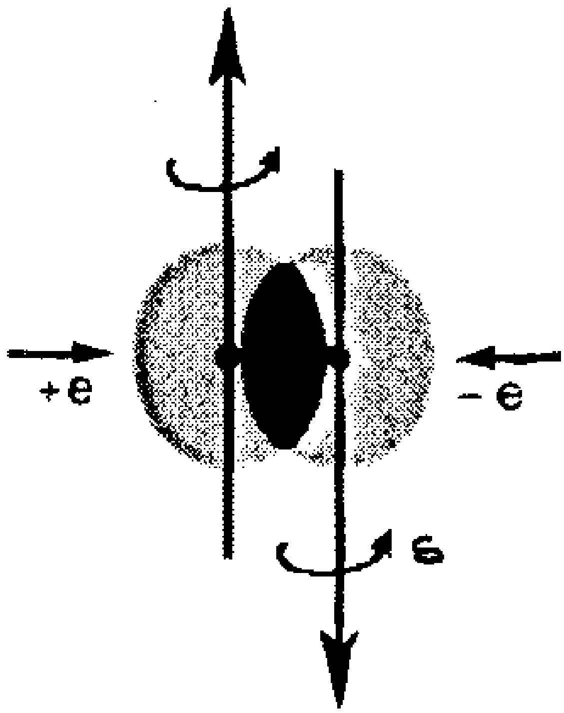

Figure 20: *According to the basic principles of quantum mechanics and chemistry, electron valence pairs in molecular structures should "repel" (rather than "attract") each other with a force of the order of* 230 N*, Eq. (107), which is extremely big for particle standards. Hadronic mathematics, mechanics and chemistry have achieved an attractive force between valence electron pairs so strong to turn them into a quasi-stable particle called the isoelectronium [30], that allowed the first known achievement of exact representation of experimental data on the hydrogen [52] and water [53] molecules that are not possible with quantum chemistry. In Santilli's view, the impossibility in quantum chemistry to achieve an attractive force between valence electron pairs as occurring in nature, is additional evidence supporting the need for the "completion" of quantum principles into covering vistas according to Einstein's legacy.*

fication of the *attractive force* bonding together neutral, dielectric and diamagnetic molecules in their liquid state, with ensuing lack of quantitative representation of the liquid state for fuels.

5. The basic axioms of quantum chemistry are reversible over time. Consequently, quantum chemistry is structurally unable to provide an exact representation of combustion, as well as all energy-releasing processes with ensuing implications for the solution of problems of large societal values, such as the improvement of fossil fuel combustion.

We cannot possibly review in this paper the ongoing process toward the solution of the above insufficiencies of quantum chemistry via the covering hadronic chemistry [30]. Nevertheless, we believe it is important to indicate the main evidence supporting the validity for molecular structures of the "completion" of quantum chemistry into a suitable covering theory according to Einstein-Podolsky-Rosen argument [1].

Insufficiencies (1 - 5) above are known to chemists, as attested by the widespread distribution of works in the field by Santilli and other scholars. For this reason, said insufficiencies are referred to as *the best kept secrets in the best Ph. D. courses in chemistry around the world.*

2.8.2. Attractive force in valence electron bonds. Recall that in the 1978 paper [19], Santilli: 1) Identified the non-potential, strongly attractive force created by the deep overlapping of the wavepackets of particles in singlet coupling; 2) Constructed the foundations of hadronic mechanics with particular reference to the iso-Schrödinger equation for quantitative treatments of non-potential interactions; and 3) Applied the emerging new methods to achieve an exact representation of all characteristics of mesons as bound states of electrons and positrons (Sections 2.5.3 to 2.5.5).

In the 1995 paper [95], A. O. E. Animalu and R. M. Santilli showed that the indicated non-potential force is essentially charge independent since it remains strongly attractive also for electron-electron pairs in superconductivity thanks to the "absorption" of the Coulomb potential by the Hulten potential irrespective of its sign, Eq. (5.1.15), page 836, Ref. [19] (reviewed in Eq. (31) above).

Thanks to the "completion" of quantum into hadronic chemistry (hc), in the 2001 monograph [30] Santilli worked out a new notion of valence bonds, today known as *strong valence bond,* which consists of valence isoelectron pairs, known as *isoelectronium* and denoted $\mathcal{I}$, with a fully identified *attractive* force with the structure (see the independent reviews [31]

[96] [97])

$$\mathcal{I} = (\tilde{e}_{\uparrow}, \tilde{e}_{\downarrow})_{hc}. \tag{113}$$

The achievement of the attractive force between the identical valence electrons, apparently done for the first time in Ref. [30], Chapter 4, is described in detail in Section 2.3, and it is essentially based on the "completion" of all products into isoproducts (1) with consequential "completion" of the local Newton's differential calculus into the non-local isodifferential calculus (7) (8), resulting in the iso-Schrödinger equation

$$\begin{aligned} i\frac{\partial}{\partial t}\psi(t,r) = \left[-\frac{\hbar^2}{m}\Delta_r + \frac{e^2}{r}\right]\psi(t,r) \ \rightarrow \\ \rightarrow i\frac{\hat{\partial}}{\hat{\partial} t}\psi(t,r) = \left[-\frac{\hbar^2}{m}\hat{\Delta}_r + \frac{e^2}{r}\right]\hat{T}(\psi,\hat{\psi},...)\hat{\psi}(t,r) \approx \\ \approx i\frac{\hat{\partial}}{\hat{\partial} t}\hat{\psi}(t,r) = \left[-\frac{\hbar^2}{m}\hat{\Delta}_r - W\frac{e^{-br}}{1-e^{-br}}\right]\hat{T}(\psi,\hat{\psi},...)\hat{\psi}(t,r), \end{aligned} \tag{114}$$

with a strong attraction characterized by the Hulten potential.

In Santilli's view, *the achievement of an attractive force in valence electron pairs via the "completion" of the quantum wavefunction $\psi(t,r)$ into the hadronic isowavefunction $\hat{\psi}(t,r)$ is a strong confirmation of the final statement by Einstein, Podolski and Rosen [1] according to which quantum wavefunctions do not represent the entire physical or chemical reality.*

The strength of the internal bond allows the reduction of four-body molecules, such as the hydrogen $H_2 = H - H$, down to a *restricted three-body system* that, as such, admit full analytic solutions with major simplification of otherwise notoriously complex elaborations.

It should be noted that, besides the existence of molecules, the biggest evidence on the existence of the isoelectronium as a particle is provided by experimental data on the photoionization of the hydrogen and helium molecules (see, e.g., Ref. [56]) in which bonded electron pairs in singlet have been systematically detected to survive molecular separation.

For comparison with preceding hadronic structures, let us recall that Ref. [30], Eqs. (4.5) to (4.23), pages 169 to 173, provided the following representation via model (29) of the rest energy $E = 1\ MeV$ and change radius $R = 6.84 \times 10^{-11}\ cm$ of the isoelectronium for the case of full stability

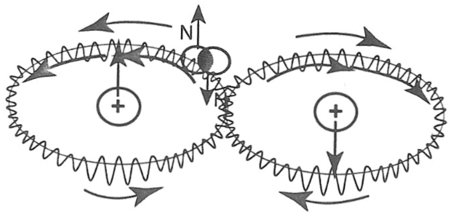

Figure 21: *An illustration of the isochemical model of the hydrogen molecule achieved by R. M. Santilli and D. D. Shillady [52] with the following main features: 1) Valence electron pairs are bonded into isoelectronia according to the principles of hadronic chemistry; 2) Due to their strong internal forces, isoelectronia are forced to have oo orbits around corresponding nuclei resulting in opposite angular momenta that represent the diamagnetic character of the hydrogen and other molecules; 3) The conventional four-body equations for the hydrogen molecule are reduced to a restricted three-body form admitting a full analytic solution.*

$\tau = \infty$

$$\left[\tfrac{1}{r^2}\left(\tfrac{d}{dr}r^2\tfrac{d}{dr}\right) + \bar{m}\left(E \pm W_1 e^{\frac{2}{r}} + W_2 \tfrac{e^{-br}}{1-e^{-br}}\right)\right] = 0,$$

$$E_{tot} = E_1 + E_2 - E = 1\ MeV, \ \ \bar{m} = \tfrac{m}{I^2} \tag{115}$$

$$\tau^{-1} = 2\pi\lambda^2|\hat{\psi}(0)|^2 \tfrac{\alpha^2 E_1}{\hbar} = \infty,$$

$$R = b^{-1} = 6.84 \times 10^{-11}\ cm,$$

with solutions for the k-parameters

$$k_1 = 0.19 \ \ k_2 = 1, \tag{116}$$

for which the binding energy is null (in first approximation) as expected from the sole use of contact non-potential interactions, resulting in the total energy of the isoelectronium $E = E_1 + E_2 = 1.022\ MeV$.

Note that, as expected from the charge independence of contact interactions, values (114) for the isoelectronium $(\tilde{e}_1, \tilde{e}_2)_{hc}$ are rather close to the corresponding values (50) for f the π^0 meson $(\tilde{e}^-, \tilde{e}^+)_{hm}$.

Interested readers should meditate a moment on the capability of contact interactions to achieve a strongly attractive force by overcoming the extremely big repulsive Coulomb force of 230 N.

This occurrence emphasizes the need in hadronic structure models such as (112) of the notion of *isoparticles* in which *all* intrinsic characteristics are mutated, thus including the charge.

This occurrence also suggests the *study of structure models of the elementary charge,* such as that of Ref. [44], capable of such a mutation in condition of deep mutual overlapping to represent the physical evidence of a strong attraction in singlet valence couplings.

In 1999, D. D. Shillady and R. M. Santilli proved that the isotopic branch of hadronic chemistry achieves the first known, numerically exact representation of the binding energy and other characteristics of the hydrogen [52] (Figure 23) and water [53] (Figure 24) molecules via isoperturbative isoseries (namely isoseries based on isoproduct $A\hat{T}B$ with $\hat{T} \ll 1$) whose convergence is at least *one thousand times faster* than the convergence of conventional quantum chemical series, thus providing an important experimental confirmation in molecular structures of both Einstein's determinism per Lemma II-3.7 as well as Corollary II-3.7.1.

In regard to the rather complex case in which the isoelectronium is partially unstable, the numerically exact representation of H_2 and H_2O data, and other aspects, we refer the interested reader to the above quoted literature.

The strong valence bond of isoelectronia evidently resolves Insufficiencies 1) and 2) of the preceding section.

Insufficiency 3) is resolved by the fact that, due to the strength of its bond, isoelectronium (112) is forced to have an oo-orbit around the respective nuclei (Figure 22) with ensuing opposite angular momenta and opposite magnetic polarities that permit a quantitative representation of the diamagnetic character of the hydrogen, water and other molecules.

The apparent resolution of Insufficiency 4) was provided by the new chemical species of *magnecules* [30] (see the U. S. patent [57], the latest experimental verification [58] and Figure 25) that allowed the identification of an actual attractive force between water molecules in their liquid state [98].

The resolution of Insufficiency 5) can be studied via the *Lie-admissible branch of hadronic chemistry,* also called *genochemistry* [30], although it does not appear that, at this writing, a consistent representation of energy releasing processes is of interest to contemporary chemists at large.

2.8.3. Industrial applications. It is a truism to state that the combustion of fossil fuels occurring in the ongoing disproportionate number of civilian, industrial and military vehicles is the same as it was at the dawn of our civilization some $50,000$ years ago because, in all cases, we strike a spark and lit the fuel.

To be "True Researchers" in Einstein's words, it is our societal duty to

Species	H_2	H_2O	HF
SCF-energy (DH) (a.u.)	−1.132800[a]	−76.051524	−100.057186
Hartree-Fock[d] (a.u.)			−100.07185[d]
Iso-energy (a.u.)	−1.174441[c]	−76.398229[c]	−100.459500[c]
Horizon R_c (Å)	0.00671	0.00038	0.00030
QMC energy[d,e] (a.u.)	−1.17447	−76.430020[e]	−100.44296[d]
Exact non-rel. (a.u.)	−1.174474[f]		−100.4595[d]
Corellation (%)	99.9[b]	91.6[b]	103.8
SCF-dipole (D)	0.0	1.996828	1.946698
Iso-dipole (D)	0.0	1.847437	1.841378
Exp. dipole (D)	0.0	1.85[g]	1.82[g]
Time[h] (min:s)	0:15.49	10:08.31	6:28.48

(DH^+) Dunning-Huzinaga (10S/6P), [6,2,1,1,1/4,1,1]+H_2P_1+3D1.
[a]LEAO-6G1S + optimized GLO-2S and GLO-2P.
[b]Relative to the basis set used here, not quite HF-limit.
[c]Iso-energy calibrated to give exact energy for HF.
[d]Hartree-Fock and QMC energies from Luchow and Anderson [33].
[e]QMC energies from Hammond *et al.* [30].
[f]First 7 sig. fig. from Kolos and Wolniewicz [34].
[g]Data from Chemical Rubber Handbook, 61st ed., p. E60.
[h]Run times on an O2 Silicon Graphics workstation (100 MFLOPS max.).

Figure 22: *A reproduction of Table 4.2 of Ref. [52] presenting the first known exact representation of the experimental data of the hydrogen molecule via isomathematics and isochemistry, by providing an experimental confirmation of the validity in molecular structres of Einstein's determinism according Lemma II-3.7, with a convergence of the isoperturbative series at least one thousand times faster than conventional series, by therefore providing an experimental confirmation of Corollary II-3.7.1.*

initiate the laborious process of trial and errors in seeking a clean combustion of fossil fuels that, to be as such, as to be *new,* namely based on new mathematical, physical and chemical principles.

In the hope of initiating the expectedly long and laborious process of trials and errors, Santilli has submitted in Ref. [59] the study of the new form of combustion of carbon and oxygen, known as *hypercombustion,* intended to achieve the full combustion of fossil fuels via a small percentage of nuclear fusions of the novel magnecular forms of $6-C-12$ and $8-O-16$ into the stable $14-Si-28$, thus without the emission of harmful radiations and without the release of radioactive waste [69] [70] (Figure 24).

We should mention in this respect the gaseous fuel *magnegas* [30] which is synthesized by hadronic reactors from a mixture of oil and water, in production and sale world wide by Magnegas Corporation, now called Taronis Corporation, whose combustion in air shows no appreciable carbon monoxide CO and hydrocarbons (HC) (Figure 25).

This environmental result was achieved via a combustion temperature more than double that of commercially available fuels, which temperature is permitted by magnecular bonds that are *weaker* than molecular bonds, and at which temperature CO and HC cannot remain unburned.

HyerCombustion aims at achieving the needed higher combustion temperature of petroleum fuels via the use of the indicated nuclear processes that are solely possible under the"completion" of quantum into hadronic mechanics.

It appears advisable to indicate that, besides the expected environmental advances, the new chemical species of magnecules appears to have significant medical applications, such as the possible killing of the corona virus in lungs (and other organs) of patients via ventilators releasing the new polarized species of *magneoxygen,* rather than conventional oxygen [62] - [65].

2.9. Einstein's determinism in gravitational collapse.

In the recent paper [10], Santilli recalls that iso-space-time metrics contain as particular cases all possible symmetric metrics in $(3+1)$-dimensions, thus including the Riemannian metric.

Ref. [10] then factorizes the space component of the Schwartzchild metric $g_s(r)$ according to isotopic rule introduced in Refs. [66] [67]

$$g_s(r) = \hat{T}(r)\delta, \tag{117}$$

where δ is the Euclidean metric.

In this way, Santilli reaches the following realization of the isotopic

	OH^+	OH^-	H_2O	HF
SCF-Energy[a]	-74.860377	-75.396624	-76.058000	-100.060379
Hartree-Fock[b]				-100.07185[b]
Iso-Energy[c]	-75.056678	-75.554299	-76.388340	-100.448029
Horizon R_c (Å)	0.00038	0.00038	0.00038	0.00030
QMC Energy[b,d]	-76.430020[d]			-100.44296[b]
Exact non-rel.				-100.4595
Iso-Dipole (D)	5.552581	8.638473	1.847437	1.8413778
Exper. Dipole			1.84	1.82

[a] Dunning-Huzinaga (10s/6p), (6,2,1,1,1/4,1,1)+H2s1+H2p1+3d1.
[b] Iso-Energy calibrated to give maximum correlation for HF.
[c] Hartree-Fock and QMC energies from Luchow and Anderson [22].
[d] QMC energies from Hammond, Lester and Reynolds [21].

Figure 23: *A reproduction of Table 5.1. of Ref. [53] presenting the first exact representation of the experimental data of the water molecule, including its electric and magnetic moments, via isomathematics and isochemistry, by therefore providing a second experimental confirmation of the validity in molecular structures of Einstein's determinism according to Lemma II-3.7, with a convergence of the isoperturbative series at least one thousand time faster than conventional series, thus providing an experimental confirmation of Corollary II-3.7.1.*

element

$$\hat{T} = \frac{1}{1 - \frac{2M}{r}} = \frac{r}{r - 2M}, \tag{118}$$

where M is the gravitational mass of the body considered with ensuing isodeterministic isoprinciple, Ref. [10], Eq. (46), page 16,

$$\Delta\hat{r}\Delta\hat{p} \approx \hat{T} = \frac{r}{r - 2M} \Rightarrow_{r\to 0} = 0, \tag{119}$$

which confirms the possible recovering of full classical determinism in the interior of gravitational collapse essentially as predicted by Einstein (see Ref. [16], Chapter 6 in particular, for a critical analysis of black holes).

It should perhaps be indicated that the 1993 paper [100] identified the universal isosymmetry of all possible (non-singular) Riemannian line elements in $3 + 1$)-dimensions formulated on iso-Minkowskian isospaces [78] over isofields. Papers [66] [67] introduced the factorization of a full Riemannian metric $g(x),\ x = (r, t)$ in $(3 + 1)$-dimensions

$$g(x) = \hat{T}_{gr}(x)\eta, \tag{120}$$

where $\hat{T}_{gr}$ is the *gravitational isotopic element*, and η is the Minkowski metric $\eta = Diag.(1, 1, 1, -1)$.

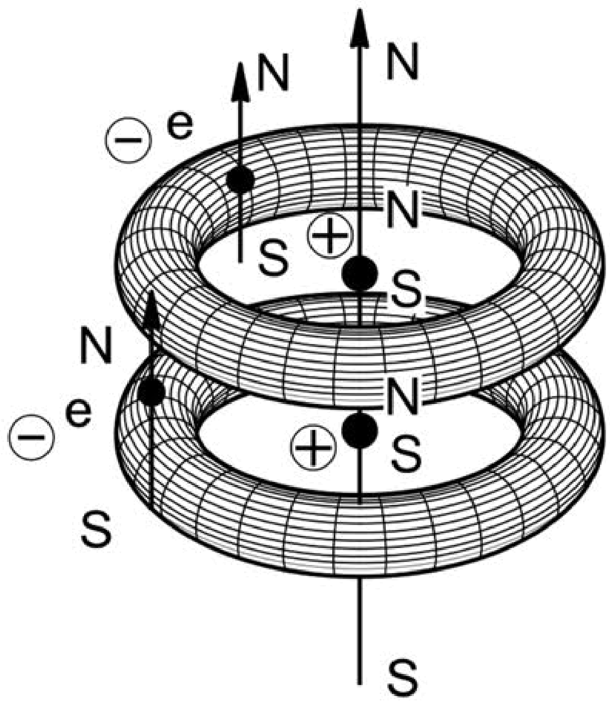

Figure 24: *In this figure, we present a conceptual rendering of the new chemical species of Santilli magnecules [30] for the case of the elementary hydrogen magnecules* $\tilde{H}_2 = H \times H$ *(where* $\times$ *denotes magnecular bond). The species is obtained via the use of special electric arcs producing electric and magnetic fields over certain minimum values [**?**], which arcs ionize the hydrogen molecule and polarize the electron orbits into toroids by therefore creating a new magnetic dipole moment which does not exist in the natural state. Polarized atoms bond together in the configuration of the figure which is stable at ambient temperatures by creating the new species of magnehydrogen (see Refs. [62] to [65]). It should be indicated that elementary magnecules between other selected atoms verify all hadronic laws for controlled nuclear fusions ([25], Section 4), for instance, of* $\tilde{CO} = 6 - C - 12 \times 8 - O - 16$ *into* $14 - Si - 28$ *without the emission of harmful radiations and without the release of radioactive waste [69] [70].*

Refs. [66] [67] then reformulated the Riemannian geometry via the transition from a formulation over the field of real numbers $\mathcal{R}$ to that over the isofield of isoreal isonumbers $\hat{\mathcal{R}}$ where the *gravitational isounit* is evidently given by

$$\hat{I}_{gr}(x) = 1/\hat{T}_{gr}(x). \tag{121}$$

The above reformulation turns the Riemannian geometry into a new geometry called iso-Minkowskian isogeometry [78] which is locally isomorphic to the *Minkowskian* geometry, while maintaining the mathematical machinery of the Riemannian geometry (covariant derivative, connection, geodesics, etc.) although reformulated in terms of the isodifferential isocalculus.

The following advantages should be mentioned for the *identical* iso-Minkowskian reformulation of general relativity, including Einstein's field equations, known as *isogravitation* [83]:

1) The achievement of a consistent operator form of gravity via the axiom-preserving embedding of the gravitational isounit $\hat{I}_{gr}(x)$ in the *unit* of relativistic quantum mechanics [33] [21];

2) The achievement of the universal *LPS isosymmetry* of *all* non-singular Riemannian metrics [100], which symmetry is locally isomorphic to the LP symmetry, (Section II-2), while being notoriously impossible in a conventional Riemannian space over the reals;

3) The achievement of clear compatibility, actually a *true isounification,* of general and special relativity since the latter can be identically recovered with the simple limit

$$\hat{I}_{gr} \rightarrow I = Diag.(1,1,1,1), \tag{122}$$

implying the transition from the universal LPS isosymmetry to the LP symmetry of special relativity with ensuing recovering of conservation and other special relativity laws;

4) The possibility of initiating systematic studies on *interior gravitational problems* along the forgotten Schwartzchild's second paper [73];

5) The achievement of axiomatic compatibility between gravitation and electroweak interactions thanks to the replacement of curvature into the covering notion of *isoflatness,* while offering realistic hopes to achieve a grand unification [38]; and other intriguing advances.

3. CONCLUDING REMARKS.

At the end of their historical paper [1], Einstein, Podolsky and Rosen state:

While we have thus shown that the wavefunction [of quantum mechanic] does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.

In the preceding Papers I and II we have provided a review and upgrade of the apparent proof of the existence of classical counterparts for

Figure 25: *A picture of the 300 kW mobile magnegas refinery built by Santilli when Chief Scientist of Magnegas Corporation (now Taronis Corporation) to process a mixture of fossil oil and water into magnegas whose combustion with the proper stochiometric oxygen has no detectable* CO *or* HC *[57]. The oxygen content of magnegas appears to have special polarizations [62] - [65] deserving tests for its possible use in ventilators to kill Corona viruses due to the emission of strong UV light in its depolarization in human lungs.*

extended particles within physical media [9] (Section II-3.7), and of the apparent progressive verification of Einstein determinism in the interior of hadrons, nuclei and stars [10] (Section II-3.8).

In this third paper, it appears we have additionally proved, apparently for the first time, the above quoted, concluding EPR statement. In fact, the "completion" of the quantum wavefunction $\psi(t, r)$ of the Schrödinger equation into the hadronic isowavefunction $\hat{\psi}(\hat{t}, \hat{r})$ of the Schrödinger-Santilli isoequation allows the achievement, otherwise impossible via quantum mechanics, of an exact representation of all characteristics of: the neutron in its synthesis from the hydrogen atom (Section 2.6); the deuteron (Section 2.7); the attractive force between the identical electrons in valence couplings (Section2.8); and the progressive achievement of Einstein's determinism in interior dynamical conditions (Section 2.9).

Acknowledgments. The writing of these papers has been possible thanks to pioneering contributions by numerous scientists we regret not having been able to quote in this paper (see Vol. I, Refs. [**?**] for comprehensive literature), including:

The mathematicians: H. C. Myung, for the initiation of iso-Hilbert spaces; S. Okubo, for pioneering work in non-associative algebras; N. Kamiya, for basic advances on isofields; M. Tomber, for an important bibliography on non-associative algebras; R. Ohemke, for advances in Lie-admissible algebtas; J. V. Kadeisvili, for the initiation of isofunctional isoanalysis; Chun-Xuan Jiang, for advances in isofield theory; Gr. Tsagas, for advances in the Lie-Santilli theory; A. U. Klimyk, for the initiation of Lie-admissible deformations; Raul M. Falcon Ganfornina and Juan Nunez Valdes, for advances in isomanifolds and isotopology; Svetlin Georgiev, for advances in the isodifferential isocalculus; A. S. Muktibodh, for advances in the isorepresentations of Lie-Santilli isoalgebras; Thomas Vougiouklis, for advances on Lie-admissible hypermathematics; and numerous other mathematicians.

The physicists and chemist: A. Jannussis, for the initiation of isocreation and isoannihilation operators; A. O. E. Animalu, for advances in the Lie-admissible formulation of hadronic mechanics; J. Fronteau, for advances in the connection between Lie-admissible mechanics and statistical mechanics; A. K. Aringazin for basic physical and chemical advances; R. Mignani for the initiation of the isoscattering theory; P. Caldirola, for advances in isotime; T. Gill, for advances in the isorelativity; J. Dunning Davies, for the initiation of the connection between irreversible Lie-admissible mechanics and thermodynamics; A. Kalnay, for the connection between Lie-admissible mechanics and Nambu mechanics; Yu. Arestov, for the experimental verification of the iso-Minkowskian structure of hadrons; A. Ahmar, for experimental verification of the iso-Minkowskian character of Earth's atmosphere; L. Ying, for the experimental verification of nuclear fusions of light elements without harmful radiation or waste; Y. Yang, for the experimental confirmation of magnecules; D. D. Shillady, for the isorepresentation of molecular binding energies; R. L. Norman, for advances in the laboratory synthesis of the neutron from the hydrogen; I. Gandzha for important advances on isorelativities; A. A. Bhalekar, for the exact representation of nuclear spins; Simone Beghella Bartoli important contributions in experimental verifications of hadronic mechanics; and numerous other physicists.

I would like to express my deepest appreciation for all the above courageous contributions written at a point in time of the history of science dominated by organized oppositions against the pursuit of new scientific knowledge due to the trillions of dollars of public funds released by governmental agencies for research on pre-existing theories.

Special thanks are due to A. A. Bhalekar, J. Dunning-Davies, S. Georgiev, R. Norman, T. Vougiouklis and other colleague for deep critical comments.

Additional special thanks are due to Simone Beghella Bartoli for a detailed critical inspection of this Paper I.

Special thanks and appreciation are due to Arun Muktibodh for a very detailed inspection and control of all three papers, although I am solely responsible for their content due to numerous revisions implemented in their final version.

Thanks are also due to Mrs. Sherri Stone for an accurate linguistic control of the manuscript.

Finally, I would like to express my deepest appreciation and gratitude to my wife Carla Gandiglio Santilli for decades of continued support of my research amidst a documented international chorus of voices opposing the study of the most important vision by Albert Einstein.

References

[1] A. Einstein, B. Podolsky , and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete?," Phys. Rev., vol. 47 , p. 777 (1935),
www.eprdebates.org/docs/epr-argument.pdf

[2] N. Bohr, "Can quantum mechanical description of physical reality be

considered complete?" Phys. Rev. Vol. 48, p. 696 (1935) www.informationphilosopher.com/solutions/scientists/bohr-/EPRBohr.pdf

[3] J.S. Bell: "On the Einstein Podolsky Rosen paradox" Physics Vol. 1, 195 (1964).
www.eprdebates.org/docs/j.s.bell.pdf

[4] J. Bell: "On the problem of hidden variables in quantum mechanics" Reviews of Modern Physics Vol. 38 No. 3, 447 (July 1966).

[5] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik,* Springer, Berlin (1951).

[6] Stanford Encyclopedia of Philosophy, "bell's Theorem" (first published 2005, revised 2019) plato.stanford.edu/entries/bell-theorem/

[7] D. Bohm, *Quantum Theory,* Dover, New Haven, CT (1989).

[8] D. Bohm, J. Bub: "A proposed solution of the measurement problem in quantum mechanics by a hidden variable theory" Reviews of Modern Physics 38 Vol. 3, 453 (1966).

[9] R. M. Santilli, "Isorepresentation of the Lie-isotopic SU(2) Algebra with Application to Nuclear Physics and Local Realism," Acta Applicandae Mathematicae Vol. 50, 177 (1998),
http://www.santilli-foundation.org/docs/Santilli-27.pdf

[10] R. M. Santilli, "Studies on the classical determinism predicted by A. Einstein, B. Podolsky and N. Rosen," Ratio Mathematica Volume 37, pages 5-23 (2019),
www.eprdebates.org/docs/epr-paper-ii.pdf

[11] R. M. Santilli, "Studies on A. Einstein, B. Podolsky, and N. Rosen argument that "quantum mechanics is not a complete theory, " I: Basic methods," submitted for publication," submitted for publication.

[12] R. M. Santilli, "Studies on A. Einstein, B. Podolsky, and N. Rosen argument that "quantum mechanics is not a complete theory, II: Apparent confirmation of the EPR argument," Submitted for publication

[13] E. Fermi *Nuclear Physics,* University of Chicago Press (1949).

[14] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics,* Wiley and Sons (1952).

[15] K. R. Popper, *Quantum Theory and the Schism in Physics,* Unwin Hyman Ltd (1982).

[16] J. Dunning-Davies, *Exploding a Myth: Conventional Wisdom or Scientific Truth?*, Horwood Publishing (2007).

[17] J. Horgan, *The End of Science,* Basic Books (2015).

[18] R. M. Santilli, "On a possible Lie-admissible covering of Galilei's relativity in Newtonian mechanics for nonconservative and Galilei form-non-invariant systems," Hadronic J. Vol. 1, 223-423 (1978), available in free pdf download from
www.santilli-foundation.org/docs/Santilli-58.pdf

[19] R. M. Santilli, "Need of subjecting to an experimental verification the validity within a hadron of Einstein special relativity and Pauli exclusion principle," Hadronic J. Vol. 1, pages 574-901 (1978),
http://www.santilli-foundation.org/docs/santilli-73.pdf

[20] R. M. Santilli, "An intriguing legacy of Einstein, Fermi, Jordan and others: The possible invalidation of quark conjectures," Found. Phys. Vol. 11, 384-472 (1981),
www.santilli-foundation.org/docs/Santilli-36.pdf

[21] R. M. Santilli, "Relativistic hadronic mechanics: non-unitary axiom-preserving completion of quantum mechanics" Foundations of Physics Vol. 27, p.625-739 (1997)
www.santilli-foundation.org/docs/Santilli-15.pdf

[22] R. M. Santilli, *Foundation of Theoretical Mechanics,* Springer-Verlag, Heidelberg, Germany, Volume I (1978) *The Inverse Problem in Newtonian Mechanics,*
www.santilli-foundation.org/docs/Santilli-209.pdf

[23] R. M. Santilli, *Foundation of Theoretical Mechanics,* Springer-Verlag, Heidelberg, Germany, Vol. II (1982) *Birkhoffian Generalization of Hamiltonian Mechanics,*
www.santilli-foundation.org/docs/santilli-69.pdf

[24] D. S. Sourlas and G. T. Tsagas, *Mathematical Foundation of the Lie-Santilli Theory,* Ukraine Academy of Sciences 91993),
www.santilli-foundation.org/docs/santilli-70.pdf

[25] R. M. Santilli, *The Physics of New Clean Energies and Fuels According to Hadronic Mechanics,* Special issue of the Journal of New Energy, 318 pages (1998)
www.santilli-foundation.org/docs/Santilli-114.pdf

[26] R. M. Santilli, "Isonumbers and Genonumbers of Dimensions 1, 2, 4, 8, their Isoduals and Pseudoduals, and "Hidden Numbers," of Dimension 3, 5, 6, 7," Algebras, Groups and Geometries Vol. 10, p. 273-295 (1993)
www.santilli-foundation.org/docs/Santilli-34.pdf

[27] Chun-Xuan Jiang, *t Foundations of Santilli Isonumber Theory,* Inter- national Academic Press (2001),
www.i-b-r.org/docs/jiang.pdf

[28] R. M. Santilli, "Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries," in Isotopies of Contemporary Mathematical Structures," Rendiconti Circolo Matematico Palermo, Suppl. Vol. 42, p. 7-82 (1996),
www.santilli-foundation.org/docs/Santilli-37.pdf

[29] S. Georgiev, *Foundation of the IsoDifferential Calculus,* Volume I, to VI, r (2014 on). Nova Academic Publishers

[30] R. M. Santilli, *Foundations of Hadronic Chemistry, with Applications to New Clean Energies and Fuels,* Kluwer Academic Publishers (2001),
www.santilli-foundation.org/docs/Santilli-113.pdf
Russian translation by A. K. Aringazin
http://i-b-r.org/docs/Santilli-Hadronic-Chemistry.pdf

[31] I. Gandzha and J. Kadeisvili, *New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli,* Sankata Printing Press, Nepal (2011),
www.santilli-foundation.org/docs/RMS.pdf

[32] R. M. Santilli, *Elements of Hadronic Mechanics*, Ukraine Academy of Sciences, Kiev, Volume I (1995), *Mathematical Foundations,*
href="http://www.santilli-foundation.org/docs/Santilli-300.pdf

[33] R. M. Santilli, *Elements of Hadronic Mechanics*, Ukraine Academy of Sciences, Kiev, Volume II (1994), *Theoretical Foundations*,
www.santilli-foundation.org/docs/Santilli-301.pdf

[34] R. M. Santilli, *Elements of Hadronic Mechanics*, Ukraine Academy of Sciences, Kiev, Volume III (2016), *Experimental verifications*, www.santilli-foundation.org/docs/elements-hadronic-mechanics-iii.compressed.pdf

[35] R. M. Santilli, "Isotopies of the spinorial covering of the Poincaré symmetry," Communication of the Joint Institute for Nuclear Research, Dubna, Russia, No. E4-93-252 (1993).

[36] R. M. Santilli, "Recent theoretical and experimental evidence on the synthesis of the neutron," Communication of the JINR, Dubna, Russia, No. E4-93-252 (1993), published in the Chinese J. System Eng. and Electr. Vol. 6, 177 (1995),
www.santilli-foundation.org/docs/Santilli-18.pdf

[37] Bijan Davvaz and Thomas Vougiouklis, *A Walk Through Weak Hyperstructures, Hv-Structures,* World Scientific (2018)

[38] R. M. Santilli, *Isodual Theory of Antimatter with Applications to Antigravity, Grand Unifications and Cosmology,* Springer (2006).
www.santilli-foundation.org/docs/santilli-79.pdf

[39] Hadronic Technologies Corporation, Directional Neutron Source,
http://hadronictechnologies.com/docs/TEC-DNS-3Za.pdf

[40] H. Rutherford, Proc. Roy. Soc. A,Vol. 97, 374 (1920).

[41] C. S. Burande, "Santilli Synthesis of the Neutron According to Hadronic Mechanics," American Journal of Modern Physics 5(2-1): 17-36 (2016)
www.santilli-foundation.org/docs/pdf3.pdf

[42] R. M. Santilli, "The etherino and/or the Neutrino Hypothesis?" Found. Phys. Vol. 37, p. 670 (2007)
www.santilli-foundation.org/docs/EtherinoFoundPhys.pdf

[43] R. M. Santilli, "Apparent consistency of Rutherford's hypothesis on the neutron as a compressed hydrogen atom, Hadronic J.Vol. 13, 513 (1990).
www.santilli-foundation.org/docs/Santilli-21.pdf

[44] R. M. Santilli, "A structure model of the elementary charge,
Hadronic J. Vol. 4, pages 770-784 (1981)
www.santilli-foundation.org/docs/Santilli-03.pdf

[45] E. A. Nersesov, *Fundamentals of atomic and nuclear physics.* Mir Publishers, Moscow (1990).

[46] R. M. Saantilli, “Nonlocal formulation of the Bose-Einstein correlation within the context of hadronic mechanics,” Hadronic J. Vol. 15, pages 1-50 and 81-133 (1992),
www.santilli-foundation.org/docs/Santilli-116.pdf

[47] F. Cardone and R. Mignani, “Nonlocal approach to the Bose-Einstein correlation, Europ. Phys. J. C 4, 705 (1998). See also “Metric description of hadronic interactions from the Bose-Einstein correlation,” JETP Vol. 83, p.435 (1996)
www.santilli-foundation.org/docs/Santilli-130.pdf

[48] R. M. Santilli, ”Apparent Experimental Confirmation of Pseudoprotons and their Application to New Clean Nuclear Energies,” International Journal of Applied Physics and Mathematics Volume 9, Number 2 (2019),
www.santilli-foundation.org/docs/pseudoproton-verification-2018.pdf

[49] S. S. Dhondge, ”Studies on Santilli Three-Body Model of the Deuteron According to Hadronic Mechanics,” American Journal of Modern Physics,Vol. 5, No. 2-1, pp. 46-55 (2016),
www.santilli-foundation.org/docs/deuteron-2018.pdf

[50] R. M. Santilli, “A quantitative isotopic representation of the deuteron magnetic moment,” in *Proceedings of the International Symposium ’Dubna Deuteron-93,* Joint Institute for Nuclear Research, Dubna, Russia (1994),
www.santilli-foundation.org/docs/Santilli-134.pdf

[51] R. M. Santilli, ”Use of relativistic hadronic mechanics for the exact representation of nuclear magnetic moments and the prediction of new recycling of nuclear waste” arXiv:physics/970401 (1996),
www.santilli-foundation.org/docs/9704015.pdf

[52] R. M. Santilli and D. D. Shillady,, “A new isochemical model of the hydrogen molecule,” Intern. J. Hydrogen Energy Vol. 24, pages 943-956 (1999)
www.santilli-foundation.org/docs/Santilli-135.pdf

[53] R. M. Santilli and D. D. Shillady, “A new isochemical model of the water molecule,” Intern. J. Hydrogen Energy Vol. 25, 173-183 (2000)
www.santilli-foundation.org/docs/Santilli-39.pdf

[54] A. A. Bhalekar and R. M. Santilli, "Studies in the reduction of s table matter to isoprotons and isoelectrons," to appear.

[55] J. N. Murrell, S. F. A . Kettle ands J. M. Tedder, *The Chemical Bond,* John Wiley and Sons (1985).

[56] R. Briown, Photoionization of thye hydrogen and helium molecules," The Astrophysics Journal, Vol. 164, pages 387-398 (1971).

[57] R. M. Santilli, "Method and Apparatus for the industrial production of new hydrogen-rich fuels, " United States Patent Number 9,700,870, B2, July 1, 2017
http://www.santilli-foundation.org/docs/Magnecule-patent.pdf

[58] Y. Yang, J. V. Kadeisvili, and S. Marton, "Experimental Confirmations of the New Chemical Species of Santilli Magnecules," The Open Physical Chemistry Journal Vol. 5, pages 1-16 (2013),
www.santilli-foundation.org/docs/Magnecules-2012.pdf

[59] R. M. Santilli, "The Novel Hyper Combustion for the Complete Combustion of Fossil Fuels", Intern. Journal of Chemical Engineering and Applications, Vol. 10, No. 1, February (2019),
www.santilli-foundation.org/docs/hypercombustion-2019.pdf

[60] R. M. Santilli, "Additional Confirmation of the "Intermediate Controlled Nuclear Fusions" without harmful radiation or waste," in *Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes,* Kathmandu University pages 163-177 (2011),
www.santilli-foundation.org/docs/ICNF-3.pdf

[61] J. V. C. Lynch and Y. Yang, "Confirmations of Santilli Intermediate Nuclear Fusions of Deuteron and Carbon into Nitrogen without Radiations." The Open Physical Chemistry Journal Vol. 5, pages 17 (2013)
www.santilli-foundation.org/docs/ICNF-Conf-2013.pdf

[62] R. M. Santilli, "The novel magnecular species of hydrogen and oxygen with increased specific weight and energy content," Intern. J. Hydrogen Energy Vol. 28, 177-196 (2003),
www.santilli-foundation.org/docs/Santilli-38.pdf

[63] Day D. TCD analysis and density measurements of Santilli Magnehydrogen. Eprida Laboratory report dated 11/10/11,

www.santilli-foundation.org/docs/Eprida-MH-Certification-10-11.pdf

[64] S. P. Zodape, "The MagneHydrogen in Hadronic Chemistry," AIP Proceedings 1558, 648 (2013); doi: 10.1063/1.4825575, www.santilli-foundation.org/docs/sangesh-Greece.pdf

[65] Y. Yang, J. V. Kadeisvili, and S. Marton, "Experimental Confirmations of the New Chemical Species of Santilli MagneHydrogen," International Journal Hydrogen Energy Vol. 38, page 5002 (2013), www.santilli-foundation.org/docs/MagneHydrogen-2012.pdf

[66] R. M. Santilli, "Isotopic quantization of gravity and its universal isopoincare' symmetry" in the *Proceedings of "The Seventh Marcel Grossmann Meeting in Gravitation,* SLAC 1992, R. T. Jantzen, G. M. Keiser and R. Ruffini, Editors, World Scientific Publishers pages 500-505(1994), www.santilli-foundation.org/docs/Santilli-120.pdf

[67] R. M. Santilli, "Unification of gravitation and electroweak interactions" in the proceedings of the Eight Marcel Grossmann Meeting in Gravitation, Israel 1997, T. Piran and R. Ruffini, Editors, World Scientific, pages 473-475 (1999), www.santilli-foundation.org/docs/Santilli-137.pdf

[68] A. K. Aringazin, "Toroidal configuration of the orbit of the electron of the hydrogen atom under strong external magnetic fields," Hadronic J. 24, 134 (2001), www.santilli-foundation.org/docs/landau.pdf

[69] R. M. Santilli, "Additional Confirmation of the "Intermediate Controlled Nuclear Fusions" without harmful radiation or waste," in *Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes,* Kathmandu University pages 163-177 (2011), www.santilli-foundation.org/docs/ICNF-3.pdf

[70] J. V. C. Lynch and Y. Yang, "Confirmations of Santilli Intermediate Nuclear Fusions of Deuteron and Carbon into Nitrogen without Radiations." The Open Physical Chemistry Journal Vol. 5, pages 17 (2013) www.santilli-foundation.org/docs/ICNF-Conf-2013.pdf

[71] R. M. Santilli, "Rudiments of isogravitation for matter and its isodual for antimatter," American Journal of Modern Physics Vol. 4(5), pages

59-75 (2015)
www.santilli-foundation.org/docs/isogravitation.pdf

[72] Schwartzchild K., " Uber das Gravitationsfeld eines Massenpunktes-nack der Einsteinshen Theorie, " Sitzber. Deut. Akad. Wiss. Berlin, Kl. Math.-Phys. Tech., 189-196 (1916).

[73] Schwartzchild K., "Uber das Gravitationsfeld einer Kugel aus inkompressibler Flussigkeit nach Einsteinshen Theorie, " Sitzber. Deut. Akad. Wiss. Berlin, Kl. Math.-Phys. Tech.,424-434 (1915).

[74] R. M. Santilli, Invited Plenary Lecture "The Novel Sustainable Hyper-Combustion And Hyper-Furnaces of Thunder Energies Corporation, International Summit on Sustainable Clean Energies, Hainan Island, China, November 6 to 10, (2016),
www.santilli-foundation.org/news.html

[75] M. Gell-Mann, Physics Letters Vol. 8, 215-215 (1964).

[76] C. Borghi, C. Giori C. and A. Dall'Olio, Communications of CEN-UFPE, Number 8 (1969) and 25 (1971), reprinted in the (Russian) Phys. Atomic Nuclei, Vol. 56, p. 205 (1993).

[77] R. M. Santilli, "Lie-isotopic Lifting of the Minkowski space for Extended Deformable Particles," Lettere Nuovo Cimento Vol. 37, p. 545-551 (1983),
www.santilli-foundation.org/docs/Santilli-50.pdf

[78] R. M. Santilli, "Isominkowskian Geometry for the Gravitational Treatment of Matter and its Isodual for Antimatter," Intern. J. Modern Phys. D Vol. 7, 351 (1998),
www.santilli-foundation.org/docs/Santilli-35.pdf

[79] R. M. Santilli, *Isotopic Generalizations of Galilei and Einstein Relativities,* International Academic Press (1991), Vols. I *Mathematical Foundations,*
http://www.santilli-foundation.org/docs/Santilli-01.pdf

[80] R. M. Santilli, *Isotopic Generalizations of Galilei and Einstein Relativities,* International Academic Press (1991), Vol. II: *Classical Formulations,*
www.santilli-foundation.org/docs/Santilli-61.pdf

[81] R. M. Santilli, *Isorelativities,* International Academic Press, (1995)

[82] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry,*, Volumes I to V, International Academic Press, (2008),
www.i-b-r.org/Hadronic-Mechanics.htm

[83] R. M. Santilli, "Rudiments of isogravitation for matter and its isodual for antimatter," American Journal of Modern Physics Vol. 4(5), pages 59-75 (2015)
www.santilli-foundation.org/docs/isogravitation.pdf

[84] R. M. Santilli, "Lie-isotopic Lifting of Special Relativity for Extended Deformable Particles," Lettere Nuovo CimentoVol. 37, 545 (1983),
www.santilli-foundation.org/docs/Santilli-50.pdf

[85] R. M. Santilli, "Lie-isotopic Lifting of Unitary Symmetries and of Wigner's Theorem for Extended and Deformable Particles," Lettere Nuovo Cimento Vol. 38, 509 (1983),
www.santilli-foundation.org/docs/Santilli-51.pdf

[86] R. M. Santilli, "Isotopies of Lie Symmetries," I: Cbasic theory," , Hadronic J. Vol. 8, 36 and 85 1(985),
www.santilli-foundation.org/docs/santilli-65.pdf

[87] R. M. Santilli, "Isotopies of Lie Symmetries," II:I sotopies of the rotational symmetry," Hadronic J. Vol. 8, 36 and 85 (1985),
www.santilli-foundation.org/docs/santilli-65.pdf

[88] R. M. Santilli, "Rotational isotopic symmetries," ICTP communication No. IC/91/261 (1991),
www.santilli-foundation.org/docs/Santilli-148.pdf

[89] R. M. Santilli, "Isotopic Lifting of the SU(2) Symmetry with Applications to Nuclear Physics," JINR rapid Comm. Vol. 6. 24-38 (1993),
www.santilli-foundation.org/docs/Santilli-19.pdf

[90] R. M. Santilli, "Lie-isotopic generalization of the Poincare' symmetry, classical formulation," ICTP communication No. IC/91/45 (1991),
www.santilli-foundation.org/docs/Santilli-140.pdf

[91] R. M. Santilli, "Nonlinear, Nonlocal and Noncanonical Isotopies of the Poincare' Symmetry," Moscow Phys. Soc. Vol. 3, 255 (1993),
www.santilli-foundation.org/docs/Santilli-40.pdf

[92] R. M. Santilli, “An introduction to the new sciences for a new era,” Invited paper, SIPS 2016, Hainan Island, China, Clifford Analysis, Clifford Algebras and their Applications ol. 6, No. 1, pp. 1-119, (2017) www.santilli-foundation.org/docs/new-sciences-new-era.pdf

[93] R. M. Santilli, ”Theory of mutation of elementary particles and its application to Rauch’s experiment on the spinorial symmetry,” ICTP release IC/91/46 (1991)
www.santilli-foundation.org/-docs/Santilli-141.pdf

[94] R. M. Santilli, ” The notion of non-relativistic isoparticle,” ICTP release IC/91/265 (1991)
www.santilli-foundation.org/docs/Santilli-145.pdf

[95] A. O. E. Animalu and R. M. Santilli, “Nonlocal isotopic representation of the Cooper pair in superconductivity,” Intern. J. Quantum Chemistry Vol. 29, 185 (1995)
www.santilli-foundation.org/docs/Santilli-26.pdf

[96] V. M. Tangde, ”Advances in hadronic chemistry and its applications,” Foundation of Chemistry, DOI 10.1007/s10698-015-9218-z (March 24, 2015),
www.santilli-foundation.org/docs/hadronic-chemistry-FC.pdf

[97] E. Trell, ”Review of Santilli Hadronic Chemistry,” International Journal Hydrogen Energy Vol. 28, p. 251 (2003),
www.scientificethics.org/Hadronic Chemistry.pdf

[98] R. M. Santilli ”A Tentative Magnecular Model of Liquid Water with an Explicit Attractive Force Between Water Molecules,” American Journal of Modern Physics Vol. 6(4-1): 46-52 (2017),
www.santilli-foundation.org/docs/santilli-liquid-water.pdf

[99] A. K. Aringazin, “Toroidal configuration of the orbit of the electron of the hydrogen atom under strong external magnetic fields,” Hadronic J. 24, 134 (2001),
www.santilli-foundation.org/docs/landau.pdf

[100] R. M. Santilli, ”Nonlinear, Nonlocal and Noncanonical Isotopies of the Poincare’ Symmetry,” Moscow Phys. Soc. Vol. 3, 255 (1993),
www.santilli-foundation.org/docs/Santilli-40.pdf