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The intertemporal choice behavior: the role of emotions in a multiagent decision problem

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Abstract

Traditional Discounted Utility Model assumes an exponential delay discount function, with a constant discount rate: this implies dynamic consistency and stationary intertemporal preferences. Contrary to the normative setting, decision neuroscience stresses a lack of rationality, i.e., inconsistency, in some intertemporal choice behaviors. We deal with both models are dealt with in the framework of some relevant decision problems.

Key words: time preference, exponential discounting, hyperbolic discounting.

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1 Introduction

The traditional Discounted Utility model (DU model) (Samuelson, 1937) [20] fails in being both normative and descriptive. Indeed several studies, especially carried out in psychology and neuroeconomics, reveal the existence of relevant anomalies violating the axioms of the traditional model (Section 3).

Bechara and colleagues [2] show that decision making processes are guided by emotional signaling, which allow people to choose advantageously before they realized the strategy that worked best. This fact justifies the presence of anomalies in intertemporal choice and the use of hyperbolic delay discounting (declining as the length of the delay increases), so, people have the tendency to increasingly choose a smaller-sooner reward over a larger-later reward as the delay occurs sooner in time. This entails intertemporal inconsistency and preferences reversal. Even so, an impatient behavior not necessarily can be considered incoherent (Section 4).

The results of some studies by Shiv et al. [21] and Naqvi et al. [17] have demonstrated that patients with lesions in specific components of a neural circuitry critical for the processing of emotions will make more advantageous decisions than normal subjects when faced with the types of positive-expected-value gambles that most people routinely shun (Section 5).

Recent neuroeconomic and econophysical studies have explored neurobiological and psychological factors, e.g. impulsivity and inconsistency that determined individual differences in intertemporal choice. Takahashi et al. [25] attempt to dissociate impulsivity and inconsistency in their econophysical studies proposing a quasi-exponential delay discount function. Other behavioral economists propose *multiple selves* models attempting to measure the strength of the internal conflict within the decision maker, best known as *quasi-hyperbolic discount* model (Laibson, 1997) [11] (Section 6).

To fight impulsivity Strotz [23] proposed two strategies that might be adopted by a person who foresees how her preferences will change over time; Thaler and Shefrin [26] built a structure in which the individual is treated as if he contained two distinct psyches denoted as *planner* and *doer* (Section 7).

In a multiagent decision context the objective for a decision group is to choose a common decision, that is an alternative which is judged the best by the majority of the decision makers. So in most strategic decisions, it is important to be able to estimate the characteristics and behaviors of others. If the characteristics of other players are unknown, estimating them is a critical task (Section 8). Moreover, psychological evidence suggests people own beliefs, values, and habits tend to bias their perceptions of how widely they are shared (*false consensus effect*). This effect demonstrates an inability of individuals to process information rationally (Section 9). Therefore when we use the aggregation of the agent preferences to assess consensus, we obtain a coefficient which includes the false consensus effect that depends on the subjectivity and also increases the degree of consensus. In order to eliminate the component of human judgment vagueness a procedure defined by ordered weighted averaging (OWA) operators, introduced by Yager [29], can be applied (Section 10).

An experiment performed by Engelmann and Strobel [8] demonstrates that a false consensus effect is present only if information about decision of other members of the group is implicit. So the consensus effect is not always false but only when people, forming expectations concerning decisions of others, weight their own decision more heavily than that of a randomly selected person from the same population (see [6], [7]), (Section 11). The result is linked with the analysis of false consensus effect in cooperative and non-cooperative decision problem. Indeed, in a cooperative decision problem, agents know choices of other members, while in a non-cooperative one they have to judge choices of others (Section 12).

2 Traditional discounting model and decision neuroscience

The standard economic model of discounted utility assumes that economic agents make intertemporal choices over consumption profiles $(c_t, \dots, c_T)$ and such preferences can be represented by an intertemporal utility function $U^t(c_t, \dots, c_T)$, which can be described by the following special functional form:

$$U^t(c_t, \dots, c_T) = \sum_{k=0}^{T-t} D(k)u(c_{t+k})$$

where

$$D(k) = \left(\frac{1}{1+\rho}\right)^k$$

So the *DU* model assumes an exponential temporal discounting function and a constant discount rate (ρ), which represents the pure rate of time preference of the individual.

An important implication of constant discount rate and exponential discounting function is that intertemporal preferences of the individual are time-consistent: if at time t a person prefers c_2 at $t+2$ to c_1 at $t+1$, then at time $t+1$ she must prefer c_2 at $t+2$ to c_1 instantly. So, with the same temporal options and the same information, later preferences confirm earlier preferences.

However, several empirical studies have documented various inadequacies of the *DU* model as a descriptive model of behavior. Behavioral economic theories on decision process have found that there are a number of behavior patterns that violate the rational choice theory [27].

Decision neuroscience is an emerging area of research whose goal is to integrate research in neuroscience and behavioral decision making. It calls into question the theories of choice that assume decisions derive from an assessment of the future outcomes of various options and alternatives through some type of cost-benefit analysis, which ignore influence of emotions on decision-making.

This investigation explores the neural road map for the physiological processes intervening between knowledge and behavior, and the potential interruptions that lead to a disconnection between what one knows and what one decides to do. Decision making studies in neurological patients, who can no longer process emotional information, normally suggest that people make judgments not only by evaluating the consequences and their probability of occurring, but also and even sometimes primarily at a gut or emotional level (see [1]).

3 Behavioral finance: empirical anomalies violating DU model

Some studies concerning the individual behavior from the psychological perspective, e.g. related with discounting real or hypothetical rewards, show the existence of violations of the DU model. A first empirical remark is that discount rates are not constant over time, but appear to decline - a pattern often referred to as *hyperbolic discounting* ([22], [23]). Furthermore, even for a given delay, discount rates vary across different locations of intertemporal choices [28].

Delay effect, *magnitude effect*, *sign effect* and *sequence effect* are among the relevant anomalies in intertemporal choice, we will deal with.

The *delay effect* rests on the evidence that as waiting time increases, the discount rates tend to be higher in the short intervals than in the longer ones. Prelec and Loewenstein [18] define this anomaly as *common difference effect* and *immediacy effect*. We can set out delay effect as:

$$(x, s) \sim (y, t) \quad \text{but} \quad (x, s + h) < (y, t + h)$$

for

$$y > x, s < t \quad \text{and} \quad h > 0$$

If two capitals, (x, s) and (y, t), are indifferent, $(x, s) \sim (y, t)$, their projections onto a common instant p have to coincide:

$$xA(s, p) = yA(t, p) \quad \text{if and only if} \quad \frac{x}{y} = \frac{A(t, p)}{A(s, p)} = v(s, t, p)$$

being $A(t, p)$ the discount function which represents the amount available at p instead of one euro available at t, and $v(s, t, p)$ the corresponding financial factor. In the same way, if $(x, s+h) \sim (y, t+h)$, this implies that

$$xA(s+h,p) = yA(t+h,p)$$

if and only if

$$\frac{x}{y} = \frac{A(t+h,p)}{A(s+h,p)} = v(s+h,t+h,p)$$

Then:

$$v(s,t,p) < v(s+h,t+h,p)$$

The magnitude effect can be described as follows. Larger outcomes are discounted at a lower rate than smaller outcomes. Let us suppose that the instantaneous discount rate is inversely proportional to the discounted amount:

$$A(c,z) = ce^{-\int_0^z \frac{k}{c}dx} = ce^{-\frac{k}{c}z}$$

Prelec and Loewenstein [18] formulate the magnitude effect as follows:

$$(x,s) \sim (y,t) \quad \text{implies} \quad (ax,s) < (ay,t)$$

for $y > x > 0$, $s < t$ and

$$(-x,s) \sim (-y,t) \quad \text{implies} \quad (-ax,s) > (-ay,t)$$

The sign effect. Gains are discounted at a higher rate than losses of the same magnitude. Prelec and Loewenstein [18] proposed the amplification loss property implying that, changing the sign of an amount from gains to losses, the weight of this amount increases:

$$(x,s) \sim (y,t) \quad \text{implies} \quad (-x,s) > (-y,t)$$

for $y > x > 0$, $s < t$.

Increasing sequences of consumption are preferred over decreasing ones even if the total amount is the same. In general, when subjects choose among different sequences of two events people tend to save the better thing for last, contradicting the standard assumption of a positive interest rate. In the *improving sequence effect*, for all s and t, and $s < t$, there is a c_0 such that, for all $y > x > c_0$, the following preference holds

$$\{(x,s),(y,t)\} >_p \{(y,s),(x,t)\}$$

in the instant p ([16], [26]).

4 Anticipation of future events and hyperbolic discounting

In contrast with the historically dominant view of emotions as a negative influence in human behavior, recent research in neuroscience and psychology has highlighted the positive roles played by emotions in decision making (Bechara et al. [2]; Damasio [5]; Loewenstein and Lerner [12]). Although strong negative emotions can lead destructive patterns of behavior, some Authors (see [2]; [5]; [21]) have shown that individuals with emotional dysfunction tend to perform poorly compared with those who have intact emotional processes.

An experiment exhibited in [2] leads to the conclusion that decision making is guided by emotional signaling generated in anticipation of future events. Without the ability to generate these emotional signals, the patients fail to avoid choices that lead to losses, and instead continue to sample from the disadvantageous choices until they go broke in a manner that is akin to how they behave in real life. In normal individuals, unconscious biases guide behavior before conscious knowledge does. Without the help of such biases, overt knowledge may be insufficient to ensure advantageous behavior.

Decision maker preferences are inconsistent and change over time, because normal people possess anticipatory indices of somatic states, that represent unconscious biases that are linked to prior experiences with reward and punishment. These biases alarm the normal subject about selecting a disadvantageous course of action, even before the subject becomes aware of the goodness or badness of the choice he is about to make [1]. Indeed, when normal people won or lost money on an investment round, they adopted a conservative strategy and became more reluctant to invest on the subsequent round [21].

Furthermore the preference for more immediate rewards per se is not always irrational, because there are opportunity costs and risk associated with non-gaining in delaying the rewards.

As a consequence there is considerable agreement among psychologists and economists that the notion of exponential discounting should be replaced by some form of hyperbolic discounting, which can represent the tendency of the individuals to increasingly choose a smaller-sooner reward over a larger-later reward as the delay occurs sooner in time (*delay effect*).

Many authors proposed different hyperbolic discount functions, in which temporal discount function δ increases with the delay to an outcome. Loewenstein and Prelec [13] proposed the form:

$$d(t) = \left(\frac{1}{1+\alpha t}\right)^{\frac{\beta}{\alpha}}$$

where $\beta > 0$ is the degree of discounting and $\alpha > 0$ is the departure from exponential discounting.

Hyperbolic discounting has been applied to a wide range of phenomena, including consumption-saving behavior. Consistent with hyperbolic discounting, people's investment behavior exhibits patience in the long run and impatience in the short run [28].

A second type of empirical support for hyperbolic discounting comes from experiments on dynamic inconsistency. Studies and empirical evidences show that delay effect can derive in preference reversal between two rewards as the time distance to these rewards diminishes. A hyperbolic discount model can demonstrate this; indeed, non-exponential time preference curves can cross [23] and consequently the preference for one future reward over another may change in time [28].

5 The negative side of emotions: impulsivity

The positive roles played by emotions when making decisions are in contrast with some contexts in which individuals deprived of normal emotional reactions might actually make better decisions than normal individuals. For instance, consider the case of a patient with ventromedial prefrontal damage (which involves severe impairments in judgment and emotion) who was driving under hazardous road conditions [5]. When other drivers reached an icy patch, they hit their brakes in panic, causing their vehicles to skid out of control, but the patient crossed the icy patch unperturbed, gently pulling away from a tailspin and driving ahead safely. The patient remembered the fact that not hitting the brakes was the appropriate behavior, and his lack of fear allowed him to perform optimally [21].

Other evidences suggest that even relatively mild negative emotions that do not result in a loss of self-control can play a counterproductive role among normal individuals in some situations. When gambles that involve some possible loss are presented one at a time, most people display extreme levels of risk aversion toward the gambles, a condition known as myopic loss aversion [3]. If myopic loss aversion does indeed have an emotional basis, then any dysfunction in neural systems subserving emotion ought to result in reduced levels of risk aversion and, thus, lead to more advantageous decisions in cases in which risk taking is rewarded. Furthermore individuals deprived of normal emotional reactions might, in certain situations, make more advantageous decisions than those not deprived of such reactions; so the lack of emotional reactions may lead to more advantageous decisions [21].

Indeed in many cases, indeed, temptations induce disadvantageous behavior, and when temptation becomes too great, what the person knows to be his best long run interests conflicts with his short run desires. Sociologists and psychologists have persistently studied impulsivity relative to its resultant behaviors such as drug addiction, suicide, aggression and violence. These studies suggests that individuals who frequently engage in impulsive behavior may fail to appropriately evaluate the consequences of their behavior [28].

6 Neuroeconomics: impulsivity and inconsistency in intertemporal choice

The greatest contradiction to rational theory, in intertemporal choice, is inconsistent preference, usually manifested as temporary preference for options that are extremely costly or harmful in the long run. This behavior can be typically seen in psychiatric disorders (alcoholism, drug abuse), but also in more ordinary phenomena (overeating, credit card debt) [28].

Some investigations in neuroeconomics, a specialized field of decision neuroscience, have found that addicts are more myopic, i.e., they have large time-discount rates, in comparison with non-addict populations [4]. It results that hyperbolic discounting may explain various human problematic behaviors [11]: loss of self-control, failure in planned abstinence from addictive substances and relapse, a deadline rush due to procrastination, failure in saving enough before retirement and risky sexual behavior. Addiction and financial mismanagement frequently co-occur, and elevated delay discounting may be a common mechanism contributing to both of these problematic behaviors.

We have noted that the preference for more immediate rewards per se is not always irrational or inconsistent (Section 4); therefore, impulsivity in intertemporal choice is rationalizable for several categories of persons. The behaviors of addicts are clinically problematic, but economically rational when their choices are time-consistent, if they have large discount rates with an exponential discount function. However, it is known that addicts also discount delayed outcomes hyperbolically, suggesting the intertemporal choices of addicts are time-inconsistent, resulting in a loss of self-control [4]:

they act more impulsively at the moment of the choice, against their own previously-intended plan. Moreover if large discount rates are due to habitual drug intake, it is expected that discount rates decreased after long term abstinence. However, recent studies on alcoholics and smokers report that abstinence does not dramatically reduce discount rates of former alcoholics and smokers [24].

Behavioral neuroeconomic and econophysical studies have proposed two discount models, in order to clarify the neural and behavioral correlates of impulsivity and inconsistency in intertemporal choice, namely, a quasi-exponential discount model and a quasi-hyperbolic discount model.

Quasi-exponential discount model. Takahashi et al. [25] have proposed and examined the following function for subjective value $V(D)$ of delayed reward:

$$V(D) = \frac{A}{\exp_q(k_q D)} = \frac{A}{[1 + (1-q)k_q D]^{\frac{1}{1-q}}}$$

where D denotes a delay until receipt of a reward, A the value of a reward at $D = 0$, and k_q a parameter of impulsivity at delay $D = 0$ (q-exponential discount rate) and the q-exponential function is defined as:

$$\exp_q(x) = (1 + (1-q)x)^{\frac{1}{1-q}}$$

This function can distinctly parametrize impulsivity and inconsistency [28].

Quasi-hyperbolic discount model. Behavioral economists have proposed that the inconsistency in intertemporal choice may be attributed to an internal conflict between multiple selves within a decision maker. As a consequence, there are at least two exponential discounting selves (with two exponential discount rates) in a single individual; and when delayed rewards are at the distant future (> 1 year), the self with a smaller discount rate wins, while delayed rewards approach to the near future (within a year), the self with a larger discount rate wins, resulting in preference reversal over time. This intertemporal choice behavior can be parametrized in a quasi-hyperbolic discount model (also as a $\beta - \delta$ model). For discrete time τ (the unit assumed is one year) the quasi-hyperbolic discount factor is defined [11] as:

$$F(\tau) = \beta\delta^t$$

for $\tau = 1, 2, 3 \ldots$ and $F(0) = 1$, $0 < \beta < \delta < 1$.

A discount factor between the present and one-time period later β is smaller than that between two future time-periods δ.

In the continuous time, the proposed model is equivalent to the linearly-weighted two-exponential functions (generalized quasi-hyperbolic discounting):

$$V(D) = A[w \exp(-k_1 D) + (1-w)\exp(-k_2 D)]$$

where $0 < w < 1$, is a weighting parameter and k_1 and k_2 are two exponential discount rates ($k_1 < k_2$). Note that the larger exponential discount rate of the two k_2, corresponds to an impulsive self, while the smaller discount rate k_1 corresponds to a patient self [28].

7 Self-control against impulsivity: Strotz model and Thaler and Shefrin model

A number of mechanisms of self-control are predicted by hyperbolic discounting. Strotz proposed two strategies that might be adopted by a person who foresees how her preferences will change over time.

1. The *strategy of precommitment*. A person commits himself to perform a plan of action. For instance, consider a consumer with an initial endowment K_0 of consumer goods which has to be allocated over the finite interval $(0, T)$. At time t he wishes to maximize his utility function:

$$J_0 = \int_0^T \lambda(t-0)U[\bar{c}(t), t]\,\mathrm{d}t$$

subject to $\int_0^T c(t)\,\mathrm{d}t = K_0$, where $[\bar{c}(t), t]$ is the instantaneous rate of consumption at time period t, and $\lambda(t-0)$ is a discount factor, whose value depends on the elapsing time between a past or future date and present. This implies that the discounted marginal utility of consumption should be the same for all periods. But, at a later date, the consumer may reconsider his consumption plan. Then the problem is to maximize

$$J_0 = \int_0^T \lambda(t-\tau)U[c(t), t]\,\mathrm{d}t$$

subject to $\int_\tau^T c(t)\,\mathrm{d}t = K_\tau = K_0 - \int_0^\tau c(t)\,\mathrm{d}t$.

The optimal pattern of consumption will change with changes in τ and if the original plan is altered, the individual is said to display *dynamic inconsistency*. Strotz showed that individuals will not alter the original plan only if $\lambda(t, \tau)$ is an exponential in $|t - \tau|$.

2. The *strategy of consistent planning*. Since precommitment is not always a feasible solution to the problem of intertemporal conflict, an individual may adopt a different strategy: take into account future changes in the utility function and reject any plan that he will not follow through. His problem is then to find the best plan among those he will actually follow.

   In the setting of multiple selves models, in order to control impulsivity, Thaler and Shefrin [26] proposed a planner-doer model which draws upon principal agent theory. They treat an individual as if he contained two distinct psyches: one *planner*, which pursue longer-run results, and multiple *doers*, which are concerned only with short-term satisfactions, so they care only about their own immediate gratification (and have no affinity for future or past doers). For instance, consider an individual with a fixed income stream $y = [y_1, y_2, \ldots, y_T]$, where

$$\sum_t y_t = Y$$

   has to be allocated over the finite interval $(0, T)$. The planner would choose a consumption plan to maximize his utility function

$$V(Z_1, Z_2 \ldots, Z_T)$$

   subject to $\sum_t c_t \leq Y$, where Z_t is a utility function of level consumption in t (c_t).

   On the other hand, the unrestrained doer 1 would borrow $Y - y_1$ on the capital market and therefore choose $c_1 = Y$; the resulting consequence is naturally $c_2 = c_3 = \ldots = c_T = 0$. Such an action would suggest a complete absence of psychic integration.

   Then the model focuses on the strategies employed by the planner to control the behavior of the doers, and it proposes two tools at his disposal.

   (a) He can impose *rules* on the doers behavior, which operate by altering the constraints imposed on any given doer; or

   (b) he can use *discretion* accompanied by some method of altering the incentives or rewards to the doer without any self-imposed constraints [28].

8 Multiagent decision problems: consensus and agreement

In a multiagent decision problem an individual needs to take his intertemporal choice considering others' preferences, in order to achieve a consensus over a common decision. Group decision problems, indeed, consist in finding the best alternative(s) from a set of feasible alternatives $A = \{a_1, \ldots, a_m\}$ according to the preferences provided by a group of agents $E = \{e_1, \ldots, e_n\}$. The objective is to obtain the maximum degree of agreement among the agents overall performance judgements on the alternatives (see [22]).

Specifically, every agent assesses each alternative in his preference system. Furthermore the group of agents has to verify if there is a possibility to rank the alternative set in a way shared by (a majority in the group). If such an operation succeeds, the group has reached a *consensus* about the ranking of the alternative set. In real situations, humans rarely come to a unanimous agreement: this has led to evaluate not only crisp degrees of consensus, but also intermediate degrees between 0 and 1, corresponding to partial agreement among all agents. However, full consensus can be considered not necessarily as a result of unanimous agreement, but it can be obtained ever in the case of agreement among a fuzzy majority of agents (see [9], [10]).

9 False consensus

It is well known, not only in the areas of social sciences, that people are egocentric. As pointed out in several experiments, in a multiagent decision problem each decision maker overestimates his own opinion. Social psychology has founded that people with a certain preference tend to make higher judgements of the popularity of that preference in others, compared to the judgements of those with different preferences. This empirical result has been termed the *false consensus effect* (see [19], [16]). It states that individuals overestimate the number of the people who possess the same attributes as they do. People often believe that others are more like themselves than they really are. Thus, their predictions about others' beliefs or behaviors, based on casual observation, are very likely to err in the direction of their own beliefs or behavior. For example, college students who preferred brown bread estimated that over 50% of all other college students preferred brown bread, while white-bread eaters estimated that 37% showed brown bread preference.

As the consequence, in multi-agent decision problem we often have to deal with different opinions, different importance of criteria and agents, who are not fully impartial objective. In this sense, the false consensus effect produces partial objectivity and incomplete impartiality, which perturbs the agreements over the evaluation.

10 Assessing consensus and false consensus

Different methods to compute a degree of a consensus in fuzzy, or imprecise, environments have been defined, and some approaches have been proposed to measure consensus in the context of fuzzy preference relations (see [9], [10]). However, as we have seen, the false consensus effect can lead to an absence of objectivity in the evaluation process. Then just a numerical indication seems not to be sufficient to synthesize the degree of consensus of agents which incorporate both the true knowledge generated in the agent opinion and the subjective component that produces false consensus outputs. The opinion of each agent is decomposed into two components: a vector, made of the ranking of the alternatives, built by means of a classical procedure, e.g., a hierarchical procedure [14], and a fuzzy component that represents the contribution of the false consensus effect, which we assume to be fuzzy in nature [15]. This allows us to consider aggregation operators, such as OWA operators, useful when synthesis among fuzzy variables is to be built [22].

A formal model considers the set N of decision makers, the set A of the alternatives, and the set C of the criteria. Let any decision maker $I \in N$ be able to assess the relevance of each criterion. Precisely, for every i, a function

$$h_i : C \to [0, 1]$$

with $\sum_{c \in C} h_i(c) = 1$, denoting the evaluation or weight that the decision maker assigns to the criterion c, is defined. Furthermore, the function

$$g_i : A \times C \to [0, 1]$$

is defined, such that $g_i(a, c)$ is the value of the alternative a with respect to the criterion c, in the perspective of i.

Let n, p, and m denote the (positive integer) numbers of the elements of the sets N, C, and A, respectively. The value $h_i(c)_{c \in C}$ denotes the evaluation of the p-tuple of the criteria by the decision maker i and the value $g_i(c, a)_{c \in C, a \in A}$, defines the matrix $p \times m$ whose elements are the evaluations, made by i, of the alternatives with respect to each criterion in C. The function: $A \to [0, 1]$, defined by

$$\big(f_i(a)\big)_{a \in A} = h_i(c)_{c \in C} \cdot g_i(c, a)_{c \in C, a \in A}$$

is the evaluation, made by i, of the alternative $a \in A$.

A Euclidean metric that acts between couples of decision makers i and j, i.e., between individual rankings of alternatives, is defined by

$$d(f_i f_j) = \sqrt{\frac{1}{|A|} \sum_{a \in A} \big(f_i(a) - f_j(a)\big)^2}$$

If the functions h_i, g_i range in $[0, 1]$, then also $0 \le d(f_i f_j) \le 1$.

If we set $\delta^\star = \max\{d(f_i, f_j) | i, j \in N\}$, then a degree of consensus $\delta^\star$ can be defined as the complement to one of the maximum distance between two positions of the agents:

$$\delta^\star = 1 - \delta^\star = 1 - \max\{d(f_i, f_j) | i, j \in N\}$$

Now to identify the portion of the false consensus effect internal to the consensus reaching process, we have to consider a vector that represents the *components of the consensus* $p(a)P + q(a)Q$. This polynomial representation of the measure of the effect is composed by a numeric component $p(a)P$, that contains all quantitative information available derived from the consensus reaching process, and $q(a)Q$ that reflects the false consensus effect. Then the measure of the effect is:

$$q(a) = \frac{1}{N(d^\star)^2} \sum_{i=1}^{N} (f_i - f_j)^2$$

with $0 \le q(a) \le 1$, $\forall i,\ j \in N$.

This component can be estimated by means of OWA operators (a class of decision support tools for providing heuristic solution to situations where several trade-offs should be taken into consideration). In Yager [29] is introduced an approach for multiple criteria aggregation, based on ordered weighted averaging (OWA) operators. By ranking the alternatives, the operators provide an enhanced methodology for evaluating actions on a qualitative basis [22].

11 Study on false consensus effect under varying information conditions. Engelmann and Strobel experiment

In Section 9, false consensus has been defined as an egocentric bias that occurs when people estimate consensus for their own behaviors. The judgements of each agent, indeed, are frequently based, in part, on intuition or subjective beliefs, rather than detailed data on the preferences of the people being predicted. However such intuitive judgements become more pervasive judgements when people lack necessary data to base their judgements. Therefore, according to Dawes (see [6], [7]), classical definition of false consensus does not justify the attribute false. He argues that it is perfectly rational to use the information about one's own decision in the same way as the information about any other randomly selected from a sample. The effect is only false if too much weight is assigned to one's own decision compared to a randomly selected person from the same population. Engelmann and Strobel [8] refer to the effect as defined above as a consensus effect and affirms that people exhibit a false consensus effect if among those with the same total information (i.e. that includes the information about their own decision) the estimates are biased in the direction of their own decision.

To demonstrate this and investigate whether a false consensus effect depends on the cognitive effort needed to retrieve information, Engelmann and Strobel compared two treatments in a simple one-shot experiment.

Results are in opposite direction to a false consensus effect when in a decision group the agents have explicit information about the choice of other members of their own group, while results are in line with a false consensus effect in all groups in which the information were implicit. This shows that most subjects are unwilling or unable to use information that is not handed to them on a silver platter. It appears to us that in the implicit information treatment it does not occur to many subjects that the other subjects' choices are valuable information and that this information is rather easily available, while the prominent information in the explicit information treatments is recognized as valuable information by virtually all subjects (or leads them to unconsciously update their beliefs).

In conclusion, Engelmann and Strobel affirm that there is no false consensus effect if representative information is highly prominent and retrievable without any effort. Indeed, there is even a significant effect in the opposite direction, indicating that subjects consider others' choices as more informative than their own.

12 False consensus effect and emotions in a multiagent decision problem

Multiagent decision problems are characterized by interplay between intertemporal considerations and strategic interactions: two or more agents could have to take a common decision for a future time and in this process they are influenced by emotional signal, which arise with impulsivity and with false or true consensus effect. Theory of games provide tools for describe strategic interaction. Indeed, in non-cooperative interaction each agent makes decisions independently, without collaboration or communication with any of the others. This can be assimilated to situations in which information about decision of other members of decision group is implicit. In this kind of strategic decision the consensus effect is false. As in Engelmann and Strobel experiment, if members of group decision do not cooperate they do not possess information about the choices of others, so the influence of psychological aspects lead to judge others in the same way that they judge themselves. Then two situations are possible:

1) each agent have the same preference and they will reach a common decision that is given by the unanimous choice,

2) the agents have different preferences and do not assign any weight to the other preferences, so it is not possible to aggregate them (see Section 10).

Then the influence of emotions has no negative consequences if the choices of the agents are unanimous, and then the final decision will be also the best decision in the Paretian sense. If this does not happen, it is impossible to achieve a common strategy without arresting impulsivity, and unanimity becomes increasingly difficult to obtain when the number of agents increases.

On the contrary in a cooperative decision problem the influence of false consensus effect is present at period-one, while the loss of self-control of each agent is fought by the imposition of a rule [26]. The rationality of the equilibrium choice of the cooperative game is saved by the possibility of making an arrangement among agents, which represents a pure rule to maintain self-control at later time in Thaler and Shefrin model (Section 7). Moreover with an arrangement the agents have explicit information about the choices of other members, so the lack of false consensus effect is in line with the result of Engelmann and Strobel experiment.

Consider the classic example of coordination game: the battle-of-the sexes. In this game an engaged couple must choose what to do in the evening: the man prefers to watch a baseball game and the women prefers to attend an opera. In terms of utility the payoff for each strategy is:

		Man	
		Opera (O)	Baseball (B)
Woman	Opera (O)	$3, 1$	$0, 0$
	Baseball (B)	$0, 0$	$1, 3$

In the example there are multiple outcomes that are equilibriums: (B, B) and (O, O). However both players would rather do something together than go to separate events, so no single individual has an incentive to deviate if others are conforming to an outcome: the man would attend the opera if he thinks the woman will be there even though he prefers the other equilibrium outcome in which both attend the baseball game.

In this context, a consensus decision making process can be considered as an instrument to choose the best strategy in a coordination game. The final decision is often not the first preference of each individual in the group and they may not even like the final result. But it is a decision to which they all consent because it is the best for the group.

Consequently, a common final decision is achievable only if the man and the woman have explicit information on the decision of other member, then only if there is cooperation.

If the man and the woman do not decide together where spend their time in the evening, probably, the result of implicit information and consequent false consensus effect will be that the man will go to the opera because he thinks that she decides to go there, and the woman will go to the baseball match to meet the man.

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