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Some improved mixed regression estimators and their comparison when disturbance terms follow Multivariate t-distribution

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Abstract

The Mean square error matrices, bias vector and risk functions of proposed improved mixed regression estimators are obtained by employing the small disturbance approximation technique under the condition, when disturbance terms follows multivariate t-distribution. Further, the risk function criterion is used to examine the efficiency of proposed improved mixed regression estimators.
Keywords: Stochastic restrictions; Mixed regression estimator; Stein- rule estimator; Multivariate t-distribution etc.
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1 Introduction

When incomplete prior information is expressible in the form of set of linear stochastic restrictions on the coefficients in a linear regression model, the method of mixed regression for the estimation of regression coefficients provides asymptotically a more efficient estimator than the least squares method that ignores the prior restrictions.
Stemming from the philosophy of stein-rule in this paper we proposed two families of improved estimators for the regression coefficients and study their properties when disturbances have multivariate t-distribution. For multivariate t - distribution see, [12], [10] and [3]. In section 2, we discuss the framework and estimators. The properties of these estimators are presented in section 3 and the results are compared in section 4. Simulation Study is carried out to support theoretical finding in Section 5.

2 Model Specification and the Estimators

Let us postulate the linear regression model

$$Y = X\beta + U \tag{1}$$

Where, Y is a $n \times 1$ vector of dependent variables; X is a $n \times p$ column rank matrix of n-observations on p explanatory non-stochastic variables; β is a $p \times 1$ non-null vector of regression coefficient and U is a $n \times 1$ vector of disturbance following multivariate student t-distribution with probability density function as:

$$f\left(\frac{U}{v}, \sigma^2\right) = \frac{\gamma^{v/2}\Gamma\left(\frac{v+n}{2}\right)}{\pi^{\frac{n}{2}}\Gamma\left(\frac{v}{2}\right)}\sigma^{-n}\left[v + \frac{U'U}{\sigma^2}\right]^{-\frac{n+v}{2}} \tag{2}$$

Where, $v > 0, \sigma > 0$ are respectively the degree of freedom and dispersion parameters; the vector U has its error components $U_i \in (-\infty, \infty), i = 1, 2, ..., n$. Here the error vector U has mean vector $E(U) = 0$ for $v > 1$, variance-covariance matrix $E(U'U) = \sigma^2\left(\frac{v}{v-2}\right)I$, for $v > 2$, measure of skewness $\gamma_1 = 0$ and measure of kurtosis $\gamma_2 = \sigma^4\left(\frac{6}{v-4}\right)I$ for $v > 4$.
Let the stochastic restrictions on β in (1) be

$$r = R\beta + V \tag{3}$$

Where, r is a $J \times 1$ vector of known elements, R is a $J \times p$ full row rank matrix of known elements and V is a $J \times 1$ vector of distribution such that

$$E(V) = 0\,;\; E(V'V) = \Omega \tag{4}$$

Where, Ω is a $J \times J$ positive definite symmetric matrix of known elements.

Further, we assume that the errors associated with the stochastic restriction are independent with the distribution in model (1).
The ordinary least square (OLS) estimator of β that ignores the prior restrictions (3) is

$$b_o = (X'X)^{-1}X'Y \tag{5}$$

If we consider the prior information (3), then the mixed regression (MR) estimator of β is given by

$$b_{MR} = [X'X + S^2R'\Omega^{-1}R]^{-1}[X'Y + S^2R'\Omega^{-1}r] \tag{6}$$

Where ,

$$S^2 = \frac{1}{n-p}((Y - Xb)'(Y - Xb)) \tag{7}$$

The Stein-rule estimator of β is

$$b_s = \left[1 - k\frac{(Y - Xb)'(Y - Xb)}{b_o'(X'X)b_0}\right] b_o \tag{8}$$

Where, k is a positive scalar characterizing the estimator.

The Stein-Mixed Regression (SMR) estimator of β is given as

$$b_{SMR} = \left[X'X + \frac{1}{n-p}[(Y - Xb_S)'(Y - Xb_S)]R'\Omega^{-1}R\right]^{-1} \left[X'Y + \frac{1}{n-p}[(Y - Xb_S)'(Y - Xb_S)]R'\Omega^{-1}r\right] \tag{9}$$

The Mixed Stein-Regression (MSR) estimator of β is

$$b_{MSR} = \left[1 - k\frac{(Y - Xb_{MR})'(Y - Xb_{MR})}{b_{MR}'(X'X)b_{MR}}\right] b_{MR} \tag{10}$$

3 Properties of the Estimators

$$P_X = X(X'X)^{-1}X' \tag{11}$$

$$M = [I - P_X] \tag{12}$$

$$M_j = [P_X - jC^{-1}X\beta\beta'X'] \qquad j = 1, 2, . \tag{13}$$

$$N_j = [(X'X)^{-1} - jC^{-1}\beta\beta'] \qquad j = 1, 2, . \tag{14}$$

$$C = \beta'X'X\beta \tag{15}$$

$$\mu = (X'X)^{-1}R'\Omega^{-1}R(X'X)^{-1} \tag{16}$$

The OLS estimator defined in (5) is found to be unbiased if $v > 1$, with variance - covariance matrix and risk function given by

$$E[(b_0 - \beta)(b_0 - \beta)'] = \sigma^2 \left(\frac{v}{v-2}\right)(X'X)^{-1}; \quad v > 2 \tag{17}$$

$$Risk(b_o) = \sigma^2 \left(\frac{v}{v-2}\right) tr(X'X)^{-1}L; \quad v > 2 \tag{18}$$

Where, L is a positive definite symmetric loss matrix.

The properties of the MR estimator are same as the SMR estimator, so we consider only the SMR estimator and present the results in the form of following theorems.

Theorem 3.1. *The asymptotic expression for the bias vector, mean squared error matrix and risk function of SMR estimator, up to order $o(\sigma^4)$ of approximations are given as*

$$B(b_{SMR}) = 0 \tag{19}$$

$$M(b_{SMR}) = \sigma^2 \left(\frac{v}{v-2}\right)(X'X)^{-1} - \sigma^4 V_1; \quad v > 4 \tag{20}$$

Where,

$$V_1 = \Bigg[\left(1 - \frac{2}{n-p} - \frac{6}{v-4}\theta\right)\mu + \frac{6}{(v-4)(n-p)} \Big(\mu X'(I_n * M)X(X'X)^{-1} + (X'X)^{-1}X'(I_n * M)X\mu\Big)\Bigg] \tag{21}$$

$$\theta = \frac{trM(I_n * M)}{(n-p)^2} \tag{22}$$

$$Risk(b_{SMR}) = \sigma^2 \left(\frac{v}{v-2}\right) tr(X'X)^{-1}L - \sigma^4 trV_1 L \tag{23}$$

Proof 3.1: To employ small disturbances asymptotic approximations. Let us write model (1) as

$$Y = X\beta + \sigma\omega \quad (U = \sigma\omega) \tag{24}$$

So that the i.i.d. elements of ω have multivariate-t distribution with mean zero for $v > 1$, variance $\left(\frac{v}{v-2}\right)$, for $v > 2$, measure of skewness $\gamma_1 = 0$ and measure of kurtosis $\gamma_2 = \left(\frac{6}{v-4}\right)$ for $v > 4$.

Now, using (24) in (5), we find

$$b_0 = \beta + \sigma(X'X)^{-1}X'\omega \tag{25}$$

So that

$$Y - Xb_0 = \sigma M\omega \tag{26}$$

Where

$$M = [I_n - X(X'X)^{-1}X'] \tag{27}$$

Using (25), we find up to order $o(\sigma)$ of approximations.

$$\frac{1}{b_o'(X'X)b_0} = C^{-1}[1 - 2\sigma C^{-1}\beta' D'\omega] \tag{28}$$

Now, using (25), (26), and (28) in (8), we get up to order $o(\sigma^2)$ of approximations.

$$b_s - \beta = \sigma(X'X)^{-1}X'\omega - \sigma^2 k\omega' M\omega C^{-1}\beta \tag{29}$$

and for the same order of approximation, we have

$$Y - Xb_s = \sigma M\omega - \sigma^2 k\omega' M\omega C^{-1}X\beta \tag{30}$$

Thus, using (30) and (3) in (9), we get

$$b_{SMR} - \beta = \sigma h_1 + \sigma^2 h_2 + \sigma^3 h_3 + \sigma^4 h_4 \tag{31}$$

Here,

$$h_1 = (X'X)^{-1}X'\omega \tag{32}$$

$$h_2 = \left(\frac{\omega' M\omega}{n-p}\right)(X'X)^{-1}R'\Omega^{-1}V \tag{33}$$

$$h_3 = \left(\frac{\omega' M\omega}{T-G}\right)\mu X'\omega \tag{34}$$

$$h_4 = \left(\frac{\omega' M\omega}{n-p}\right)^2 [k^2(n-p)C^{-1}(X'X)^{-1}R'\Omega^{-1}V - \mu R'\Omega^{-1}V] \tag{35}$$

It is easy to see that

$$E(h_1) = E(h_2) = E(h_3) = E(h_4) = 0 \tag{36}$$

Utilizing (36) in (31), we obtain the result (19) of the Theorem 1.

Now using (31), we get

$$(b_{SMR} - \beta)(b_{SMR} - \beta)' = \sigma^2 h_1 h_1' + \sigma^3(h_1 h_2' + h_2 h_1') + \sigma^4(h_1 h_3' + h_2 h_2' + h_3 h_1') \tag{37}$$

Here,

$$E(h_1 h_1') = (X'X)^{-1} \tag{38}$$

$$E(h_1 h_2') = E(h_2 h_1') = 0 \tag{39}$$

$$E(h_1 h_3') = \frac{1}{n-p}\left[\frac{6}{v-4}(X'X)^{-1}X'(I_n * M)X\mu + (n-p)\mu\right] \tag{40}$$

$$E(h_2 h_2') = \left[\left(\frac{6}{v-4}\right)\theta + \left(\frac{n-p+2}{n-p}\right)\right]\mu \tag{41}$$

Utilizing (38), (39), (40) and (41) in (37), we obtain the result (20) of the **Theorem 1**.

$$Risk(b_{SMR}) = trM(b_{SMR})L \tag{42}$$

Thus, result (23) of the Theorem 1 follows from (42).

Theorem 3.2. *The asymptotic expression for bias vector, mean squared error matrix and risk function of MSR estimator, up to order $o(\sigma^4)$ of approximations are given as*

$$B(b_{MSR}) = -\sigma^2 \frac{kv(n-p)}{v-2}C^{-1}\beta + \sigma^4\Big[\frac{6k}{v-4}C^{-2} \Big((trM_4(I_n * M))I + 2(X'X)^{-1}X'(I_n * M)X - C\theta\mu(X'X)\Big)\beta + kC^{-2}\Big((n-p)(p-2)I - \frac{n-p+2}{n-p}C\mu(X'X)\Big)\beta\Big] \tag{43}$$

Where $*$ *denotes Hadamard product.*

$$M(b_{MSR}) = \sigma^2 \left(\frac{v}{v-2}\right)(X'X)^{-1} - \sigma^4\Big[V_1 + \frac{12k}{v-4}C^{-1} \Big[(X'X)^{-1}X'(I_n * M)X(X'X)^{-1} - C^{-1}\Big((X'X)^{-1}X'(I_n * M)X\beta\beta' + \beta\beta'X'(I_n * M)X(X'X)^{-1} + \left(\frac{k}{2}\right)(trM(I_n * M))\beta\beta'\Big)\Big] + 2k(n-p)N_{(2+\frac{k}{2}(n-p+2))}\Big] \quad (44)$$

$$Risk(b_{MSR}) = \sigma^2 \left(\frac{v}{v-2}\right) tr(X'X)^{-1}L - \sigma^4\Big[trV_1L + 12\frac{k}{v-4}C^{-1}\Big(tr(X'X)^{-1}X'(I_n * M)X(X'X)^{-1}L - C^{-1}\Big(2\beta'X'(I_n * M)X(X'X)^{-1}L\beta + \frac{k}{2}(trM(I_n * M))\beta'L\beta\Big)\Big) + 2k(n-p)trN_{(2+\frac{k}{2}(n-p+2))}L\Big] \quad (45)$$

Proof 3.2: Using (3), (24) and (26) in (6), we obtain up to order $o(\sigma^2)$ of approximations.

$$b_{MR} = \beta + \sigma(X'X)^{-1}X'\omega + \sigma^2\left(\frac{\omega'M\omega}{n-p}\right)(X'X)^{-1}R'\Omega^{-1}V \quad (46)$$

Thus, for the same order of approximation, we have

$$\frac{1}{b'_{MR}(X'X)b_{MR}} = C^{-1}\Big[1 - 2\sigma C^{-1}\beta'X'\omega - \sigma^2C^{-1}\Big(\frac{2}{n-p}\omega'M\omega V'\Omega^{-1}R\beta + \omega'M_D\omega\Big)\Big] \quad (47)$$

Using (46), we get up to order $o(\sigma^2)$of approximations.

$$Y - Xb_{MR} = \sigma M\omega - \sigma^2\left(\frac{\omega'M\omega}{n-p}\right)X(X'X)^{-1}R'\Omega^{-1}V \quad (48)$$

Using (46), (47) and (48) in (10), we obtain up to order $o(\sigma^4)$, we get

$$b_{MSR} - \beta = \sigma h_1^* + \sigma^2 h_2^* + \sigma^3 h_3^* + \sigma^4 h_4^* \quad (49)$$

Where

$$h_1^* = (X'X)^{-1}X'\omega \quad (50)$$

$$h_2^* = \left(\frac{\omega' M\omega}{n-p}\right)\left[(X'X)^{-1}R'\Omega^{-1}V - kC^{-1}\beta\right] \tag{51}$$

$$h_3^* = -\left(\frac{\omega' M\omega}{n-p}\right)\left[\mu + k(n-p)C^{-1}N_2)\right]X'\omega \tag{52}$$

$$\begin{aligned} h_4^* = k(\omega' M\omega)C^{-2}\Big(&\frac{2}{n-p}\omega' M\omega\beta\beta' R'\Omega^{-1}V + \omega' M_4\omega\beta \\ &+ 2(X'X)^{-1}X'\omega\omega' X\beta\Big) - \left(\frac{\omega' M\omega}{n-p}\right)^2 \\ &\left[\mu R'\Omega^{-1}V + kC^{-1}(\beta' V'\Omega^{-1}R + (n-p)I)(X'X)^{-1}R'\Omega^{-1}V\right] \end{aligned} \tag{53}$$

Here, it is easy to verify that

$$E(h_1^*) = 0 \tag{54}$$

$$E(h_2^*) = -k(n-p)C^{-1}\beta \tag{55}$$

$$E(h_3^*) = 0 \tag{56}$$

$$\begin{aligned} E(h_4^*) = &\frac{6k}{v-4}C^{-2}\Big[(trM_4(I_n * M))I + 2(X'X)^{-1}X'(I_n * M)X \\ &- C\theta\mu(X'X)\Big]\beta + kC^{-2}\Big[(n-p)(p-2)I - \left(\frac{n-p+2}{n-p}\right)C\mu(X'X)\Big]\beta \end{aligned} \tag{57}$$

Utilizing (54), (55), (56) and (57) in (53), we obtain the result (43) of the Theorem 2.
Now, using (53) we get

$$\begin{aligned} (b_{MSR} - \beta)(b_{MSR} - \beta)' = &\sigma^2 h_1^* h_1^{*'} + \sigma^3(h_1^* h_2^{*'} + h_2^* h_1^{*'}) \\ &+ \sigma^4(h_1^* h_1^{*'} + h_2^* h_2^{*'} + h_3^* h_1^{*'}) \end{aligned} \tag{58}$$

Here, we see that

$$E(h_1^* h_1^{*'}) = \left(\frac{v}{v-2}\right)(X'X)^{-1} \tag{59}$$

$$E(h_1^* h_2^{*'}) = 0 \tag{60}$$

$$\begin{aligned} E(h_1^* h_3^{*'}) = &\frac{6}{(v-4)(n-p)}\Big[(X'X)^{-1}X'(I_n * M)X\mu \\ &+ k(n-p)C^{-1}(X'X)^{-1}X'(I_n * M)XN_2\Big] - \mu - k(n-p)C^{-1}N_2 \end{aligned} \tag{61}$$

$$E(h_2^* h_2^{*'}) = \mu \left[\frac{6}{v-4}\theta + \left(\frac{n-p+2}{n-p} \right) I \right] + k^2 C^{-2} \beta\beta' \Big[\frac{6}{v-4} trM(I_n * M) + (n-p)(n-p+2) \Big] \quad (62)$$

Utilizing (59), (60), (61) and (62) in (58), we obtain the result (44) of the Theorem 2. Similarly, we can obtain the result (45) of the **Theorem 2**.

4 Comparison of the Estimators

4.1 The comparison the risk functions of OLS and SMR estimators

On comparison the risk functions of OLS and SMR estimators. We observe that up to order $o(\sigma^2)$ of approximations, both the estimators have same risk and for higher order of approximation, we see that

$$Risk(b_0) - Risk(b_{SMR}) = \sigma^4 \Big[\frac{6}{v-2} \Big(\frac{2}{n-p} tr(X'X)^{-1} X'(I_n * M) X \mu L - \theta tr \mu L \Big) + \Big(\frac{n-p-2}{n-p} \Big) tr \mu L \Big] \quad (63)$$

If we choose $L = (X'X)$, then expression (63) becomes

$$Risk(b_0) - Risk(b_{SMR}) = \sigma^4 \Big[\frac{6}{v-2} \Big(\frac{2}{n-p} tr(X'X)^{-1} X'(I_n * M) X (X'X)^{-1} R'\Omega^{-1} R - \theta tr(X'X)^{-1} R'\Omega^{-1} R \Big) + \left(\frac{n-p-2}{n-p} \right) tr(X'X)^{-1} R'\Omega^{-1} R \Big] \quad (64)$$

Since, the expression (64) is positive semi-definite, so b_{SMR} dominates b_0 and as $v \to \infty$, expression (64) reduces to

$$Risk(b_0) - Risk(b_{SMR}) = \sigma^4 \left(\frac{n-p-2}{n-p} \right) tr(X'X)^{-1} R'\Omega^{-1} R \quad (65)$$

Which is positive semi-definite. Thus, b_{SMR} dominates b_0, so long as $n-p > 2$.

4.2 The comparison the risk functions of OLS and MSR estimators

On comparison the risks of OLS and MSR, we see that up to order $o(\sigma^2)$ of approximations, both the estimators have same risk and for higher order of approximations, we find that b_{MSR} dominates b_o so long as (65) is positive semi-definite and if we choose k to satisfy,

$$0 < k < \frac{2(n-p)}{T}\frac{C}{\beta' A\beta}\Big[tr(X'X)^{-1}L - 2C^{-1}\beta' L\beta + \frac{6}{(n-p)(v-4)} \Big(tr(X'X)^{-1}X'(I_n * M)X(X'X)^{-1}L - 2C^{-1}\beta' X'(I_n * M)X(X'X)^{-1}L\beta\Big)\Big] \quad (66)$$

Where

$$T = \left[\frac{6}{v-4}(trM(I_n * M)) + (n-p)(n-p+2)\right] \quad (67)$$

If we choose $L = (X'X)$, then the above condition of dominance becomes

$$0 < k < \frac{2(n-p)}{T}\Big[p - 2 + \frac{6}{(n-p)(v-4)}\Big(tr(X'X)^{-1}X'(I_n * M)X - 2C^{-1}\beta' X'(I_n * M)X\beta\Big)\Big] \quad (68)$$

And as $v \to \infty$, condition (68) reduces to

$$0 < k < \frac{2}{n-p+2}(p-2); p > 2 \quad (69)$$

Which is well known condition of dominance of stein-rule estimator over the least squares estimator.

4.3 The comparison the risk functions of SMR and MSR estimators

On comparing the risk function associated with the estimators SMR and MSR respectively, we observe that the estimator MSR dominates the estimator SMR so long as (30)holds and as $v \to \infty$ and again by choosing $L = (X'X)$, the condition of dominance becomes (69).

5 Simulation Results

The proposed estimator b_{SMR} is more efficient than OLSE under given linear model. Although, theoretically the results are drawn in equation (65), the proposed Stein-mixed Regression (SMR) estimator b_{SMR} is more efficient than ordinary least square estimator b_0 under condition $n - p > 2$. In this section, we perform simulations for exact equation (65) under conditions $n-p > 2, n > p > j$ with sigma equal to one.

Each result is based on 100,000 simulations runs using MATLAB. The result shown for $n = 10, 11, 12, 13, 14, 15$ in Table 1, 2, 3 & 4. The main finding of our numerical evaluation is following:-

1. The simulation results strongly support the theoretical findings.
2. The simulation result also explains the strength keep on increasing as we go for large value of n, p and j.
3. The results are independent of value of sigma.
4. Hence, b_{SMR} is more efficient than b_0 under condition $n - p > 2$.
5. The simulation results also reveals that b_{MSR} is also more efficient over b_0 (as it also depends on (65) under condition at (69)).

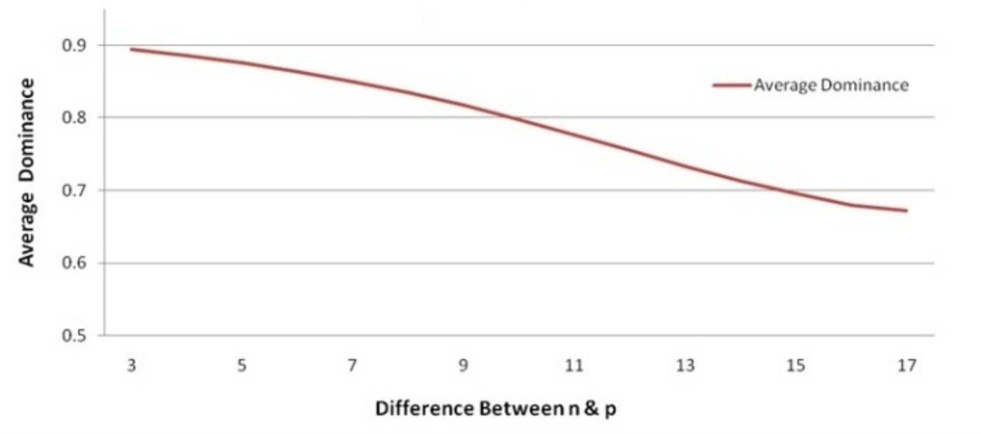

Figure 1: Average dominance condition for difference between n & p

Based on simulation study, the dominance of b_{SMR} has been proven over b_0 under certain set of conditions. Further, the behavior of dominance is studied for various combination of different values of n, p and j. The average dominance is derived based on probability for different combination of n, p and j; when $\sigma = 1$. The *figure 1* depicts average dominance keeps on decreasing with increase in gap

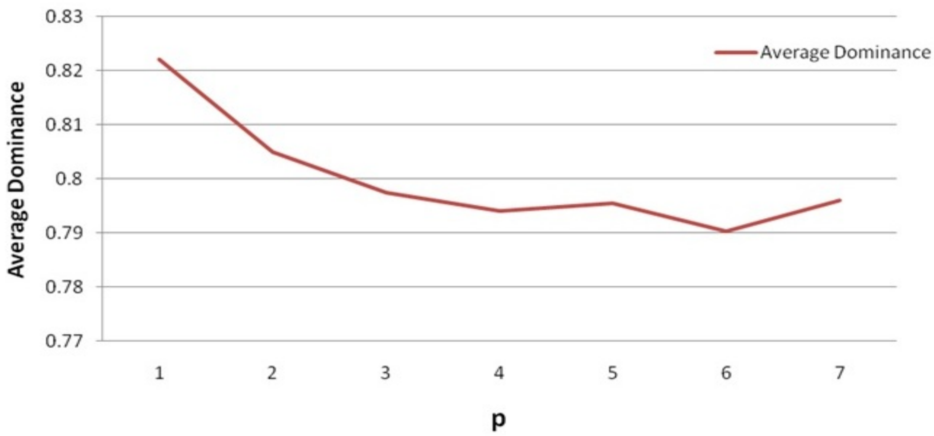

Figure 2: Dominance behavior for different values of p; when n=10 & j=5

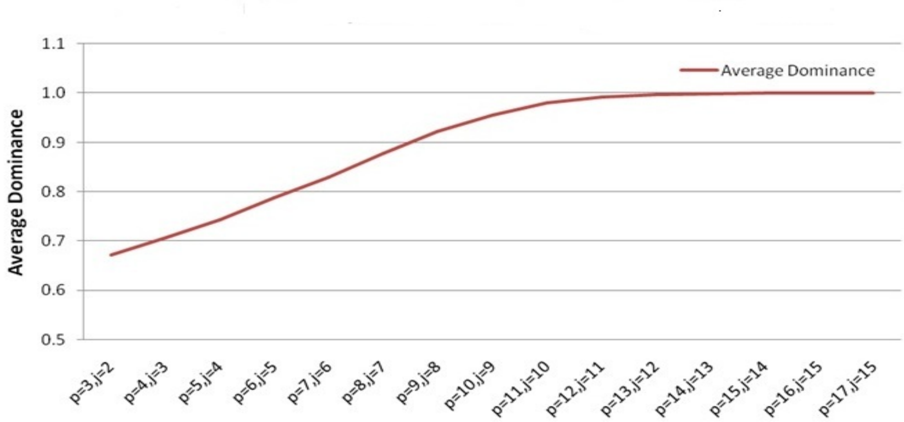

Figure 3: Average dominance condition for given value of p & j for n=20

between n and p. The *figure 2* also depicts a decreasing trend with increase in value of p, when $n = 10$ and $j = 5$. Similarly, *figure 3* shows the behavior of dominance condition for different value of p and j for fixed value of n equal to 20.

Table 1: Average Value of Dominance for different values of n and p for $j = 2$ $sigma = 1$.

j=2	n=10	n=11	n=12	n=13	n=14	n=15
p=3	0.67823	0.67492	0.67793	0.67688	0.67390	0.67539
p=4	0.67300	0.67079	0.66877	0.66903	0.66816	0.66654
p=5	0.67187	0.66562	0.66558	0.66668	0.66416	0.66691
p=6	0.66660	0.66642	0.66755	0.66509	0.66370	0.66186
p=7	0.66755	0.66497	0.66408	0.65994	0.66305	0.65982
p=8	-	0.66816	0.66611	0.66010	0.66261	0.66156
p=9	-	-	0.66861	0.66352	0.66143	0.66036

Remark: No value for dominance where $n - p \leqslant 2$.

Table 2: Average Value of Dominance for different values of n and p for $j = 3$ $sigma = 1$.

j=3	n=10	n=11	n=12	n=13	n=14	n=15
p= 4	0.70809	0.70806	0.70722	0.70725	0.70806	0.70611
p= 5	0.70570	0.70449	0.70581	0.70385	0.70381	0.70162
p= 6	0.70612	0.70319	0.70252	0.69998	0.70281	0.70205
p= 7	0.70651	0.70266	0.70281	0.70253	0.69810	0.69816
p= 8	-	0.70735	0.70167	0.70011	0.70183	0.69960
p= 9	-	-	0.70592	0.70486	0.70001	0.70028
p=10	-	-	-	0.70784	0.70426	0.70033
p=11	-	-	-	-	0.70692	0.70500

Remark: No value for dominance where $n - p \leqslant 2$.

Table 3: Average Value of Dominance for different values of n and p for $j = 4$ $sigma = 1$.

j=4	n=10	n=11	n=12	n=13	n=14	n=15
p= 5	0.74738	0.74891	0.74713	0.74538	0.74603	0.74668
p= 6	0.74694	0.74586	0.74771	0.74337	0.74335	0.74346
p= 7	0.75060	0.74841	0.74539	0.74412	0.74366	0.74381
p= 8	-	0.74740	0.74561	0.74405	0.74213	0.74607
p= 9	-	-	0.74752	0.74623	0.74547	0.74207
p=10	-	-	-	0.74601	0.74774	0.74091
p=11	-	-	-	-	0.74832	0.74426
p=12	-	-	-	-	-	0.74612

Remark: No value for dominance where $n - p \leqslant 2$.

Table 4: Average Value of Dominance for different values of n and p for $j = 5$ $sigma = 1$.

j=5	n=20	n=25	n=30	n=35	n=40	n=45	n=50
p = 5	0.78748	0.78793	0.78713	0.78373	0.78562	0.78336	0.78686
p=10	0.78359	0.78228	0.78284	0.78158	0.77898	0.77644	0.78005
p=15	0.78586	0.78143	0.77689	0.78072	0.77643	0.77454	0.77655
p=20	-	0.78350	0.78017	0.77976	0.77657	0.77576	0.77628
p=25	-	-	0.78453	0.78062	0.77884	0.77852	0.77563
p=30	-	-	-	0.78408	0.78044	0.77743	0.77703
p=35	-	-	-	-	0.78478	0.77772	0.77486
p=40	-	-	-	-	-	0.78208	0.77728

Remark: No value for dominance where $n - p \leqslant 2$.

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