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# NON LINEAR EVOLUTIVE MODELS FOR A STOCK-MARKET WITH VARIABLE FUNDAMENTAL VALUES AND ASYMMETRY OF INFORMATION[1]

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## 1. INTRODUZIONE

There has recently been an increased interest of the economic as well as financial literature in the laws governing the functioning of stock-markets.

The actual performance of stock-markets has pointed to the fact that many of the existing theoretical models, which are typically based on efficiency and other idealistic hypotheses, are insatisfactory when confronted with empirical reality. This is particularly true for thin markets and in the presence of information asymmetries.

Lately, several authors [8], [9], [14], [15], [6], [7], [4] have advanced an alternative approach dealing with simplified situations and suggesting models for a market with asymmetries of information and "irrationality".

They have studied a market where just one type of financial asset is traded and where the agents involved, although all being "fundamentalists", may be heterogeneus due to differences in information, possibilities of intervention and objectives.

In this paper the models advanced in [4] are generalized in that we abandon some of their linearity assumptions, and we investigate equilibrium and its stability.

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In this case the systems describing the market evolution consist of quadratic nonlinear difference equations.

Moreover the assumptions made on the behavioural functions are very general, such as monotonicity and the passage through (0,0). The conclusions reached about the solutions of equilibrium and their stability are, accordingly, valid for a large class of evolutive models.

In order to explain our presentation, we give the basic hypotheses and the notations used in § 2. In § 3, we study the market equilibrium and its stability within a given session.

In § 4, we suggest a model for the market evolution frcm one session to the other, and we investigate the equilibrium and the conditions for its stability. In § 5, we examine the consequences of the imitative behaviour on the suggested models.

## 2. BASIC HYPOTHESES AND NOTATIONS

A detailed analisys of the laws governing a market where not all the brockers can negotiate in real time can be carried out only by suggesting two different evolutive models: one which deals with the behaviour of agents within a session, and another one which treats the behaviour from one session to the next.

We denote with T=1, ..., n the index of each session and $t_n(T)$, $n \in \aleph$, the sequence of the calls in the T-th session.

Moreover we assume that agents can be partitioned into three classes. Each class includes agents who are homogeneus both for information received and for their possibilities of intervention in the market.

Non professional brokers (NP) constitute the first class. They can give irrevocable orders only at the beginning of each session; these orders are based on information from previous sessions. NPs have no direct access to negotiations.

Professional agents (P) form the second class and pass on the orders received from NPs. Since they receive informations regarding the ongoing session, they can negotiate on their own behalf when they think it convenient.

The last class of agents is made up by specialists (S); they can get confidential informations and can price stocks on the basis of their fundamental values, as well as on the basis of the market trend. Since all agents within one class are homogeneous, it is not too restrictive to assume that each class consists of one representative agent only.

Our notations will be the following:
- Q(T) the orders given by NPs in the T-th session
- B(T) the orders given by the Ps in the T-th session
- r(T) the fundamental value given to a stock by the NP at time T
- h(T) the fundamental value given to a stock by the P agent in the T-th session

- k(T) the fundamental value given to a stock by S at time T
- P(T) the price of a stock at the end of the T-th session.

The symbols $h_n(T)$, $k_n(T)$ e $P_n(T)$ stand for the values reached by the corresponding quantities at the n-th call in the T-th session; $B_n(T)$ indicates the orders given by the P agent up to the n-th call in the T-th session.

Assuming that the length of each session is constant and equal to D, we get the following relations

$$\lim_{n\to\infty} t_n = D, \quad \lim_{n\to\infty} B_n(T) = B(T), \quad \lim_{n\to\infty} h_n(T) = h(T),$$
$$\lim_{n\to\infty} k_n(T) = k_n, \quad \lim_{n\to\infty} P_n(T) = P(T).$$

## 3. MARKET EVOLUTION WITHIN A SESSION

The agents negotiating directly within a session are called P and S. P gives orders according to being a fundamentalist; his transactions at the n-th call can thus be described by

$$B_n(T)-B_{n-1}(T)=-\beta[P_{n-1}(T)-h_n(T)] \qquad (3.1)$$

were $\beta(\bullet)$ is a stricly increasing function, with $\beta(0)=0$.

Contrary to what been assumed in [8], [9], [14], [15], [6], [7], we suppose here that the fundamental value given by P be variable within a session, according to

$$h_n(T)=h_{n-1}(T)+\delta[B_{n-1}(T)-B_{n-2}(T)] \qquad (3.2)$$

where $\delta(\bullet)$ is a stricly increasing function, with $\delta(0)=0$.

Such a hypothesis allows P to change his mind according to the stock exchange trend within the session. The relationship connecting the stock price fixed by the S at the n-th call with the stock fundamental value and the market trend is

$$P_n(T)=k_n(T)+\eta[Q(T)+B_n(T)] \qquad (3.3)$$

where $\eta(\bullet)$ is a stricly increasing function, with $\eta(0)=0$.

Moreover we assume that the fundamental value fixed by the S may change in a session because of factors outside the market, such as confidential information regarding the stock at issue. This value can be expressed by means of

$$k_n(T)=f[k_{n-1}(T)] \qquad (3.4)$$

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where $f(\bullet)$ is a nonnegative function.

Delating the dependence on T when not essential, the system governing the market trend in the T-th session in the following

$$
\begin{cases}
B_n = B_{n-1} - \beta[P_{n-1} - h_n] \\
P_n = k_n + \eta[Q(T) + B_n] \\
h_n = h_{n-1} + \delta(B_{n-1} - B_{n-2}) \\
k_n = f(k_{n-1})
\end{cases}
\tag{3.5}
$$

The initial values for the beginning of the session are

$$
B_0(T) = 0, \qquad h_0(T) = h_0 \qquad P_0(T) = k_0 + \eta[Q(T)]
$$

where $h_0$ e $k_0$ are known.

## 3.1

We define the system to be in equilibrium if $B^*, h^*, k^*, P^*$ are solutions of

$$
\begin{cases}
-\beta[P^* - h^*] = 0 \\
P^* = k^* = \eta[Q(T) + B^*] \\
k^* = f(k^*)
\end{cases}
\tag{3.6}
$$

If $f(\bullet)$ fulfils the Lipschitz condition with the constant $L \in (0,1)$ there is one solution only for (3.6) given by:

$$
B^* = \eta^{-1}[P^* - k^*] - Q(T)
$$
$$
h^* = P^*
$$
$$
k^* = f(k^*)
$$

From the (3.5), we obtain

$$
\begin{cases}
B_n = B_{n-1} - \beta[P_{n-1} - h_{n-1} + \delta(B_{n-1} - Z_{n-1})] \\
h_n = h_{n-1} - \delta(B_{n-1} - Z_{n-1}) \\
P_n = k_n + \eta\big[Q(T) + B_{n-1} - \beta[P_{n-1} - h_{n-1} + \delta(B_{n-1} - Z_{n-1})]\big] \\
k_n = f(k_{n-1}) \\
Z_n = B_{n-1}
\end{cases}
\tag{3.7}
$$

Using vector notation allows to write the system as follows

$$y_n = G(y_{n-1}) \tag{3.8}$$

where

$$y_n = \begin{pmatrix} B_n \\ h_n \\ P_n \\ k_n \\ Z_n \end{pmatrix} \text{ and } G: \mathfrak{R}^5 \to \mathfrak{R}^5.$$

At this point local stability of the equilibrium in $(y^*)=(B^*, h^*, P^*, K^*, Z^*)$ can be asserted if the linear approximation of G in $y^*$ is globally stable [13]. If we call A the Jacobian matrix of G evaluated at $y^*$ we get

$$A = \begin{bmatrix} 1-\beta'\delta' & \beta' & -\beta' & 0 & +\beta'\delta' \\ -\delta' & 1 & 0 & 0 & +\delta' \\ \eta'(\cdot)(1-\beta'\delta') & \eta'(\cdot)\beta' & -\eta'(\cdot)\beta' & f'(k^*) & \eta'(\cdot)\beta'\delta' \\ 0 & 0 & 0 & f'(k^*) & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.9}$$

where we have assumed $\eta'(\cdot)=\eta'(Q(T)+B^*)$, $\beta'=\beta'(0)$ and $\delta'=\delta'(0)$ for the sake of brevity.

The eigenvalues of A are

$\lambda=0$ with multiplicity two

$\lambda=f'(k^*)$

$\lambda=1$

$\lambda=1+\beta'(0)[-\delta'(0)-\eta'(Q(T)+B^*)]$.

Considered that the necessary and sufficient condition for stability, namely $|\lambda|<1$, is not verified, we can conclude that the equilibrium found is not locally stable. It is known, however, that, since $\lambda=1$ is the simple root of the characteristic equation of A, in case that

$$|f'(k^*)|<1 \quad \text{and} \quad \beta'(0) [\delta'(0)+\eta'(Q(T)+B^*)]<2 \tag{3.10}$$

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the linear approximation of G is stable at y* in the sense of Ljapounov. For this reason, if the equilibrium found satisfied (3.10), it will be locally stable in the sense of Ljapounov.

# 4. MARKET EVOLUTION FROM ONE SESSION TO THE OTHER

The dynamic evolution of the market from one sssion to the other results from the equations governing the behaviour of each representative agent.

The NP agent behaves as a fundamentalist and in the T-th session he will negotiate a quantity of stocks equal to

$$Q(T) = -\alpha[P(T-1)-r(T)] \tag{4.1}$$

where $\alpha(\bullet)$ is a strictly increasing function, with $\alpha(0)=0$. The transactions within the same session of agent P are given by

$$B(T) = -\beta[P(T-1)-h(T)]-Q(T) \tag{4.2}$$

where $\beta(\bullet)$ is a strictly increasing function passing through (0,0).

Note that the two terms in the second member of (4.2) indicate, respectively, the orders carried out autonomously and the orders carried out on the NP's behalf.

In addition we assume that the NP's and P's fundamental values can vary according to the outcomes regarding the market trend in the last two sessions.

As a consequence we obtain the following relations

$$r(T) = (1-\omega)r(T-1)+\omega P(T-1)+\gamma[\Delta P(T-1)] \tag{4.3}$$

$$h(T) = h(T-1)-\Omega[Q(T-1)+B(T-1)]-\varphi[\Delta Q(T-1)+\Delta B(T-1)] \tag{4.4}$$

where

$$\Delta P(T-1) = P(T-1)-P(T-2)$$
$$\Delta Q(T-1) = Q(T-1)-Q(T-2)$$
$$\Delta B(T-1) = B(T-1)-B(T-2)$$

and where $\gamma(\bullet)$, $\Omega(\bullet)$ and $\varphi(\bullet)$ are strictly increasing functions passing through (0,0) and $\omega \in (0,1)$ is a parameter.

Analogously to what we have learned from the analysis regarding a single session, also in the present case the specialist's price and fundamental value

vary according to

$$P(T) = k(T)+\eta[Q(T)+B(T)] \tag{4.5}$$

$$k(T) = f[k(T-1)] \tag{4.6}$$

where $\eta(\cdot)$ is again a strictly increasing function, with $\eta(0)=0$ and $f(\cdot)$ is a nonnegative function.

The market evolution from one session to the other is therefore governed by the following system

$$\begin{cases} Q(T)=-\alpha[P(T-1)-r(T)] \\ B(T)=-\beta[P(T-1)-h(T)]-Q(T) \\ P(T)=k(T)+\eta[Q(T)+B(T)] \\ r(T)=(1-\omega)r(T-1)+\omega P(T-1)+\gamma(\Delta P) \\ h(T)=h(T-1)-\Omega[Q(T-1)+B(T-1)]-\varphi(\Delta Q+\Delta B) \\ k(T)=f[k(T-1)] \end{cases} \tag{4.7}$$

## 4.1

A market is said to be in equilibrium if the values $Q^*, B^*, P^*, r^*, h^*, k^*$ are solutions of

$$\begin{cases} Q^*=-\alpha(P^*-r^*) \\ B^*=-\beta(P^*-h^*)-Q^* \\ P^*=k^*+\eta(Q^*+B^*) \\ r^*=(1-\omega)r^*+\omega P^* \\ h^*=h^*-\Omega[Q^*+B^*] \\ k^*=f(k^*) \end{cases} \tag{4.8}$$

If $f(\cdot)$ is Lipschitzian with constant $L\in(0,1)$, equilibrium is unique and given by

$$P^* = k^* = h^* = r^* \qquad B^* = Q^* = 0 \tag{4.9}$$

Therefore, the long run equilibrium is characterized by common views of the three agents, regarding the fundamental value, by its coincidence with the

price, and by the absence of transactions.
Introducing auxiliary variables

$$z(T) = P(T-1)$$
$$w(T) = B(T-1)$$
$$v(T) = Q(T-1)$$

the sistem (4.7) can be written equivalently as

$$X(T) = G(X(T-1)) \qquad (4.10)$$

where $(X(T))' = (Q(T), B(T), P(T), r(T), h(T), k(T), z(T), w(T), v(T))$ and $G{:}\mathfrak{R}^9 \to \mathfrak{R}^9$ and where the apex indicates the transposition operation.

Also in this case the local stability of the equilibrium given by (4.5) is guaranteed by the stability of the linear approximation of G at $(X^*)' = 0, 0, k^*$, $k^*$ ,$k^*$ ,$k^*$ ,$k^*$ ,$0$ ,$0$).

If we call B the Jacobian matrix of G evaluated at $X^*$, we get:

$$B = \begin{bmatrix} 0 & 0 & -\alpha(1-\omega-\gamma') & \alpha'(1-\omega) & 0 & 0 & -\alpha'\gamma' & 0 \\ -\beta'(\Omega'+\varphi') & -\beta'(\Omega'+\varphi') & \alpha'(1-\omega-\gamma')-\beta' & -\alpha'(1-\omega) & \beta' & 0 & \alpha'\gamma' & \beta'\varphi' \\ -\beta'\eta'(\Omega'+\varphi') & -\beta'\eta'(\Omega'+\varphi') & -\eta'\beta' & 0 & \eta'\beta' & f'(k^*) & 0 & \eta'\beta'\varphi' \\ 0 & 0 & \gamma'+\omega & 1-\omega & 0 & 0 & -\gamma' & 0 \\ -(\Omega'+\varphi') & -(\Omega'+\varphi') & 0 & 0 & 1 & 0 & 0 & \varphi' \\ 0 & 0 & 0 & 0 & 0 & f'(k^*) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (4.11)$$

where, for the sake of brevity, we have written $\beta'=\beta'(0)$, $\Omega'=\Omega'(0)$, $\varphi'=\varphi'(0)$, $\gamma'=\gamma'(0)$, $\alpha'=\alpha'(0)$.

The eigenlavues of B are real equal to

$\lambda = 0$ with multiplicity five

$\lambda = f'(k^*)$

$\lambda = 1-\omega$

$$\lambda = \left[1 - \beta'(0)[\eta'(0) + \Omega'(0) + \varphi'(0)] \pm \right.$$

$$\left. \pm \sqrt{\{1 - \beta'(0)[\eta'(0) + \Omega'(0) + \varphi'(0)]\}^2 + 4\beta'(0)(\eta'(0) + \varphi'(0))} \right] / 2$$

If the conditions

$$|f'(k^*)| < 1, \quad 0 < \omega < 2, \quad \beta'(0)[\Omega'(0) + 2\varphi'(0) + 2\eta'(0)] < 2 \qquad (4.12)$$

are satisfied, (4.9) gives a locally stable equilibrium solution.

The first two relations in (4.12) are verified because of the hypotheses on $\omega$ and $f(\bullet)$. The third relation allows the following interpretation: in order to get a locally stable equilibrium, the P agent's behaviour must not be random; rather, if the pays great attention to small variations in the market's performance, he must intervene with autonomous, but no too large, transactions.

## 5. IMITATIVE BEHAVIOUR

The fact that we have assumed that agents are subject to asymmetric information justifies "irrational" behaviour in the sense that the less informed agents try to imitative those better informed.

We will consider here specifically two types of imitative behaviours:
i) imitation of P agents by NP agents
ii) imitation of P by NP agents, and imitation of NP agents among each other.

### 5.1

The first type of imitation still allows to identify the three classes of agents (NP, P, S) by means of a single representative agent.

The NP agent's transactions are influenced by P's behaviours in the previous session; therefore

$$Q(T) = -\alpha[P(T-1)-r(T)]+\mu[B(T-1)] \qquad (5.1)$$

price, and by the absence of transactions.

Introducing auxiliary variables

$$z(T) = P(T-1)$$
$$w(T) = B(T-1)$$
$$v(T) = Q(T-1)$$

the sistem (4.7) can be written equivalently as

$$X(T) = G(X(T-1))$$
(4.10)

where $(X(T))' = (Q(T), B(T), P(T), r(T), h(T), k(T), z(T), w(T), v(T))$ and $G:\mathfrak{R}^9$ $\to\mathfrak{R}^9$ and where the apex indicates the transposition operation.

Also in this case the local stability of the equilibrium given by (4.5) is guaranteed by the stability of the linear approximation of G at $(X^*)' = 0, 0, k^*,$ $k^*, k^*, k^*, k^*, 0, 0)$.

If we call B the Jacobian matrix of G evaluated at $X^*$, we get:

$$B = \begin{bmatrix}
0 & 0 & -\alpha(1-\omega-\gamma') & \alpha'(1-\omega) & 0 & 0 & -\alpha'\gamma' & 0 \\
-\beta'(\Omega'+\varphi') & -\beta'(\Omega'+\varphi') & \alpha'(1-\omega-\gamma')-\beta' & -\alpha'(1-\omega) & \beta' & 0 & \alpha'\gamma' & \beta'\varphi' \\
-\beta'\eta'(\Omega'+\varphi') & -\beta'\eta'(\Omega'+\varphi') & -\eta'\beta' & 0 & \eta'\beta' & f'(k^*) & 0 & \eta'\beta'\varphi' \\
0 & 0 & \gamma'+\omega & 1-\omega & 0 & 0 & -\gamma' & 0 \\
-(\Omega'+\varphi') & -(\Omega'+\varphi') & 0 & 0 & 1 & 0 & 0 & \varphi' \\
0 & 0 & 0 & 0 & 0 & f'(k^*) & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$
(4.11)

- $E = [e_{ij}]$ the matrix of the imitation coefficients among NP agents
- $\overline{\alpha}$ the diagonal matrix having the functions $\alpha_i(\bullet)$ as elements
- $\overline{\omega}$ the diagonal matrix having the parameters $\omega_i(\bullet)$ as elements
    Under the above assumptions the quantity traded by the i-th NP agent is

$$q_i(T) = -\alpha_i[P(T-1) - r_i(T)] + \mu_i[B(T-1)] + \sum_j e_{ij}q_j(T-1). \quad (5.2)$$

If we indicate with **1** the column vector all of whose m components are one and with **I** the identity matrix mxm, the system governing the market evolution can be written as follows

$$\begin{cases} \underline{q}(T) = -\overline{\alpha}[1P(T-1) - r(t)] + \underline{\mu}[B(T-1)] + E\underline{q}(T-1) \\ B(T) = -\beta[P(T-1) - h(T)] - 1'\underline{q}(T) \\ P(T) = k(T) + \eta[B(T) + 1'\underline{q}(T)] \\ r(T) = (I - \overline{\omega})\underline{r}(T-1) + \overline{\omega}1P(T-1) + \gamma(1\Delta P) \\ h(T) = h(T-1) - \Omega[1'\underline{q}(T-1) + B(T-1)] - \varphi[1'\Delta\underline{q} + \Delta B] \\ k(T) = f[k(T-1)] \end{cases} \quad (5.3)$$

The market is said to be in equilibrium $\underline{q}^*, B^*, \underline{r}^*, h^*, P^*, k^*$ if these values satisfy the system

$$\begin{cases} \underline{q}^* = -\overline{\alpha}[1P^* - r^*] + \mu B^* + E\underline{q} \\ B^* = -\beta[P^* - h^*] - 1'\underline{q}^* \\ P^* = k^* + \eta[B^* + 1'\underline{q}^*] \\ \underline{r}^* = (I - \overline{\omega})\underline{r}^* + \overline{\omega}1P^* \\ 0 = -\Omega[1'\underline{q}^* + B^*] \\ k^* = f(k^*) \end{cases} \quad (5.4)$$

If $f(\bullet)$ is a contraction, we get through simple manipulations from system (5.4) the following relations

$$h^* = P^* = k^*, \quad \underline{r}^* = 1 \bullet P^*, \quad B^* = -1'\underline{q}^*, \quad \underline{q}^*(I-E) = \underline{\mu}(B^*). \quad (5.5)$$

If it is assumed that the matrix (I-E) admits its inverse, the last relation of (5.5) gives

$$\underline{q}^*=(I-E)^{-1}\underline{\mu}(B^*).\qquad (5.6)$$

Substituting (5.6) for the last equation in (5.5) we obtain

$$B^*+1'(I-E)^{-1}\underline{\mu}(B^*)=0.\qquad (5.7)$$

This last relation allows to say that the solution

$$B^*=0\quad 1'\underline{q}^*=0\quad \underline{r}^*=1P^*=1h^*=1k^*$$

is the equilibrium solution, although nothing can be said as to its uniqueness and stability.

In fact for particular imitation structures with appropriate funcions $\mu_i(\bullet)$ we can get

$$B^*=-1'(I-E)^{-1}\underline{\mu}(B^*)\qquad (5.8)$$

with $B^*\neq0$.

In case that (5.7) gives $B^*=0$ as the only solution, the results obtained as to the equilibrium are not different from the results obtained without imitation.

# 6. CONCLUSION

This paper points out how models, built on very general assumptions regarding the equations governing stock-market performance, can contribute to the understanding and explanation of instability in these markets. This is in particular the case in markets where asymmetry of information generates incontrollable imitative behaviour and where insider information is used for speculative purposes.

We can finally observe that, although the market's performance is conditional on the professional agents' behaviour, it is not that behaviour as such that we were mainly interested in; rather it is its influence on the decisions of the other types of agents.

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