

INTERSECTION PROBLEMS FOR STSs AND SQSs: A SHORT SURVEY (*)

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Abstract. We give a brief survey of the latest results on the block intersection problem for $S(t, t+1, v)$ for $t=2, 3$.

1. Definitions.

A t -*design* on v point is a pair (V, B) where V is a finite set of size v (called the *order* of the t -design) and B is a collection of k -subsets of V (called *blocks*) such that every t -subset of V is contained in exactly λ blocks of B . A t -design on v point is called an $S_\lambda(t, k, v)$. In the case $\lambda=1$, it is called a Steiner system $S(t, k, v)$.

It is well-known that an $S(2, 3, v)$, or *Steiner triple system* $STS(v)$, exists if and only if $v \equiv 1$ or $3 \pmod{6}$ and an $S(3, 4, v)$, or *Steiner quadruple system* $SQS(v)$, exists if and only if $v \equiv 2$ or $4 \pmod{6}$ [27].

In an $S(t, k, v)$ (V, B) , $|B| = \binom{v}{t} / \binom{k}{t}$. If (V, B) is an $STS(v)$ ($SQS(v)$) we will denote by $t_v = \frac{v(v-1)}{6}$ $\left(\begin{array}{l} q_v = \\ = \frac{v(v-1)(v-2)}{24} \end{array} \right)$ the cardinality of B .

(*) Research supported by M.P.I.

A *parallel class* in an $S(t,k,v)$ is a set of blocks that between them contain every point of V exactly once. A Steiner system is called *resolvable* if one can partition its blocks into parallel classes. Such a partition is called a *resolution*. Clearly, the number of blocks in a parallel class must equal v/k , and therefore k must divide v in any resolvable design. It is well-known that not every design with k a divisor of v is resolvable.

A resolvable $S(2,3,v)$, called a *Kirkman triple system* $KTS(v)$, exists if and only if $v \equiv 3 \pmod{6}$ [76].

Hartman [32] proved that the necessary condition for the existence of a resolvable $S(3,4,v)$ (i.e. $v \equiv 4$ or $8 \pmod{12}$) is also sufficient with the possible exception of twenty-three values of v .

Given an $S(3,4,v)$ (V,B) , if one chooses any point $x \in P$ and deletes that point from the set P and from all blocks which contain it then the resulting system $(V(x),B(x))$, where $V(x) = V - \{x\}$ and $B(x) = \{b' = b - \{x\} / b \in B \text{ and } x \in b\}$, will be an $S(2,3,v-1)$. Such a triple system is said to be *derived* from the quadruple system and is called a *derived triple system* (DTS). It is easy to see that there exists a DTS of every possible order. However it is unknown whether or not every triple system is a DTS. See [6,68] for results on this topic.

A *partial system* $S(t,k,v)$ is a pair (P,C) where P is a finite set of size n (called the order of the partial system) and C is a collection of k -subsets of P (called *blocks*) such that every

t -subset of P is contained in *at most* one block of C .

Two partial systems (P, C_1) and (P, C_2) are said to be *mutually balanced* if any given t -subset of P is contained in a block of C_1 if and only if it is contained in a block of C_2 . Two mutually balanced partial systems are *disjoint* if they have no block in common.

A partial system (P, C) is *maximal* if there is no partial system (P, C') with CCC' .

A *maximum partial system* is a maximal partial system with a maximum number of blocks.

A partial system with $t=2$ and $k=3$ ($t=3$ and $k=4$) is called a *partial triple system* (*partial quadruple system*).

The interested reader should consult the books [1,3,81] concerning design theory and [17,45] for more detailed results on Steiner systems.

2. The block intersection problem for STS and SQS.

Two Steiner systems (V, B_1) and (V, B_2) are said to *intersect* in k blocks provided $|B_1 \cap B_2| = k$. If $k=0$, (V, B_1) and (V, B_2) are said to be *disjoint*, and if $|B_1 \cap B_2| = 1$ they are said to be *almost disjoint*. The existence of a pair of disjoint $S(2,3,v)$ s of every order $v \geq 7$ has been shown by Doyen in [7]. Teirlinck [80] proved that if (V_1, B_1) and (V_2, B_2) are any two $S(2,3,v)$ s, $v \geq 7$, and if V is any v -set, then there exist two disjoint $S(2,3,v)$ s (V, B'_1) , (V, B'_2) such that $(V_1, B_1) \cong (V, B'_1)$ and $(V_2, B_2) \cong (V, B'_2)$. Lindner [37,38] proved that there exists for every order $v \geq 3$ a pair of almost

disjoint $S(2,3,v)$ s. In [43] the existence of large sets of mutually almost disjoint $S(2,3,v)$ s is considered. A large set of $S(2,3,v)$ s is a set $\{(V, B_i) / i \in I\}$ of $S(2,3,v)$ s such that $\bigcup_{i \in I} B_i = \binom{V}{3}$, the set of all 3-subsets of V .

Etzion and Hartman have proved the existence of a pair of disjoint $S(3,4,v)$ s for every admissible $v \geq 8$.

It is possible to consider the above results as particular cases of the following *block intersection problem for Steiner systems*: For every admissible v determine the set $J(v)$ of all integers k such that there exists a pair of $S(t, t+1, v)$ intersecting in k blocks.

For $t=2$ the set $J(v)$ is completely determined [42]. More precisely, if $I(v) = \{0, 1, \dots, t_v - 6\} \cup \{t_v - 4, t_v - v(v-1)/6\}$, it is $J(v) = I(v)$ for every $v \equiv 1, 3 \pmod{6}, v \geq 13$, $J(3) = \{1\}$, $J(7) = \{0, 1, 3, 7\}$, and $J(9) = \{0, 1, 2, 3, 4, 6, 12\}$.

Many authors have studied the intersection problem for Steiner quadruple systems. The following results are known: Let $I(v) = \{0, 1, 2, \dots, q_v - 14\} \cup \left\{ q_v - 12, q_v - 8, q_v - \frac{v(v-1)(v-2)}{24} \right\}$, $v \equiv 2, 4 \pmod{6}$.

Then:

- (1) $J(4) = 1$, $J(2^n) = I(2^n)$ for every $n \geq 3$ [19, 12, 22, 51].
- (2) $J(10) = \{0, 2, 4, 6, 8, 12, 14, 30\}$ [35], and $J(5 \cdot 2^n) = I(5 \cdot 2^n)$ for every $n \geq 2$ [13, 16, 22, 51].
- (3) $J(14) \supseteq I(14) - \{48, 50, 52, 57, 58, 60, 62, 63, \dots, 79, 83\}$ [54]; $76, 77, 79, 83 \notin J(14)$ [21]; and $J(7 \cdot 2^n) = I(7 \cdot 2^n)$ for every $n \geq 2$ [50, 51].
- (4) $J(v) = I(v)$ for every $v \equiv 4, 8 \pmod{12}$ [52] (and independently

[10]).

The proof techniques of the above results use well-known constructions of $S(3,4,2v)$. Very interesting constructions for $S(3,4,v)$ s are known [27,29,30,31,36].

Recently a new construction (hextupling construction) for $S(3,4,v)$ s is given in [25]. Applying the hextupling construction to the intersection problem, Colbourn and Hartman [33]: can show

$$I(v) - \left\{ 0, \dots, \frac{(v-2)(v-14)}{6} - 1 \right\} \subseteq J(v) \text{ for } v \equiv 2 \pmod{12} \geq 38 \text{ and}$$

$$I(v) - \left\{ 0, \dots, \frac{(v-10)}{6} - 1 \right\} \subseteq J(v) \text{ for } v \equiv 10 \pmod{12} \geq 46.$$

At last in a remarkable paper, Hartman and Yehudai [33A] complete the determination of the sets of possible intersection sizes for Steiner quadruple systems of all admissible orders v except possibly $v=14,16$.

Let (V, B_1) and (V, B_2) be two Steiner systems such that $B_1 \cap B_2 = B \neq \emptyset$. If $X = \bigcup_{b \in B_1 - B} b$, then $(X, B_1 - B)$ and $(X, B_2 - B)$ are two disjoint and mutually balanced (DMB) partial systems. If there does not exist a pair of DMB partial systems with k blocks, then $k \notin J(v)$.

As a consequence, many papers [11,14,15,18,22,23,24,46,47,48,49] have studied the existence of DMB partial systems.

If $J_R(v)$ denotes the set of all integers k such that there exists a pair of Kirkman triple systems intersecting in k blocks then $0,1 \in J_R(v)$ for every admissible $v \geq 3$ [26,80] (see also [77]) and $I(v) - \{t_v - 13, t_v - 7, t_v - 4\} \subseteq J_R(v)$ for $v = 3^{n+1}, 5 \cdot 3^n, 7 \cdot 3^n$ ($n \geq 2$)

[53].

To determine $J_R(v)$ for the other values of v is an open problem.

Since it is unknown whether or not every triple system is a DTS [68], the following question is of interest: Do the intersection problems for DTSs and for $S(2,3,v)$ s have the same solution? The answer is yes for every admissible $v \leq 15$ [6] and for every $v \equiv 3, 7 \pmod{12}$ [73]; but it is an open problem for $v \equiv 1, 9 \pmod{12}$, $v \geq 21$.

Given an integer k such that $0 \leq k \leq \frac{v-1}{2}$, let us denote by $D(v,k)$ the maximum number of $STS(v)$ s that can be constructed on a v -set in such a way that any two of them have exactly k blocks in common, these k blocks being moreover in each of the $D(v,k)$ systems. In [7] Doyen posed the problem of determining $D(v,k)$. Clearly $D(v,k) \geq 2$ for every $k \in J(v)$. For $k=0$, $D(v,0)$ denotes the maximum number of pairwise disjoint $S(2,3,v)$ s. Then $D(v,0) \leq v-2$. If the equality sign holds, the $v-2$ Steiner triple systems form a *large set of disjoint* $S(2,3,v)$ s. Clearly $D(7,0)=2$. Many papers have studied the parameter $D(v,0)$ [77]. The best results on this topic are due to Lu Jia-Xi [55] who has shown $D(v,0)=v-2$ for all admissible v with the possible exception of $v=141, 283, 506, 789, 1051, 2365$. However Teirlinck writes in [80A] that Lu [55A] has completed these cases, so that $D(v,0)=v-2$ for all admissible $v \geq 9$.

In [59,61,62,63,70,71] the following values of the parameter $D(v,k)$ are determined:

(1) If $m=6$ and $v=9,13$; or $m=7$ and $v \geq 7$; or $m=8,9,11$ and $v=13,15$; or $m=12$ and $v=13$; $D(v, t_v - m) = 2$.

(2) If $m=6$ and $v \geq 13$; or $m=9$ and $v \geq 9$ $v \neq 13,15$; or $m=10$ and $v \geq 19$; or $m=11$ and $v \geq 9$ $v \neq 13,15$; or $m=12$ and $v=15$; or $m=13$ and $v \geq 15$; or $m=14$ and $v=13,15,19$; $D(v, t_v - m) = 3$.

(3) If $m=8$ and $v \geq 9$ $v \neq 13,15$ or $m=14$ and $v \geq 21$, $D(v, t_v - m) = 4$.

(4) $D(v, t_v - m) \leq 2 \left\lceil 3m / \left\lceil \left(1 + \sqrt{1 + 24m} \right) / 2 \right\rceil \right\rceil$; so that $D(v, t_v - t_w) = D(w, 0)$ for any admissible w such that $v \geq 2w + 1$. Therefore from [55,55A] it follows that $D(v, t_v - t_w) = w - 2$ for $w \geq 9$.

It is possible to define the parameter $D(v, k)$ for Steiner quadruple systems. For $k=0$ the following results are known: $D(2v, 0) \geq v$ [39], $D(4v, 0) \geq 3v$ [38], $D(2 \cdot 5^m, 0) \geq 5^m$ [67], $D(2n, 0) \geq n$ where $n \equiv 1$ or $5 \pmod{6}$ [69], $D(2^k n, 0) \geq (2^k - 1)n$ if $k \geq 2$ and there exists a set of $3n$ pairwise disjoint Steiner quadruple systems of order $4n$ with a certain structure [9A]. For $k > 0$ it is proved in [20] that:

$$(1) D\left(v, q_v - \frac{v^2(v-2)}{32}\right) \geq \frac{v}{2} - 1 \text{ for every } v \equiv 4, 8 \pmod{12}.$$

(2) For every $k \in \mathbb{N}$, $k \geq 2$, let $w = \min(\lambda \in \mathbb{N} / \lambda \geq 4k, \lambda \equiv 2, 4 \pmod{6})$. It follows that $D(2w, q_{2w} - k^2(2k-1)) \geq 2k-1$ and $D(2v, q_v - k^2(2k-1)) \geq 2k-1$, for $v \geq w$, $v \equiv 2, 4 \pmod{6}$.

(3) $D(v, q_v - 8) = D(v, q_v - 14) = D(v, q_v - 15) = 2$ for every $v = 2^{n+2}, 5 \cdot 2^n, 7 \cdot 2^n$ and $n \geq 2$.

The block intersection problem can be generalized in the following way: Determine the sets $J^m(v)$ ($J^m(v)$) of all integers k

such that there exists a collection of m (≥ 2) Steiner systems mutually intersecting in k blocks (in the same set of k blocks).

Clearly $J^2(v) = J^2(v) = J(v)$ and $J^m(v) \subseteq J^m(v) \subseteq J(v)$. Let $I^3(v) = \{0, 1, \dots, t_v - 8\} \cup \{t_v - 6, t_v\}$. The following results are proved in [64]: $J^3(v) = I^3(v)$ for every $v \equiv 1, 3 \pmod{6}$, $v \geq 19$, $J^3(7) = \{1, 7\}$, $J^3(9) = \{0, 1, 3, 4, 12\}$, $J^3(13) = I_{13}^3 - \{14, \dots, 18, 20\}$ and $J^3(15) = I_{15}^3 - \{24, \dots, 27\}$. I am not aware of any further results in this direction.

Let (V, B_1) and (V, B_2) be two $S(t, t+1, v)$ such that $|B_1 \cap B_2| = k$. Clearly $(V, B_1 \cup B_2)$ is an $S_2(t, t+1, v)$ having exactly k repeated blocks. Therefore the solution of the intersection problem proves the existence of $S_\lambda(t, t+1, v)$ (at least for certain values of v), with repeated blocks. See [4, 61, 65, 72, 78].

3. Steiner systems intersecting in a set with additional properties.

Let (V, B_1) and (V, B_2) be two $S(t, t+1, v)$ s intersecting in exactly k blocks. One could require, as is done in [34], that $B_1 \cap B_2$ contains an opportunely defined block set F and that $h = |F| + k$.

The *flower* [34] or *star* at a point x of a Steiner system is the set of all blocks containing x . The *flower intersection problem* for Steiner systems is the determination for each admissible v of the set $J^F(v)$ of all integers k such that there exists a pair of Steiner systems (V, B_1) and (V, B_2) of order v such that $|B_1 \cap B_2| = |F| + k$, where F is the flower of a point $x \in V$. The following results are proved:

(1) Case $t=2$ ($v \equiv 1, 3 \pmod{6}$). Let $I^F(v) = (0, 1, \dots, f_v - 6) \cup \left\{ f_v - 4, f_v - \frac{(v-1)(v-3)}{6} \right\}$. Then $J^F(3) = I^F(3) = (0)$, $J^F(7) = I^F(7) = (0, 4)$, $J^F(9) = I^F(9) = (1, 4)$, and $J^F(v) = I^F(v)$ for every $v \geq 13$ [34].

(2) Case $t=3$ ($v \equiv 2, 4 \pmod{6}$). Let $I^F(v) = (0, 1, \dots, f_v - 14) \cup \left\{ f_v - 12, f_v - 8, f_v - \frac{(v-1)(v-2)(v-4)}{24} \right\}$. Then $J^F(4) = (0)$, $J^F(8) = (7)$, $J^F(10) = (0, 18)$, $I^F(16) = (16) \subsetneq J^F(16)$ ($16 \in J^F(16)$ is an open problem), and $J^F(v) = I^F(v)$ for every $v \equiv 4, 8 \pmod{12}$, $v \geq 20$ [66, 73].

We remark that the solution of the flower intersection problem gives a solution of the block intersection problem for other incidence structures. For example let (V, B) be a triple system and write $B = FUA$ where F is the flower at the point $x \in V$. Let $X = V - \{x\}$ and $G = (\{a, b \mid (a, b, x) \in F)$. Then (X, G, A) is a *group divisible design* (GDD) with group size 2 and block size 3, and $(X, B - F)$ is a maximum partial system (MPT) of order $v \equiv 0, 2 \pmod{6}$.

In these terms the flower intersection problem becomes the intersection problem for GDDs with group size 2 and block size 3. Moreover we can see the same flower intersection problem as the intersection problem for MPTs of order $v \equiv 0, 2 \pmod{6}$.

These problems are completely solved in [2] and [74] respectively. Also the flower intersection problem for MPTs of order $v \equiv 5 \pmod{6}$ is solved in [74].

Lo Faro and Marino [54A, 56] determine those pairs (k, v) $v \equiv 4$ or $8 \pmod{12}$ (with some possible exception for $v = 20, 28$) for which

there exists a pair of Steiner quadruple systems on the same v -set, the quadruples in one system containing two particular distinct points are the same as those in the other system containing that pair of points, and the two systems have otherwise exactly k triples in common.

A slightly different intersection problem was posed by Micale [58]: Determine the set $J_0(v)$ of all integers k such that there exists a pair of $S(3,4,v)$ s having exactly k pairwise disjoint blocks in common.

Let $I_0(v) = \left\{ 0, 1, \dots, \left\lfloor \frac{v}{4} \right\rfloor \right\}$ ($\left\lfloor \frac{v}{4} \right\rfloor$ denotes the maximum integer $\leq \frac{v}{4}$), $v \equiv 2, 4 \pmod{6}$. It is known [35] that $J_0(10) = \{0\}$. In [58] it is proved that $J_0(v) = -I_0(v)$ for every $v = m \cdot 2^n$ with $n \geq 2$ and $m = 4, 5, 7$; $J_0(4) = \{1\}$, $J_0(8) = \{0, 2\}$ and $\{0, 1, 2\} \subseteq J_0(14) \subseteq \{0, 1, 2, 3\}$.

In [75] it is proved that $J_0(v) = I_0(v)$ for every $v \equiv 4, 8 \pmod{12}$, $v \geq 16$.

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