Robust Fuzzy Graph

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Abstract

The idea of fuzzy graphs is crucial for dealing with uncertainty in day-to-day living situations. In this work, we have a present a work fuzzy graph associates with robustness model that's easy, flexible, and simple. With this model, we have a tendency to address the examination planning downside below uncertainty supported. Randomness and unclearness are two very separate types of uncertainty in knowledge. This study bridges resilient fuzzy graphs and addresses every type of uncertainty in higher cognitive processes. In this paper we introduce a type of fuzzy graph relating with robustness is called robust fuzzy graph and some of its properties. Robust fuzzy graph becomes a best tool in evidence theory for calculating belief functions, plausibility functions, spanning functions etc.

Keywords: Robust; fuzzy graph; Belief measure; Plausibility measure; spanning.
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1 Introduction

Graphs are commonly thought to be nothing more than relational representations. A graph can be used to express information about item connections. Edges represent connections, whereas vertices represent things. It is common to need to develop a "fuzzy graph model" when the description of an object, its relationships, or both are imprecise.
Particularly in the fields of cluster analysis, neural networks, computer networks, pattern recognition, decision making, and professional systems, the use of fuzzy connections is pervasive and essential. Every one of them has a fuzzy graph as its fundamental mathematical structure.
A graph is a symmetric binary relation on a nonempty set V, as we are well aware. A fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. In 1973, A.Kaufmann [1965] initially proposed the idea of a fuzzy graph, which was based on Zadeh's fuzzy relations Zadeh [1980], Zadeh [1968]. Goguen is recognised with creating the idea of fuzzy graphs and taking into account fuzzy relations on fuzzy sets in 1968, however. Ravi.J et.al.,Ravi [2022] discovered several connection notions in fuzzy graphs at the same time.
There are few high-quality papers on fuzzy graphs. According to our data, a strength-associated degreed fuzzy graph and a degreed fuzzy graph have never been combined in a programming task. The works M. Blue and Puckett [2002] and D.E.Goldberg [1989], address programming flaws A.Lim and Wang [2005], Sunitha [2001], Sunitha and Vijayakumar [2002],Wang and Xu [2013]and Y´a˜nez and Ramırez [2003] contain works on fuzzy graph colouring.The structure of the paper is presented below. The section 2 briefly reviews a few essential definitions of fuzzy graphs. Section 3 describes the resilient fuzzy graph model and Section 4 concludes our discussion.

2 Preliminaries

Definition 2.1. *The two functions: Q $\rightarrow$ [0,1] and : Q x Q $\rightarrow$ [0,1] and a non-empty set V. such that the fuzzy graph G = (Q) is formed for each x, y in V by $\rho(.)$ $\leq$ $\mu(.)$ Λ $\mu(.)$. Fuzzy edge set and fuzzy vertex set for G are represented by and, respectively.*

Definition 2.2. *Let G = (μ,ρ) to have ρ^* = (u, v, w) and x. Let (u,v) = 0.2, (v, w) = 0.2, (w, x) = 0.3, (x, u) = 0.5, and (u, w) = 0.4..*
Due to the presence of the two weakest arcs in G, arcs (u, v) and (v, w), C1 = u, v, w, x, u is a fuzzy cycle, but C2 = u, w, x, u is not..

Definition 2.3. *Assume that the graph G=(μ,ρ) has more fuzziness. The word* *"connectedness strength" refers to the total strength of all paths linking two vertices, x and y, and is denoted by the sign $CONN_G$ (.). The strongest x-y route is one in which P = $CONN_G$ (.)..*

Definition 2.4. *This is a definition of a mathematical objectIf $CONN_G(.) > 0$ for each pair of (.) in ρ*, an f-graph G = (.) is connected. When G is not present, components are maximally connected fuzzy graphs. If G is linked, any two vertices can be connected by a route. The arc (x,y) of a fuzzy graph G is considered normal if and only if $(.) = CONN_G(.)$ >0.*

Definition 2.5. *Based on judgments of plausibility and belief, the evidence hypothesis is developed. The function Bel.: (.) $\rightarrow$ [0,1] represents a belief measure for a finite universal set X such that Bel(.) =0, Bel(.) =1, and Bel(.)$_j$ Bel(.$_j$) - jk Bel(.,.) +..+ (-1)n+1j Bel(.) $\subseteq$ X.. According to the data that is now available, Bel understands for each A(.) the degree of confidence that a certain member of (.) belongs to the set A. Belief measures are superadditive when (.) is infinite and continuous above..*

Definition 2.6. *A plausibility measure is a function P_I: (.) $\rightarrow$ [0,1] such that PI (.) =0, P_I(.) =1, and P_I(.) $\leq \sum$ (.) $-\sum <$ (.$\cup$.)+..+ (-1)$^{n+1}\sum$ (.) $\subseteq$ X.*

Definition 2.7. *A function m:(.)$\rightarrow$[0,1] that has the properties m(.)=0 and m(.)=1 characterises belief and plausibility measurements. The term "basic probability assignment" refers to this function..*
The value m(.) for each A indicates the strength of the argument that a specific member of (.), whose categorization in terms of relevant attributes is lacking, belongs to the set A(.)..

3 Robust fuzzy graph (RFG)

The kth regular super fuzzy matrix is examined in this section.

Definition 3.1. *Let $\{X\}$ and A Q$\neq$ P(.) and two functions m: Q$\rightarrow$ [0,1] and : Q x Q$\rightarrow$ [0,1] A,B$\in$ Q, (.) $\in \delta$, whenever A $\subseteq$B and (.) =m(.) m are present, constitute a resilient fuzzy graph (.). Also $\sum$.$\in$Q (.)=1. RFG is represented by the formula G=(.) where m is referred to as the assignment function and is referred to as the edge function.*

Definition 3.2. *If n(.) m(.) for all A and (.) (.) for all (.) such that the FG, H=(.) is referred to as a partial RFG of G=(.). If P V, n(.) = m(.), (.) = (.) for any (.) such that the FG, H=(.) is referred to as a RFG of G = (.) induced by P.*

Definition 3.3. *The vertex in a RFG which is adjacent from every other vertex is called complete vertex.*

Definition 3.4. *Consider the robust fuzzy matrix MG=(.) of G= (.), where mAB= (.,.) (.)⊆ (.) 0 h.*
The matrix M^{G^k} such that M^{G^k} =$M^{G^{k+1}}$, where k is a positive integer is called the evidence reachability matrix of G denoted by R_G=(r_{AB}).

Definition 3.5. *Two vertices A and B of a RFG is said to be mutually disconnected if there is neither an edge (.,.) nor (.,.).*
The vertices A and B are mutually disconnected. The Belief and plausibility. The measures of a complete vertex are always one.

Theorem 3.1. *Fuzzy graphs (FG) include robust fuzzy graphs (RFG).*

Proof. Proof is evident from the definition of RFG.

Theorem 3.2. *Number of vertices of a RFG corresponding to a crispest X with n elements is, 2n-1.*

Proof. Let G = (m,δ)be the RFG. Then by the definition, V= ()$\setminus \psi$ is the vertex set and vertices is 2n -1.

Theorem 3.3. *Number of edges of a robust fuzzy graph corresponding to a crisp set with n elements is*
($\sum^{-1}$($-$ 1))$_{-1}$+($\sum^{-2}$($-$ 2))$_{-2}$+ + n.

Proof. Proof is the consequence of set theory.
Let us start with singleton sets. Every vertex corresponding to singleton sets is adjacent to all the vertices corresponding to their supersets - 2-element sets, 3-element sets etc.
{.}→{l,m},{l,m,n},{l,m,n,o}........
This can be done in $\sum^{-1}(- 1)$ = nCn-1
So the total cases corresponding to singleton sets is $(\sum^{-1}(- 1))_{-1}$
The vertices corresponding to 2-element sets is adjacent to all vertices corresponding to all vertices corresponding to their super sets - 3-element sets, 4-element sets etc.
{.}→{l,m},{l,m,n},{l,m,n,o}......... □

Theorem 3.4. *Robust fuzzy graph is complete.*

Proof. A complete fuzzy graph (CFG) is a FG, G=(.) such that δ(.)=m(.)Λ m(.) for all (.,.). So by definition every RFG is complete.□

Theorem 3.5. *There does not exist an edge (.), such that δ (.) =1 in a RFG, G=(.).*

Proof. If possible, suppose that there exist an edge (.) such that δ (.) =1
$\Rightarrow$ m(.)=m(.)=1 by the definition of RFG.
$\Rightarrow \sum$ (.)$\neq$1, a contradiction.□

Theorem 3.6. *The partial RFs and RFs of an RFG need not be a RFG*

Proof. For a partial RFG and RFG, $\sum$ () equal to 1.
But $\sum$ () $\leq$ 1.□

Theorem 3.7. *The partial RFs of an RFG is an RFG if and only if m(.) = n(.) for all A and τ(.,.) = δ (.,.) for all (.).*

Proof. By definition $\sum$ () = 1.
Let H=(.) be a partial RFs of G.
For a partial RFs, H=(n, τ) of G=(.), n(.) $\leq$ m(.) for all A and τ(.,.) $\leq$ δ (.,.).
But $\sum$ (.) $\neq$ 1 if n(.) $<$ m(.).So n(.) = m(.) for all A which implies τ(.,.) = δ (.,.) for all (.,.).
Converse is obvious.□

Theorem 3.8. *Maximum length of a path P in a RFG with n vertices is n-1.*

Proof. Consider a RFG with n vertices A0,A1,.An. Start from an arbitrary vertex Ai. Since there are only n-1 vertices remaining, choose a vertex Aj such that ρ(Ai,Aj) $>$0, Ai$\subseteq$Aj, i $\neq$ .Similarly choose a vertex Ar from the remaining n-2 such that ρ(Aj,Ar) $>$0, Aj$\subseteq$Ar, and so on. Since there are only n distinct vertices the process must terminate at a vertex Ap such that,
ρ(Ap-1,Ap) $>$0, Ap-1$\subseteq$Ap, p$<$ n.
We get the sequence Ai,Aj,—-Ap which is of length less than n and equal to n-1 only if every vertex is ordered by the relation $\subseteq$. □

Theorem 3.9. *RFG does not contain cycles and so fuzzy cycles.*

Proof. Since in a RFG G = (.), for all (.,) $\in$ V, (.,.) $\in$ δ whenever (.)$\subseteq$ (.) there will not be an edge (.,.) and so a cycle.□

Theorem 3.10. *The RFG is always disconnected.*

Proof. For X= {.,.} there does not exist a path between x and y.□

4 Conclusions

In this paper we introduce new type of fuzzy graph called robust fuzzy graph. We find some of its properties like completeness, paths, connectivity etc. We also present fuzzy graph's application in robust theory for finding belief measure plausibility measure etc in uncertain situations. We can calculate belief measure using RFG as Bel(.) = m(.) + $\sum$ (.), (., .) is an edge; where m(.) is the degree of the vertex (.) in RFG. Our proposed method RFG perform is well even it is uncertainty situation. In future this concept is applying in computer vision concept.

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