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Solving fuzzy linear programming problems by using the fuzzy exponential barrier method

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Abstract

In order to resolve the fuzzy linear programming problem, the fuzzy exponential barrier approach is the major strategy employed in this article. To overcome the problems with fuzzy linear programming, this method uses an algorithm. In this concept, a fuzzy inequality constraint is produced since the objective functions are convex.numerical examples are provided.
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1 Introduction

Fuzzy set theory has aided mathematical modelling and control theory. Unsolved decision-making problems can be applied in real-world situations using a variety of techniques. The fuzzy set theory developed by A. et al. [2012] has presented in 1965. Fuzzy linear programming was first established in 1997 by Tanaka and Asai [1984], and fuzzy numbers were published in Dubois [1983]. A function was proposed by Hsieh [1999] to cope with fuzzy arithmetical operations. Using a technique that divides, adds, and multiplies fuzzy integers in addition to subtracting and adding them. The fuzzy linear programming problem, as formulated by Mahadevi et al. [2009], A. and Yogarani [2021] and Fiacco and V. [1990] Zimmermann [1978] likewise has a duality. The process of defuzzifying involves converting fuzzy values into clear crispvalues. Since a few years ago, these techniques have been thoroughly researched and applied in fuzzy systems Mahadevi et al. [2006]. A representative value from a given set, according to some characters, was the main objective of these processes. The defuzzification method establishes a link between all fuzzy sets and all real numbers. Given the high cost ofan infeasible solution, Moengin and Parwadi [2011] et al. Fiacco and Anthony [1976] developed the exponential barrier function technique. The fuzzy exponential barrier approach is a different way to solve fuzzy linear programming problems. The fuzzy exponential barrier technique includes a fuzzy objective function that specifies a severe penalty for violating the fuzzy requirements in order to approximate fuzzy constraints. a fuzzy barrier function in fuzziness optimization problems that has a value that grows exponentially with the separation from the viable zone. The fuzzy exponential barrier parameter, a positive decreasing parameter used in this process, establishes how close to the original fuzzy unconstrained problem is. In section 2 of this study, some fundamental ideas of fuzzy set theory and the algebraic operation of triangular fuzzy numbers are provided. A fuzzy exponential barrier technique is used to build an algorithm in section 3 to tackle the problem of fuzzy linear programming problem. Section 4 includes an example.

2 Preliminaries

2.1 : Fuzzy Set

If $\widetilde{M} = \{(x, \mu_M(x)) : x \in M, \mu_{\widetilde{M}}(x) \in [0,1]\}$ defines a fuzzy set $\widetilde{M}$ The pair $(x, \mu_{\widetilde{M}}(x))$that makes up the membership function has two elements: x, which is a member of the classical set $\widetilde{M}$,and the $\mu_{\widetilde{M}}(x)$ belongs to the interval [0,1].

2.2 : Triangular fuzzy number

The triangular fuzzy number is the fuzzy set $\widetilde{M} = (M_1, M_2, M_3), M_1 \leq M_2 \leq$

$M_3 \in R$, if the membership function of $\widetilde{M}$ is defined by

$$\mu_{\widetilde{M}}(y) = \begin{cases} \frac{y-M_1}{M_2-M_1}, & M_1 \leq y \leq M_2 \\ \frac{m_3-y}{m_3-M_2}, & M_2 \leq y \leq M_3 \\ 0, & otherwise \end{cases}$$

3 : Fuzzy exponential barrier method

Primal-dual of the fuzzy linear programming problems
$\max \widetilde{Z} = \widetilde{f}^t(\widetilde{y})$
subject to $M_j\widetilde{y} \geq \widetilde{N}_k, j = 1, 2.$
Where $M_1 \in R^{m\times n}, \widetilde{f}, \widetilde{y} \in R^n, \widetilde{N}_k \in R^m$

$$\widetilde{f}^t = (f_1, f_2, f_3), \widetilde{N} = (n_1, n_2, n_3) \tag{1}$$

$\min \widetilde{Z} = \widetilde{N}(\widetilde{y})$
$M_j^T(\widetilde{x}) \leq \widetilde{f}, M_2 = M_1^T \in R^{m\times n}, \widetilde{N}_k^T, \widetilde{f}^{t^t} = \widetilde{f},$
We may consider the rank of the matrix M_1 without losing generality. The triangular fuzzy number is used to specify the fuzzy components in the preceding problem. Consider that every problem has at least one feasible solution (1)
The fuzzy exponential barrier function is considered by
$\widetilde{E}(\widetilde{y}, \gamma) : R^n \to R$for every scalar $\delta > 0$ as follows.

$$\widetilde{E}(\widetilde{y}, \gamma) = \widetilde{f}^t(\widetilde{y}) - \gamma \sum_{i=1}^{m} e^{(B_i(\widetilde{y}))^\beta} \tag{2}$$

Consider $\widetilde{E} : R^n \to (-\infty, \infty)$ is a function given by

$$\begin{aligned} B_i(\widetilde{y}) =& M_j\widetilde{y} - \widetilde{N}_k \leq 0, M_j\widetilde{y} - \widetilde{N}_k = 0 forallj, \\ & M_j\widetilde{y} - \widetilde{N}_k > 0, M_j\widetilde{y} - \widetilde{N}_k \neq 0 forallj. \end{aligned}$$

3.1 : Fuzzy exponential barrier lemma

This lemma is based on the local and global behaviour of the unconstraintsmaximizer of the fuzzy exponential barrier function. Statement: An exponential barrier parameter γ^k is a fuzzy sequence that increased in size. The fuzzy linear programming problems and fuzzy exponential barrier function then for each k condition are below.

$$\begin{aligned} &(i).\widetilde{E}(\widetilde{y}^{k+1}, \gamma^{k+1}) \geq \widetilde{E}(\widetilde{y}^k, \gamma^k) \\ &(ii).\widetilde{E}(\widetilde{y}^k \leq \widetilde{E}(\widetilde{y}^{k+1}) \\ &(iii).\widetilde{f}^t(\widetilde{y}^k) \geq \widetilde{f}^t(\widetilde{y}^{k+1}) \\ &(iv).\widetilde{f}^t(\widetilde{y}^*) \leq \widetilde{E}(\widetilde{y}^k, \gamma^k) \leq \widetilde{f}^t(\widetilde{y}^k) \end{aligned}$$

Proof :

$$(i).\widetilde{E}(\widetilde{y}^{k+1},\gamma^{k+1}) = \widetilde{f}^t(\widetilde{y}^{k+1}) - \gamma^{k+1}\sum_{i=1}^{m} e^{(\beta(M_j\widetilde{y}^{k+1}-\widetilde{N}_k))} \geq \widetilde{f}^t(\widetilde{y}^k) - \gamma^k\sum_{i=1}^{m} e^{(\beta(M_j\widetilde{y}^{k}-\widetilde{N}_k))} = \widetilde{E}(\widetilde{x}^k,\gamma^k)$$

$$(ii).\widetilde{f}^t(\widetilde{y}^k) - \gamma^k\sum_{i=1}^{m} e^{(\beta(M_j\widetilde{y}^{k+1}-\widetilde{N}_k))} \leq \widetilde{f}^t(\widetilde{y}^{k+1})\gamma^{k+1}\sum_{i=1}^{m} e^{(\beta(M_j\widetilde{y}^{k+1}-\widetilde{N}_k))} \quad (3)$$

$$\widetilde{f}^t(\widetilde{y}^{k+1}) - \gamma^{k+1}\sum_{i=1}^{m} e^{(\beta(M_j\widetilde{y}^{k+1}-\widetilde{N}_k))} \leq \widetilde{f}^t(\widetilde{y}^k) - \gamma^k\sum_{i=1}^{m} e^{(\beta(M_j\widetilde{y}^{k}-\widetilde{N}_k))} \quad (4)$$

The inequalities adding (3) and (4), we get

$$\widetilde{f}^t(\widetilde{y}^k) - \gamma^k\sum_{i=1}^{m} e^{(\beta(M_j\widetilde{y}^{k}-\widetilde{N}_k))} + \widetilde{f}^t(\widetilde{y}^{k+1}) - \sum_{i=1}^{m} e^{(\beta^{k+1}(M_j\widetilde{y}^{k}-\widetilde{N}_k))} \leq \widetilde{f}^t(\widetilde{y}^{k+1}) - \sum_{i=1}^{m} e^{(\beta^{k+1}(M_j\widetilde{y}^{k+1}-\widetilde{N}_k))} + \widetilde{f}^t(\widetilde{y}^k) - \gamma^k\sum_{i=1}^{m} e^{(\beta(M_j\widetilde{y}^{k}-\widetilde{N}_k))} ...(\widetilde{f}^t(\widetilde{y}^k) + \widetilde{f}^t(\widetilde{y}^{k+1})) - (\widetilde{f}^t(\widetilde{y}^{k+1}) + (\widetilde{f}^t(\widetilde{y}^k)) + \gamma^k\sum_{i=1}^{m} e^{(\beta^k(M_j\widetilde{y}^{k}-\widetilde{N}_k))} + \gamma^{k+1}\sum_{i=1}^{m} e^{(\beta^{k+1}(M_j\widetilde{y}^{k}-\widetilde{N}_k))}) \leq \gamma^k\sum_{i=1}^{m} e^{(\beta^k(M_j\widetilde{y}^{k}-\widetilde{N}_k))} + \gamma^{k+1}\sum_{i=1}^{m} e^{(\beta^{k+1}(M_j\widetilde{y}^{k}-\widetilde{N}_k))}).$$

However $(\widetilde{f}^t(\widetilde{y}^k) + \widetilde{f}^t(\widetilde{y}^{k+1})) - (\widetilde{f}^t(\widetilde{x}^{k+1}) + \widetilde{f}^t(\widetilde{x}^k)) = 0.$

Then $(\gamma^k+\gamma^{k+1})\gamma^k\sum_{i=1}^{m} e^{(\beta^k(M_j\widetilde{y}^{k}-\widetilde{N}_k))} \geq (\gamma^k+\gamma^{k+1})\sum_{i=1}^{m} e^{(\beta^{k+1}(M_j\widetilde{y}^{k}-\widetilde{N}_k))}$

Since $\delta^k \leq \delta^{k+1}, \widetilde{E}(\widetilde{y}^{k+1}) \geq \widetilde{E}(\widetilde{y}^k)$.

(iii). From the proof of (i), it can be obtained that

$\widetilde{f}^t(\widetilde{y}^k) - \gamma^k \sum_{i=1}^m e^{(\beta(M_j\widetilde{y}^k - \widetilde{N}_k))} \geq \widetilde{f}^t(\widetilde{y}^{k+1}) - \gamma^{k+1} \sum_{i=1}^m e^{(\beta(M_j\widetilde{y}^{k+1} - \widetilde{N}_k))}$. $\widetilde{E}(\widetilde{y}^k) \leq \widetilde{E}(\widetilde{y}^{k+1})$. Then $\widetilde{f}^t(\widetilde{y}^{k+1}) \geq \widetilde{f}^t(\widetilde{y}^k)$.

(iv). From the proof of (ii) $\sum_{i=1}^m e^{(\beta(M_j\widetilde{y}^k - \widetilde{N}_k))} \leq \sum_{i=1}^m e^{(\beta(M_j\widetilde{y}^k - \widetilde{N}_k))}, \widetilde{f}^t(\widetilde{y}^k) \geq \widetilde{f}^t(\widetilde{y}^{k+1})$.

$$\widetilde{f}^t(\widetilde{y}^*) \geq \widetilde{f}^t(\widetilde{y}^k) - \gamma^k \sum_{i=1}^m e^{(\beta(M_j\widetilde{y}^k - \widetilde{N}_k))} = \widetilde{E}(\widetilde{y}^k, \gamma^k) \geq \widetilde{f}^t(\widetilde{y}^k).$$

3.2 : A Fuzzy exponential barrier convergence theorem

Suppose that the fuzzy linear programming problem and the fuzzy exponential barrier functions are both continuous functions. In a fuzzy linear programming problem, an increasing series of positive fuzzy exponential barrier parameters k is necessary such that $\{\gamma^k\} \to \infty, k \to \infty$ to $\widetilde{y}$ of $\widetilde{z}$there is an optimal solution. If the limit point$x$is within the boundaries of the range of $\widetilde{y}$of $\widetilde{z}$, then it exists.

Proof :

Let $\widetilde{y}$ be a limit point of $\{\widetilde{y}^k\}$

$$\widetilde{E}(\widetilde{y}^k, \widetilde{\gamma}^k) = \widetilde{f}^t(\widetilde{y}^k) - \gamma^k \sum_{i=1}^m e^{(\beta(M_j\widetilde{y}^k - \widetilde{N}_k))} \leq \widetilde{f}^t(\widetilde{y}^*)$$

$$\lim_{k\to\infty} \widetilde{f}^t(\widetilde{y}^k) = \widetilde{f}^t(\widetilde{y}), \lim_{k\to\infty} e^{(\beta(M_j\widetilde{y}^k - \widetilde{N}_k))} \leq 0$$

From the lemma 3.2 (iv) we get,

$$\lim_{k\to\infty} \widetilde{E}(\widetilde{y}^k, \widetilde{\gamma}^k) = \widetilde{f}^t(y^*)$$

$\widetilde{y}$ is feasible.

3.2 fuzzy exponential barrier function algorithm

1. Acknowledge the fuzzy objective function and constraints of the problem and repharse the problem standard forms to reflect them. write $\max \widetilde{z} = \widetilde{f}^t(\widetilde{y})$. Subject to $M_j\widetilde{y}^k - \widetilde{N}_k \leq 0$.

2.The fuzzy exponential barrier function defined by

$$\widetilde{E}(\widetilde{y},\widetilde{\gamma}) = \widetilde{f}^t(\widetilde{y}) - \gamma \sum_{i=1}^{m} e^{(\beta(M_j\widetilde{y}^k - \widetilde{N}_k))}.$$

3.3. Maxinequality constraints with the fuzzy exponential barrier functions

$$\max \widetilde{E}(\widetilde{y},\widetilde{\gamma}) = \widetilde{f}^t(\widetilde{y}) - \gamma \sum_{i=1}^{m} e^{(\beta(M_j\widetilde{y}^k - \widetilde{N}_k))}.$$

4.First-order needed conditions for optimality $\gamma \to \infty$, which are used to find the best solution to the specified fuzzy linear programming problems.

5.Find $\widetilde{E}(\widetilde{y}^k,\widetilde{\gamma}^k) = \max_{x\geq 0} \widetilde{E}(\widetilde{y},\widetilde{\gamma})$, then minimize $\widetilde{y}^k$ and $\gamma = 1, k = 1, 2, ..., k = I$ then stop.
Alternatively, cancontinue to step 5.
The fuzzy exponential barrier approach employs the same procedure as the earlier algorithm for the dual fuzzy linear programming problems.

4. Numerical example The primal fuzzy linear programming problem

$$\begin{aligned} \min \widetilde{z} = (3.75, 4, 4.25)\widetilde{y}_1 + (2.75, 3, 3.25)\widetilde{y}_2 \\ 2\widetilde{y}_1 + 3\widetilde{y}_2 \geq (5.75, 6, 6.25), \\ 4\widetilde{y}_1 + \widetilde{y}_2 \geq (3.75, 4, 4.25). \end{aligned}$$

Solution : Figure 1: Finding an infeasible solution for primal fuzzy linear programming problem Using the fuzzy exponential barrier function algorithm we get, The fuzzy exponential barrier method is given by

$$\widetilde{E}(\widetilde{y},\gamma) = ((3.75, 4, 4.25)\widetilde{y}_1 + (2.75, 3, 3.25)\widetilde{y}_2 - \gamma \sum_{i=1}^{m} e^{(B_i(y))^\beta}$$

According to this strategy, you can convert fuzzy linear programming problems into a standard form of unconstrained problems.

$$\widetilde{y}_1 = (0.45, 0.8, 1.15) - \frac{1.2104}{\gamma}, \widetilde{y}_2 = (0.0727, 0.6, 1.1273) - \frac{1.3951}{\gamma}$$

$$\widetilde{x}_1 = \frac{0.0060}{\gamma} + (0.35, 0.60, 0.85), \widetilde{x}_2 = (0.9518, 1.60, 2.25) + \frac{0.0180}{\gamma},$$

For different values of $\gamma^j, j \to \infty$ the optimal values of $\widetilde{y}_1(\gamma), \widetilde{y}_2(\gamma)$ are calculated as listed in Tables (1) and (2) we get,

Table1: Primal fuzzy linear programming problem solution(i)

No	δ^k	$\widetilde{y}_1$	$\widetilde{y}_2$
1	10	(0.3290,0.6790,1.0290)	(-0.0668,0.4605,0.9878)
2	10^2	(0.4379,0.7879,1.1379)	(0.0587,0.5860,1.1133)
3	10^3	(0.4488,0.7988,1.1488)	(0.0713,0.5986,1.1259)
4	10^4	(0.4499,0.7999,1.1499)	(0.0726,0.5999,1.1272)
5	10^5	(0.45,0.8,1.15)	(0.0727,0.6,1.1273)

Table 2: Dual fuzzy linear programming problem solution(i)

No	γ^j	$\widetilde{y}_1$	$\widetilde{y}_2$
1	10	(0.3506,0.6006,0.8506)	(0.9518,1.6018,2.2518)
2	10^2	(0.3501,0.6001,0.8501)	(0.9502,1.6002,2.2502)
3	10^3	(0.3500,0.600,0.8500)	(0.9500,1.600,2.2500)
4	10^4	(0.3500,0.600,0.8500)	(0.9500,1.600,2.2500)
5	10^5	(0.3500,0.600,0.8500)	(0.9500,1.600,2.2500)

The optimal value of the given problem (1) can be obtained by

$$\widetilde{y}_1 = (0.45, 0.8, 1.15), \widetilde{y}_2 = (0.0727, 0.6, 1.1273), \min \widetilde{z} = (2.9908, 7.2, 11.4092).$$

The corresponding given problem of the optimal values for $\widetilde{x}_1, \widetilde{x}_2$ are to be obtained by

$$\widetilde{x}_1 = (0.35, 0.6, 0.85), \widetilde{x}_2 = (0.95, 1.6, 2.25), \max \widetilde{z} = (4.25, 7.2, 10.15).$$

3 Conclusions

This article outlines a method for solving the primal-dual fuzzy linear programming problem more effectively by combining the fuzzy exponential barrier function and the fuzzy exponential barrier parameter.The table for the primal-dual problem demonstrates how quickly the best solution can be reached using the fuzzy exponential barrier primal-dual algorithm we developed when the Fuzzy Exponential barrier parameter is used.

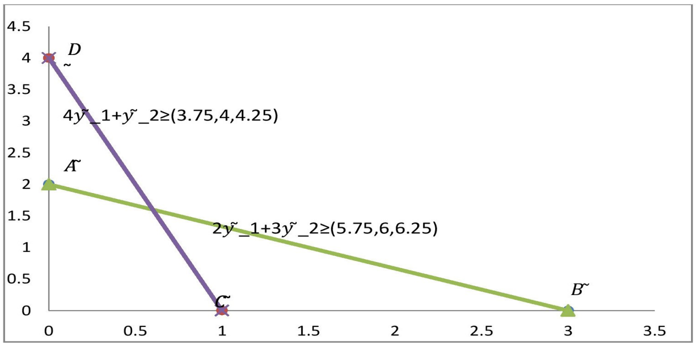

Figure 1: Finding an infeasible solution for primal fuzzy linear programming problem

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