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Super fuzzy matrix of inverse in kth Order

R.Deepa*
P. Sundararajan†

Abstract

In normal matrix and fuzzy matrix working well but sometimes fail to work for classical model problems. Because, it is not satisfied the consistency conditions and other parameter. Our proposed method to satisfy the all condition including consistency and perform well compared to the other existing models. Unexpected event modelling is a affluent area of study in fuzzy matrix (FM) modelling. Every fuzzy matrix may be shown as a multi-dimensional concept, but standard matrices cannot achieve this without the proper scale. To solve this issue, a certain kind of classical fuzzy matrix is required. In this study, the idea of an inverse of a k-regular fuzzy matrix is introduced, and some of its key characteristics are listed. As a result, the same regularity indicator is used to describe a matrix. Investigation is also done on the relationship between the regular, k-regular, and consistency of fuzzy matrices powers.

Keywords: inverse; k-regular; super fuzzy matrix; Fuzzy matrix; regular

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*Excel Engineering College(Autonomous), Komarapalayam, Tamilnadu, India. deepssengo @gmail.com.

†Department of Mathematics, Arignar Anna Govt. Arts College, Namakkal. Tamilnadu, India. ponsundar03@gmail.com.

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1 Introduction

Each component of a matrix known as a boolean matrix has a value of 0 or 1. Basic values for a fuzzy matrix fall between [0, 1]. By Kim and Roush, the idea of fuzzy matrix sections was introduced. In all auothers contributed significantly to fuzzy matrix augmentation. Later, utilising modified regular inverses of the coefficient matrix, Zheng and Wang developed the widely used m x n fuzzy linear machine and the inconsistent fuzzy linear system. Using matrix-modified common inverse theory, Abbasbandy .S and . M [2005] investigated the minimal response of the overall twin fuzzy linear device. We provide a unique way for finding the inverse of a fantastic fuzzy matrix by changing a well-known idea in this research D and H [1978],P [1963], Ravi.J and et. al. [2022], Kandasamy W.B.V. and llanthenral K. [2007], M.Z. and E.G [1994], M.G [1977], K and B [2006], B and K [2006].
The super fuzzy matrix's initial and core notions are covered in Section 2 along with an illustration. The method of our suggested notion is presented in Section 3. The main findings, theorems, and numerical examples of super fuzzy inverse matrices of order k are presented in Section 4, and the conclusion is derived in Section 5.

2 Preliminaries

noindent The core concepts and notations of a super fuzzy matrix will be described in this section. This article concentrates on the same old number ordering, underlying maxmin (min max) operations, and fuzzy matrices in super (SFM) with [0,1] support. By (SF)mxn and (SF)n, respectively, all fuzzy matrices in super of order m x n and n x n are represented. Space created by the row (or) column is indicated by the letters R(A) or C(A) (A) , Kaufmann and Gupta [1985], J.B [1983], .

Definition 2.1. *Deepa.R and Sundararajan.P [2020] A matrix containing entries that fall between [0, 1] is referred to as a fuzzy matrix. In the case of matrices, we have a fuzzy related different matrix.*

Definition 2.2. *Deepa.R and Sundararajan.P [2020] Let us deem C fuzzy matrix*
$C = \begin{bmatrix} C_{11} & C_{12} & C \\ C_{21} & C_{22} & C_{23} \end{bmatrix}$

The fuzzy submatrices C_{11} and C_{21} have the same number of columns because they are fuzzy submatrices, along with C_{12}, C_{13}, C_{21}, C_{22}, and C_{23}. The fuzzy matrices C13 and C23, as well as the fuzzy submatrices C_{12} and C_{22}, all have equal columns. The second index of the fuzzy sub-matrices reveals this. The number of rows in fuzzy submatrices C_{11}, C_{12}, and C_{13} is the same. There are exactly the same number of rows in the C_{21}, C_{22}, and C_{23} fuzzy submatrices. as a result, we currently possess C generic super fuzzy matrix.

$$C = \begin{bmatrix} C_{11} & C_{12}\dots & C_{1n} \\ C_{21} & C_{22\dots} & C_{2n} \\ & & . \\ & & . \\ & & . \\ C_{m1} & C_{m2\dots} & C_{mn} \end{bmatrix}$$

where C_{ij}'s i = 1, 2, …, m and j =1, 2, …, n.

Definition 2.3. *The modified regular inverse of matrix solution of a positive set of equations can be found by altering the regular inverse of a non-singular matrix. The modified regular inverse of any (potentially square) matrix with complex components may be found using any method. It is used here and in other programmes to immediately solve linear matrix issues as well as to get an expression for the principal idempotent components of the matrix.*

3 Methodology

The k^{th} regular super fuzzy matrix is examined in this section.

Definition 3.1. *Let C =* $\begin{bmatrix} c_{11} & c_{12}\dots & c_{1n} \\ c_{21} & c_{22\dots} & c_{2n} \\ & & . \\ & & . \\ & & . \\ c_{m1} & c_{m2\dots} & c_{mn} \end{bmatrix}$

It is a matrix of fuzzy that is C^k each a_{ij} is 0$\leq a_{ij} \leq$1. The exact strength of each ingredient is examined here. It is the G-name. inverse's.

$$C^k = \begin{bmatrix} 1-kc_{11} & 1-kc_{12}\dots & 1-kc_{1n} \\ 1-kc_{21} & 1-kc_{22\dots} & 1-kc_{2n} \\ & & . \\ & & . \\ & & . \\ 1-kc_{m1} & 1-kc_{m2\dots} & 1-kc_{mn} \end{bmatrix}$$

Here, k is the smallest positive integer (Based on Probability). It is possible to multiply two fuzzy matrices without requiring that they both be fuzzy. Every entry that is positive is changed to a 1, and every entry that is negative is changed to a 0 or a tendency toward zero.

Definition 3.2. *The Properties of Super fuzzy matrix is,*

$$\left(A^k\right)^k = A$$

$AxA^k = A^k$,
$(A+B)^k = A^k + B^k$,
$(\lambda A)^k = \lambda A^k$,
$(BA)^k = A^k B^k$,
$AA^k=0$ *implies* $A=0$.

Definition 3.3. *The condition of the supper fuzzy matrix are given below,*
$AXA=A$
$XAX=X$
$(AX)^k = AX$
$(XA)^k = XA$
$XX^kA^k = X$
$XAA^k=A^k$
$BA^kAA^k=A^k$
$X=XX^kA^k=XX^kA^kAY=XAY=XAA^kY^kY=A^kY^kY=Y.$

4 Result & application

The C^k of super fuzzy matrix met all of the requirements for convergence.

Theorem 4.1. *Let A be the inverse of the k by KRSFM (k regular super fuzzy matrix), with non-zero rows representing the norm. If A meets the maximin conditions for the matrix equation ASA=A for some super fuzzy matrix S, then C is k regular.*

Proof. The non-zero rows of the inverse of the KRSFM of C serve as the standard basis in this case. If SB=Q, then Q's rows are permutations of C's rows. Then, X is an idempotent of KRSFM, with the same row space as C and non-zero Z rows functioning as a standard basis..
Since the standard foundation is different for each S, B=GQ.
Then
$CS^TC=SQS^TSQ=SQQ=SQ=C^k$

$\Rightarrow$CSC=C^k.
So, C is k regular.

Theorem 4.2. *Let C,D (SF)nxm and k=p be two inverse of p regular super fuzzy matrix. If C is p regular, then we prove that*

Hence Di* = $\sum x_{ij}A^*$.
$\Rightarrow$D=XCp
$\Rightarrow$D=XCC'C (since CC'C=Cp)
$\Rightarrow$D=DC'Cp
$\Rightarrow$D=CC'C
$\Rightarrow$D=CC'CY.
$\Rightarrow$D=CpC'D.

Theorem 4.3. *For $C \in (SF)_n$ and for any $G^- \in (SF)_n$, if $C^kX=C^kG^-$, where X is a $\{1^K,3^K\}$ inverse of C then, G^- is a $\{1^K,3^K\}$ inverse of C.*

Proof. Since X is a $\{1^K,3^K\}$
$\Rightarrow C^kXC=C^k$ and $(C^kX)^T=C^kX$.
The Post is multiplied by A on both sides of $C^kX=C^kG^-$, $C^kG^-C=C^kXC=C^k$.
$(C^kG^-)^T=(C^kX)^T=C^kX=C^kG^-$.
Hence G^- is a $\{1^K,3^K\}$inverse of C.

Theorem 4.4. *For $C \in (SF)_n$, X is a $\{1^P,3^P\}$ inverse of C and H^- is a $\{1^P,3^P\}$ inverse of C then, $C^pX=C^pH^-$.*

Proof. Since X is a $\{1^P,3^P\}$ inverse of C.
$\Rightarrow C^pXC=C^p$ and $(C^pX)^T=C^pX$.
H^- is a $\{1^P,3^P\}$ inverse of C,
$\Rightarrow CH^-C^p=C^p$ and $(CH^-)^T=CH^-$.
$C^pH^-=(C^pXC)H^-=(C^pX)^T(CH^-)=(C^pX)^T(CH^-)^T=X^T(C^T)^p(H^-)^TC^T$
$=X^T(CH^-C^p)^T=X^T(C^p)^T=(C^pX)^T=C^pX$.

Theorem 4.5. *For $D \in (SF)_n$, if D^TD is D right KRSFM and $R(D^p) \subseteq R(D^TD)^p$ then D has a $\{1^P,3^P\}$ inverse. In particular for p=1, $U=(D^TD)^-D^T$ is a $\{1, 3\}$ inverse of D.*

Proof. let $(D^TD)^p(D^TD)^-(D^TD)=(D^TD)^p$
Since $R(D^p) \subseteq R((D^TD)^p)$,
$D^p=X(D^TD)^p$ for some $X \in (SF)_n$ Dnd tape $U=(D^TD)^-D^T$
$D^pUD=(D^p)(UD)=(X(D^TD)^p)((D^TD)^-D^TD)$
$=X((D^TD)^p(D^TD)^-(D^TD))$
$=X(D^TD)^p=(D^p)$.
TDpe $V=(D^TD)^-(D^p)^T$.

$D^pV=(D^p)V$
$=(X(D^TD)^p)((D^TD)^-(D^p)^T)$
$= X(D^TD)^p(D^TD)^-(D^TD)^pD^T$
$=X(D^TD)^p(D^TD)^-(D^TD)(D^TD)^{p-1}X^T$
$= X(D^TD)^p(D^TD)^{p-1}X^T$
$=X(D^TD)^{2p-1}X^T=(X(D^TD)^{2p-1}X^T)^T = (D^pV)^T$.
Hence D has D $\{1^P,3^P\}$inverse.
In particular for p=1,Y= $(D^TD)^-D^T$ is D $\{1, 3\}$ inverse of D.

Theorem 4.6. *Let $C\in(SF)_n$ be a right KRSFM and $R(C^TC)^k\subseteq R(C^k)$ then C^TC has a $\{3^K\}$inverse.*

Proof. Since, $C^kXC=C^k$.
Since $R((C^TC)^k)\subseteq R(C^k)$, $(C^TC)^k=ZC^k$ for some $Z\in(SF)_n$ and take Y=XC.
$(C^TC)^kY=(ZC^k)(XC)=Z(C^kXC)=ZC^k=(C^TC)^k=((C^T C)^k)^T$
$=((C^TC)^kY)^T$
Hence C^T C has a $\{3^K\}$ inverse.

The findings for the other associated fuzzy matrices improve, and the theorem is fulfilled. While some techniques (regular and inverse) meet some conditions, our proposed KRSFM meets all of them (regular & inverse).

Example 4.1. *Let us consider IKSFM*

$$A=\begin{bmatrix} 0.5 & 0.7 & 0.6 \\ 0.3 & 0.2 & 0.2 \\ 0.4 & 0.3 & 0.1 \end{bmatrix}$$

The rows of A are separate and make up the foundation. IKSFPM is a form that

$$P=\begin{bmatrix} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We know that APA=A. Now for IKSFM

$$R=\begin{bmatrix} 0.1 & 0.4 & 0.2 \\ 0.3 & 0.4 & 0.2 \\ 0.3 & 0.1 & 0.1 \end{bmatrix}$$

There for RA=A. So g inverse of A sis

$$PR=\begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.1 & 0.4 & 0.2 \\ 0.3 & 0.1 & 0.1 \end{bmatrix}$$

Which is satisfy the relation AXA=A.

Example 4.2. *Let* $A = \begin{bmatrix} <0.4, 0.2> & <0.4, 0.5> \\ <0.8,\ 0.1> & <0.3, 0.4> \end{bmatrix}$ *and*

$B = \begin{bmatrix} <0.6, 0.2> & <0.5, 0.4> \\ <0.,7\ 0.1> & <0.4, 0.4> \end{bmatrix}$

For which B=BA^{T}A holds.

Example 4.3. *Let IKSFM is,*

$$C = \begin{bmatrix} <0.8, 0.2> & <0.4, 0.2> & <0.3, 0.2> \\ <0.4, 0.2> & <0.4, 0.2> & <0.3, 0.2> \\ <0.6, 0.2> & <0.6, 0.2> & <0.8, 0.2> \end{bmatrix}$$

$$D = \begin{bmatrix} <0.8, 0.2> & <0.4, 0.3> & <0.2, 0.2> \\ <0.4, 0.3> & <0.4, 0.2> & <0.2, 0.2> \\ <0.4, 0.2> & <0.4, 0.2> & <0.6, 0.2> \end{bmatrix}$$

$$E = \begin{bmatrix} <0.7, 0.2> & <0.4, 0.3> & <0.3, 0.3> \\ <0.4, 0.3> & <0.4, 0.2> & <0.4, 0.2> \\ <0.4, 0.2> & <0.6, 0.2> & <0.8, 0.2> \end{bmatrix}$$

Be two of its g inverse of A.

Then $X = CDE = \begin{bmatrix} <0.8, 0.2> & <0.4, 0.2> & <0.3, 0.2> \\ <0.4, 0.2> & <0.4, 0.2> & <0.3, 0.2> \\ <0.6, 0.2> & <0.6, 0.2> & <0.8, 0.2> \end{bmatrix}$

Forth above X, CXC=C and XCX=X holds. So X is a semi inverse of the IKSFM.

Example 4.4. *Let us consider the symmetric IKSFM*

$A = \begin{bmatrix} <0.4, 0.2> & <0.4, 0.5> \\ <0.8,\ 0.1> & <0.3, 0.4> \end{bmatrix}$

Now, A^2 is,

$A^2 = \begin{bmatrix} <0.4, 0.2> & <0.4, 0.5> \\ <0.8,\ 0.1> & <0.3, 0.4> \end{bmatrix} \begin{bmatrix} <0.4, 0.2> & <0.4, 0.5> \\ <0.8,\ 0.1> & <0.3, 0.4> \end{bmatrix}$

$$= \begin{bmatrix} <0.4, 0.2> & <0.4, 0.5> \\ <0.8,\ 0.1> & <0.3, 0.4> \end{bmatrix}$$

$= A$

The above matrix is symmetric and idempotent.

Example 4.5. *Let us consider the symmetric IKSFM*
$A = \begin{bmatrix} <0.4, 0.2> & <0.4, 0.5> \\ <0.8,\ 0.1> & <0.3, 0.4> \end{bmatrix}$ *and*
$B = \begin{bmatrix} <0.4, 0.2> & <0.4, 0.5> \\ <0.8,\ 0> & <0, 0.4> \end{bmatrix}$
Now, A^2 *is,*
$B^2 = \begin{bmatrix} <0.4, 0.2> & <0.4, 0.5> \\ <0.8,\ 0> & <0, 0.4> \end{bmatrix} \begin{bmatrix} <0.4, 0.2> & <0.4, 0.5> \\ <0.8,\ 0> & <0, 0.4> \end{bmatrix}$

$$= \begin{bmatrix} <0.4, 0.2> & <0.4, 0.5> \\ <0.8,\ 0.1> & <0.3, 0.4> \end{bmatrix}$$

$\neq B$
The above matrix is not idempotent.

5 Conclusions

The main conclusions of the study may be presented in a short Conclusions section, which may stand alone or form a subsection of a Discussion or Results. In this paper, the inverse of the KRSFM is used to provide several innovative proposals and theorems. The original machine is replaced with matrix coefficient A by two distinct n m matrix equation systems. As a result, FM must resolve the issue. The k-regular, the regularity of the fuzzy matrix powers, and the relationship between each regular are all examples of regularities. Our approach addresses any issues that need the inverse of k regular super fuzzy matrices. A appropriate theorem is also shown. In the next essay, we will look at a variety of fundamental properties as well as computer vision applications. In our proposed classical model to proved, it is a consistent. Also in future, we are trying this concept in neural networking system and IOT.

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