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Properties of nano generalized pre c-interior in a nano topological space.

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Abstract

The aim of this paper is to introduce and study the properties the nano generalized pre c- interior of a set such as nano generalized pre c-border and nano generalized pre c-exterior in a nano topological space.

Keywords:Nano generalized pre c-border, Nano generalized pre c-exterior.

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1 Introduction

The concept of generalized-semi closed sets to characterize the S-normality axiom was introduced by S.P.Arya et.al. The semi-generalized mappings and generalized-semi mappings were studied. In 2013 , Govindappa Navalagi investigated some of the regularity axioms, normality axioms and continuous functions through gs-open sets and sg-open sets. Also, Govindappa Navalagi continued the study of gs-continuous and sg-continuous functions to introduce the new notions like generalized semiclosure and generalized semi-interior operators. Lellis Thivagar [1] obtained the notion of nano topology and he studied the various forms of nano sets, their closures and interiors and their homeomorphisms Lellis Thivagar et al introduced nano topological space with respect to a subset of a Universe which is defined in terms of approximations and boundary region. In this paper, I have introduced the properties of nano generalized pre c-interior in a nano topological space.

2 Preliminaries

Definition 2.1. *[3] Let $\Im$ be a non empty finite set of objects called the universe and $\Re$ be an equivalence relation on $\Im$ named as indiscernibility relation. Then $\Im$ is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\Im, \Re)$ is said to be approximation space. Let $\aleph \subseteq \Im$. Then*

(i) The lower approximation of $\aleph$ with respect to $\Re$ is the set of all objects, which can be for certain classified as $\aleph$ with respect to $\Re$ and it is denoted by $\gamma_{\Re}(\aleph)$. $\gamma_{\Re}(\aleph) = \Im_{x\in\Im}\Re(x) : \Re(x) \subseteq \aleph$ by $\gamma_{\Re}(\aleph)$.
where $\Re(\aleph)$ denotes the equivalence class determined.

(ii) The upper approximation of $\aleph$ with respect to $\Re$ is the set of all objects which can be possibly classified as $\aleph$ with respect to $\Re$ and it is denoted by $\tau_{\Re}(\aleph)$. $\tau_{\Re}(\aleph) = \Im_{x\in\Im}\Re(x) : \Re(x) \bigcap \aleph \neq 0$.

(iii) The boundary region of $\aleph$ with respect to $\Re$ is the set of all objects which can be classified neither as $\aleph$ nor as not-X with respect to $\Re$ and it is denoted by $B_{\Re}(\aleph)$.$B_{\Re}(\aleph) = \tau_{\Re}(\aleph) - \gamma_{\Re}(\aleph)$.

Proposition 2.1. [3] If $(\Im, \Re)$ is an approximation space and $\aleph, Y \subseteq \Im$,then

1. $\gamma_{\Re}(\aleph) \subseteq \aleph \subseteq \tau_{\Re}(\aleph)$
2. $\gamma_{\Re}(\phi) = \tau_{\Re}(\aleph) = \phi$

3. $\gamma_{\Re}(U) = \tau_{\Re}(\Im) = \Im$
4. $\tau_{\Re}(\aleph \cup Y) = \tau_{\Re}(\aleph) \cup \tau_{\Re}(Y)$
5. $\tau_{\Re}(\aleph \cap Y) \subseteq \tau_{\Re}(\aleph) \cap \tau_{\Re}(Y)$
6. $\gamma_{\Re}(\aleph \cup Y) \supseteq \gamma_{\Re}(\aleph) \cup \gamma_{\Re}(Y)$
7. $\gamma_{\Re}(\aleph \cap Y) = \gamma_{\Re}(\aleph) \cap \gamma_{\Re}(Y)$
8. $\gamma_{\Re}(\aleph) \subseteq \gamma_{\Re}(Y) and \tau_{\Re}(\aleph) \subseteq \tau_{\Re}(Y)$, whenever $\aleph \subseteq Y$.
9. $\tau_{\Re}(\aleph^c) = [\gamma_{\Re}(\aleph)]^c and \gamma_{\Re}(\aleph^c) = [\tau_{\Re}(\aleph)]^c$
10. $\tau_{\Re}[\tau_{\Re}(\aleph)] = \gamma_{\Re}[\tau_{\Re}(\aleph)] = \tau_{\Re}(\aleph)$
11. $\gamma_{\Re}[\gamma_{\Re}(\aleph)] = \tau_{\Re}[\gamma_{\Re}(\aleph)] = \gamma_{\Re}(\aleph)$

Definition 2.2. [1] Let $\Im$ be the universe, $\Re$ be an equivalence relation on $\Im$ and $r_{\Re}(\aleph) = \{\Im, \phi, \gamma_{\Re}(\aleph), \tau_{\Re}(\aleph), B_{\Re}(\aleph)\}$ where $\aleph \subseteq \Im$. Then $r_{\Re}(\aleph)$ satisfies the following axioms.

1. $\Im$ and $\phi \in r_{\Re}(\aleph)$.
2. The union of all the elements of any sub-collection of $r_{\Re}(\aleph)$ is in $r_{\Re}(\aleph)$.
3. The intersection of the elements of any finite sub collection of $r_{\Re}(\aleph)$ is in $r_{\Re}(\aleph)$.Then $r_{\Re}(\aleph)$ is a topology on $\Im$ called the nano topology on $\Im$ with respect to $\aleph$. The elements of $r_{\Re}(\aleph)$ are called as nano open sets in $\Im$ and $(\Im, r_{\Re}(\aleph))$ is called as a nano topological space. The complement of the nano open sets are called nano closed sets.

Definition 2.3. *[1] If $(\Im, r_{\Re}(\aleph))$ is a nano topological space with respect to $\aleph$, where $\aleph \subseteq \Im$ and if $A \subseteq \Im$, then*

1. *The nano interior of A is defined as the union of all nano open subsets contained in A and is denoted by $Nint(A)$. That is $Nint(A)$ is the largest nano open subset of A .*
2. *The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$. That is $Ncl(A)$ is the smallest nano closed set containing A.*

Definition 2.4. *[2] A subset A of a nano topological space $(\Im, r_{\Re}(\aleph))$ is called a nano generalized pre c-closed set (briefly $Ngpc-closed$ set) if $Npcl(A) \subseteq G$ whenever $A \subseteq G$ and C is nano c-set.*
The complement of a $Ngpc-closed$ set is called $Ngpc-open$ set.

Definition 2.5. *[2] The Nano generalized pre c-interior of a set* A *in* $(\Im, r_{\Re}(\aleph))$ *is defined as the union of all* $Ngpc-$*open sets of* U *contained in* A *and it is denoted by* $Ngpc-int(A)$. *That is* $Ngpc-int(A)$ *is the largest* $Ngpc-$*open subset of* A.

Definition 2.6. *[2] The Nano generalized pre c-closure of a set* A *in* $(\Im, r_{\Re}(\aleph))$ *is defined as the intersection of all* $Ngpc-$*closed sets of* U *containing* A *and it is denoted by* $Ngpc-cl(A)$. *That is* $Ngpc-cl(A)$ *is the smallest* $Ngpc-$*closed superset of* A *in* IU.

Remark 2.1. *[2]*

1. *A subset* A *of* $(\Im, r_{\Re}(\aleph))$ *is* $Ngpc-$*open if and ony if* $Ngpc-int(A) = A$.
2. *A subset* A *of* $(\Im, r_{\Re}(\aleph))$ *is* $Ngpc-$*closed if and only if* $Ngpc-cl(A) = A$.

Theorem 2.1. *[2] Let* A *and* B *be subsets of* $(\Im, r_{\Re}(\aleph))$. *Then*

1. $Ngpc-int(\Im) = \Im$ *and* $Ngpc-int(\phi) = \phi$.
2. $Ngpc-int(A) \subset A$.
3. *If* B *is any* $Ngpc-$*open set contained in* A, *then* $B \subset Ngpc-int(A)$.
4. *If* $A \subset B$ *then* $Ngpc-int(A) \subseteq Ngpc-int(B)$.
5. $Ngpc-int(Ngpc-int(A)) = Ngpc-int(A)$.

Theorem 2.2. *[2] If* A *and* B *are subsets of* $\Im$, *then the following statements are true.*

1. $Ngpc-int(A) \cup Ngpc-int(B) \subset Ngpc-int(A \cup B)$.
2. $Ngpc-int(A \cap B) = Ngpc-int(A) \cap Ngpc-int(B)$.

Theorem 2.3. *[2] If* A *is a subset of* $(\Im, r_{\Re}(\aleph))$, *then* $Nint(A) \subset Ngpc-int(A)$.

Theorem 2.4. *[2] For the subsets* A *and* B *of* $\Im$, *the following statements are true.*

1. $\Im - Ngpc-cl(A) \subset Ngpc-cl(\Im - A)$.
2. *If* A *is* $Ngpc-$*closed then* $Ngpc-cl(A)-Ngpc-cl(B) \subset Ngpc-cl(A-B)$.

3 Properties of nano generalized pre c-interior

In this section the nano generalized pre c-border and nano generalized pre c-exterior of a set are defined in terms of nano generalized pre c-interior and some of their properties are derived.

Definition 3.1. *The nano generalized pre c-border of a set* A *in* $(\Im, r_{\Re}(\aleph))$ *is defined as* $A - Ngpc - int(A)$ *and it is denoted by* $Ngpc - Bd(A)$*.*

Definition 3.2. *The nano generalized pre c-exterior of a set* A *in* $(\Im, r_{\Re}(\aleph))$ *is defined as* $Ngpc - int(\Im - A)$ *and it is denoted by* $Ngpc - ext(A)$*.*

Example 3.1. *Let* $\Im = \{a, b, c, d\}$ *with* $\Im/\Re = \{\{a\}, \{b\}, \{c, d\}\}$ *and* $\aleph = \{b, d\}$*. Then* $r_{\Re}(\aleph) = \{\Im, \phi, \{b\}, \{c, d\}, \{b, c, d\}\}$ *is a nano topology on* U *with respect to* $\aleph$*. The complement of* $r_{\Re}(\aleph)$ *is given by* $r_C(\aleph) = \{U, \phi, \{a\}, \{a, b\},$
$\{a, c, d\}$*.* $Ngpc-$*closed sets are* $\{\phi, \Im, \{a\}, \{c\}, \{d\},$
$\{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}$*.* $Ngpc-$*open sets are* $\phi, \Im,$
$\{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}$*.*
Here $Ngpc - int(\{a\}) = \phi$*,* $Ngpc - int(\{b\}) = \{b\}$*,* $Ngpc - int(\{a, c, d\}) = \{c, d\}$*,* $Ngpc - int(\{c, d\}) = \{c, d\}$ *and* $Ngpc - int(\{a, b, d\}) = \{a, b, d\}$*.*
Then $Ngpc - Bd(\{a\}) = \{a\}, Ngpc - Bd(\{b\}) = \phi$ *,*
$Ngpc - Bd(\{a, b, d\}) = \phi$ *and* $Ngpc - Bd(\{a, c, d\}) = \{a\}$*.*
$Ngpc - ext(\{a\}) = \{b, c, d\}$*,* $Ngpc - ext(\{b\}) = \{c, d\}$*,* $Ngpc - ext(\{a, b\}) = \{c, d\}$ *and* $Ngpc - ext(\{a, c, d\}) = \{b\}$*.*

Theorem 3.1. *For a subset* A *of* $\Im$ *the following statements hold.*

1. $Ngpc - Bd(\phi) = Ngpc - Bd(\Im) = \phi$.
2. $Ngpc - Bd(A) \subset NBd(A)$.
3. $A = Ngpc - int(A) \cup Ngpc - Bd(A)$.
4. $Ngpc - int(A) \cap Ngpc - Bd(A) = \phi$.
5. $Ngpc - int(A) = A - Ngpc - Bd(A)$.
6. $Ngpc - int(Ngpc - Bd(A)) = Ngpc - Bd(Ngpc - int(A)) = \phi$.
7. *A is* $Ngpc-$*open if and only if* $Ngpc - Bd(A) = \phi$.
8. $Ngpc - Bd(Ngpc - Bd(A)) = Ngpc - Bd(A)$.

Proof. 1. The proof is an immediate consequence of definition (3.1).

2. Let $x \in Ngpc - Bd(A)$. $\Rightarrow x \in A - Ngpc - int(A)$. By theorem (2.3), $Nint(A) \subset Ngpc - int(A) \Rightarrow A - Ngpc - int(A) \subset A - Nint(A)$.
   Hence $x \in A - Ngpc - int(A) \Rightarrow x \in A - Nint(A)$.
   $\Rightarrow x \in NBd(A)$. Therefore $Ngpc - Bd(A) \subset NBd(A)$.

3. $Ngpc - int(A) \cup Ngpc - Bd(A) = Ngpc - int(A) \cup (A - Ngpc - int(A)) = A$.

4. $Ngpc - int(A) \cap Ngpc - Bd(A) = Ngpc - int(A) \cap (A - Ngpc - int(A)) = \phi$.

5. The proof directly follows from definition (3.1).

6. Let $x \in Ngpc - int(Ngpc - Bd(A))$. Then $x \in Ngpc - Bd(A)$ as $Ngpc - Bd(A) \subset A$.
   Also $x \in Ngpc - int(Ngpc - Bd(A)) \subset Ngpc - int(A)R$.
   Therefore $x \in Ngpc - int(A) \cap Ngpc - Bd(A)$ which is a contradiction to (d).
   Hence $Ngpc - int(Ngpc - Bd(A)) = \phi$.

7. By result (2.8), A is $Ngpc - open \Leftrightarrow Ngpc - int(A) = A \Leftrightarrow A - Ngpc - int(A) = \phi$
   $\Leftrightarrow Ngpc - Bd(A) = \phi$. (by definition (3.1))

8. In definition (3.1) let $A = Ngpc - Bd(A)$.
   Then $Ngpc - Bd(Ngpc - Bd(A)) = Ngpc - Bd(A) - Ngpc - int(Ngpc - Bd(A)) = Ngpc - Bd(A) - \phi$
   $= Ngpc - Bd(A)$. (Using (6)).

□

Theorem 3.2. *For the subsets A and B of $\Im$ the following statements hold.*

1. $Ngpc - ext(\phi) = \Im$ *and* $Ngpc - ext(\Im) = \phi$.

2. $Next(A)Ngpc - ext(A)$.

3. *If* $A \subset B$ *then* $Ngpc - ext(B) \subset Ngpc - ext(A)$.

4. $Ngpc - ext(A)$ *is* $Ngpc-$*open.*

5. $Ngpc - ext(A) = \Im - Ngpc - cl(A)$.

6. *A is* $Ngpc-$*closed if and only if* $Ngpc - ext(A) = \Im - A$.

7. $Ngpc - ext(Ngpc - ext(A)) = Ngpc - int(Ngpc - cl(A))$

8. $Ngpc - ext(Ngpc - ext(A)) = Ngpc - ext(Ngpc - int(\Im - A)) = Ngpc - ext(\Im - Ngpc - cl(A))$.

9. $Ngpc - ext(A \cup B) \subset Ngpc - ext(A) \cup Ngpc - ext(B)$.

10. $Ngpc - ext(A \cup B) = Ngpc - ext(A) \cap Ngpc - ext(B)$.

11. $Ngpc - ext(A) \cap Ngpc - ext(B) \subset Ngpc - ext(A \cap B)$.

Proof. 1. The proof is immediate from definition (3.2).

2. $Next(A) \subset Ngpc - ext(A)$ follows from theorem (2.3).

3. If $A \subset B$ then $\Im - B \subset \Im - A$. By (iv) of theorem (2.2), $Ngpc - int(\Im - B) \subset Ngpc - int(\Im - A)$. Hence $Ngpc - ext(B) \subset Ngpc - ext(A)$.

4. Consider $Ngpc - int(Ngpc - ext(A)) = Ngpc - int(Ngpc - int(\Im - A))$ $= Ngpc - int(\Im - A) = Ngpc - ext(A)$. (by (v) of theorem (2.9)) By remark (2.1),
$Ngpc - ext(A)$ is $Ngpc-$open.

5. $Ngpc - ext(A) = Ngpc - int(\Im - A) = \Im - Ngpc - cl(A)$. (from (ii) of theorem (2.4)).

6. By remark (2.1), A is $Ngpc - closed \Leftrightarrow Ngpc - cl(A) = A \Leftrightarrow \Im - Ngpc - cl(A) = \Im - A - Ngpc - int(\Im - A)$
$= \Im - A \Leftrightarrow Ngpc - ext(A) = \Im - A$.

7. In definition let $A = Ngpc - ext(A)$.Then $Ngpc - ext(Ngpc - ext(A)) = Ngpc - int(\Im - Ngpc - ext(A)$
$= Ngpc - int(Ngpc - cl(A))$. (Using (5)).

8. It follows from definition (3.2) and (5).

9. We know that $A \subset A \cup B$ and $B \subset A \cup B$. From (c) $Ngpc - ext(A \cup B) \subset Ngpc - ext(B)$ and $Ngpc - ext(A \cup B) \subset Ngpc - ext(B)$. Hence $Ngpc - ext(A \cup B) \subset Ngpc - ext(A) \subset Ngpc - ext(B)$.

10. $Ngpc - ext(A \cup B) = Ngpc - int(\Im - (A \cup B))$. (by definition (3.2))
$= Ngpc - int((\Im - A) \cap (\Im - B))$.
$= Ngpc - int(\Im - A) \cap Ngpc - int(\Im - B)$. (by (ii) of theorem(2.4))
$= Ngpc - ext(A) \cap Ngpc - ext(B)$.
Hence $Ngpc - ext(A \cup B) = Ngpc - ext(A) \cap Ngpc - ext(B)$.

11. We know that $A \cap B \subset A$ and $A \cap B \subset B$. From (c) $Ngpc - ext(A) \subset Ngpc - ext(A \cap B)$ and $Ngpc - ext(B) \subset Ngpc - ext(A \cap B)$. Hence $Ngpc - ext(A) \cap Ngpc - ext(B) \subset Ngpc - ext(A \cap B)$.

□

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