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A dynamical analysis of a mathematical model on type-2 diabetic from obestiy

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Abstract

A model for type-2 diabetic from obesity with two control variables diet with physical activity and medication is formulated.The disease free and endemic equilibrium points of the model are obtained. The existence of optimal controls is verified through Pontryagainaximum principle. The local stability is analyzed using Routh-Hurwitz criteria. The global stability is studied using Lyapunov function. The parameters are chosen based on the female population in India. The aim of this research is to construct a model for type-2 diabetic from obesity using parameters based on the female population in India. We have introduced two control variables as diet with physical activity and medication. The positive endemic equilibrium is obtained. The local and global stability of the model are analyzed with some specific conditions. Numerical simulations are carried out to exhibit the flow of variables with controls. Our study mainly highlights the awareness of metabolic risk by healthy diet, physical activities and medications.

Keywords: Obesity, Diabetic, Pontryagain's maximum principle, Lyapunov function, Stability.
2020 AMS subject classifications: 34D20, 34D23,81T80 [1]

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1 Introduction

Obesity and Diabetes are two epidemics that are wreaking havoc around the world. Obesity puts you at a much increased chance of having diabetes throughout the course of your life. Obesity is more than a cosmetic issue. It is a long-term medical condition that can progress to diabetes. Obese people account for over 90 percent of people with type 2 diabetes which is characterised by the body's failure to regulate and control blood sugar levels. Obesity allows diabetes to take up residence in the body. Obesity increases the risk of developing diabetes which is characterised by an excess of glucose in the bloodstream. Obesity is linked to a higher risk of non communicable diseases such as diabetes, heart disease and hypertension. Obesity is one of the primary causes of early death and 4.7 million deaths worldwide was recorded in 2017 due to obesity. The key to understanding the link between obesity and diabetes is to look at insulin resistance. Physical inactivity, overeating, heredity, medications and psychological factors are the most prevalent causes of obesity. Type 2 diabetes has no known treatment however, it can be in control. Inactivity, family history, high blood pressure and obese are the most common causes of type 2 diabetics. Obesity reduction is a significant objective for the treatment and prevention of type 2 diabetes. Many authors studied a mathematical models for obesity and diabetic. A Boutayed Boutayed et al. [2010] developed a mathematical model for the burdern of diabetes and its complications. Wiam Boutayeb et al. [2014] introduced a mathematical model for the impact of obesity on predisposed people to type 2 diabetes. Nikhil Kumar et al. [2017] studied a diabetsity an epidemic with its causes, prevention and control with special focus on dietary regime. Miguel A. Ortega Ortega et al. [2020] illustrated a technique for type 2 diabetes mellitus associated with obesity. Abdelfatah Kouidere K.Abdelfatah et al. [2020] developed a mathematical model for the dynamics of population of diabetics and its complications with effect of behavioral factors. Wellars Banzi et al. [2021] studied a mathematical model for Glucose-Insulin System and test of abnormalities of type 2 diabetic patients. In this paper, we formulated and anlaysed a mathematical model for type-2 diabetic from obesity with control variables.

2 Formulation of model

The system of Ordinary Differential Equation represents the Obesity and Diabetic model as follows::

$$\begin{aligned}\frac{dS}{dt} &= \omega N - (\beta + \mu)S \\ \frac{dO}{dt} &= \beta S - (\alpha + \mu + u)O \\ \frac{dD}{dt} &= \alpha O - (\mu + u + v)D \\ \frac{dR}{dt} &= (u + v)D + uO - \mu R\end{aligned} \tag{1}$$

With the initial conditions as $S(t), O(t), D(t), R(t) \geq 0$. Also $\beta, \mu, \omega, \alpha, u, v > 0$.

Where $S(t), O(t), D(t), R(t)$ are the Susceptible, Infected by Obesity, Infected by Diabetic and Recovery state respectively and ω - Average birth rate of female, μ - Average death rate of female, β - Obesity rate , α - Diabetic rate, $N(t)$ - Female Population. The control variables are u - Diet with Physical Activity and v - Medication.
The following figure shows the considered Obesity and Diabetic model:

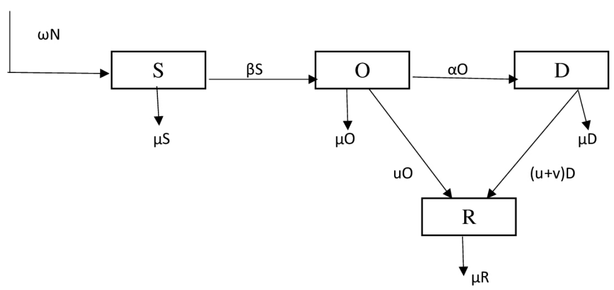

Figure 1: Obesity and Diabetic model

3 Equilibrium analysis

The steady states are $G_0(0,0,0,0)$, $G_1(\overline{S},0,0,0)$, $G_2(\tilde{S},\tilde{O},0,0)$, $G_3(S',O',D',0)$ and $G_4(S^*,O^*,D^*,R^*)$.

Case(1): $G_0(0,0,0,0)$ exists always.
Case(2): In $G_1(\overline{S},0,0,0)$,
Let $\overline{S}$ be the positive solution of $\frac{dS}{dt}=0$
From (1),

$$\overline{S}=\frac{\omega N}{(\beta+\mu)}>0$$

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$\therefore G_1(\overline{S},0,0,0)=(\frac{\omega N}{(\beta+\mu)},0,0,0)$
Case(3): In $G_2(\tilde{S},\tilde{O},0,0)$,
Let $\tilde{S},\tilde{O}$ be the positive solutions of $\frac{dS}{dt}=0,\frac{dO}{dt}=0.$
From (1),

$$\tilde{S}=\frac{\omega N}{(\beta+\mu)}>0$$
$$\tilde{O}=\frac{\beta\omega N}{(\beta+\mu)(\alpha+\mu+u)}>0$$

$\therefore G_2(\tilde{S},\tilde{O},0,0)=G_2\left(\frac{\omega N}{(\beta+\mu)},\frac{\beta\omega N}{(\beta+\mu)(\alpha+\mu+u)},0,0\right)$
Case(4): In $G_3(S',O',D',0)$,
Let S',O' and D' be the positive solutions of $\frac{dS}{dt}=0,\frac{dO}{dt}=0,\frac{dD}{dt}=0$
From (1),

$$S'=\frac{\omega N}{(\beta+\mu)}$$
$$O'=\frac{\beta\omega N}{(\beta+\mu)(\alpha+\mu+u)}$$
$$D'=\frac{\alpha\beta\omega N}{(\beta+\mu)(\alpha+\mu+u)(\mu+u+v)}$$

$\therefore G_3(S',O',D',0)=\left(\frac{\omega N}{\beta+\mu},\frac{\beta\omega N}{(\beta+\mu)(\alpha+\mu+u)},\frac{\alpha\beta\omega N}{(\beta+\mu)(\alpha+\mu+u)(\mu+u+v)},0\right)$
Case(5): For $G_4(S^*,O^*,D^*,R^*)$,
Let S^*, O^*, D^* and R^* be the positive solutions of $\frac{dS}{dt}=0,\frac{dO}{dt}=0,\frac{dD}{dt}=0,\ \frac{dR}{dt}=0$
From (1),

$$S^*=\omega N\left(\frac{1}{\beta+\mu}\right)$$

$$O^*=\omega N\left(\frac{\beta}{(\alpha+\mu+u)(\beta+\mu)}\right)$$

$$D^* = \omega N \left(\frac{\alpha\beta}{(\alpha+\mu+u)(\beta+\mu)(\mu+u+v)} \right)$$

$$R^* = \omega N \left[\frac{1}{\mu} \left(\frac{\alpha\beta(u+v)}{(\alpha+\mu+u)(\beta+\mu)(\mu+u+v)} + \frac{\beta u}{(\alpha+\mu+u)(\beta+\mu)} \right) \right]$$

$\therefore$ the diabesity equilibrium is given by

$$\begin{aligned} & G_4(S^*, O^*, D^*, R^*) \\ & = (\omega N \left(\frac{1}{\beta+\mu} \right), \omega N \left(\frac{\beta}{(\alpha+\mu+u)(\beta+\mu)} \right), \\ & \quad \omega N \left(\frac{\alpha\beta}{(\alpha+\mu+u)(\beta+\mu)(\mu+u+v)} \right), \\ & \quad \omega N \left[\frac{1}{\mu} \left(\frac{\alpha\beta(u+v)}{(\alpha+\mu+u)(\beta+\mu)(\mu+u+v)} + \frac{\beta u}{(\alpha+\mu+u)(\beta+\mu)} \right) \right]) \end{aligned}$$

4 Pontryagain's maximum principle

We compute the optimal controls of the control variables by using Pontryagain's maximum principle. We prove the following four properties :

I: The set of controls and state variables that correspond to them is non empty.

II: U is a closed and convex control set.

III: The state system's R.H.S is bounded by a linear function in the state and control variables.

IV: The integrand of the objective functional is convex on U and is bounded below by $k_2 + k_1|U|^\eta$, with $k_1 > 0, k_2 > 0, \eta > 1$.

To Prove I:

From (1),

$$\begin{aligned} F_1 &= \omega N - (\beta+\mu)S \\ F_2 &= \beta S - (\alpha+\mu+u)O \\ F_3 &= \alpha O - (\mu+u+v)D \\ F_4 &= (u+v)D + uO - \mu R \end{aligned} \tag{2}$$

From (2)

$$\begin{aligned} &\left|\frac{\partial F_1}{\partial S}\right| < \infty, \left|\frac{\partial F_1}{\partial O}\right| < \infty, \left|\frac{\partial F_1}{\partial D}\right| < \infty, \left|\frac{\partial F_1}{\partial R}\right| < \infty \\ &\left|\frac{\partial F_2}{\partial S}\right| < \infty, \left|\frac{\partial F_2}{\partial O}\right| < \infty, \left|\frac{\partial F_2}{\partial D}\right| < \infty, \left|\frac{\partial F_2}{\partial R}\right| < \infty \\ &\left|\frac{\partial F_3}{\partial S}\right| < \infty, \left|\frac{\partial F_3}{\partial O}\right| < \infty, \left|\frac{\partial F_3}{\partial D}\right| < \infty, \left|\frac{\partial F_3}{\partial R}\right| < \infty \\ &\left|\frac{\partial F_4}{\partial S}\right| < \infty, \left|\frac{\partial F_4}{\partial O}\right| < \infty, \left|\frac{\partial F_4}{\partial D}\right| < \infty, \left|\frac{\partial F_4}{\partial R}\right| < \infty \end{aligned}$$

As it is continuous and bounded then there exist a unique solution of (1) by Existence and uniqueness theorem.
Hence, I is satisfied.
To prove II:
The control set is given by,

$$U = \{(u(t), v(t))/0 \leq u \leq 1, 0 \leq v \leq 1, t \in [0, 1]\}$$

U is closed, from our definition.
Let u,v $\in$ U, $\alpha \in [0, 1]$
Then$\alpha u + (1 - \alpha)v \geq 0$
As u ≤ 1 and v≤ 1,
$0 \leq \alpha u + (1 - \alpha)v \leq (1 - \alpha)$, for all u,v $\in$ U
Hence II is satisfied.
To prove III:
From (2),

$$\begin{aligned} F_1 &\leq \omega N \\ F_2 &\leq \beta S - uO \\ F_3 &\leq \alpha O - (u+v)D \\ F_4 &\leq (u+v)D + uO \end{aligned}$$

It can be rewritten as follows:

$$\overline{F}(t, \overline{X}, U) \leq \begin{bmatrix} 0 & 0 & 0 & 0 \\ \beta & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} S \\ O \\ D \\ R \end{bmatrix} + \begin{bmatrix} 0 \\ -O \\ -D \\ D+O \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Which can be written as the linear combination of controls

$$\begin{aligned} \left|\overline{F}(t, \overline{X}, U)\right| &\leq \|\bar{m}\| \left|\bar{X}\right| + \|D + O\| \left|u(t), v(t)\right| \\ &\leq K \left[\left|\bar{X}\right| + \left|u(t), v(t)\right|\right] \end{aligned}$$

As $\overline{S}$,$\overline{O}$,$\overline{D}$ are bounded and K is upper bound.
$\therefore$ III is satisfied.

To Prove IV:
The control variables and state variables are non negative $u, v \in U$.
u,v is convex and closed.
Then from (3),

$$J = AI(t) + \frac{\omega_1 u^2 + \omega_2 v^2}{2}$$

$$\geq -AI(t) + \frac{\omega}{2}(u^2 + v^2)$$
$$= -k_2 + k_1(u, v)^2$$

where $\omega = \omega_1 + \omega_2$, $k_1 = \frac{\omega}{2}$.
Here $k_2 > 0$, $k_1 > 0$ and $\eta = 2 > 1$
Hence IV is satisfied.
Hence there exists an optimal control variables which is found in the following section.

5 Existence of optimal control

The objective functional is defined as in (2) as follows:

$$J = minAI(t) + \frac{1}{2}\int_0^T (\omega_1 u^2 + \omega_2 v^2)dt \tag{3}$$

Where AI(t) is the number of individuals, ω_1, ω_2 are the weight parameters for the cost of diet with physical activity and the cost of medication respectively.
From previous section, our system (1) converts into a problem of minimizing.
An Hamiltonian H is defined as,

$$\begin{aligned} H(S, O, D, R) = AI(t) + \frac{\omega_1 u^2}{2} + \frac{\omega_2 v^2}{2} + \lambda_S\left(\omega N - (\beta + \mu)S\right) \\ + \lambda_O\left(\beta S - (\alpha + \mu + u)O\right) + \lambda_D\left(\alpha O - (\mu + u + v)D\right) \\ + \lambda_R\left((u + v)D + uO - \mu R\right) \end{aligned} \tag{4}$$

The adjoint system is given by

$$\begin{aligned} \tfrac{d\lambda_S}{dt} &= \lambda_S(\beta + \mu) - \lambda_O\beta \\ \tfrac{d\lambda_O}{dt} &= \lambda_O(\alpha + \mu + u) - \alpha\lambda_D - \lambda_R u \\ \tfrac{d\lambda_D}{dt} &= \lambda_D(\mu + u + v) - \lambda_R(u + v) \\ \tfrac{d\lambda_R}{dt} &= \lambda_R\mu \end{aligned}$$

with the final condition $\lambda_{S(T)} = \lambda_{O(T)} = \lambda_{D(T)} = \lambda_{R(T)}$=0 for free problem.
The optimality is found from $\frac{\partial H}{\partial u} = 0$ and $\frac{\partial H}{\partial v} = 0$.
$\therefore$ The optimal controls are,

$$u^* = min\left(1, max\left(0, \tfrac{D(\lambda_D - \lambda_R) - O\lambda_R}{\omega_1}\right)\right), v^* = min\left(1, max\left(0, \tfrac{D(\lambda_D - \lambda_R)}{\omega_2}\right)\right)$$

Here u^*, v^* are positive when $\lambda_D > \lambda_R$ and $D(\lambda_D - \lambda_R) > O\lambda_R$.

6 Local stability

The Jacobian matrix for the system (1) is

$$\begin{pmatrix} -(\beta+\mu) & 0 & 0 & \delta \\ \beta & -(\alpha+\mu+u) & 0 & 0 \\ 0 & \alpha & -(\mu+u+v) & 0 \\ 0 & u & u+v & -\mu \end{pmatrix} \tag{5}$$

At the interior equlilibrium (5) becomes

$$\begin{pmatrix} -\frac{\omega N}{S} & 0 & 0 & 0 \\ \beta & -\frac{\beta S}{O} & 0 & 0 \\ 0 & \alpha & -\frac{\alpha O}{D} & 0 \\ 0 & u & u+v & -\frac{(u+v)D+uO}{R} \end{pmatrix} \tag{6}$$

The characteristic equation of (6) is given by

$$\begin{vmatrix} -\frac{\omega N}{S}-\lambda & 0 & 0 & 0 \\ \beta & -\frac{\beta S}{O}-\lambda & 0 & 0 \\ 0 & \alpha & -\frac{\alpha O}{D}-\lambda & 0 \\ 0 & u & (u+v) & -\frac{(u+v)D+uO}{R}-\lambda \end{vmatrix} = 0$$

$$\begin{aligned}
&\lambda^4+\left(\frac{\omega N}{S}+\frac{\beta S}{O}+\frac{\alpha S}{D}+\frac{uO}{R}+\frac{(u+v)}{R}\right)\lambda^3 \\
&+(\frac{\omega N\beta}{O}+\frac{(u+v)D\omega N}{SR}+\frac{uO\omega N}{SR}+\frac{\omega N\alpha}{D}+\frac{(u+v)D\beta S}{R}+\frac{u\beta S}{R}+\frac{\beta\alpha S^2}{DO} \\
&+\frac{(u+v)\alpha S}{R}+\frac{uO\alpha S}{RD})\lambda^2+(\frac{(u+v)\alpha\omega ND\beta}{R}+\frac{u\beta\omega N}{R}+\frac{\omega N\beta\alpha S}{DO} \\
&+\frac{(u+v)\alpha\omega N}{R}+\frac{\omega NuO\alpha}{RD}+\frac{(u+v)\alpha\beta S^2}{RO}+\frac{u\alpha\beta S^2}{RD})\lambda \\
&+\frac{(u+v)\alpha\beta\omega NS}{RO}+\frac{u\alpha\beta\omega NS}{RD}=0
\end{aligned} \tag{7}$$

Comparing (4) with $S^4 + AS^3 + BS^2 + CS + D = 0$
where

$$\begin{aligned}
A &= \frac{\omega N}{S} + \frac{\beta S}{O} + \frac{\alpha S}{D} + \frac{uO}{R} + \frac{(u+v)}{R},\\
B &= \frac{\omega N \beta}{O} + \frac{(u+v)D\omega N}{SR} + \frac{uO\omega N}{SR} + \frac{\omega N \alpha}{D} + \frac{(u+v)D\beta S}{R}\\
&\quad + \frac{u\beta S}{R} + \frac{\beta\alpha S^2}{DO} + \frac{(u+v)\alpha S}{R} + \frac{uO\alpha S}{RD},\\
C &= \frac{(u+v)\alpha\omega N D\beta}{R} + \frac{u\beta\omega N}{R} + \frac{\omega N\beta\alpha S}{DO} + \frac{(u+v)\alpha\omega N}{R}\\
&\quad + \frac{\omega N u O\alpha}{RD} + \frac{(u+v)\alpha\beta S^2}{RO} + \frac{u\alpha\beta S^2}{RD},\\
D &= \frac{(u+v)\alpha\beta\omega N S}{RO} + \frac{u\alpha\beta\omega N S}{RD}
\end{aligned}$$

Here $A > 0; D > 0; AB - C > 0; C(AB - C) - A^2 D > 0$ when $1 - S > 0$ and the system (1) is locally stable by the Routh-Hurwitz criteria.

7 Global stability

We establish the Lyapunov function given below,

$$\begin{aligned}
V(S,O,D,R) = &\left((S - S^*) - S^* ln\frac{S}{S^*}\right) + l_1\left((O - O^*) - O^* ln\frac{O}{O^*}\right)\\
&+ l_2\left((D - D^*) - D^* ln\frac{D}{D^*}\right) + l_3\left((R - R^*) - R^* ln\frac{R}{R^*}\right)
\end{aligned} \tag{8}$$

Differentiate (8) with respect to t,

$$\frac{dV}{dt} = (\frac{S - S^*}{S})\frac{dS}{dt} + (\frac{O - O^*}{O})\frac{dO}{dt} + (\frac{D - D^*}{D})\frac{dD}{dt} + (\frac{R - R^*}{R})\frac{dR}{dt}$$

From the model equations (1),

$$\begin{aligned}
\frac{dV}{dt} &= (\frac{S - S^*}{S})\left(\omega N - (\beta + \mu)S\right) + (\frac{O - O^*}{O})\left(\beta S - (\alpha + \mu + u)O\right)\\
&+(\frac{D - D^*}{D})\left(\alpha O - (\mu + u + v)D\right) + (\frac{R - R^*}{R})\left((u+v)D + uO - \mu R\right)\\
&= (S - S^*)\left(\frac{\omega N}{S} - (\beta + \mu)\right) + l_1(O - O^*)\left(\frac{\beta S}{O} - (\alpha + \mu + u)\right)
\end{aligned}$$

$$+l_2(D-D^*)\left(\frac{\alpha O}{D}-(\mu+u+v)\right)+l_3(R-R^*)\left(\frac{(u+v)D+uO}{R}-\mu\right)$$

At (S^*, O^*, D^*, R^*), we have

$$\frac{dV}{dt}=(S-S^*)[\frac{\omega N}{S}-(\frac{\omega N}{S^*})]+l_1(O-O^*)[\frac{\beta S}{O}-(\frac{\beta S^*}{O^*})]$$

$$+l_2(D-D^*)[\frac{\alpha O}{D}-(\frac{\alpha O^*}{D^*})]+l_3(R-R^*)[(u+v)(\frac{D+uO}{R}-(\frac{(u+v)D^*+uO^*}{R^*})]$$

$$=(S-S^*)\omega N(\frac{1}{S}-\frac{1}{S^*})+l_1(O-O^*)\beta(\frac{S}{O}-\frac{S^*}{O^*})$$

$$+l_2(D-D^*)\alpha(\frac{O}{D}-\frac{O^*}{D^*})+l_3(R-R^*)[(u+v)(\frac{D}{R}-\frac{D^*}{R^*})+u(\frac{O}{R}-\frac{O^*}{R^*})]$$

Choosing $l_1=\frac{1}{\beta}, l_2=\frac{1}{\alpha}, l_3=\frac{1}{(u+v)u}$

$$\frac{dV}{dt}=(S-S^*)\omega N\left(\frac{1}{S}-\frac{1}{S^*}\right)+\frac{(O-O^*)\beta}{\beta}\left(\frac{S}{O^*}-\frac{S^*}{O^*}\right)+\frac{(D-D^*)\alpha}{\alpha}$$
$$\left(\frac{O}{D}-\frac{O^*}{D^*}\right)+\frac{(R-R^*)}{(u+v)u}\left(\frac{(u+v)D}{R}-\frac{D^*}{R^*}\right)+\frac{(R-R^*)}{(u+v)u}\left(u(\frac{D}{R}-\frac{D^*}{R^*})\right)$$
$$=-\frac{\omega N(S-S^*)^2}{SS^*}+\frac{1}{OO^*}[SOO^*-S^*O^2-SO^*2+S^*O^*O]$$
$$+\frac{1}{DD^*}[OD^*D-D^2O^*-OD^*2+D^*O^*D]+\frac{1}{uRR^*}[DR^*R-R^2D^*-R^*2D$$
$$+R^*RD^*]+\frac{1}{(u+v)RR^*}[DR^*R-R^2D^*-R^*2D+R^*RD^*]$$
$$=-(S-S^*)\omega N\left(\frac{S-S^*}{SS^*}\right)+(O-O^*)\left(\frac{SO^*-S^*O}{OO^*}\right)+(D-D^*)$$
$$\left(\frac{OD^*-DO^*}{DD^*}\right)+\frac{R-R^*}{u}\left(\frac{DR^*-RD^*}{RR^*}\right)+\frac{R-R^*}{u+v}\left(\frac{DR^*-RD^*}{RR^*}\right)$$
$$=\frac{-\omega N(S-S^*)^2}{SS^*}+\left(S-\frac{SO^*}{O}-\frac{S^*O}{O^*}+S^*\right)+\left(O-\frac{OD^*}{D}+\frac{DO^*}{D^*}+O^*\right)$$
$$+\frac{1}{u}\left(R-\frac{DR^*}{R}+\frac{D^*R}{R^*}+R^*\right)+\frac{1}{u+v}\left(R-\frac{DR^*}{R}+\frac{D^*R}{R^*}+R^*\right)$$

$\therefore \frac{dV}{dt}<0$, when $\frac{R}{S}<\frac{R^*}{S^*}, \frac{S}{O}<\frac{S^*}{O^*}, \frac{O}{D}<\frac{O^*}{D^*}, \frac{D}{R}<\frac{D^*}{R^*}, \frac{O}{R}<\frac{O^*}{R^*}$
Here the system (1) is globally asymptotically stable by Lyapunov theorem.

8 Numerical simulations

We have choose the values of parameters as $\omega = 0.0234$, $\mu = 0.008$, $\alpha = 0.2792$, $\beta = 0.23$ based on a female population.
Figure (2) shows the flow of variables with control parameters. The obesity rate

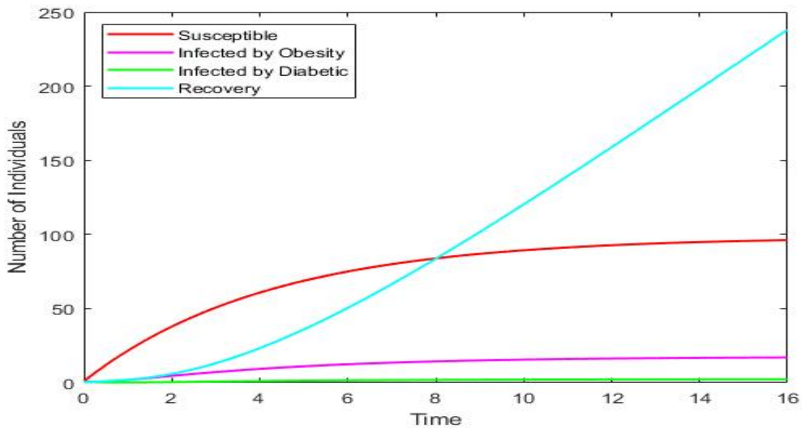

Figure 2: Obesity and Diabetic with control parameters

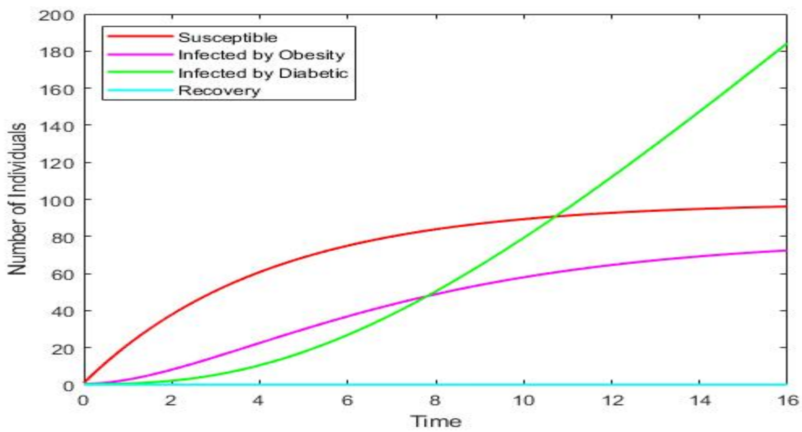

Figure 3: Obesity and Diabetic without control parameters

and diabetic rate are stable with high recovery rate with respect to time . Figure (3) shows the flow of variables without control parameters. The diabetic rate is high due to no recovery rate.
Figure (4) shows the flow of individuals of susceptible class for various values of β. If β values are increasing, the number of individuals of susceptible class move

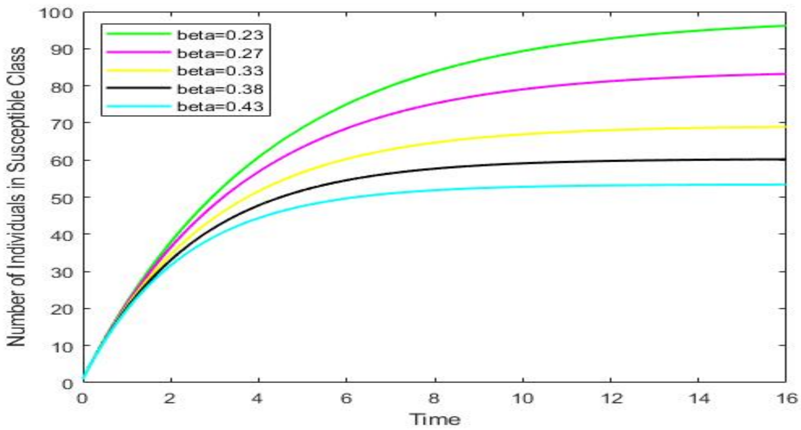

Figure 4: Flow of individuals in Susceptible Class for various values of β

towards the infected by obesity class which becomes stable with respect to time.

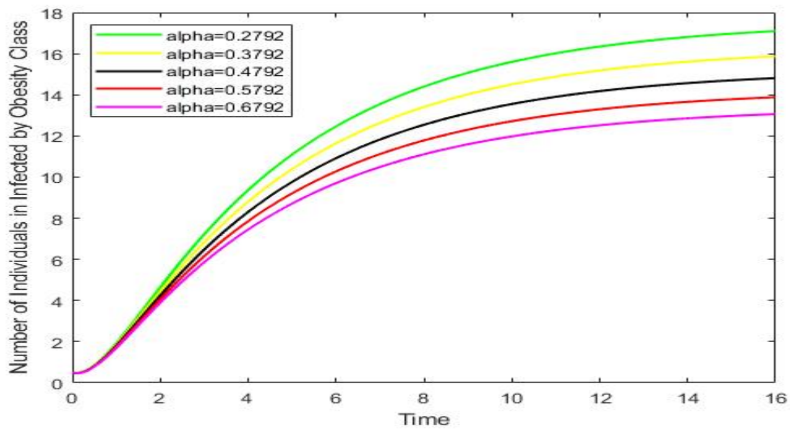

Figure 5: Infected by Obesity class for different values of α with control parameters

Figure(5) and figure(6) show the flow of individuals of infected by obesity class for various values of α with and without control parameters respectively. If we use the control parameters by giving different values to α, then the number of individuals in obesity class will be stable with respect to time. Otherwise it will increase the diabetic rate. While comparing, figures (5) and (6) the obesity rate of figure (5) is higher than the obesity rate of figure(6). So we can understand that the obesity rate can be managed with control parameters. Which gives that the

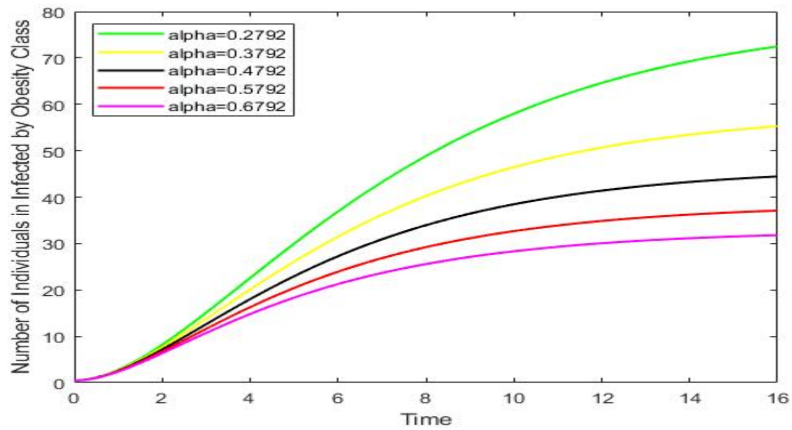

Figure 6: Infected by Obesity class for different values of α without control parameters

obesity rate is controlled with the control parameters.

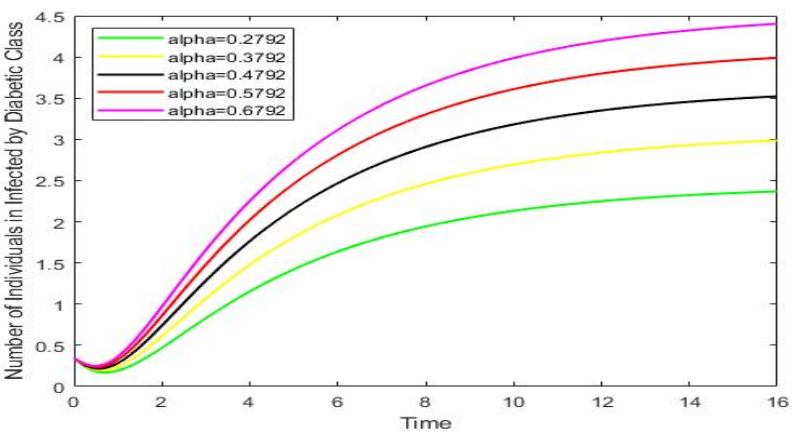

Figure 7: Infected by Diabetic class for different values of α with control parameters

Figure(7) and figure(8) show the flow of individuals in infected by diabetic class for various values of α with and without control parameters respectively. If we use the control parameters by giving different values to α, then the number of individuals in diabetic class will be stable with respect to time. Otherwise it will reduce the recovery rate. While comparing, figures (7) and (8) the diabetic rate of figure (7) is higher than the diabetic rate of figure(8). So we can understand that

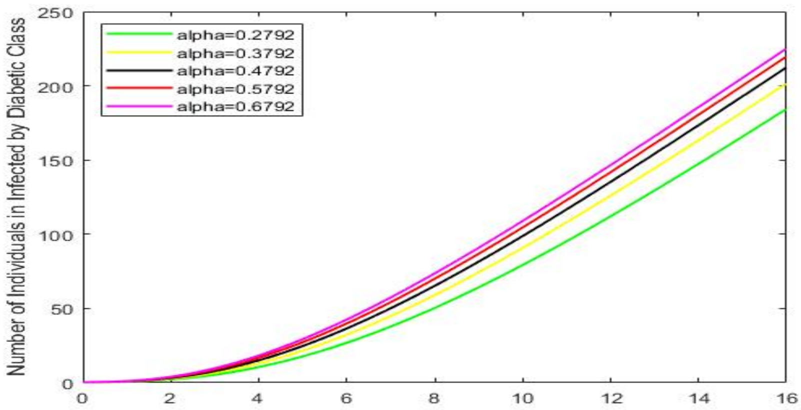

Figure 8: Infected by Diabetic class for different values of α without control parameters

the diabetic rate can be managed with control parameters. Which gives that the diabetic rate is controlled with the control parameters.

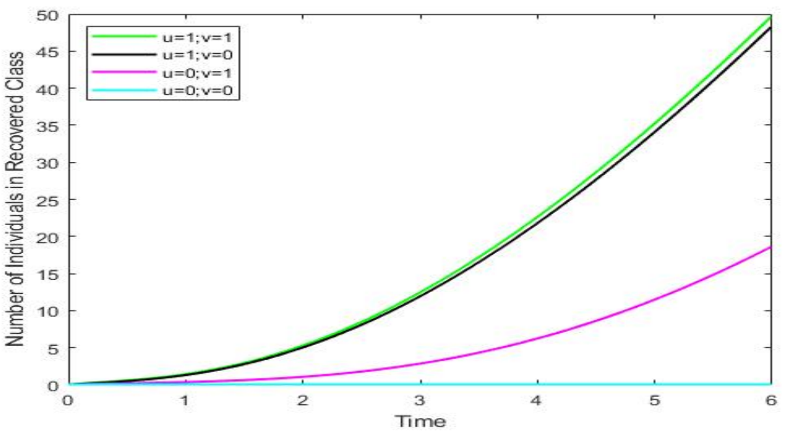

Figure 9: Recovered class for various values of control parameters

Figure(9) shows the flow of individuals in recovered state for the various values of control parameters. We can observe the following cases:
Case 1 : The recovery rate increases when both the control parameters exist.
Case 2 : The recovery rate is slightly decrease when the control parameter diet with physical activity is involved.
Casse 3 : The recovery rate decreases from case 2, when the control parameter medication is involved.
Case 4 : There is no recovery rate, when no control parameters are involved.

Hence, both the control parameters are required to control the obesity and diabetic rates and to increase the recovery rate.

9 Conclusions

This paper presents a mathematical model for type 2 diabetic from obesity with controls diet with physical activity and medication. By the Pontryagin's maximum principle, the optimal levels of two control variables are attained with some specific conditions. An optimal control approach is proposed in order to reduce the burden of obesity to diabetic. The positive equilibrium points have been found. The system is locally asymptotically stable with a condition on susceptible population. The system is globally asymptotically state in the identified parametric domain. The numerical simulations show the effectiveness of proposed control strategies. In the numerical simulations, we can observe that the flow of diabetic state was dropped down, when the two control parameters are involved. We can also observe that the obesity rate can be managed at moderate levels when the controls parameters are at optimal levels. Hence the two introduced control parameters healthy diet with physical activity and medication are effectively reduce the rates of obesity and diabetic and increase the recovery rate.

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