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Strong interval – valued Pythagorean fuzzy soft graphs

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Abstract

A Strong interval – valued Pythagorean fuzzy soft sets (SIVPFSS) an extending the theory of Interval-valued Pythagorean fuzzy soft set (IVPFSS). Then we Propose Strong interval valued Pythagorean fuzzy soft graphs (SIVPFSGs). We also present several different types of operations on Strong interval- valued Pythagorean fuzzy soft graphs and explore of their analysis.

Keywords: Strong Interval-valued Pythagorean fuzzy graph; Strong Interval-valued Pythagorean fuzzy soft graph;
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1 Introduction

Fuzzy set is a analytical imitation to grips the exciting and insufficient details. consider a differentiating that uncertainty is also independently, FS was continued to intuitionistic fuzzy set (IFS) by Atanassov and Gargov [1989]. If assigned a membership value α and a non membership value β to the conditions, satisflying this results $\alpha + \beta \leq 1$ and uncertainty elements, $\gamma = 1 - \alpha - \beta$. In decision-making problems, the membership value 0.7 and non membership value 0.4 for some information, then IF fails in this situation because $0.7 + 0.4 > 1$, but $(0.7)^2 + (0.4)^2 \leq 1$. To overcome this situation, the notion of Pythagorean fuzzy set (PFS) was satisfying the condition $\alpha^2 + \beta^2 \leq 1$. A PFS has more potential as compared to IFS is solving decision-making problems. The Pythagorean fuzzy number (PFG) was determinate by Zhang (see S.Shahzadi and Akram [2020]). Zhang provided the Pythagorean fuzzy weighted averaging operator.

The theory of IVFS was introduced by Zadeh [1965] as a perpetuation of fuzzy sets. Because they present more adequate description for uncertainty, interval-valued fuzzy sets more useful than conventional fuzzy sets. Soft set theory was started by Molodstov [1999] for the parameterized point of view for uncertainty modeling and soft computing. The iterpretation of IFSGs was given by Akram [2011]. The explanation of novel intuitionistic fuzzy soft multiple – decision-making methods of grips by Akram. Pythagorean fuzzy soft graphs with applications was proposed by S.Shahzadi and Akram [2020].The SIVPFSG is defined and some results on SIVPFSG are studied. Also explore of their analysis.

2 Preliminaries

Definition 2.1. *An IVFSG over the set V is given by ordered 4 tuple* $\tilde{\xi} = (\xi^*, X, Y, A)$ *such that*
(i) A *is of parameters.*
(ii) (X, A) *is an IVFSS over* V.
(iii) (Y, A) *is an IVFSS over* E.
(iv) $(X(e), Y(e))$ *is an IVFSG for all* $e \in A$.
That is,
$\alpha^-_{Y(e)}((pq)) \leq min(\alpha^-_{X(e)}(p), \alpha^-_{X(e)}(q))$ *and*
$\alpha^+_{Y(e)}((pq)) \leq min(\alpha^+_{X(e)}(p), \alpha^+_{X(e)}(q)) forall\, pq \in E.$
We denote $\xi^* = (V, E)$ *a crisp graph* $H(e) = (X(e), Y(e))$ *an IVFSG and* $\tilde{\xi} = (\xi^*, X, Y, A)$ *an IVFSG.*

Definition 2.2. *An IVFSG over the set* V *is defined to be a pair* $\xi = (X, Y)$ *where*
1) The conditions $\widetilde{\alpha_X} : V \to D[0, 1]$ *and* $\widetilde{\beta_X} : V \to D[0, 1]$ *denote the degree of*

membership and non membership of the element $p \in V$. *such that*

$$0 \leq \widetilde{\alpha_X}(p) + \widetilde{\beta_X}(p) \leq 1 \forall (p,q) \in V.$$

2) The conditions $\widetilde{\alpha_Y} : E \subseteq V \times V \to D[0,1]$ *and* $\widetilde{\beta_Y} : E \subseteq V \times V \to D[0,1]$ *defined by*
$\alpha^-_{YL}((p,q)) \leq min(\alpha^-_{XL}(p), \alpha^-_{XL}(q))$ *and* $\beta^-_{YL}((p,q)) \geq max(\beta^-_{XL}(p), \alpha^-_{XL}(q))$,
$\alpha^+_{YU}((p,q)) \leq min(\alpha^+_{XU}(p), \alpha^+_{XU}(q))$ *and* $\beta^+_{YU}((p,q)) \geq max(\beta^+_{XU}(p), \alpha^+_{XU}(q))$,
such that $0 \leq \alpha^2_{YU}(p,q) + \beta^2_{YU}(p,q) \leq 1 \forall (p,q) \in E$.
We the notation pq *for* (p,q) *an element of* E.

Definition 2.3. *An IVPFSG over the set* V *is given by* $\tilde{\xi} = (\xi^*, X, Y, A)$ *such that*
1) The conditions $\widetilde{\alpha_X} : V \to D[0,1]$ *and* $\widetilde{\beta_X} : V \to D[0,1]$ *standered for the degree of membership and non membership of the element* $p \in V$. *such that*

$$0 \leq \widetilde{\alpha_X}(p,q) + \widetilde{\beta_X}(p,q) \leq 1 \forall (p,q) \in V.$$

2)(i) A *is set of parameters*
(ii) (X, A) *is an IVPFSS over* V.
(iii) (Y, A) *is an IVPFSS over* E.
(iv) $(X(e), Y(e))$ *is an IVPFSG for all* $e \in A$.

The conditions $\widetilde{\alpha_Y} : E \subseteq V \times V \to D[0,1]$ *and* $\widetilde{\beta_Y} : E \subseteq V \times V \to D[0,1]$ *defined by*
$\alpha^+_{YU}((p,q)) \leq min(\alpha^+_{XU}(p), \beta^+_{XU}(q))$ *and* $\beta^+_{YU}((p,q)) \geq max(\beta^+_{XU}(p), \beta^+_{XU}(q))$,
$\alpha^-_{YL}((p,q)) \leq min(\alpha^-_{XL}(p), \beta^-_{XL}(q))$ *and* $\beta^-_{YL}((p,q)) \geq max(\beta^-_{XL}(p), \beta^-_{XL}(q))$,
such that $0 \leq \alpha^2_{YU}(p,q) + \beta^2_{YU}(p,q) \leq 1 \forall (p,q) \in E$.

3 Strong intervel-valued Pythagorean fuzzy Graphs

Definition 3.1. *An SIVPFSG over the set* V *is given by* $\tilde{\xi} = (\xi^*, X, Y, A)$ *such that*
1) The conditions $\widetilde{\alpha_X} : V \to D[0,1]$ *and* $\widetilde{\beta_X} : V \to D[0,1]$ *denote the degree of membership and non membership of the element* $x \in V$. *such that*

$$0 \leq \widetilde{\alpha_X}(p,q) + \widetilde{\beta_X}(p,q) \leq 1 \forall (p,q) \in V.$$

2)(i) A *is set of parameters*
(ii) (X, A) *is an SIVPFSS over* V.
(iii) (Y, A) *is an SIVPFSS over* E.
(iv) $(X(e), Y(e))$ *is an SIVPFSG for all* $e \in A$.

The conditions $\widetilde{\alpha_Y} : E \subseteq V \times V \to D[0,1]$ *and* $\widetilde{\beta_Y} : E \subseteq V \times V \to D[0,1]$ *defined by*

$\alpha^{+}_{YU}((p,q)) = min(\alpha^{+}_{XU}(p), \beta^{+}_{XU}(q))$ *and* $\beta^{+}_{YU}((p,q)) = max(\beta^{+}_{XU}(p), \beta^{+}_{XU}(q))$, $\alpha^{-}_{YL}((p,q)) = min(\alpha^{-}_{XL}(p), \alpha^{-}_{XL}(q))$ *and* $\beta^{-}_{YL}((p,q)) = max(\alpha^{-}_{XL}(p), \beta^{-}_{XL}(q))$, *such that* $0 \leq \alpha^{2}_{YU}(p,q) + \beta^{2}_{YU}(p,q) \leq 1 \forall (p,q) \in E$.

Example 3.1. *If* $\xi^* = (X, Y)$ *is a simple graph with* $X = \{a, b, c, d\}$ *and* $Y = \{ab, bc, cd, ad\}$. *Let* $A = \{e_1, e_2\}$ *be a parameter set and* (X, A) *be an SIVPFSS* V *determine*

$$X_1(e) = \Big\{ \langle a, [0.3, 0.4][0.2, 0.7] \rangle, \langle b, [0.2, 0.5][0.3, 0.7] \rangle, \langle c, [0.1, 0.6][0.2, 0.5] \rangle, \\ and \langle D[0.2, 0.7][0.3, 0.5] \rangle \Big\}$$

$$X_2(e) = \Big\{ \langle a, [0.2, 0.7][0.3, 0.5] \rangle, \langle b[0.1, 0.6][0.2, 0.5] \rangle, \langle c, [0.3, 0.4][0.2, 0.7] \rangle \Big\}$$

Take (Y, A) *be an SIVPFSS* E *determine*

$$Y_1(e) = \Big\{ \langle ab[0.2, 0.5][0.3, 0.7] \rangle, \langle bc[0.1, 0.6][0.3, 0.7] \rangle, \langle ad[0.2, 0.7][0.3, 0.7] \rangle, \\ and \langle cd[0.1, 0.6][0.3, 0.5] \rangle \Big\}$$

$$Y_2(e) = \Big\{ \langle ab[0.1, 0.5][0.4, 0.6] \rangle, \langle bc[0.1, 04][0.4, 0.8] \rangle, \langle ac[0.1, 0.3][0.4, 0.8] \rangle \Big\}$$

It is clearly seen that $H(e_1) = (X(e_1), Y(e_1))$ *and* $H(e_2) = (X(e_2), Y(e_2))$ *are SIVPFSGs comparable to the parameters* e_1 *and* e_2 *accordingly, by Figure 1. Hence* $\tilde{\xi} = (\xi^*, X, Y, A)$ *SIVPFSGs.*

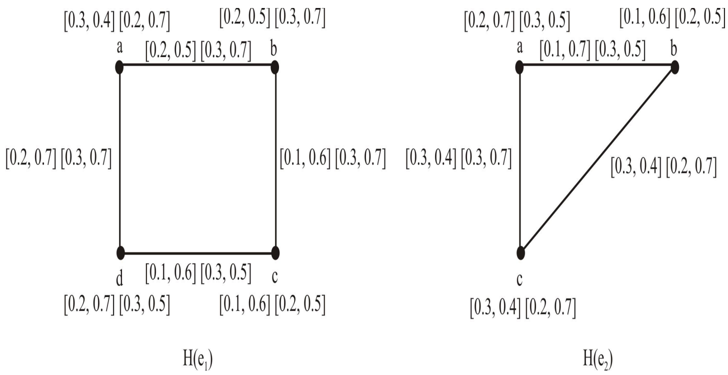

Figure 1: SIVPFSGs $\breve{G}$.

Definition 3.2. *If* $\tilde{\xi}_1 = (\xi^*_1, X_1, Y_1, A)$ *and* $\tilde{\xi}_2 = (\xi^*_2, X_2, Y_2, B)$ *be double SIVPF-SGs of* $\xi^*_1 = (X_1, Y_1)$ *and* $\xi^*_2 = (X_2, Y_2)$ *accordingly. The cross product of* $\tilde{\xi}_1$ *and*

$\tilde{\xi}_2$ *is denoted by* $\tilde{\xi}_1 \times \tilde{\xi}_2 = (X_1 \times X_2, Y_1 \times Y_2)$ *and is defined by*
1) $(\alpha_{X_1L} \times \alpha_{X_2L})(p_1, p_2) = min(\alpha_{X_1L}(p_1), \beta_{X_2L}(p_2))$,
$(\alpha_{X_1U} \times \alpha_{X_2U})(p_1, p_2) = min(\alpha_{X_1U}(p_1), \alpha_{X_2U}(p_2))$,
$(\beta_{X_1L} \times \beta_{X_2L})(p_1, p_2) = min(\beta_{X_1L}(p_1), \beta_{X_2L}(p_2))$,
$(\beta_{X_1U} \times \beta_{X_2U})(p_1, p_2) = max(\beta_{X_1U}(p_1), \beta_{X_2U}(p_2)), \forall p_1 \in V_1, p_2 \in V_2$.
2) $(\alpha_{Y_1L} \times \alpha_{Y_2L})(p, p_2)(p, q_2) = min(\alpha_{Y_1L}(p), \alpha_{Y_2L}(p_2, q_2))$,
$(\alpha_{Y_1U} \times \alpha_{Y_2U})(p, p_2)(p, p_2) = min(\alpha_{Y_1U}(p), \alpha_{Y_2U}(p_2, q_2))$,
$(\beta_{Y_1L} \times \beta_{Y_2L})(p, p_2)(p, q_2) = max(\beta_{Y_1L}(p), \beta_{Y_2L}(p_2, q_2))$,
$(\beta_{Y_1U} \times \beta_{Y_2U})(p, p_2)(p, q_2) = max(\beta_{Y_1U}(p), \beta_{Y_2U}(p_2, q_2)), \forall p \in V_1, p_2q_2 \in E_2$.
3) $(\alpha_{Y_1L} \times \alpha_{Y_2L})(p_1, r)(q_1, r) = min(\alpha_{Y_1L}(p_1q_1), \alpha_{Y_2L}(r))$,
$(\alpha_{Y_1U} \times \alpha_{Y_2U})(p_1, r)(q_1, r) = min(\alpha_{Y_1U}(p_1q_1), \alpha_{Y_2U}(r))$,
$(\beta_{Y_1L} \times \beta_{Y_2L})(p_1, r)(q_1, r) = max(\beta_{Y_1L}(p_1q_1), \beta_{Y_2L}(r))$,
$(\beta_{Y_1U} \times \beta_{Y_2U})(p_1, r)(q_1, r) = max(\beta_{Y_1U}(p_1q_1), \beta_{Y_2U}(r)), \forall r \in V_2, p_1q_1 \in E_1$.

Example 3.2. *Let Consider a graph* $\xi_1^* = (X_1, Y_1)$ *and* $\xi_2^* = (X_2, Y_2)$ *be two graphs such that* $X_1 = \{a_1, b_1, c_1, d_1\}, Y_1 = \{a_1b_1, c_1d_1\}$ *and* $X_2 = \{a_2, b_2, c_2, d_2\}$, $Y_2 = \{a_2b_2, c_2d_2\}$. *Let* $A = e_1$ *be a set of parameters and let* (X_1, A) *and* (Y_1, A) *be two SIVPFSSs over* X_1 *and* Y_1 *accordingly, defined by*

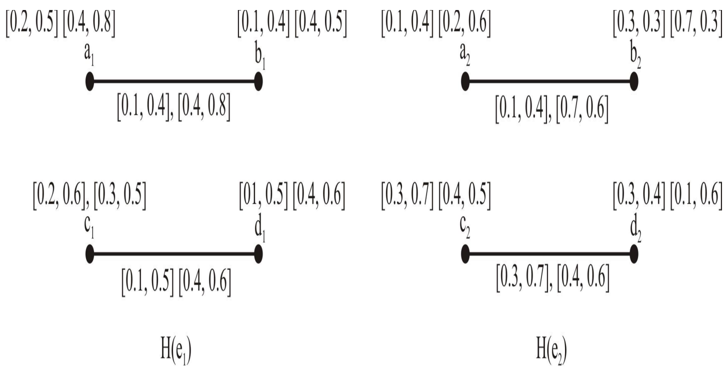

Figure 2: SIVPFSGs $\tilde{\xi}_1$ and $\tilde{\xi}_2$.

$$X_1(e) = \Big\{ \langle a_1[0.2, 0.5][0.4, 0.8]\rangle, \langle b_1[0.1, 0.4][0.4, 0.5]\rangle, \langle c_1[0.2, 0.6][0.3, 0.5]\rangle, and \langle d_1[0.1, 0.5][0.4, 0.6]\rangle \Big\}$$

$$Y_1(e) = \Big\{ \langle a_1b_1[0.1, 0.4][0.4, 0.8]\rangle, \langle c_1d_1[0.1, 0.5][0.4, 0.6]\rangle \Big\}$$

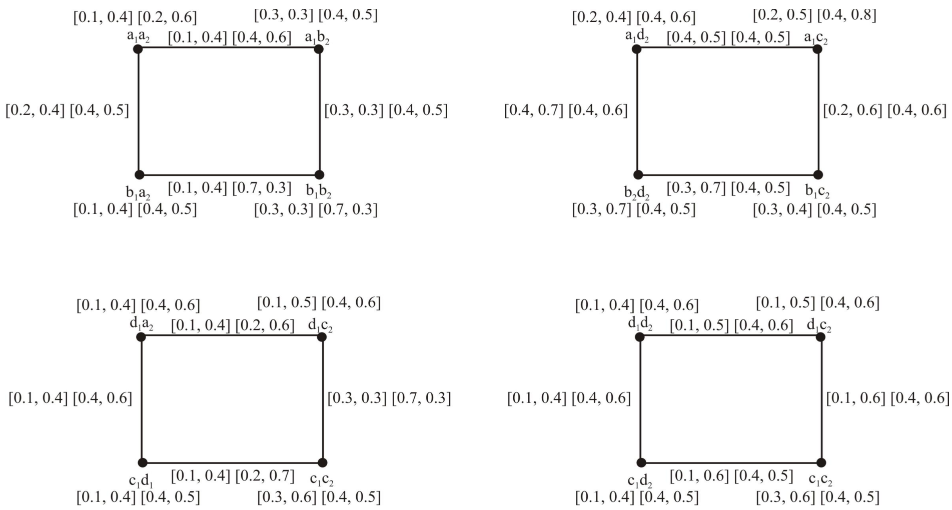

Figure 3: Cross product of $\tilde{\xi_1}$ and $\tilde{\xi_2}$.

Take $B = e_2$ be a set of parameters and let (X_2, B) and (Y_2, B) be two SIVPFSSs over X_2 and Y_2 accordingly, Find out

$$X_2(e) = \Big\{ \langle a_2[0.1, 0.4][0.2, 0.6] \rangle, \langle b_2[0.3, 0.3][0.7, 0.3] \rangle, \langle c_2[0.3, 0.7][0.4, 0.5] \rangle, \\ and \langle d_2[0.3, 0.4][0.1, 0.6] \rangle \Big\}$$

$$Y_2(e) = \Big\{ \langle a_2b_2[0.1, 0.4][0.7, 0.6] \rangle, \langle c_2d_2[0.3, 0.7][0.4, 0.6] \rangle \Big\}$$

Clearly $H(e_1) = (X(e_1), Y(e_1))$ and $H(e_2) = (X(e_2)), Y(e_2))$ are SIVPFSGs. Hence $\tilde{\xi_1} = (\xi_1^*, X_1, Y_1, A)$ and $\tilde{\xi_2} = (\xi_2^*, X_2, Y_2, B)$ are SIVPFSGs ξ_1^* and ξ_2^*, accordingly, as shown in the Figure 2.

Definition 3.3. *If $\tilde{\xi_1} = (\xi_1^*, X_1, Y_1, A)$ and $\tilde{G_2} = (\xi_2^*, X_2, Y_2, B)$ be two SIVPF-SGs of $\xi_1^* = (X_1, Y_1)$ and $\xi_2^* = (X_2, Y_2)$ accordingly. The composition of $\tilde{\xi_1}$ and $\tilde{\xi_2}$ is standed by $\tilde{\xi_1} \circ \tilde{\xi_2} = (X_1 \circ X_2, Y_1 \circ Y_2)$ and is defined by*
1) $(\alpha_{X_1L} \circ \alpha_{X_2L})(p_1, p_2) = min(\alpha_{X_1L}(p_1), \beta_{X_2L}(p_2)),$
$(\alpha_{X_1U} \circ \alpha_{X_2U})(p_1, p_2) = min(\alpha_{X_1U}(p_1), \alpha_{X_2U}(p_2)),$
$(\beta_{X_1L} \circ \beta_{X_2L})(p_1, p_2) = min(\beta_{X_1L}(p_1), \beta_{X_2L}(p_2)),$
$(\beta_{X_1U} \circ \beta_{X_2U})(p_1, p_2) = max(\beta_{X_1U}(p_1), \beta_{X_2U}(p_2)), \forall p_1 \in V_1, p_2 \in V_2.$
2) $(\alpha_{Y_1L} \circ \alpha_{Y_2L})(p, p_2) = min(\alpha_{Y_1L}(p), \alpha_{Y_2L}(p_2, q_2)),$
$(\alpha_{Y_1U} \circ \alpha_{Y_2U})(p, p_2) = min(\alpha_{Y_1U}(p), \alpha_{Y_2U}(p_2, q_2)),$
$(\beta_{Y_1L} \circ \beta_{Y_2L})(p, q_2) = max(\beta_{Y_1L}(p), \beta_{Y_2L}(p_2, q_2)),$
$(\beta_{Y_1U} \circ \beta_{Y_2U})(p, q_2) = max(\beta_{Y_1U}(p), \beta_{Y_2U}(p_2, q_2)), \forall p_1 \in V_1, p_2q_2 \in E_2.$

3) $(\alpha_{Y_1L} \circ \alpha_{Y_2L})(p_1, r)(q_1, r) = min(\alpha_{Y_1L}(p_1q_1), \alpha_{Y_2L}(r)),$
$(\alpha_{Y_1U} \circ \alpha_{Y_2U})(p_1, r)(q_1, r) = min(\alpha_{Y_1U}(p_1q_1), \alpha_{Y_2U}(r)),$
$(\beta_{Y_1L} \circ \beta_{Y_2L})(p_1, r)(q_1, r) = max(\beta_{Y_1L}(p_1q_1), \beta_{Y_2L}(r)),$
$(\beta_{Y_1U} \circ \beta_{Y_2U})(p_1, r)(q_1, r) = max(\beta_{Y_1U}(p_1q_1), \beta_{Y_2U}(r)), \forall r \in V_2, p_1q_1 \in E_1.$
4) $(\alpha_{Y_1L} \circ \alpha_{Y_2L})(p_1, p_2)(q_1, q_2) = min(\alpha_{X_2L}(p_2), \alpha_{X_2L}, \alpha_{X_1L}(p_1, q_1)),$
$(\alpha_{Y_1U} \circ \alpha_{Y_2U})(p_1, r)(q_1, r) = min(\alpha_{X_2U}(p_2), \alpha_{X_2U}(q_2), \alpha_{Y_1U}(p_1, q_1)),$
$(\beta_{Y_1L} \circ \beta_{Y_2L})(p_1, r)(q_1, r) = max(\beta_{X_2L}(p_2), \beta_{X_2L}(q_2), \beta_{Y_1L}(p_1, q_1)),$
$(\beta_{Y_1U} \circ \beta_{Y_2U})(p_1, r)(q_1, r) = max(\beta_{X_2U}(p_2), \beta_{X_2U}(q_2), \beta_{Y_1U})(p_1, p_2)(q_1, q_2)),$
$\forall (p_1, p_2)(q_1, q_2) \in E^\circ - E.$
where $E^\circ = E \cup \{(p_1, p_2)(q_1, q_2) | p_1q_1 \in E_1, p_2 \neq q_2\}.$

Definition 3.4. *Let* $\tilde{\xi}_1 = (\xi_1^*, X_1, Y_1, A)$ *and* $\tilde{\xi}_2 = (\xi_2^*, X_2, Y_2, B)$ *be two SIVPF-SGs of* $\xi_1^* = (X_1, Y_1)$ *and* $\xi_2^* = (X_2, Y_2)$ *accordingly. If* $\tilde{\xi}_1$ *and* $\tilde{\xi}_2$ *is standed by* $\tilde{\xi}_1 \cup \tilde{\xi}_2 = (G^*, X, Y, A \cup B)$ *where* $(X_1 \cup X_2, Y_1 \cup Y_2)$ *and is replace*
1) (i) $(\alpha_{X_1L} \cup \alpha_{X_2L})(p) = max(\alpha_{X_1L})(p), \alpha_{X_2L})(p)) if p \in V_1 \cap V_2$
$(\alpha_{X_1U} \cup \alpha_{X_2U})(p) = max(\alpha_{X_1U})(p), \alpha_{X_2U})(p)) if p \in V_1 \cap V_2$
(ii) $(\beta_{X_1L} \cup \beta_{X_2L})(p) = max(\beta_{X_1U})(p), \beta_{X_2L})(p)) if p \in V_1 \cap V_2$
$(\beta_{X_1U} \cup \beta_{X_2U})(p) = max(\beta_{X_1U})(p), \beta_{X_2U})(p)) if p \in V_1 \cap V_2$
2) (i) $(\alpha_{Y_1L} \cup \alpha_{Y_2L})(p, q) = max(\alpha_{X_1L}(p, q), \alpha_{X_2L}(p, q)) if \quad pq \in E_1 \cap E_2$
$(\alpha_{Y_1U} \cup \alpha_{Y_2U})(p, q) = max(\alpha_{X_1U}(p, q), \alpha_{X_2U}(p, q)) if \quad pq \in E_1 \cap E_2$
(ii) $(\beta_{Y_1L} \cup \beta_{Y_2L})(p, q) = max(\beta_{X_1L}(p), \beta_{X_2L}(q)) if \quad pq \in E_1 \cap E_2$
$(\beta_{Y_1U} \cup \beta_{Y_2U})(p, q) = max(\beta_{X_1U}(p), \beta_{Y_2U}(q)) if \quad pq \in E_1 \cap E_2$

Definition 3.5. *Let* $\tilde{G}_1 = (\xi_1^*, X_1, Y_1, A)$ *and* $\tilde{\xi}_2 = (\xi_2^*, X_2, Y_2, B)$ *be two SIVPF-SGs of* $\xi_1^* = (X_1, Y_1)$ *and* $\xi_2^* = (X_2, Y_2)$ *accordingly. If* $\tilde{\xi}_1$ *and* $\tilde{\xi}_2$ *is standed by* $\tilde{\xi}_1 + \tilde{\xi}_2 = (\xi_1^*, X_1, Y_1, A + B)$*. Where* $\xi^* = (X_1 + X_2, Y_1 + Y_2)$ *and is defined by*
1) $(\alpha_{X_1L} + \alpha_{X_2L})(p) = (\alpha_{X_1L} \cup \alpha_{X_2L}))(p)$
$(\alpha_{X_1U} + \alpha_{X_2U})(p) = (\alpha_{X_1U} \cup \alpha_{X_2U})(p) if \quad p \in V_1 \cup V_2$
$(\beta_{X_1L} + \beta_{X_2L})(p) = (\beta_{X_1L} \cup \beta_{X_2L})(p)$
$(\beta_{X_1U} + \beta_{X_2U})(p) = (\beta_{X_1U} \cup \beta_{X_2U})(p) if \quad p \in V_1 \cup V_2$
2)$(\alpha_{Y_1L} + \alpha_{Y_2L})(p, q) = (\alpha_{Y_1L} \cup \alpha_{Y_2L})(p, q)$
$(\alpha_{Y_1U} + \alpha_{Y_2U})(p, q) = (\alpha_{Y_1U} \cup \alpha_{Y_2U})(p, q) if \quad p \in E_1 \cap E_2$
$(\beta_{Y_1L} + \beta_{Y_2L})(p, q) = (\beta_{Y_1L} \cup \beta_{Y_2L})(p, q)$
$(\beta_{Y_1U} + \beta_{Y_2U})(p, q) = (\beta_{Y_1U} \cup \beta_{Y_2U})(p, q) if \quad (p, q) \in E_1 \cap E_2.$
3)$(\alpha_{Y_1L} + \alpha_{Y_2L})(p, q) = min(\alpha_{X_1L}(p), \alpha_{X_2L}(q))$
$(\alpha_{Y_1U} + \alpha_{Y_2U})(p, q) = min(\alpha_{X_1U}(p), \alpha_{X_2U}(q))$
$(\beta_{YL} + \beta_{Y_2L})(p, q) = max(\beta_{X_1L}(p), \beta_{X_2L}(q))$
$(\beta_{Y_1U} + \beta_{Y_2U})(p, q) = max(\beta_{X_1U}(p), \beta_{X_2U}(q)) if \quad pq \in E$
Where E *is the set of all edges joining the vertices of* V_1 *and* V_2.

Theorem 3.1. *If* $\tilde{\xi}_1$ *and* $\tilde{\xi}_2$ *are SIVPFSGs, then so is* $\tilde{\xi}_1 \times \tilde{\xi}_2$.

proof Let $\tilde{\xi_1} = (\xi_1^*, X_1, Y_1, A)$ and $\tilde{\xi_2} = (\xi_1^*, X_1, Y_1, B)$ be two SIVPFSGs of simple graphs $\xi_1^* = (X_1, Y_1)$ and $\xi_2^* = (X_2, Y_2)$ accordingly. for all $e_1 \in A$ and $e_2 \in B$, there are some results. Let ξ_1 and ξ_2 be SIVPFSGs
Let $E = \{(p, p_2)(p, q_2)/p \in V_1, p_2q_2 \in E_2\} \cup \{(p_1, r)(q_1, r)/r \in V_2, p_1q_1 \in E_1\}$.
Consider $(p, p_2)(p, q_2) \in E$, we have

$$(\alpha_{Y_1L} \times \alpha_{Y_2L})(p, p_2)(p, q_2) = min(\alpha_{X_1L}(p), \alpha_{Y_2L}(p_2q_2))$$
$$=min(\alpha_{X_1L}(p), \alpha_{X_2L}(p_2).\alpha_{X_2L}(q_2))$$
$$=min(min(\alpha_{X_1L}(p), \alpha_{X_2L}(p_2))min(\alpha_{X_1L}(p), \alpha_{X_2L}(q_2)))$$
$$(\alpha_{Y_1L} \times \alpha_{Y_2L})(p, p_2)(p, q_2) = min((\alpha_{X_1L} \times \alpha_{X_2L})(p, p_2), (\alpha_{X_1L} \times \alpha_{X_2L})(p, q_2))$$

Similarly,

$$(\alpha_{Y_1U} \times \alpha_{Y_2U})(p, p_2)(p, q_2) = min((\alpha_{X_1U} \times \alpha_{X_2U})(p, p_2), (\alpha_{Y_1U} \times \alpha_{Y_2U})(p, q_2))$$

Now,

$$(\beta_{Y_1L} \times \beta_{Y_2L})(p, p_2)(p, q_2) = max((\beta_{X_1L} \times \beta_{X_2L})(p, p_2), (\beta_{X_1L} \times \beta_{X_2L})(p, q_2))$$

Similarly,

$$(\beta_{Y_1U} \times \beta_{Y_2U})(p, p_2)(p, q_2) = max((\beta_{X_1U} \times \beta_{X_2U})(p, p_2), (\beta_{X_1U} \times \beta_{X_2U})(p, q_2))$$

Consider, $(p_1, r)(q_1, r) \in E$, we have

$$(\alpha_{Y_1L} \times \alpha_{Y_2L})(p_1, r)(q_1, r) = min(\alpha_{Y_1L}(p_1q_1), (\alpha_{X_2L}(r))$$
$$=min(\alpha_{X_1L}(p_1), \alpha_{X_2L}(q_1).\alpha_{X_2L}(r))$$
$$=min(min(\alpha_{X_1L}(p_1), \alpha_{X_2L}(r))min(\alpha_{X_1L}(y_1), \alpha_{X_2L}(r)))$$

$$(\alpha_{Y_1L} \times \alpha_{Y_2L})(p_1, r)(q_1, r) = min((\alpha_{X_1L} \times \alpha_{X_2L})(p_1, r), (\alpha_{X_1L} \times \alpha_{X_2L})(q_1, r))$$

Similarly,

$$(\alpha_{Y_1U} \times \alpha_{Y_2U})(p_1, r)(q_1, r) = min((\alpha_{X_1U} \times \alpha_{X_2U})(p_1, r), (\alpha_{X_1U} \times \alpha_{X_2U})(q_1, r))$$

Now,

$$(\beta_{Y_1L} \times \beta_{Y_1U})(p_1, r)(q_1, r) = max((\beta_{X_1L} \times \beta_{X_2L})(p_1, r), (\beta_{X_1L} \times \beta_{X_2L})(q_1, r))$$

Similarly,

$$(\beta_{Y_1U} \times \beta_{Y_2U})(p_1, r)(q_1, r) = max((\beta_{X_1U} \times \beta_{X_2U})(p_1, r), (\beta_{X_1U} \times \beta_{X_2U})(q_1, r))$$

Hence $\xi_1 \times \xi_2$ is an SIVPFSGs.

Theorem 3.2. *If $\tilde{\xi}_1[\tilde{\xi}_2]$ be SIVPFSGs $\tilde{\xi}_1$ and $\tilde{\xi}_2$ of ξ_1^* and ξ_2^* is an SIVPFSGs.*

Proof Take $(p, p_2)(p, q_2) \in E$, we get

$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((p, p_2)(p, q_2)) = min((\alpha_{X_1L}(p), \alpha_{Y_2L})(p_2q_2)$
$=min(\alpha_{X_1L}(p), \alpha_{X_2L}(p_2), \alpha_{X_2L}(q_2))$
$=min(min(\alpha_{X_1L}(p), \alpha_{X_2L}(p_2)), min(\alpha_{X_1L}(p), \alpha_{X_2L}(q_2)))$
$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((p, p_2)(p, q_2) = min(\alpha_{X_1L} \circ \alpha_{X_2L})(p, p_2), (\alpha_{X_1L} \circ \alpha_{X_2L})(p, q_2)).$

Similarly,

$(\alpha_{Y_1U} \circ \alpha_{Y_2U})((p, p_2)(p, q_2) = min(\alpha_{X_1U} \circ \alpha_{X_2U})(p, p_2), (\alpha_{X_1U} \circ \alpha_{X_2U})(p, q_2))$

Consider $(p_1, r)(q_1, r) \in E$,

$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((p_1, r)(q_1, r)) = min(\alpha_{Y_1L}(p_1, q_1), \alpha_{X_2L}(r))$
$=min(\alpha_{X_1L}(p_1), \alpha_{X_1L}(q_1), \alpha_{X_2L}(r))$
$=min(min(\alpha_{X_1L}(p_1), \alpha_{X_2L}(r)), min(\alpha_{X_1L}(q_1), \alpha_{X_2L}(r)))$
$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((p_1, r)(q_1, r)) = min(\alpha_{X_1L} \circ \alpha_{X_2L})(p_1, r), (\alpha_{X_1L} \circ \alpha_{X_2L})((q_1, r))$

Similarly,

$(\alpha_{X_1U} \circ \alpha_{X_2U})((p_1, r)(q_1, r) = min(\alpha_{X_1U} \circ \alpha_{X_2U})(p_1, r), (\alpha_{X_1U}\alpha_{X_2U})((q_1, r))$

Consider $(p_1, p_2)(q_1, q_2) \in E$,

$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((p_1, p_2)(q_1, q_2)) = min(\alpha_{X_2L}(p_2), \alpha_{X_2L}(q_2), \alpha_{Y_1L}(p_1q_1))$
$=min(\alpha_{X_2L}(p_2), \alpha_{X_2L}(q_2)), min(\alpha_{X_1L}(p_1), \alpha_{X_1L}(q_1))$
$=min(min(\alpha_{X_1L}(p_1), \alpha_{X_2L}(p_2)), min(\alpha_{X_1L}(q_1), \alpha_{X_2L}(q_2)))$
$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((p_1, p_2)(q_1, q_2)) = min(\alpha_{X_1L} \circ \alpha_{X_2L})(p_1, p_2), (\alpha_{X_1L} \circ \alpha_{X_2L})$
$((q_1, q_2))$

Hence $\tilde{\xi}_1[\tilde{\xi}_2]$ be SIVPFSG .

Theorem 3.3. *If $\tilde{\xi}_1 \cup \tilde{\xi}_2$ be SIVPFSGs $\tilde{\xi}_1$ and $\tilde{\xi}_2$ of ξ_1^* and ξ_2^* is an SIVPFSGs.*

Proof Take $\tilde{\xi}_1$ and $\tilde{\xi}_2$ be the SIVPFSGs of $\tilde{\xi}_1$ and $\tilde{\xi}_2$ accordingly. Since all conditions for $X_1 \cup X_2$ are obviously satisfied. It is enough to verify the conditions for $Y_1 \cup Y_2$, Consider $(p, q) \in E_1 \cup E_2$. Then

$(\alpha_{Y_1L} \cup \alpha_{Y_2L})(p, q) = max(\alpha_{Y_1L}(p, q), \alpha_{Y_2L}(p, q))$
$=max(min(\alpha_{X_1L}(p), \alpha_{X_1L}(q)), (min(\alpha_{X_2L})(p), \alpha_{X_2L}(q))$
$=min(max(\alpha_{X_1L}(p), \alpha_{X_2L}(p)), (max(\alpha_{X_1L}(p), \alpha_{X_2L}(q))))$
$=min((\alpha_{Y_1L} \cup \alpha_{Y_2L})(p), (\alpha_{Y_1L} \cup \alpha_{Y_2L})(q))$
$(\alpha_{Y_1L} \cup \alpha_{Y_2L})(p, q) = min((\alpha_{Y_1L} \cup \alpha_{Y_2L})(p), (\alpha_{Y_1L} \cup \alpha_{Y_2L})(q)).$

Similarly,

$$(\alpha_{Y_1U} \cup \alpha_{Y_2U})(p,q) = min((\alpha_{Y_1U} \cup \alpha_{Y_2U})(p), (\alpha_{Y_1U} \cup \alpha_{Y_2U})(q))$$

If $(x,y) \in E_1$ and $(x,y) \notin E_2$,

$$\begin{aligned}(\alpha_{Y_1L} \cup \alpha_{Y_2L})(p,q) =& min((\alpha_{Y_1L} \cup \alpha_{Y_2L})(p), (\alpha_{Y_1L} \cup \alpha_{Y_2L})(q))\\(\alpha_{Y_1U} \cup \alpha_{Y_2U})(p,q) =& min((\alpha_{Y_1U} \cup \alpha_{Y_2U})(p), (\alpha_{Y_1U} \cup \alpha_{Y_2U})(q)).\end{aligned}$$

If $(p,q) \in E_2$ and $(p,q) \in E_1$,

$$\begin{aligned}(\alpha_{Y_1L} \cup \alpha_{Y_2L})(p,q) =& min((\alpha_{Y_1L} \cup \alpha_{Y_2L})(p), (\alpha_{Y_1L} \cup \alpha_{Y_2L})(q))\\(\alpha_{Y_1U} \cup \alpha_{Y_2U})(p,q) =& min((\alpha_{Y_1U} \cup \alpha_{Y_2U})(p), (\alpha_{Y_1U} \cup \alpha_{Y_2U})(q)).\end{aligned}$$

Theorem 3.4. *If $\tilde{\xi}_1 + \tilde{\xi}_2$ be SIVPFSGs $\tilde{\xi}_1$ and $\tilde{\xi}_2$ of ξ_1^* and ξ_2^* is an SIVPFSGs.*

Proof Take $\tilde{\xi}_1 + \tilde{\xi}_2$ be the SIVPFSGs of ξ_1^* and ξ_2^* accordingly. , it is enough to find that $\tilde{\xi}_1 + \tilde{\xi}_2 = (X_1 + X_2, Y_1 + Y_2)$ is an SIVPFSGs. Then Let $(p,q) \in E$

$$\begin{aligned}(\alpha_{Y_1L} + \alpha_{Y_2L})(p,q) =& min(\alpha_{X_1L}(p), \alpha_{X_2L}(q))\\=& min((\alpha_{X_1L} \cup \alpha_{X_2L})(p), ((\alpha_{X_1L} \cup \alpha_{X_2L})(q)))\\(\alpha_{Y_1L} + \alpha_{Y_2L})(p,q) =& min((\alpha_{X_1L} + \alpha_{X_2L})(p), ((\alpha_{X_1L} + \alpha_{X_2L})(q))).\end{aligned}$$

Similarly,

$$(\alpha_{Y_1U} + \alpha_{Y_2U})(p,q) = min((\alpha_{X_1U} + \alpha_{X_2U})(p), ((\alpha_{X_1U} + \alpha_{X_2U})(q))).$$

4 Conclusions

Graph theory is a very helpful mathematical tool for tackling challenging issues in a variety of disciplines. The IVPFSs model is appropriate for modeling issues involving uncertainty and inconsistent data when human understanding and evaluation are required. In contrast to IVFS models, IVIFS models, and, IVPFS models provide systems with sensitivity, flexibility, and conformance. SIVPFSGs are a novel idea that is introduced in this work. We also defined for the Cartesian product as well as some information about its composition on SIVPFSGs. We plan to use this data to create some algorithms and models shortly soon.

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