

Some New $\tilde{\theta}$ - \mathcal{J} -Locally Closed sets with Respect to an Ideal Topological Spaces

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Abstract

In this paper, we introduce the new notions called $\tilde{\theta}$ - \mathcal{J} -locally closed sets, $\tilde{\theta}$ - \mathcal{J} -locally closed sets and $\tilde{\theta}$ - \mathcal{J} -closed functions and investigated their properties and also we have studied their relations to the other types of locally closed sets with suitable examples. Finally we introduce the notion $\tilde{\theta}$ - \mathcal{J} -submaximal spaces and also investigated the properties with examples.

Keywords: $\tilde{\theta}$ - \mathcal{J} -cld, $\tilde{\theta}$ - \mathcal{J} -lc, $\tilde{\theta}$ - \mathcal{J} -lc*, $\tilde{\theta}$ - \mathcal{J} -lc**.

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1. Introduction

In 1970, Levine [11] introduced the generalized closed sets and after that many authors introduced and studied many types of generalized closed sets in topological spaces. In the continuity, locally closedness was done by Bourbaki [4]. He defined a set A to be locally closed if it is the intersection of an open set and a closed set. In literature many general topologists introduced the studies of locally closed sets. Extensive research on locally closedness and generalizing locally closedness were done in recent years. Stone [16] used the term FG for a locally closed set. Ganster and Reilly used locally closed sets in [6] to define LC-continuity and LC-irresoluteness. Balachandran et al [3] introduced the concept of generalized locally closed sets. Veera Kumar [18] (Sheik John [15]) introduced \hat{g} -locally closed sets (ω -locally closed sets) respectively.

In this paper, we introduce the new notions called $\check{\theta}$ - \mathcal{J} -locally closed sets, $\check{\theta}$ - \mathcal{J} -locally closed sets and $\check{\theta}$ - \mathcal{J} -closed functions and investigated their properties and also we have studied their relations to the other types of locally closed sets with suitable examples. Finally we introduce the notion $\check{\theta}$ - \mathcal{J} -submaximal spaces and also investigated the properties with examples.

2. Preliminaries

An ideal \mathcal{J} on a topological space (briefly, TPS) (X, τ) is a nonempty collection of subsets of X which satisfies

- (1) $A \in \mathcal{J}$ and $B \subseteq A \Rightarrow B \in \mathcal{J}$ and
- (2) $A \in \mathcal{J}$ and $B \in \mathcal{J} \Rightarrow A \cup B \in \mathcal{J}$.

Given a topological space (X, τ) with an ideal \mathcal{J} on X if $\wp(X)$ is the set of all subsets of X , a set operator $(\bullet)^*$: $\wp(X) \rightarrow \wp(X)$, called a local function [10] of A with respect to τ and \mathcal{J} is defined as follows: for $A \subseteq X$, $A^*(\mathcal{J}, \tau) = \{x \in X : U \cap A \notin \mathcal{J} \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$. A Kuratowski closure operator $cl^*(\bullet)$ for a topology $\tau^*(\mathcal{J}, \tau)$, called the \star -topology and finer than τ , is defined by $cl^*(A) = A \cup A^*(\mathcal{J}, \tau)$ [10]. We will simply write A^* for $A^*(\mathcal{J}, \tau)$ and τ^* for $\tau^*(\mathcal{J}, \tau)$. If \mathcal{J} is an ideal on X , then (X, τ, \mathcal{J}) is called an ideal topological space (briefly, ITPS). A subset A of an ideal topological space (X, τ, \mathcal{J}) is \star -closed (briefly, \star -cld) [10] if $A^* \subseteq A$. The interior of a subset A in $(X, \tau^*(\mathcal{J}))$ is denoted by $int^*(A)$.

Definition 2.1 A subset K of a TPS X is called:

- (i) semi-open set [9] if $K \subseteq cl(int(K))$;
- (ii) α -open set [9] if $K \subseteq int(cl(int(K)))$;
- (iii) regular open set [12] if $K = int(cl(K))$;

The complements of the above mentioned open sets are called their respective closed sets.

Definition 2.2 A subset K of a TPS X is called

- (i) g -closed set (briefly, g -cld) [11] if $cl(K) \subseteq V$ whenever $K \subseteq V$ and V is open.

- (ii) semi-generalized closed (briefly, sg-cld)[7] if $scl(K) \subseteq V$ whenever $K \subseteq V$ and V is semi-open.
- (iii) generalized semi-closed (briefly, gs-cld)[18] if $scl(K) \subseteq V$ whenever $K \subseteq V$ and V is open.
- (iv) \hat{g} -closed set [18] (ω -cld set [18]) if $cl(K) \subseteq V$ whenever $K \subseteq V$ and V is semi-open in X .

The complements of the above mentioned closed sets are called their respective open sets.

Definition 2.3 A subset K of a ITPS X is called

- (i) \mathcal{J}_g -closed (briefly, \mathcal{J}_g -cld) set [9] if $K^* \subseteq V$ whenever $K \subseteq V$ and V is open.
- The complements of the above mentioned closed sets are called their respective open sets.

Definition 2.4 A subset K of a space X is called a regular generalized closed (briefly, rg-closed) set [12] if $cl(K) \subseteq V$ whenever $K \subseteq V$ and V is regular open in X . The complement of rg-closed set is called rg-open set;

Remark 2.5

The collection of all rg-closed sets in X is denoted by $RG C(X)$.

The collection of all rg-open sets in X is denoted by $RG O(X)$.

Definition 2.6 A subset K of a space X is called

- i. generalized locally closed (briefly, glc) [2] if $A = V \cap F$, where V is g-open and F is g-closed in X .
- ii. semi-generalized locally closed (briefly, sglc) [13] if $K = V \cap F$, where V is sg-open and F is sg-closed in X .
- iii. regular-generalized locally closed (briefly, rg-lc) [1] if $K = V \cap F$, where V is rg-open and F is rg-closed in X .
- iv. generalized locally semi-closed (briefly, glsc) [8] if $K = V \cap F$, where V is g-open and F is semi-closed in X .
- v. locally semi-closed (briefly, lsc) [8] if $K = V \cap F$, where V is open and F is semi-closed in X .
- vi. α -locally closed (briefly, α -lc) [8] if $K = V \cap F$, where V is α -open and F is α -closed in X .
- vii. ω -locally closed (briefly, ω -lc) [15] if $K = V \cap F$, where V is ω -open and F is ω -closed in X .

The class of all generalized locally closed (resp. generalized locally semi-closed, locally semi-closed, ω -locally closed) sets in X is denoted by $GLC(X)$ (resp. $GLSC(X)$, $LSC(X)$, ω -LC(X)).

Definition 2.7 A topological space X is called

- i. submaximal [5, 18] if every dense subset is open.

- ii. \hat{g} (or ω)-submaximal [15, 18] if every dense subset is ω -open.
- iii. g -submaximal [2] if every dense subset is g -open.
- iv. rg -submaximal [12] if every dense subset is rg -open.

Theorem 2.8 Let X be a topological space

- i. If X is submaximal, then X is \hat{g} -submaximal.
- ii. If X is \hat{g} -submaximal, then X is g -submaximal.
- iii. If X is g -submaximal, then X is rg -submaximal.

3. $\check{\theta}$ - \mathcal{J} -Locally Closed Sets

We introduce the following definition.

Definition 3.1 A subset K of X is called

- (i) $\check{\theta}$ - \mathcal{J} -closed (briefly, $\check{\theta}$ - \mathcal{J} -cld) if $K^* \subseteq V$ whenever $K \subseteq V$ and V is sg -open. The complement of $\check{\theta}$ - \mathcal{J} -cld is called $\check{\theta}$ - \mathcal{J} -open. The family of all $\check{\theta}$ - \mathcal{J} -cld in X is denoted by $\check{\theta}\mathcal{J}C(X)$.
- (ii) $\check{\theta}$ - \mathcal{J} -locally closed (briefly, $\check{\theta}$ - \mathcal{J} -lc) if $K = S \cap G$, where S is $\check{\theta}$ - \mathcal{J} -open and G is $\check{\theta}$ - \mathcal{J} -cld.
- (iii) A function $f : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma)$ is called $\check{\theta}$ - \mathcal{J} -continuous if the inverse image of every closed set in Y is $\check{\theta}$ - \mathcal{J} -cld set in X . The class of all $\check{\theta}$ - \mathcal{J} -locally closed sets in X is denoted by $\check{\theta}\mathcal{J}LC(X)$.

Proposition 3.2 Every $\check{\theta}$ - \mathcal{J} -cld (resp. $\check{\theta}$ - \mathcal{J} -open) set is $\check{\theta}$ - \mathcal{J} -lc set but not conversely.

Proof

It follows from Definition 3.1 (i) and (ii).

Example 3.3 Let $X = \{p, q, r\}$ and $\tau = \{\phi, \{q\}, X\}$ with $\mathcal{J} = \{\phi\}$. Then the set $\{q\}$ is $\check{\theta}$ - \mathcal{J} -lc set but it is not $\check{\theta}$ - \mathcal{J} -closed and the set $\{p, r\}$ is $\check{\theta}$ - \mathcal{J} -lc set but it is not $\check{\theta}$ - \mathcal{J} -open in X .

Proposition 3.4 Every lc set is $\check{\theta}$ - \mathcal{J} -lc set but not conversely.

Proof

It follows from Proposition 3.2.

Example 3.5 Let $X = \{p, q, r\}$ and $\tau = \{\phi, \{q, r\}, X\}$ with $\mathcal{J} = \{\phi\}$. Then the set $\{q\}$ is $\check{\theta}$ - \mathcal{J} -lc set but it is not lc set in X .

Proposition 3.6 Every $\check{\theta}$ - \mathcal{J} -lc set is a (i) ω -lc set, (ii) glc set and (iii) $sglc$ set. However the separate converses are not true.

Proof

It is obviously.

Example 3.7 Let $X = \{p, q, r\}$ and $\tau = \{\emptyset, \{p\}, X\}$ with $\mathcal{J} = \{\emptyset\}$. Then the set $\{b\}$ is glc set but it is not $\check{\theta}$ - \mathcal{J} -lc set in X . Moreover, the set $\{r\}$ is sglc set but it is not $\check{\theta}$ - \mathcal{J} -lc set in X .

Example 3.8 Let $X = \{p, q, r\}$ and $\tau = \{\emptyset, \{q\}, \{p, r\}, X\}$ with $\mathcal{J} = \{\emptyset\}$. Then the set $\{p\}$ is ω -lc set but it is not $\check{\theta}$ - \mathcal{J} -lc set in X .

Remark 3.9 The concepts of α -lc sets and $\check{\theta}$ - \mathcal{J} -lc sets are independent of each other.

Example 3.10 The set $\{q, r\}$ in Example 3.3 is α -lc set but it is not a $\check{\theta}$ - \mathcal{J} -lc set in X and the set $\{p, q\}$ in Example 3.5 is $\check{\theta}$ - \mathcal{J} -lc set but it is not an α -lc set in X .

Remark 3.11 The concepts of lsc sets and $\check{\theta}$ - \mathcal{J} -lc sets are independent of each other.

Example 3.12 The set $\{p\}$ in Example 3.3 is lsc set but it is not a $\check{\theta}$ - \mathcal{J} -lc set in X and the set $\{p, q\}$ in Example 3.5 is $\check{\theta}$ - \mathcal{J} -lc set but it is not a lsc set in X .

Remark 3.13 The concepts of $\check{\theta}$ - \mathcal{J} -lc sets and glsc sets are independent of each other.

Example 3.14 The set $\{q, r\}$ in Example 3.3 is glsc set but it is not a $\check{\theta}$ - \mathcal{J} -lc set in X and the set $\{p, q\}$ in Example 3.5 is $\check{\theta}$ - \mathcal{J} -lc set but it is not a glsc set in X .

Remark 3.15 The concepts of $\check{\theta}$ - \mathcal{J} -lc sets and $sglc^*$ sets are independent of each other.

Example 3.16 The set $\{q, r\}$ in Example 3.3 is $sglc^*$ set but it is not a $\check{\theta}$ - \mathcal{J} -lc set in X and the set $\{p, q\}$ in Example 3.5 is $\check{\theta}$ - \mathcal{J} -lc set but it is not a $sglc^*$ set in X .

Theorem 3.17 For a $T\check{\theta}$ - \mathcal{J} -space X , the following properties hold

- (i) $\check{\theta}$ - \mathcal{J} - $LC(X) = LC(X)$.
- (ii) $\check{\theta}$ - \mathcal{J} - $LC(X) \subseteq GLC(X)$.
- (iii) $\check{\theta}$ - \mathcal{J} - $LC(X) \subseteq GLSC(X)$.
- (iv) $\check{\theta}$ - \mathcal{J} - $LC(X) \subseteq \omega$ - $LC(X)$.

Proof

(i) Since every $\check{\theta}$ - \mathcal{J} -open set is open and every $\check{\theta}$ - \mathcal{J} -closed set is closed in (X, τ) , $\check{\theta}$ - \mathcal{J} - $LC(X) \subseteq LC(X)$ and hence $\check{\theta}$ - \mathcal{J} - $LC(X) = LC(X)$.

(ii), (iii) and (iv) follows from (i), since for any space X , $LC(X) \subseteq GLC(X)$, $LC(X) \subseteq GLSC(X)$ and $LC(X) \subseteq \omega$ - $LC(X)$.

Corollary 3.18 If $G O(X) = \tau$, then $\check{\theta}$ - \mathcal{J} - $LC(X) \subseteq GLSC(X) \subseteq LSC(X)$.

Proof

$GO(X) = \tau$ implies that X is a $T\check{\theta}\mathcal{J}$ -space and hence by Theorem 3.17, $\check{\theta}\mathcal{J}\text{-LC}(X) \subseteq GLSC(X)$. Let $K \in GLSC(X)$. Then $K = V \cap F$, where V is g -open and F is semi-closed. By hypothesis, V is open and hence K is a lsc-set and so $K \in LSC(X)$.

Definition 3.19 A subset K of a space X is called:

(i) $\check{\theta}\mathcal{J}\text{-lc}^*$ set if $K = S \cap G$, where S is $\check{\theta}\mathcal{J}$ -open in X and G is closed in X .

(ii) $\check{\theta}\mathcal{J}\text{-lc}^{**}$ set if $K = S \cap G$, where S is open in X and G is $\check{\theta}\mathcal{J}$ -closed in X .

The class of all $\check{\theta}\mathcal{J}\text{-lc}^*$ (resp. $\check{\theta}\mathcal{J}\text{-lc}^{**}$) sets in a ideal topological space X is denoted by $\check{\theta}\mathcal{J}\text{-LC}^*(X)$ (resp. $\check{\theta}\mathcal{J}\text{-LC}^{**}(X)$).

Proposition 3.20 Every lc-set is $\check{\theta}\mathcal{J}\text{-lc}^*$ set but not conversely.

Proof

It follows from Definition 3.19 (i) and Definition of locally closed set.

Example 3.21 The set $\{q\}$ in Example 3.5 is $\check{\theta}\mathcal{J}\text{-lc}^*$ set but it is not a lc set in X .

Proposition 3.22 Every lc-set is $\check{\theta}\mathcal{J}\text{-lc}^{**}$ set but not conversely.

Proof

It follows from Definition 3.19 (ii) and Definition of locally closed set.

Example 3.23 The set $\{p, r\}$ in Example 3.5 is $\check{\theta}\mathcal{J}\text{-lc}^{**}$ set but it is not a lc set in X .

Proposition 3.24 Every $\check{\theta}\mathcal{J}\text{-lc}^*$ set is $\check{\theta}\mathcal{J}\text{-lc}$ set but not conversely.

Proof

It follows from Definitions 3.1 and 3.19 (i).

Example 3.25 The set $\{p, q\}$ in Example 3.5 is $\check{\theta}\mathcal{J}\text{-lc}$ set but it is not a $\check{\theta}\mathcal{J}\text{-lc}^*$ set in X .

Proposition 3.26 Every $\check{\theta}\mathcal{J}\text{-lc}^{**}$ set is $\check{\theta}\mathcal{J}\text{-lc}$ set but not conversely.

Proof

It follows from Definitions 3.1 and 3.19 (ii).

Remark 3.27 The concepts of $\check{\theta}\mathcal{J}\text{-lc}^*$ sets and lsc sets are independent of each other.

Example 3.28 The set $\{r\}$ in Example 3.5 is $\check{\theta}\mathcal{J}\text{-lc}^*$ set but it is not a lsc set in X and the set $\{p\}$ in Example 3.3 is lsc set but it is not a $\check{\theta}\mathcal{J}\text{-lc}^*$ set in X .

Remark 3.29 The concepts of $\check{\theta}\mathcal{J}\text{-lc}^{**}$ sets and α -lc sets are independent of each other.

Example 3.30 The set $\{p, q\}$ in Example 3.5 is $\check{\theta}$ - \mathcal{J} - lc^{**} set but it is not a α - lc set in X and the set $\{p, q\}$ in Example 3.3 is α - lc set but it is not a $\check{\theta}$ - \mathcal{J} - lc^{**} set in X .

Remark 3.31 From the above discussions we have the following implications where $A \rightarrow B$ (resp. $A \not\rightarrow B$) represents A implies B but not conversely (resp. A and B are independent of each other).

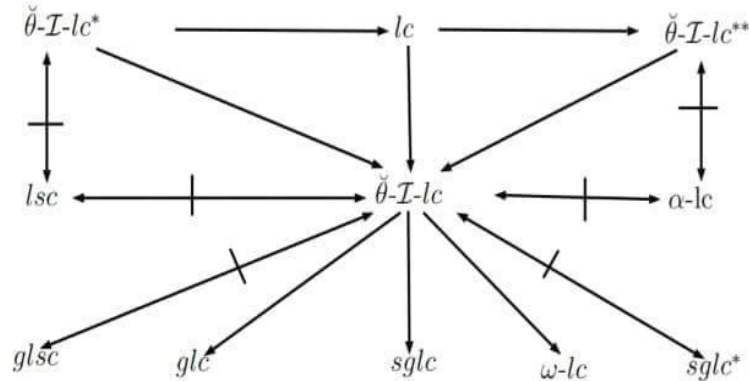


Figure 1: Relations between some of generalized closed sets

Proposition 3.32 If $G O(X) = \tau$, then $\check{\theta}\text{-}\mathcal{J}\text{-}LC(X) = \check{\theta}\text{-}\mathcal{J}\text{-}LC^*(X) = \check{\theta}\text{-}\mathcal{J}\text{-}LC^{**}(X)$.

Proof

For any space (X, τ) , $\tau \subseteq \check{\theta}\text{-}\mathcal{J}\text{-}O(X) \subseteq G O(X)$. Therefore by hypothesis, $\check{\theta}\text{-}\mathcal{J}\text{-}O(X) = \tau$. i.e., (X, τ) is a $T\check{\theta}\text{-}\mathcal{J}$ -space and hence $\check{\theta}\text{-}\mathcal{J}\text{-}LC(X) = \check{\theta}\text{-}\mathcal{J}\text{-}LC^*(X) = \check{\theta}\text{-}\mathcal{J}\text{-}LC^{**}(X)$.

Remark 3.33 The converse of Proposition 3.32 need not be true.

For the ideal topological space X in Example 3.3. $\check{\theta}\text{-}\mathcal{J}\text{-}LC(X) = \check{\theta}\text{-}\mathcal{J}\text{-}LC^*(X) = \check{\theta}\text{-}\mathcal{J}\text{-}LC^{**}(X)$. However, $G O(X) = \{\phi, \{p\}, \{q\}, \{r\}, \{p, q\}, \{q, r\}, X\} \neq \tau$.

Proposition 3.34 Let X be an ideal topological space. If $G O(X) \subseteq LC(X)$, then $\check{\theta}\text{-}\mathcal{J}\text{-}LC(X) = \check{\theta}\text{-}\mathcal{J}\text{-}LC^{**}(X)$.

Proof

Let $K \in \check{\theta}\text{-}\mathcal{J}\text{-}LC(X)$. Then $K = S \cap G$ where S is $\check{\theta}\text{-}\mathcal{J}$ -open and G is $\check{\theta}\text{-}\mathcal{J}$ -closed. Since $\check{\theta}\text{-}\mathcal{J}\text{-}O(X) \subseteq G O(X)$ and by hypothesis $G O(X) \subseteq LC(X)$, S is locally closed. Then $S = P \cap Q$, where P is open and Q is $*$ -closed. Therefore, $K = P \cap (Q \cap G)$. We have, $Q \cap G$ is $\check{\theta}\text{-}\mathcal{J}$ -closed and hence $K \in \check{\theta}\text{-}\mathcal{J}\text{-}LC^{**}(X)$. i.e., $\check{\theta}\text{-}\mathcal{J}\text{-}LC(X) \subseteq \check{\theta}\text{-}\mathcal{J}\text{-}LC^{**}(X)$. For any ideal topological space, $\check{\theta}\text{-}\mathcal{J}\text{-}LC^{**}(X) \subseteq \check{\theta}\text{-}\mathcal{J}\text{-}LC(X)$ and so $\check{\theta}\text{-}\mathcal{J}\text{-}LC(X) = \check{\theta}\text{-}\mathcal{J}\text{-}LC^{**}(X)$.

Remark 3.35 The converse of Proposition 3.34 need not be true in general.

For the ideal topological space X in Example 3.3, then $\check{\theta}\text{-}\mathcal{J}\text{-LC}(X) = \check{\theta}\text{-}\mathcal{J}\text{-LC}^{**}(X) = \{\emptyset, \{q\}, \{p, r\}, X\}$. But $G O(X) = \{\emptyset, \{p\}, \{q\}, \{r\}, \{p, q\}, \{q, r\}, X\} \not\subseteq LC(X) = \{\emptyset, \{q\}, \{p, r\}, X\}$.

Corollary 3.36 Let X be an ideal topological space. If $\omega O(X) \subseteq LC(X)$, then $\check{\theta}\text{-}\mathcal{J}\text{-LC}(X) = \check{\theta}\text{-}\mathcal{J}\text{-LC}^{**}(X)$.

Proof

It follows from the fact that $\omega O(X) \subseteq G O(X)$ and Proposition 3.34.

Remark 3.37 The converse of Corollary 3.36 need not be true in general.

For the ideal topological space X in Example 3.8, then $\check{\theta}\text{-}\mathcal{J}\text{-LC}(X) = \check{\theta}\text{-}\mathcal{J}\text{-LC}^{**}(X) = \{\emptyset, \{q\}, \{p, r\}, X\}$. But $\omega O(X) = P(X) \not\subseteq LC(X) = \{\emptyset, \{q\}, \{p, r\}, X\}$.

The following results are characterizations of $\check{\theta}\text{-}\mathcal{J}\text{-lc}$ sets, $\check{\theta}\text{-}\mathcal{J}\text{-lc}^*$ sets and $\check{\theta}\text{-}\mathcal{J}\text{-lc}^{**}$ sets.

Theorem 3.38 Assume that $\check{\theta}\text{-}\mathcal{J}\text{-C}(X)$ is closed under finite intersection. For a subset K of X , the following statements are equivalent:

- (i) $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}(X)$.
- (ii) $K = S \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$ for some $\check{\theta}\text{-}\mathcal{J}\text{-open}$ set S .
- (iii) $\check{\theta}\text{-}\mathcal{J}\text{-cl}(K) - K$ is $\check{\theta}\text{-}\mathcal{J}\text{-closed}$.
- (iv) $K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c$ is $\check{\theta}\text{-}\mathcal{J}\text{-open}$.
- (v) $K \subseteq \check{\theta}\text{-}\mathcal{J}\text{-int}(K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c)$.

Proof

(i) \Rightarrow (ii). Let $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}(X)$. Then $K = S \cap G$ where S is $\check{\theta}\text{-}\mathcal{J}\text{-open}$ and G is $\check{\theta}\text{-}\mathcal{J}\text{-closed}$. Since $K \subseteq G$, $\check{\theta}\text{-}\mathcal{J}\text{-cl}(K) \subseteq G$ and so $S \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K) \subseteq K$. Also $K \subseteq S$ and $K \subseteq \check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$ implies $K \subseteq S \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$ and therefore $K = S \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$.

(ii) \Rightarrow (iii). $K = S \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$ implies $\check{\theta}\text{-}\mathcal{J}\text{-cl}(K) - K = \check{\theta}\text{-}\mathcal{J}\text{-cl}(K) \cap S^c$ which is $\check{\theta}\text{-}\mathcal{J}\text{-closed}$ since S^c is $\check{\theta}\text{-}\mathcal{J}\text{-closed}$ and $\check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$ is $\check{\theta}\text{-}\mathcal{J}\text{-closed}$.

(iii) \Rightarrow (iv). $K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c = (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K) - K)^c$ and by assumption, $(\check{\theta}\text{-}\mathcal{J}\text{-cl}(K) - K)^c$ is $\check{\theta}\text{-}\mathcal{J}\text{-open}$ and so is $K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c$.

(iv) \Rightarrow (v). By assumption, $K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c = \check{\theta}\text{-}\mathcal{J}\text{-int}(K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c)$ and hence $K \subseteq \check{\theta}\text{-}\mathcal{J}\text{-int}(K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c)$.

(v) \Rightarrow (i). By assumption and since $K \subseteq \check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$, $K = \check{\theta}\text{-}\mathcal{J}\text{-int}(K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c) \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$. Therefore, $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}(X)$.

Theorem 3.39 For a subset K of X , the following statements are equivalent:

- i. $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}^*(X)$.
- ii. $K = S \cap K^*$ for some $\check{\theta}\text{-}\mathcal{J}\text{-open}$ set S .
- iii. $K^* - K$ is $\check{\theta}\text{-}\mathcal{J}\text{-closed}$.
- iv. $K \cup (K^*)^c$ is $\check{\theta}\text{-}\mathcal{J}\text{-open}$.

Proof

- (i) \Rightarrow (ii). Let $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}^*(X)$. There exist an $\check{\theta}$ - \mathcal{J} -open set S and a $*$ -closed set G such that $K = S \cap G$. Since $K \subseteq S$ and $K \subseteq K^*$, $K \subseteq S \cap K^*$. Also, since $K^* \subseteq G$, $S \cap K^* \subseteq S \cap G = K$. Therefore $K = S \cap K^*$.
- (ii) \Rightarrow (i). Since S is $\check{\theta}$ - \mathcal{J} -open and K^* is a $*$ -closed set, $K = S \cap K^* \in \check{\theta}\text{-}\mathcal{J}\text{-LC}^*(X)$.
- (ii) \Rightarrow (iii). Since $K^* - K = K^* \cap S^c$, $K^* - K$ is $\check{\theta}$ - \mathcal{J} -closed.
- (iii) \Rightarrow (ii). Let $S = (K^* - K)^c$. Then by assumption S is $\check{\theta}$ - \mathcal{J} -open in X and $K = S \cap K^*$.
- (iii) \Rightarrow (iv). Let $G = K^* - K$. Then $G^c = K \cup (K^*)^c$ and $K \cup (K^*)^c$ is $\check{\theta}$ - \mathcal{J} -open.
- (iv) \Rightarrow (iii). Let $S = K \cup (K^*)^c$. Then S^c is $\check{\theta}$ - \mathcal{J} -closed and $S^c = K^* - K$ and so $K^* - K$ is $\check{\theta}$ - \mathcal{J} -closed.

Theorem 3.40 Let K be a subset of X . Then $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}^{**}(X)$ if and only if $K = S \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$ for some open set S .

Proof Let $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}^{**}(X)$. Then $K = S \cap G$ where S is open and G is $\check{\theta}$ - \mathcal{J} -closed. Since $K \subseteq G$, $\check{\theta}\text{-}\mathcal{J}\text{-cl}(K) \subseteq G$. We obtain $K = K \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K) = S \cap G \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K) = S \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$.

Converse part is trivial.

Corollary 3.41 Let K be a subset of X . If $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}^{**}(X)$, then $\check{\theta}\text{-}\mathcal{J}\text{-cl}(K) - K$ is $\check{\theta}$ - \mathcal{J} -closed and $K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c$ is $\check{\theta}$ - \mathcal{J} -open.

Proof

Let $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}^{**}(X)$. Then by Theorem 3.40, $K = S \cap \check{\theta}\text{-}\mathcal{J}\text{-cl}(K)$ for some open set S and $\check{\theta}\text{-}\mathcal{J}\text{-cl}(K) - K = \check{\theta}\text{-}\mathcal{J}\text{-cl}(K) \cap S^c$ is $\check{\theta}$ - \mathcal{J} -closed in X . If $G = \check{\theta}\text{-}\mathcal{J}\text{-cl}(K) - K$, then $G^c = K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c$ and G^c is $\check{\theta}$ - \mathcal{J} -open and so is $K \cup (\check{\theta}\text{-}\mathcal{J}\text{-cl}(K))^c$.

4. $\check{\theta}$ - \mathcal{J} -Submaximal Spaces

Definition 4.1

- i. A subset K of a space X is called \mathcal{J} -dense if $K^* = X$.
- ii. A subset K of a space X is called \mathcal{J}_g -submaximal if every \mathcal{J} -dense subset is \mathcal{J}_g -open.

Proposition 4.2 Every $\check{\theta}$ - \mathcal{J} -dense set is \mathcal{J} -dense.

Proof Let K be an $\check{\theta}$ - \mathcal{J} -dense set in X . Then $\check{\theta}\text{-}\mathcal{J}\text{-cl}(K) = X$. Since $\check{\theta}\text{-}\mathcal{J}\text{-cl}(K) \subseteq \text{cl}(K)$, we have $K^* = X$ and so K is \mathcal{J} -dense.

The converse of Proposition 4.2 need not be true as can be seen from the following example.

Example 4.3 The set $\{p, r\}$ in Example 3.5 is a \mathcal{J} -dense in X but it is not $\check{\theta}$ - \mathcal{J} -dense in X .

We introduce the following definition.

Definition 4.4 An ideal topological space X is called $\check{\theta}$ - \mathcal{J} -submaximal if every \mathcal{J} -dense subset in it is $\check{\theta}$ - \mathcal{J} -open in X .

Proposition 4.5 Every submximal space is $\check{\theta}$ - \mathcal{J} -submaximal.

Proof Let X be a submximal space and K be a \mathcal{J} -dense subset of X . Then K is open. But every open set is $\check{\theta}$ - \mathcal{J} -open and so K is $\check{\theta}$ - \mathcal{J} -open. Therefore, X is $\check{\theta}$ - \mathcal{J} -submaximal. The converse of Proposition 4.5 need not be true as can be seen from the following example.

Example 4.6 For the ideal topological space X of Example 3.5, every \mathcal{J} -dense subset is $\check{\theta}$ - \mathcal{J} -open and hence X is $\check{\theta}$ - \mathcal{J} -submaximal. However, the set $K = \{p, q\}$ is \mathcal{J} -dense in X , but it is not open in X . Therefore, X is not submaximal.

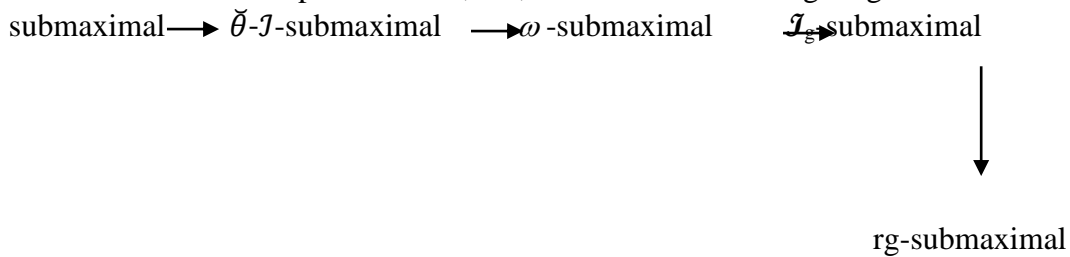
Proposition 4.7 Every $\check{\theta}$ - \mathcal{J} -submaximal space is ω -submaximal.

Proof Let X be an $\check{\theta}$ - \mathcal{J} -submaximal space and K be a \mathcal{J} -dense subset of X . Then K is $\check{\theta}$ - \mathcal{J} -open. But every $\check{\theta}$ - \mathcal{J} -open set is ω -open and so K is ω -open. Therefore, X is ω -submaximal.

The converse of Proposition 4.7 need not be true as can be seen from the following example.

Example 4.8 Consider the ideal topological space X in Example 3.8. Then X is ω -submaximal but it is not $\check{\theta}$ - \mathcal{J} -submaximal, because the set $K = \{q, r\}$ is a \mathcal{J} -dense set in X but it is not $\check{\theta}$ - \mathcal{J} -open in X .

Remark 4.9 From Propositions 4.5, 4.7, we have the following diagram:



Theorem 4.10 A space X is $\check{\theta}$ - \mathcal{J} -submaximal if and only if $P(X) = \check{\theta}\text{-}\mathcal{J}\text{-LC}^*(X)$.

Proof

Necessity. Let $K \in P(X)$ and let $V = K \cup (K^*)^c$. This imply that $V^* = K^* \cup (K^*)^c = X$. Hence $V^* = X$. Therefore, V is a dense subset of X . Since X is $\check{\theta}$ - \mathcal{J} -submaximal, V is $\check{\theta}$ - \mathcal{J} -open. Thus $K \cup (K^*)^c$ is $\check{\theta}$ - \mathcal{J} -open and by Theorem 3.39, we have $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}^*(X)$.

Sufficiency. Let K be a \mathcal{J} -dense subset of X . This implies $K \cup (K^*)^c = K \cup X^c = K \cup \emptyset = K$. Now $K \in \check{\theta}\text{-}\mathcal{J}\text{-LC}^*(X)$ implies that $K = K \cup (K^*)^c$ is $\check{\theta}$ - \mathcal{J} -open by Theorem 3.39. Hence X is $\check{\theta}$ - \mathcal{J} -submaximal.

Remark 4.11 Union of two $\check{\theta}$ - \mathcal{J} -lc sets (resp. $\check{\theta}$ - \mathcal{J} -lc* sets, $\check{\theta}$ - \mathcal{J} -lc** sets) need not be an $\check{\theta}$ - \mathcal{J} -lc set (resp. $\check{\theta}$ - \mathcal{J} -lc* set, $\check{\theta}$ - \mathcal{J} -lc** set) as can be seen from the following examples.

Example 4.12 Let $X = \{p, q, r\}$ with $\tau = \{\emptyset, \{p\}, \{p, q\}, X\}$. Then $\check{\theta}$ - \mathcal{J} -LC(X) = $\{\emptyset, \{p\}, \{q\}, \{r\}, \{p, q\}, \{q, r\}, X\}$. Then the sets $\{p\}$ and $\{r\}$ are $\check{\theta}$ - \mathcal{J} -lc sets, but their union $\{p, r\} \notin \check{\theta}$ - \mathcal{J} -LC(X).

Example 4.13 Let $X = \{p, q, r\}$ and $\tau = \{\emptyset, \{q\}, \{p, q\}, X\}$ with $\mathcal{J} = \{\emptyset\}$. Then $\check{\theta}$ - \mathcal{J} -LC*(X) = $\{\emptyset, \{p\}, \{q\}, \{r\}, \{p, q\}, \{p, r\}, X\}$. Then the sets $\{q\}$ and $\{r\}$ are $\check{\theta}$ - \mathcal{J} -lc* sets, but their union $\{q, r\} \notin \check{\theta}$ - \mathcal{J} -LC*(X).

Example 4.14 Let $X = \{p, q, r\}$ and $\tau = \{\emptyset, \{q\}, \{q, r\}, X\}$ with $\mathcal{J} = \{\emptyset\}$. Then $\check{\theta}$ - \mathcal{J} -LC**(X) = $\{\emptyset, \{p\}, \{q\}, \{r\}, \{p, r\}, \{q, r\}, X\}$. Then the sets $\{p\}$ and $\{q\}$ are $\check{\theta}$ - \mathcal{J} -lc** sets, but their union $\{p, q\} \notin \check{\theta}$ - \mathcal{J} -LC**(X).

We introduce the following definition.

Definition 4.15 Let K and B be subsets of X . Then K and B are said to be $\check{\theta}$ - \mathcal{J} -separated if $K \cap \check{\theta}$ - \mathcal{J} -cl(B) = \emptyset and $\check{\theta}$ - \mathcal{J} -cl(K) \cap B = \emptyset .

Example 4.16 For the ideal topological space X of Example 3.5. Let $K = \{q\}$ and let $B = \{r\}$. Then $\check{\theta}$ - \mathcal{J} -cl(K) = $\{p, q\}$ and $\check{\theta}$ - \mathcal{J} -cl(B) = $\{p, r\}$ and so the sets K and B are $\check{\theta}$ - \mathcal{J} -separated.

Proposition 4.17 Assume that $\check{\theta}$ - \mathcal{J} -O(X) forms an ideal topology. For the ideal topological space X , the followings are true

- i. Let $K, B \in \check{\theta}$ - \mathcal{J} -LC(X). If K and B are $\check{\theta}$ - \mathcal{J} -separated then $K \cup B \in \check{\theta}$ - \mathcal{J} -LC(X).
- ii. Let $K, B \in \check{\theta}$ - \mathcal{J} -LC*(X). If A and B are separated (i.e., $K \cap B^* = \emptyset$ and $K^* \cap B = \emptyset$), then $K \cup B \in \check{\theta}$ - \mathcal{J} -LC*(X).
- iii. Let $K, B \in \check{\theta}$ - \mathcal{J} -LC**(X). If K and B are $\check{\theta}$ - \mathcal{J} -separated then $K \cup B \in \check{\theta}$ - \mathcal{J} -LC**(X).

Proof

(i) Since $K, B \in \check{\theta}$ - \mathcal{J} -LC(X), by Theorem 3.38, there exist $\check{\theta}$ - \mathcal{J} -open sets U and V of X such that $K = U \cap \check{\theta}$ - \mathcal{J} -cl(A) and $B = V \cap \check{\theta}$ - \mathcal{J} -cl(B). Now $G = U \cap (X - \check{\theta}$ - \mathcal{J} -cl(B)) and $H = V \cap (X - \check{\theta}$ - \mathcal{J} -cl(K)) are $\check{\theta}$ - \mathcal{J} -open subsets of X . Since $K \cap \check{\theta}$ - \mathcal{J} -cl(B) = \emptyset , $K \subseteq (\check{\theta}$ - \mathcal{J} -cl(B))^c. Now $K = U \cap \check{\theta}$ - \mathcal{J} -cl(K) becomes $K \cap (\check{\theta}$ - \mathcal{J} -cl(B))^c = $G \cap \check{\theta}$ - \mathcal{J} -cl(K). Then $K = G \cap \check{\theta}$ - \mathcal{J} -cl(K). Similarly $B = H \cap \check{\theta}$ - \mathcal{J} -cl(B). Moreover $G \cap \check{\theta}$ - \mathcal{J} -cl(B) = \emptyset and $H \cap \check{\theta}$ - \mathcal{J} -cl(K) = \emptyset . Since G and H are $\check{\theta}$ - \mathcal{J} -open sets of X , $G \cup H$ is $\check{\theta}$ - \mathcal{J} -open. Therefore $K \cup B = (G \cup H) \cap \check{\theta}$ - \mathcal{J} -cl(K \cup B) and hence $A \cup B \in \check{\theta}$ - \mathcal{J} -LC(X).

(ii) and (iii) are similar to (i), using Theorems 3.39 and 3.40.

Remark 4.18 The assumption that K and B are $\check{\theta}$ - \mathcal{J} -separated in (i) of Proposition 4.17 cannot be removed. In the ideal topological space X in Example 4.12, the sets $\{p\}$ and $\{r\}$ are not $\check{\theta}$ - \mathcal{J} -separated and their union $\{p, r\} \notin \check{\theta}$ - \mathcal{J} -LC(X).

Lemma 4.19 For an $x \in X$, $x \in \check{\theta}$ - \mathcal{J} -cl(K) if and only if $V \cap K \neq \emptyset$ for every $\check{\theta}$ - \mathcal{J} -open set V containing x .

Proof

Let $x \in \check{\theta}$ - \mathcal{J} -cl(K) for any $x \in X$. To prove $V \cap K \neq \emptyset$ for every $\check{\theta}$ - \mathcal{J} -open set V containing x . Prove the result by contradiction. Suppose there exists a $\check{\theta}$ - \mathcal{J} -open set V containing x such that $V \cap K = \emptyset$. Then $K \subseteq V^c$ and V^c is $\check{\theta}$ - \mathcal{J} -cld. We have $\check{\theta}$ - \mathcal{J} -cl(K) $\subseteq V^c$. This shows that $x \notin \check{\theta}$ - \mathcal{J} -cl(K) which is a contradiction. Hence $V \cap K \neq \emptyset$ for every $\check{\theta}$ - \mathcal{J} -open set V containing x .

Conversely, let $V \cap K \neq \emptyset$ for every $\check{\theta}$ - \mathcal{J} -open set V containing x . To prove $x \in \check{\theta}$ - \mathcal{J} -cl(K). We prove the result by contradiction. Suppose $x \notin \check{\theta}$ - \mathcal{J} -cl(K). Then there exists a $\check{\theta}$ - \mathcal{J} -cld set F containing K such that $x \notin F$. Then $x \in F^c$ and F^c is $\check{\theta}$ - \mathcal{J} -open. Also, $F^c \cap K = \emptyset$, which is a contradiction to the hypothesis. Hence $x \in \check{\theta}$ - \mathcal{J} -cl(K).

Theorem 4.20 Suppose that A is $\check{\theta}$ - \mathcal{J} -open in X and that B is $\check{\theta}$ - \mathcal{J} -open in Y . Then $A \times B$ is $\check{\theta}$ - \mathcal{J} -open in $X \times Y$.

Proof

Suppose that F is \star -cld and hence sg-cld in $X \times Y$ and that $F \subseteq A \times B$. It suffices to show that $F \subseteq \text{int}(A \times B)$.

Let $(x, y) \in F$. Then, for each $(x, y) \in F$, $(\{x\})^* \times (\{y\})^* = (\{x\} \times \{y\})^* = (\{x, y\})^* \subseteq F^* = F \subseteq A \times B$. Two \star -cld sets $(\{x\})^*$ and $(\{y\})^*$ are contained in A and B respectively. It follows from the assumption that $(\{x\})^* \subseteq \text{int}(A)$ and that $(\{y\})^* \subseteq \text{int}(B)$. Thus $(x, y) \in (\{x\})^* \times (\{y\})^* \subseteq \text{int}(A) \times \text{int}(B) \subseteq \text{int}(A \times B)$. It means that, for each $(x, y) \in F$, $(x, y) \in \text{int}(A \times B)$ and hence $F \subseteq \text{int}(A \times B)$. Therefore, $A \times B$ is $\check{\theta}$ - \mathcal{J} -open in $X \times Y$.

Theorem 4.21 The following are equivalent for a function $f : (X, \tau, I) \rightarrow (Y, \sigma)$

- i. f is $\check{\theta}$ - \mathcal{J} -continuous.
- ii. The inverse image of a regular closed set of Y is $\check{\theta}$ - \mathcal{J} -open in X .
- iii. $f^{-1}(\text{int}(V^*))$ is $\check{\theta}$ - \mathcal{J} -closed in X for every open subset V of Y .
- iv. $f^{-1}((\text{int}(F))^*)$ is $\check{\theta}$ - \mathcal{J} -open in X for every closed subset F of Y .
- v. $f^{-1}(U^*)$ is $\check{\theta}$ - \mathcal{J} -open in X for every $U \in \beta O(Y)$.
- vi. $f^{-1}(U^*)$ is $\check{\theta}$ - \mathcal{J} -open in X for every $U \in SO(Y)$.
- vii. $f^{-1}(\text{int}(U^*))$ is $\check{\theta}$ - \mathcal{J} -closed in X for every $U \in PO(Y)$.

Proof

(i) \Leftrightarrow (ii). Obvious.

(i) \Leftrightarrow (iii). Let V be an open subset of Y . Since $\text{int}(V^*)$ is regular open, $f^{-1}(\text{int}(V^*))$ is $\check{\theta}$ - \mathcal{J} -closed. The converse is similar.

(ii) \Leftrightarrow (iv). Similar to (i) \Leftrightarrow (iii).

(ii) \Rightarrow (v). Let U be any β -open set of Y . We have U^* is regular closed. Then by (ii) $f^{-1}(U^*)$ is $\check{\theta}$ - \mathcal{J} -open in X .

(v) \Rightarrow (vi). Obvious from the fact that $SO(Y) \subseteq \beta O(Y)$.

(vi) \Rightarrow (vii). Let $U \in PO(Y)$. Then $Y \setminus \text{int}(U^*)$ is regular closed and hence it is semi-open. Then, we have $X \setminus f^{-1}(\text{int}(U^*)) = f^{-1}(Y \setminus \text{int}(U^*)) = f^{-1}((Y \setminus \text{int}(U^*))^*)$ is $\check{\theta}$ - \mathcal{J} -open in X . Hence $f^{-1}(\text{int}(U^*))$ is $\check{\theta}$ - \mathcal{J} -closed in X .

(vii) \Rightarrow (i). Let U be any regular open set of Y . Then $U \in PO(Y)$ and hence $f^{-1}(U) = f^{-1}(\text{int}(U^*))$ is $\check{\theta}$ - \mathcal{J} -closed in X .

Proposition 4.22 A function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is $\check{\theta}$ - \mathcal{J} -continuous if and only if $f^{-1}(U)$ is $\check{\theta}$ - \mathcal{J} -open in X , for every open set U in Y .

Proof

Let $f : (X, \tau, I) \rightarrow (Y, \sigma)$ be $\check{\theta}$ - \mathcal{J} -continuous and U be an open set in Y . Then U^c is closed in Y and since f is $\check{\theta}$ - \mathcal{J} -continuous, $f^{-1}(U^c)$ is $\check{\theta}$ - \mathcal{J} -cld in X . But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is $\check{\theta}$ - \mathcal{J} -open in X .

Conversely, assume that $f^{-1}(U)$ is $\check{\theta}$ - \mathcal{J} -open in X , for each open set U in Y . Let F be a closed set in Y . Then F^c is open in Y and by assumption, $f^{-1}(F^c)$ is $\check{\theta}$ - \mathcal{J} -open in X . Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is $\check{\theta}$ - \mathcal{J} -cld in X and so f is $\check{\theta}$ - \mathcal{J} -continuous.

Theorem 4.23 If $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is $\check{\theta}$ - \mathcal{J} -continuous and pre-sg-closed and if A is an $\check{\theta}$ - \mathcal{J} -open (or $\check{\theta}$ - \mathcal{J} -cld) subset of Y , then $f^{-1}(A)$ is $\check{\theta}$ - \mathcal{J} -open (or $\check{\theta}$ - \mathcal{J} -cld) in X .

Proof

Let H be an $\check{\theta}$ - \mathcal{J} -open set in Y and F be any sg-closed set in X such that $F \subseteq f^{-1}(H)$. Then $f(F) \subseteq H$. By hypothesis, $f(F)$ is sg-closed and H is $\check{\theta}$ - \mathcal{J} -open in Y . Therefore, $f(F) \subseteq \text{int}(H)$ and so $F \subseteq f^{-1}(\text{int}(H))$. Since f is $\check{\theta}$ - \mathcal{J} -continuous and $\text{int}(H)$ is open in Y , $f^{-1}(\text{int}(H))$ is $\check{\theta}$ - \mathcal{J} -open in X . Thus $F \subseteq \text{int}(f^{-1}(\text{int}(H))) \subseteq \text{int}(f^{-1}(H))$. i.e., $F \subseteq \text{int}(f^{-1}(H))$ and $f^{-1}(H)$ is $\check{\theta}$ - \mathcal{J} -open in X . By taking complements, we can show that if H is $\check{\theta}$ - \mathcal{J} -cld in Y , $f^{-1}(H)$ is $\check{\theta}$ - \mathcal{J} -cld in X .

5. Conclusion

The notions of sets and functions in ideal topological spaces and fuzzy topological spaces are extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences.

By researching generalizations of closed sets, some new separation axioms have been founded and they turn out to be useful in the study of digital topology. Therefore, all topological functions defined in this thesis will have many possibilities of applications in digital topology and computer graphics.

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