Ratio Mathematica Volume 45, 2023

On δ-open sets in ideal nano topological spaces

N. Sekar *
R. Asokan †
I. Rajasekaran ‡

Abstract

Aim of this paper, the new notions of introduce δ-open sets in ideal nano topological spaces and investigate some of their properties. A comparison between these types of ideal nano continuity will be discussed. Finally, we introduce application examples in ideal nano topological spaces.

Keywords: nano-open (resp. n-open), semi-n-open (resp. ns-open) semi-nI-open, pre-nI-open and strong β-nI-open and nI_δ-open.
2020 AMS subject classifications: 54B05, 54C60, 54E55.
[1]

*Research Scholar, Department of Mathematics, School of Mathematics, Madurai Kamaraj University, Madurai, Tamil Nadu, India; sekar.skrss@gmail.com.
†Department of Mathematics, School of Mathematics, Madurai Kamaraj University, Madurai, Tamil Nadu, India; rasoka_mku@yahoo.co.in.
‡Department of Mathematics, Tirunelveli Dakshina Mara Nadar Sangam College, T. Kallikulam-627 113, Tirunelveli District, Tamil Nadu, India; sekarmelakkal@gmail.com.
[1]Received on September 15, 2022. Accepted on December 15, 2022. Published on January 30, 2023. DOI: 10.23755/rm.v45i0.1013. ISSN: 1592-7415. eISSN: 2282-8214.

1 Introduction

An ideal I (16) on a space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.
1. $A \in I$ and $B \subset A$ imply $B \in I$ and
2. $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a space (X, τ) with an ideal I on X if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ (2). The closure operator defined by $cl^\star(A) = A \cup A^\star(I, \tau)$ (15) is a Kuratowski closure operator which generates a topology $\tau^\star(I, \tau)$ called the $\star$-topology which is finer then τ. We will simply write $A^\star$ for $A^\star(I, \tau)$ and $\tau^\star$ for $\tau^\star(I, \tau)$. If I is an ideal on X, then (X, τ, I) is called an ideal topological space or an ideal space.

Rajasekaran and Nethaji, introduced pre-nI-open sets and α-nI-open sets in the concept of ideal nano topological spaces.

In this paper, we introduce the notions of δ-open sets in ideal nano topological spaces and investigate some of their properties. A comparison between these types of ideal nano continuity will be discussed. Finally, we introduce application examples in ideal nano topological spaces.

2 Preliminaries

Definition 2.1. *(9) Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let* $X \subseteq U$.

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by* $L_R(X)$. *That is,* $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, *where R(x) denotes the equivalence class determined by x.*
2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by* $U_R(X)$. *That is,* $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.
3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by* $B_R(X)$. *That is,* $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. *(3) Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then R(X) satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$,*
2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*
3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset O of U are denoted by $I_n(O)$ and $C_n(O)$, respectively.

A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called (6) an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \,|x \in G_n, G_n \in \mathcal{N}\}$, denotes (6) the family of nano open sets containing x.

In future an ideal nano topological spaces $(U, \mathcal{N}, I)$ is referred as a space.

Definition 2.3. *(6) Let $(U, \mathcal{N}, I)$ be a space with an ideal I on U. Let $(.)^\star_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U).*

For a subset $O \subseteq U$, $O^\star_n(I, \mathcal{N}) = \{x \in U : G_n \cap O \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $O^\star_n$ for $O^\star_n(I, \mathcal{N})$.

Theorem 2.1. *(6) Let $(U, \mathcal{N}, I)$ be a space and O and B be subsets of U. Then*

1. *$O \subseteq B \Rightarrow O^\star_n \subseteq B^\star_n$,*
2. *$O^\star_n = C_n(O^\star_n) \subseteq C_n(O)$ ($O^\star_n$ is a n-closed subset of $C_n(O)$),*
3. *$(O^\star_n)^\star_n \subseteq O^\star_n$,*
4. *$(O \cup B)^\star_n = O^\star_n \cup B^\star_n$,*
5. *$V \in \mathcal{N} \Rightarrow V \cap O^\star_n = V \cap (V \cap O)^\star_n \subseteq (V \cap O)^\star_n$,*
6. *$J \in I \Rightarrow (O \cup J)^\star_n = O^\star_n = (O - J)^\star_n$.*

Theorem 2.2. *(6) Let $(U, \mathcal{N}, I)$ be a space with an ideal I and $O \subseteq O^\star_n$, then $O^\star_n = C_n(O^\star_n) = C_n(O)$.*

Definition 2.4. *(8) A subset A of a space $(U, \mathcal{N}, I)$ is $n\star$-dense in itself (resp. $n\star$-perfect and $n\star$-closed) if $O \subseteq O^\star_n$ (resp. $O = O^\star_n$, $O^\star_n \subseteq O$).*

The complement of a $n\star$-closed set is said to be $n\star$-open.

Definition 2.5. *(5) A subset O of U in a nano topological space $(U, \mathcal{N})$ is called nano-codense (briefly n-codense) if $U - O$ is n-dense.*

Theorem 2.3. *(6) Let $(U, \mathcal{N}, I)$ be an ideal nano space. Then is $\mathcal{I}$ is n-codense $\Longleftrightarrow$ $O \subseteq O^{\star}$ for every n-open set O.*

Definition 2.6. *(6) Let $(U, \mathcal{N}, I)$ be a space. The set operator $C_n^{\star}$ called a nano $\star$-closure is defined by $C_n^{\star}(O) = O \cup O_n^{\star}$ for $O \subseteq U$.*

It can be easily observed that $C_n^{\star}(O) \subseteq C_n(O)$.

Theorem 2.4. *(7) In a space $(U, \mathcal{N}, I)$, if O and B are subsets of U, then the following results are true for the set operator n-$cl^{\star}$.*

1. *$O \subseteq C_n^{\star}(O)$,*
2. *$C_n^{\star}(\phi) = \phi$ and $C_n^{\star}(U) = U$,*
3. *$IfO \subset B$, then $C_n^{\star}(O) \subseteq C_n^{\star}(B)$,*
4. *$C_n^{\star}(O) \cup C_n^{\star}(B) = C_n^{\star}(O \cup B)$.*
5. *$C_n^{\star}(C_n^{\star}(O)) = C_n^{\star}(O)$.*

Definition 2.7. *A subset O of a nano space $(U, \mathcal{N})$, is called a*

1. *nano pre-open (resp. np-open) set (3) if $O \subseteq I_n(C_n(O))$.*
2. *nano semi-open (resp. ns-open) set (3) if $O \subseteq C_n(I_n(O))$.*
3. *nano ε-open (resp. $n\varepsilon$-open) set (13) if $I_n(C_n(O)) \subseteq C_n(I_n(O))$.*
4. *nano nowhere dense (resp. n-nowhere dense) (4)if $I_n(C_n(O)) = \phi$.*

Definition 2.8. *A subset O of an ideal nano space $(U, \mathcal{N}, I)$, is called a*

1. *nano pre-I-open (resp. pre-nI-open) (10) if $O \subseteq I_n(C_n^{\star}(O))$.*
2. *nano semi-I-open (resp. semi-nI-open) (10) if $O \subseteq C_n^{\star}(I_n(O))$.*
3. *nano α-I-open (resp. α-nI-open) (10) if $O \subseteq I_n(C_n^{\star}(I_n(O)))$.*
4. *strongly nano β-I-open (resp. $\mathcal{S}\beta$-nI-open) (11) if $O \subseteq C_n^{\star}((I_n(C_n^{\star}(O)))$.*

Theorem 2.5. *(10) In a nano space $(U, \mathcal{N}, I)$, if O is α-nI-open, then O is semi-nI-open.*

Definition 2.9. *(1) A function $f : (U, \mathcal{N}, I) \longrightarrow (V, \mathcal{N}')$ is said to be*

1. *α-nI-continuous if $f^{-1}(H)$ is α-nI-open set in $(U, \mathcal{N}, I)$ for every n-open set H in $(V, \mathcal{N}')$.*
2. *pre-nI-continuous if $f^{-1}(H)$ is pre-nI-open set in $(U, \mathcal{N}, I)$ for every n-open set H in $(V, \mathcal{N}')$.*
3. *semi-nI-continuous if $f^{-1}(H)$ is semi-nI-open set in $(U, \mathcal{N}, I)$ for every n-open set H in $(V, \mathcal{N}')$.*

3 On δ-open sets in ideal nano space

Definition 3.1. *A subset O of an ideal nano space $(U, \mathcal{N}, I)$, is called a nano I_δ-open (resp. nI_δ-open) set if $I_n(C_n^\star(O)) \subseteq C_n^\star(I_n(O))$.*

Example 3.1. *Let $U = \{A_1, A_2, A_3, A_4\}$ with $U/R = \{\{A_2\}, \{A_4\}, \{A_1, A_3\}\}$ and $X = \{A_3, A_4\}$.*
Then the nano topology $\mathcal{N} = \{\phi, \{A_4\}, \{A_1, A_3\}, \{A_1, A_3, A_4\}, U\}$ and $I = \{\phi, \{A_3\}\}$. Clear that $\{\phi, \{A_2\}, \{A_3\}, \{A_4\}, \{A_1, A_3\}, \{A_2, A_3\}, \{A_2, A_4\}, \{A_1, A_2, A_3\}, \{A_1, A_3, A_4\}, U\}$ is nI_δ-open.

Proposition 3.1. *Let $(U, \mathcal{N}, I)$ be an ideal nano space. Then a subset of U is semi-nI-open $\iff$ if it is both nI_δ-open and $\mathcal{S}\beta$-nI-open.*

Proof.
Necessity. Let O be a semi-nI-open, then we have $O \subseteq C_n^\star(I_n(O)) \subseteq C_n^\star(I_n(C_n^\star(O)))$. This show that O is $\mathcal{S}\beta$-nI-open.

Moreover, $I_n(C_n^\star(O)) \subseteq C_n^\star(O) \subseteq C_n^\star(C_n^\star(I_n(O))) = C_n^\star(I_n(O))$. Therefore O is nI_δ-open.

Sufficiency. Let O be nI_δ-open and $\mathcal{S}\beta$-nI-open, then we have $I_n(C_n^\star(O)) \subseteq C_n^\star(I_n(O))$. Thus we obtain that $C_n^\star(I_n(C_n^\star(O))) \subseteq C_n^\star(C_n^\star(I_n(O))) = C_n^\star(I_n(O))$. Since O is $\mathcal{S}\beta$-nI-open, we have $O \subseteq C_n^\star(I_n(C_n^\star(O))) \subseteq C_n^\star(I_n(O))$ and $O \subseteq C_n^\star(I_n(O))$. Hence O is semi-nI-open. □

Proposition 3.2. *Let $(U, \mathcal{N}, I)$ be an ideal nano space. Then a subset of U is α-nI-open $\iff$ if it is both nI_δ-open and pre-nI-open.*

Proof.
Necessity. Let O be a α-nI-open, since every α-nI-open set is semi-nI-open (10), by Proposition 3.1 O is a nI_δ-open set. Now we prove that $O \subseteq I_n(C_n^\star(O))$. Since O is α-nI-open set, we have $O \subseteq I_n(C_n^\star(I_n(O))) \subseteq I_n(C_n^\star(O))$. Hence O is a pre-nI-open set.

Sufficiency. Let O be a nI_δ-open and pre-nI-open. Then we have $I_n(C_n^\star(O)) \subseteq C_n^\star(I_n(O))$ and hence $I_n(C_n^\star(O)) \subseteq I_n(C_n^\star(I_n(O)))$. Since O is pre-nI-open, we have $O \subseteq I_n(C_n^\star(O))$. Therefore we obtain that $O \subseteq I_n(C_n^\star(I_n(O)))$ and hence O ia α-nI-open. □

Remark 3.1. *In $(U, \mathcal{N}, I)$ ideal nano space.*

1. *$\mathcal{S}\beta$-nI-open and nI_δ-open are independent.*
2. *pre-nI-open and nI_δ-open are independent.*

Example 3.2. *In Example 3.1,*

1. *the set $\{A_1\}$ is $\mathcal{S}\beta$-nI-open set but not nI_δ-open.*
2. *the set $\{A_2\}$ is not $\mathcal{S}\beta$-nI-open but nI_δ-open.*
3. *the set $\{A_1, A_4\}$ is pre-nI-open but not nI_δ-open.*
4. *the set $\{A_2, A_3\}$ is not pre-nI-open but nI_δ-open.*

Proposition 3.3. *Let O, P be subsets of an ideal nano space $(U, \mathcal{N}, I)$. If $O \subseteq P \subseteq C_n^\star(O)$ and O is nI_δ-open, then P is nI_δ-open.*

Proof.

Suppose that $O \subseteq P \subseteq C_n^\star(O)$ and O is nI_δ-open. Then, since O is nI_δ-open, we have $I_n(C_n^\star(O)) \subseteq C_n^\star(I_n(O))$. Since, $O \subseteq P$, $C_n^\star(I_n(O)) \subseteq C_n^\star(I(P))$ and $I_n(C_n^\star(O)) \subseteq C_n^\star(I_n(P))$. Since $P \subseteq C_n^\star(O)$, we have $C_n^\star(P) \subseteq C_n^\star(C_n^\star(O)) = C_n^\star(O)$ and $I_n(C_n^\star(P)) \subseteq I_n(C_n^\star(O))$. Therefore, we obtain that $I_n(C_n^\star(P)) \subseteq C_n^\star(I_n(P))$. This show that P is a nI_δ-open. □

Proposition 3.4. *let O, P and Q be subsets of an ideal nano space $(U, \mathcal{N}, I)$. If O is nI_δ-open, then $O = P \cup Q$, where P is α-nI-open, $I_n(C_n^\star(Q)) = \phi$ and $P \cap Q = \phi$.*

Proof.

Suppose that O is nI_δ-open. Then we have $I_n(C_n^\star(O)) \subseteq C_n^\star(I_n(O))$ and $I_n(C_n^\star(O)) \subseteq I_n(C_n^\star(I_n O)))$.

Now we have $O = \big(I_n(C_n^\star(O)) \cap O\big) \cup \big(O - I_n(C_n^\star(O))\big)$. Now, we set $P = I_n(C_n^\star(O)) \cap O$ and $Q = O - I_n(C_n^\star(O))$.

We first show that P is α-nI-open, that is, $P \subseteq I_n(C_n^\star(I_n(P))$. Now we have $I_n(C_n^\star(I_n(P)) = I_n(C_n^\star(I_n(I_n(C_n^\star(O)) \cap O))) = I_n(C_n^\star(I_n(C_n^\star(O)) \cap I_n(O))) = I_n(C_n^\star(I_n(O)))$. Since O is nI_δ-open, $I_n(C_n^\star(I_n(O))) \supseteq I_n(C_n^\star(O)) \supseteq P$ and thus P is α-nI-open.

Next we show that $I_n(C_n^\star(Q)) = \phi$. Since $\mathcal{N} \subseteq \mathcal{N}^\star$, $C_n^\star(K) \subseteq C_n(K)$ for any subset K of U. Therefore, we have $I_n(C_n^\star(Q)) = I_n(C_n^\star(O \cap (U - I_n(C_n^\star(O))))) \subseteq I_n(C_n^\star(O)) \cap I_n(C_n^\star(U - I_n(C_n^\star(O)))) \subseteq I_n(C_n^\star(O)) \cap I_n(C_n(U - I_n(C_n^\star(O)))) \subseteq I_n(C_n^\star(O)) \cap (U - I_n(C_n^\star(O))) = \phi$.

It is obvious that $P \cap Q = (I_n(C_n^\star(O)) \cap O) \cap (O - I_n(C_n^\star(O))) = \phi$.

□

Remark 3.2. *In an ideal nano space $(U, \mathcal{N}, I)$, $n\varepsilon$-open and nI_δ-open sets are independent.*

Example 3.3. *In Example 3.1,*

1. *the set $\{A_1, A_2\}$ is $n\varepsilon$-open but not nI_δ-open.*
2. *the set $\{A_3\}$ is not $n\varepsilon$-open but nI_δ-open.*

Proposition 3.5. *Let $(U, \mathcal{N}, I)$ be an ideal nano space and $O \subseteq U$. If O is both nI_δ-open and ns-closed, then O is a $n\varepsilon$-open.*

Proof.

Since O is ns-closed, $I_n(C_n(O)) \subseteq O$ and hence $I_n(C_n(O)) = I_n(O)$. Thus, $I_n(C_n(O)) \subseteq I_n(O) \subseteq I_n(C_n^\star(O)) \subseteq C_n^\star(I_n(O)) = I_n(O) \cup (I_n(O))_n^\star$. $(I_n(O))_n^\star \subseteq C_n(I_n(O))$ and hence we obtain $I_n(C_n(O)) \subseteq C_n(I_n(O))$. This show that O is $n\varepsilon$-open. $\square$

Proposition 3.6. *Let $(U, \mathcal{N}, I)$ be an ideal nano space. Let $I = \{\phi\}$ or $I = H$, where H is the ideal of n-nowhere dense. Then a subset O of U is nI_δ-open $\Longleftrightarrow$ O is a $n\varepsilon$-open.*

Proof.

1. Let $I = \{\phi\}$. Then for every subset O of U, $O_n^\star = C_n(O)$ and $C_n^\star(O) = O \cup O_n^\star = O \cup C_n(O) = C_n(O)$. Therefore, the statement holds obviously.
2. Let $I = H$, we have $O_n^\star = C_n(I_n(C_n(O)))$.
   First, let O be a $n\varepsilon$-open. Then $I_n(C_n(O)) \subseteq C_n(I_n(O))$ and $I_n(C_n^\star(O)) = I_n(O_n^\star \cup O) \subseteq I_n(C_n(O) \cup O) = I_n(C_n(O)) \subseteq C_n(I_n(O)) = C_n(I_n(C_n(I_n(O)))) = (I_n(O))_n^\star \subseteq C_n^\star(I_n(O))$.
   Therefore, O is nI_δ-open set. Next, let O be a nI_δ-open set. Then $I_n(C_n^\star(O)) \subseteq C_n^\star(I_n(O))$. We have $I_n(C_n(O)) = I_n(C_n(I_n(C_n(O)))) = I_n(O_n^\star) \subseteq I_n(O_n^\star \cup O) = I_n(C_n^\star(O)) \subseteq C_n^\star(I_n(O)) = (I_n(O))_n^\star \cup I_n(O) = C_n(I_n(C_n(I_n(O)))) \cup I_n(O) = C_n(I_n(O))$. Therefore, $n\varepsilon$-open. $\square$

4 On semi-δ-nI-continuous

Definition 4.1. *A function $f : (U, \mathcal{N}, I) \longrightarrow (V, \mathcal{N}')$ is called a*

1. *semi-δ-nI-continuous if every $H \in \mathcal{N}'$, $f^{-1}(H) \in nI_\delta$-open.*
2. *strong β-nI-continuous (resp. $\mathcal{S}\beta$-nI-continuous) if every $H \in \mathcal{N}'$, $f^{-1}(H) \in \mathcal{S}\beta$-$nI$-open.*

Theorem 4.1. *For a function $f : (U, \mathcal{N}, I) \longrightarrow (V, \mathcal{N}')$, the following properties are equivalent.*

1. *f is semi-nI-continuous.*
2. *f is $\mathcal{S}\beta$-nI-continuous and semi-δ-nI-continuous.*

Proof.

The proof is obvious by Proposition 3.1. $\square$

Theorem 4.2. *For a function $f : (U, \mathcal{N}, I) \longrightarrow (V, \mathcal{N}')$, the following properties are equivalent.*

1. *f is α-nI-continuous.*
2. *f is pre-nI-continuous and semi-nI-continuous.*
3. *f is pre-nI-continuous and semi-δ-nI-continuous.*

Proof.
The proof is obvious by Proposition 3.1 and Proposition 3.2. □

5 Conclusion

Because of the spaces is stripped of the geometric form and its is used to measure thing that are difficult to measure, such as intelligence, beauty and goodness. In this paper, different type of ideal nano continuity and ideal nano closed sets are introduced and studied. Also, we introduce an applications example in ideal nano topology. Some applications on them are given in some real life branches such as medicine and physics.

References

[1] V. Inthumathi, R. Abinprakash and M. Parveen Banu, *Some weaker form of continuous and irresolute mapping in nano ideal topological spaces,* Journal of new results in science, 8(1)(2019), 14-25.

[2] K. Kuratowski, *Topology*, Vol I. Academic Press (New York) 1966.

[3] M. Lellis Thivagar and Carmel Richard, *On nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention,1(1)(2013), 31-37.

[4] M. Lellis Thivagar, Saeid Safari and V. Sutha Devi, *On new class of contra continuity in nano topology*, Available on researchgate in https://www.researchgate.net/ publication /315892547.

[5] O. Nethaji, R. Asokan and I. Rajasekaran, *New generalized classes of an ideal nano topological spaces*, Bull. Int. Math. Virtual Inst., 9(3)(2019), 543-552.

[6] M. Parimala, T. Noiri and S. Jafari, *New types of nano topological spaces via nano ideals* (to appear).

[7] M. Parimala and S. Jafari, *On some new notions in nano ideal topological spaces*, International Balkan Journal of Mathematics(IBJM), 1(3)(2018), 85-92.

[8] M. Parimala, S. Jafari and S. Murali, *Nano ideal generalized closed sets in nano ideal topological spaces*, Annales Univ. Sci. Budapest., 60(2017), 3-11.

[9] Z. Pawlak, *Rough sets*, International journal of computer and Information Sciences, 11(5)(1982), 341-356.

[10] I. Rajasekaran and O. Nethaji, *Simple forms of nano open sets in an ideal nano topological spaces*, Journal of New Theory, 24(2018), 35-43.

[11] I. Rajasekaran, *Weak forms of strongly nano open sets in ideal nano topological spaces*, Asia Mathematika, 5(2)(2021), 96-102.

[12] I. Rajasekaran and O. Nethaji, *Unified approach of several sets in ideal nanotopological spaces*, Asia Mathematika, 3(1)(2019), 70-78.

[13] I. Rajasekaran, M. Meharin and O. Nethaji, *On new classes of some nano open sets*, International Journal of Pure and Applied Mathematical Sciences, 10(2)(2017, 147-155.

[14] A. Revathy and G. Ilango, On nano β-open sets, *Int. Jr. of Engineering, Contemporary Mathematics and Sciences*, 1(2)2015, 1-6.

[15] R. Vaidyanathaswamy, *The localization theory in set topology*, Proc. Indian Acad. Sci., 20(1945), 51-61.

[16] R. Vaidyanathaswamy, *Set topology*, Chelsea Publishing Company, New York, 1946.