Tri- $b\widehat{g}$ Closed sets in Tri- Topological Spaces

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Abstract

In this paper, we introduce a new class of sets called tri- $b\widehat{g}$ closed sets and tri- $b\widehat{g}$ open sets via the concept of tri- $\widehat{g}$ closed sets in tri topological spaces. Also, we investigate the relationship with other existing closed sets in tri-topological space.

Keywords: Tri- $b\widehat{g}$ closed sets, tri- $b\widehat{g}$ open sets, tri- $b\widehat{g}$ closure, tri- $b\widehat{g}$ interior.

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1. Introduction

The concept of tri- topological space was first initiated by M. Kovar [6] in 2000, in 2003, R. Subasree and M. Maria Singam [10] defined $b\hat{g}$- closed sets in topological spaces. In [3], we introduced tri- $\hat{g}$ closed sets in tri- topological spaces and studied their properties. In this paper, we define tri- $b\hat{g}$ closed sets and tri- $b\hat{g}$ open sets via the concept of tri- $\hat{g}$ closed sets. Also, we investigate the relationship with other existing closed sets in tri- topological space.

2. Preliminaries

Throughout this paper (X, τ_1, τ_2, τ_3) (or simply X) represents tri- topological spaces on which no separation axioms are assumed unless other wised mentioned. For a subset A of (X,τ_1,τ_2,τ_3), tri- cl(A), tri- int(A) and A^c denote the tri- closure of A, tri- interior of A and compliment of A respectively.

Definition 2.1 Let X be a non-empty set. A family τ of subsets of X is said to be a topology on X, if τ satisfies the following axioms.

a) ϕ, X ∈τ,

b) If $A_i \in \tau$ for i = 1,2,…..,n, then $\bigcap_{i=1}^{n} A_i \in \tau$,

c) If $A_\alpha \in \tau$ for α ∈ I, then $\bigcup_{\alpha} A_\alpha \in \tau$.

The pair (X, τ) is called a topological space and any set A in τ is called an open set. The complement of an open set A is called closed set.

Definition 2.2 Let X be a non-empty set. A family G of subsets of X is said to be a generalized topology on X, if G satisfies the followings axioms.

a) ϕ ∈ G,

b) If $A_\alpha \in G$ for α ∈I, then $\bigcup_{\alpha} A_\alpha \in G$.

The pair (X, G) is called a generalized topological space.

Definition 2.3 Let X be a non-empty set. A family τ* of subsets of X is said to be a Supra topology on X, if τ* satisfies the following axioms.

a) ϕ, X ∈ τ*,

b) If $A_\alpha \in \tau^*$ for α ∈ I, then $\bigcup_{\alpha} A_\alpha \in \tau^*$.

The pair (X, τ*) is called a Supra topological space.

Definition 2.4 Let X be a non-empty set. A family τ_{iX} of subsets of X is said to be a Infra topology on X, if τ_{iX} satisfies the following axioms.

a) ϕ, X ∈ τ_{iX},

b) If $A_i \in \tau_{iX}$ for i = 1, 2… n, then $\bigcap_{i=1}^{n} A_i \in \tau_{iX}$.

The pair (X, τ_{iX}) is called Infra topological space.

Definition 2.5 Let (X, τ) be a topological space then τ is said to be indiscrete topology if τ is a collection of only X and ϕ. Indiscrete topology is also known as trivial topology.

Definition 2.6 Let (X, τ) be a topological space then τ is said to be discrete topology if τ is a collection of all subsets of X.

Definition 2.7 Let (X, τ) be a topological space then a subset A of X is said to be $b\hat{g}$- closed set if bcl (A) ⊆ U whenever A ⊆ U, U is ĝ- open in X.

Definition 2.8 Let X be a nonempty set and τ_1, τ_2 and τ_3 are topologies on X. Then a subset A of X is said to be tri- open set if A ∈ $\tau_1 \cup \tau_2 \cup \tau_3$ and its complement is said to be tri- closed set and X with three topologies called tri- topological spaces (X, τ_1, τ_2, τ_3).

Definition 2.9 Let (X, τ_1, τ_2, τ_3) be a tri- topological space and let A ⊆ X. The union of all tri- open sets contained in A is called the tri- interior of A. The intersection of all tri- closed sets containing A is called the tri- closure of A.

Definition 2.10 Let (X, τ_1, τ_2, τ_3) be a tri- topological space. A ⊆ X is said to be

1) A tri- α open set if A ⊆ tri- int (tri- cl (tri- int (A))).
2) A tri- b open set if A ⊆ [tri- cl (tri- int (A))] ∪ [tri- int (tri- cl (A))].
3) A tri- semi closed set if tri- int (tri- cl (A)) ⊆ A.
4) A tri- g closed set if tri- cl (A) ⊆ U whenever A ⊆ U and U is tri- open set in X.
5) A tri- gs closed set if tri- scl (A) ⊆ U whenever A ⊆ U and U is tri- open set in X.
6) A tri- bτ closed set if tri- cl_b (A) ⊆ U whenever A ⊆ U and U is tri- open set in X.
7) A tri- g*bw closed set if tri- bcl (A) ⊆ U whenever A ⊆ U, U is tri- gs open in X.
8) A tri- ĝ closed set if tri- cl (A) ⊆ U whenever A ⊆U, U is tri- semi open in X.

The complement of tri- α open set, tri- b open set, tri- semi closed set, tri- g closed set, tri- gs closed set, tri- bτ closed set, tri- g*bw closed set and tri- ĝ closed setis called tri- α closed set, tri- b closed set, tri- semi open set, tri- g open set, tri- gs open set, tri- bτ open set, tri- g*bw open set and tri- ĝ open set respectively.

Theorems 2.11

1) Every tri- closed set is tri- semi closed.
2) Every tri- closed set is tri- b closed.
3) Every tri- closed set is tri- gs closed.
4) Every tri- closed set is tri- bτ closed.
5) Every tri- closed set is tri- g*bω closed.
6) Every tri- closed set is tri- ĝclosed set.
7) Every tri- semi closed set is tri- gs closed.

8) Every tri- semi closed set is tri- b closed.
9) Every tri- semi closed set is tri- g*bω closed.
10) Every tri- b closed set is tri- bτ closed.
11) Every tri- semi closed set is tri- bτ closed.
12) Every tri- α closed set is tri- b closed set.
13) Every tri- g*bω closed set is tri- bτ closed.
14) Every tri- $\hat{g}$ closed set is tri- g closed.
15) Every tri- $\hat{g}$ closed set is tri- gs closed.

3. Tri- $b\hat{g}$ Closed Sets in Tri- Topological Space

We introduce the following definitions

Definition 3.1 Let (X,τ_1,τ_2,τ_3) be a tri- topological space then a subset A of X is said to be tri- $b\hat{g}$ closed set if tri- bcl (A) $\subseteq$ U whenever A $\subseteq$ U, U is tri- $\hat{g}$ open in X. The family of all tri- $b\hat{g}$ closed sets of X is denoted by tri- $b\hat{g}$ C(X).

Example 3.2 Let X = {a, b, c} with the topologies τ_1 = {X, ϕ, {a, b}}, τ_2 = {X, ϕ, {b, c}}, τ_3 = {X, ϕ, {a, c}}, Open sets in tri- topological spaces are union of all three topologies. $\tau_1 \cup \tau_2 \cup \tau_3$ = {X, ϕ, {a, b}, {b, c}, {a, c}}; Tri- $\hat{g}$O(X) = {X, ϕ, {a, b}, {b, c}, {a, c}}; Hence tri- $b\hat{g}$ C(X) = {X, ϕ, {a}, {b}, {c}}.

Remark 3.3 ϕ and X are always tri- $b\hat{g}$ closed set.

Remark 3.4 Intersection of tri- $b\hat{g}$ closed sets need not be tri- $b\hat{g}$ closed set.

Example 3.5 Let X = {a, b, c}, τ_1 = {X, ϕ}, $\tau_2 = \tau_3$ = {X, ϕ, {a}}, tri- $b\hat{g}$ C(X) = {X, ϕ, {b}, {c}, {a, b}, {b, c}, {a, c}}. Here, {a, b}, {a, c} are tri- $b\hat{g}$ closed sets but {a, b} ∩ {a, c} = {a} is not a tri- $b\hat{g}$ closed set.

Remark 3.6 Union of tri- $b\hat{g}$ closed sets need not be tri- $b\hat{g}$ closed set.

Example 3.7 Let X = {a, b, c}, τ_1 = {X, ϕ, {a, c}}, τ_2 = {X, ϕ, {b}, {b, c}}, τ_3 = {X, ϕ, {c}, {a, b}}, tri- $b\hat{g}$ C(X) = {X, ϕ, {a}, {b}, {c}, {a, b}, {a, c}}. Here, {b}, {c} are tri- $b\hat{g}$ closed sets but {b} ∪ {c} = {b, c} ∉ tri- $b\hat{g}$ C(X).

Remark 3.8 Difference of two tri- $b\hat{g}$ closed sets need not be tri- $b\hat{g}$ closed set.

Example 3.9 In previous example – 3.7, tri- $b\hat{g}$ C(X) = {X, ϕ, {a}, {b}, {c}, {a, b}, {a, c}}. Let A = X and B = {a}, Also A and B are tri- $b\hat{g}$ closed sets. But A \ B = X \ {a} = {b, c} is not a tri- $b\hat{g}$ closed set.

Remark 3.10

1) (X, Tri- $b\hat{g}$ C(X)) need not be Topological space.
2) (X, Tri- $b\hat{g}$ C(X)) need not be Generalized topological space.
3) (X, Tri- $b\hat{g}$ C(X)) need not be Supra topological space.
4) (X, Tri- $b\hat{g}$ C(X)) need not be Infra topological space.

Example 3.11 In examples – 3.5, 3.7 we get the results.

Definition 3.12 Let (X, τ_1, τ_2, τ_3) be a tri- topological space. The intersection of all tri- $b\hat{g}$ closed sets of X containing a subset A of X is called tri- $b\hat{g}$ closure of A and is denoted by tri- $b\hat{g}$ cl(A). (i.e) tri- $b\hat{g}$ cl (A) = ∩ {B ⊆ X: B ⊇ A and B is tri- $b\hat{g}$ closed set}.

Remark 3.13

1) tri- $b\hat{g}$ cl(ϕ) = ϕ,
2) tri- $b\hat{g}$ cl(X) = X,
3) A ⊆ tri- $b\hat{g}$ cl(A),
4) tri- $b\hat{g}$ cl(A) = tri- $b\hat{g}$ cl(tri- $b\hat{g}$ cl(A)).

Proposition 3.14 Let (X,τ_1,τ_2,τ_3) be a tri- topological space. Let A ⊆ X, Then A = tri- $b\hat{g}$ cl (A) if A is tri- $b\hat{g}$ closed set.

Proof. Suppose A is a tri- $b\hat{g}$ closed set in X then, tri- bcl (A) ⊆ U whenever A⊆U, U is tri- ĝ open in X. Since, A⊇A and A is tri- $b\hat{g}$ closed set. Let B ⊆ X then A ∈{B ⊆ X : B ⊇ A and B is tri- $b\hat{g}$ closed} ⇒ A = ∩ {B ⊆ X : B ⊇ A and B is tri- $b\hat{g}$ closed}. Hence A= tri- $b\hat{g}$ cl(A).

Remark 3.15 The tri- $b\hat{g}$ closure of a set A is not always tri- $b\hat{g}$ closed set.

Example 3.16 Let X = {a, b, c}, τ_1 = {X, ϕ}, $\tau_2 = \tau_3$ = {X, ϕ,{a}}, tri- $b\hat{g}$ C(X) = {X, ϕ, {b},{c},{a, b},{b, c}, {a, c}}.Here, tri- $b\hat{g}$ cl({a}) = {a} is not a tri- $b\hat{g}$ closed set.

Proposition 3.17 Every tri- b closed set is tri- $b\hat{g}$ closed set.

Proof: Let A be any tri- b closed set in X and U be any tri- ĝ open set in X such that A⊆U. Since, A is tri- b closed then tri- bcl (A) =A for every subset A of X. tri- bcl(A) = A ⊆ U. Hence A is tri- $b\hat{g}$ closed set.

Converse of the above proposition need not be true as seen in the following example.

Example 3.18 Let X = {a, b, c}, τ_1 = {X, ϕ}, $\tau_2 = \tau_3$ = {X, ϕ,{a}}, tri- b C(X) = {X, ϕ,{b}, {c},{b, c}; tri- $b\hat{g}$ C(X) = {X, ϕ, {b}, {c}, {a, b}, {b, c}, {a, c}}; here {a, b}, {a, c} are tri- $b\hat{g}$ closed sets but not a tri- b closed set.

Proposition 3.19 Every tri- closed set is tri- $b\hat{g}$ closed set.
Proof: Let A be any tri- closed set in X. Since every tri- closed set is tri- b closed set. Therefore, A is tri- b closed set in X. By proposition 3.17, A is tri- $b\hat{g}$ closed set.
Converse of the above proposition need not be true as seen in the following example.

Example 3.20 Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {b}}, τ_3 = {X, ϕ, {a, c}}, tri- C(X) = {X, ϕ, {b}, {a, c}, {b, c}}; tri- $b\hat{g}$ C(X) = {X, ϕ, {b}, {c}, {b, c}, {a, c}}; here {c} is tri- $b\hat{g}$ closed set but not a tri- closed set.

Proposition 3.21 Every tri- semi closed set is tri- $b\hat{g}$ closed set.
Proof: Let A be any tri- semi closed set in X. Since every tri- semi closed set is tri- b closed set. Therefore, A is tri- b closed set in X. By proposition 3.17, A is tri- $b\hat{g}$ closed
Converse of the above proposition need not be true as seen in the following example.

Example 3.22 Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {a, b}}, τ_3 = {X, ϕ, {b, c}}, tri- sC(X) = {X, ϕ, {a}, {c}, {b, c}}; tri- $b\hat{g}$ C(X) = {X, ϕ, {a}, {b}, {c}, {b, c}, {a, c}}; here {b},{a, c} are tri- $b\hat{g}$ closed sets but not a tri-semi closed set.

Proposition 3.23 Every tri- α closed set is tri- $b\hat{g}$ closed set.
Proof: Let A be any tri- α closed set in X. Since every tri- α closed set is tri- b closed set. Therefore, A is tri- b closed set in X. By proposition 3.17, A is tri- $b\hat{g}$ closed set.
Converse of the above proposition need not be true as seen in the following example.

Example 3.24 Let X = {a, b, c}, $\tau_1 = \tau_2$ = {X, ϕ, {a}}, τ_3 = {X, ϕ, {b, c}}, tri- α C(X) = {X, ϕ, {a}, {b, c}}; tri- $b\hat{g}$ C(X) = {X, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}; here {b}, {c}, {a, b}, {b, c} are tri- $b\hat{g}$ closed sets but not a tri-α closed set.

Proposition 3.25 Every tri-g*bω closed set is tri- $b\hat{g}$ closed set.
Proof: Let A be any tri-g*bω closed set in X and A ⊆ U, where U is tri- $\hat{g}$ open set in X. Since, every tri- $\hat{g}$ open set is tri- gs open. Therefore, U is tri- gs open in X. Since, A is tri- g*bω closed set in X then tri- bcl(A) ⊆ U. Hence A is tri- $b\hat{g}$ closed set in X.
Converse of the above proposition need not be true as seen in the following example.

Example 3.26 Let X = {a, b, c}, τ_1 = {X, ϕ}, τ_2 = {X, ϕ, {a}}, τ_3 = {X, ϕ, {a, b}}, tri- g*bω C(X) = {X, ϕ, {b}, {c}, {b, c}}; tri- $b\hat{g}$ C(X) = {X, ϕ, {b}, {c}, {b, c}, {a, c}}; here {a, c} is tri- $b\hat{g}$ closed set but not a tri- g*bω closed set.

Proposition 3.27 Every tri- $b\hat{g}$ closed set is tri- bτ closed set.
Proof: Let A be any tri- $b\hat{g}$ closed set in X and A ⊆ U, where U is tri- open set in X. Since, every tri- open set is tri- $\hat{g}$ open. Therefore, U is tri- $\hat{g}$ open in X. Since, A is tri- $b\hat{g}$ closed set in X then tri- bcl(A) ⊆ U. Hence A is tri- bτ closed set in X.
Converse of the above proposition need not be true as seen in the following example.

Example 3.28 Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {b}}, τ_3 = {X, ϕ, {a, c}}, tri- $b\hat{g}$ C(X) = {X, ϕ, {b}, {c}, {b, c}, {a, c}}; tri- bτ C(X) = {X, ϕ, {b}, {c}, {a, b}, {b, c}, {a, c}}; here {a, b} is tri- bτ closed set but not a tri- $b\hat{g}$ closed set.

Remark 3.29 Tri- g closed sets and tri- $b\hat{g}$ closed sets are independent.

Example 3.30 Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {b}}, τ_3 = {X, ϕ, {a, b}}, tri- gC(X) = {X, ϕ, {c}, {b, c}, {a, c}}; tri- $b\hat{g}$ C(X) = {X, ϕ, {a}, {b}, {c}, {b, c}, {a, c}}; here {a} and {b} are tri- $b\hat{g}$ closed sets but not a tri-g closed sets.

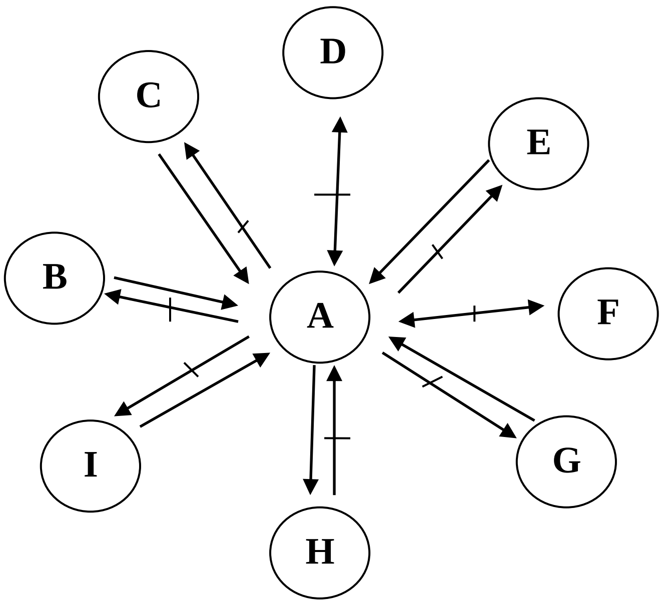
Example 3.31 Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {b}}, τ_3 = {X, ϕ, {a, c}}, tri- $b\hat{g}$ C(X) ={X, ϕ, {b}, {c}, {b, c}, {a, c}}; tri- gC(X) ={X, ϕ, {b}, {c}, {a, b}, {b, c}, {a, c}}; here {a, b} is tri- g closed set but not a tri- $b\hat{g}$ closed set.

Remark 3.32 Tri- gs closed sets and tri- $b\hat{g}$ closed sets are independent.

Example 3.33 Let X = {a, b, c}, τ_1 = {X, ϕ}, τ_2 = {X, ϕ, {a, b}}, τ_3 = {X, ϕ, {b, c}}, tri- gsC(X) = {X, ϕ, {a}, {c}, {a, c}}; tri- $b\hat{g}$ C(X) = {X, ϕ, {a}, {b}, {c}, {a, c}}; here {b} is tri- $b\hat{g}$ closed set but not a tri- gs closed set.

Example 3.34 Let X ={a, b, c}, τ_1 = {X, ϕ}, τ_2 = {X, ϕ, {a}}, τ_3 = {X, ϕ, {b}}, tri- $b\hat{g}$ C(X) = {X, ϕ, {a}, {b}, {c}, {b, c}, {a, c}}; tri- gsC(X) = P(X); here {a, b} is tri-gs closed set but not a tri- $b\hat{g}$ closed set.

Remark 3.35 The following diagram shows the relationship of tri- $b\hat{g}$ closed sets with other known existing closed sets in tri- topological space.



A → Tri- $b\hat{g}$ closed set
B → Tri- closed set
C → Tri- b closed set
D → Tri- g closed set
E → Tri- α closed set
F → Tri- gs closed set
G → Tri- g*bw closed set
H → Tri- bτ closed set
I → Tri- semi closed set

Remark 3.36 If (X, Tri- C(X)) is indiscrete topology then (X, Tri- $b\hat{g}$ C(X)) is discrete topology but converse part need not be true.

Example 3.37 Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, $\tau_2 = \tau_3$ = {X, ϕ, {b, c}; Tri- C(X) = {X, ϕ, {a}, {b, c}}; Tri- $b\hat{g}$ C(X) = {X, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}} = P(X). Here, (X, Tri- $b\hat{g}$ C(X)) is discrete topology but (X, Tri- C(X)) is not an indiscrete topology.

Remark 3.38 If (X, Tri- C(X)) is discrete topology then (X, Tri- $b\hat{g}$ C(X)) is discrete topology but converse part need not be true.

Example 3.39 In example – 3.7, (X, Tri- $b\hat{g}$ C(X)) is discrete topology but (X, Tri- C(X)) is not a discrete topology.

Remark 3.40 If (X, Tri- C(X)) is indiscrete topology then,
1) Every tri- $b\hat{g}$ closed set is tri- b closed set.
2) Every tri- $b\hat{g}$ closed set is tri- g closed set.
3) Every tri- $b\hat{g}$ closed set is tri- gs closed set.
4) Every tri- $b\hat{g}$ closed set is tri- $g^*b\omega$ closed set.
5) Every tri- g closed set is tri- $b\hat{g}$ closed set.
6) Every tri- gs closed set is tri- $b\hat{g}$ closed set.
7) Every tri- bτ closed set is tri- $b\hat{g}$ closed set.

Example 3.41 Let X be any non-empty set, $\tau_1 = \tau_2 = \tau_3$ = {X, ϕ} are topologies of X. Tri- C(X) ={X, ϕ}; Tri- bC (X) = Tri- gC(X) = Tri- gsC(X) = Tri- bτC(X) = Tri- $g^*b\omega$C(X) = Tri- $b\hat{g}$ C(X) = P(X).

Remark 3.42 If (X, Tri- C(X)) is discrete topology then,
1) Every tri- $b\hat{g}$ closed set is tri- closed set.
2) Every tri- $b\hat{g}$ closed set is tri- semi closed set.
3) Every tri- $b\hat{g}$ closed set is tri- α closed set.
4) Every tri- $b\hat{g}$ closed set is tri- b closed set.
5) Every tri- $b\hat{g}$ closed set is tri- g closed set.
6) Every tri- $b\hat{g}$ closed set is tri- gs closed set.
7) Every tri- $b\hat{g}$ closed set is tri- $g^*b\omega$ closed set.
8) Every tri- g closed set is tri- $b\hat{g}$ closed set.
9) Every tri- gs closed set is tri- $b\hat{g}$ closed set.
10) Every tri- bτ closed set is tri- $b\hat{g}$ closed set.

Example 3.43 Let X be any non-empty set, $\tau_1 = \tau_2 = \tau_3 = P(X)$ are topologies of X. Tri- C(X) = Tri- sC(X) = Tri- αC(X) = Tri- bC(X) = Tri- gC(X) = Tri- gsC(X) = Tri- bτC(X) = Tri- g^*bωC(X) = Tri- $b\hat{g}$ C(X) = P(X).

4. Tri- $b\hat{g}$ Open Sets In Tri- Topological Space

Definition 4.1 The complement of a tri- $b\hat{g}$ closed set is called the tri- $b\hat{g}$ open set. The family of all tri- $b\hat{g}$ open sets of X is denoted by tri- $b\hat{g}$ O(X).

Example 4.2 In example 3.2, tri- $b\hat{g}$ O(X) = {X, ϕ, {a, b}, {b, c}, {a, c}}.

Remark 4.3 ϕ and X are always tri- $b\hat{g}$ open set.

Remark 4.4 Intersection of tri- $b\hat{g}$ open sets need not be tri- $b\hat{g}$ open set.

Example 4.5 In example – 3.2, tri- $b\hat{g}$ O(X) = {X, ϕ, {a, b}, {b, c}, {a, c}}. Here, {a, b}, {b, c} are tri- $b\hat{g}$ open sets but {a, b} ∩ {b, c} = {b} ∉ tri- $b\hat{g}$ O(X).

Remark 4.6 Union of tri- $b\hat{g}$ open sets need not be tri- $b\hat{g}$ open set.

Example 4.7 In example – 3.16, tri- $b\hat{g}$ O(X) = {X, ϕ, {a}, {b}, {c}, {a, b}, {a, c}}. Here, {b} and {c} are tri- $b\hat{g}$ open sets but {b} ∪ {c} = {b, c} ∉ tri- $b\hat{g}$ O(X).

Remark 4.8 Difference of two tri- $b\hat{g}$ open sets need not be tri- $b\hat{g}$ open set.

Example 4.9 In previous example – 4.7, tri- $b\hat{g}$ O(X) = {X, ϕ, {a}, {b}, {c}, {a, b}, {a, c}}. Let A = X and B = {a}, Also A and B are tri- $b\hat{g}$ open sets. But A\B = X\{a} = {b, c} is not a tri- $b\hat{g}$ open set.

Definition 4.10 Let (X, τ_1, τ_2, τ_3) be a tri- topological space. The union of all tri- $b\hat{g}$ open sets of X contained in A is called the tri- $b\hat{g}$ interior of A and is denoted by tri- $b\hat{g}$ int(A). (i.e) tri- $b\hat{g}$ (A) = ∪ {B ⊆ X / B ⊆ A and A is tri- $b\hat{g}$ open set}.

Remark 4.11

1) tri- $b\hat{g}$ int(ϕ) = ϕ,
2) tri- $b\hat{g}$ int(X) = X,
3) tri- $b\hat{g}$ int(A) ⊆ A,
4) tri- $b\hat{g}$ int(A) = tri- $b\hat{g}$ int(tri- $b\hat{g}$ int(A)).

Proposition 4.12 For any A ⊆X, (tri- $b\hat{g}$ int(A))^c = tri- $b\hat{g}$ cl(A^c).

Proof: (tri- $b\hat{g}$ int(A))^c = [∪ {G / G ⊆ A & G is tri- $b\hat{g}$ open set}]^c = ∩ { G^c / G^c ⊇ A^c & G^c is tri- $b\hat{g}$ closed set} = ∩{ F / F ⊇ A^c& F is tri- $b\hat{g}$ closed set} where F = G^c. Hence, (tri- $b\hat{g}$ int(A))^c = tri- $b\hat{g}$ cl(A^c).

Proposition 4.13 Let (X,τ_1,τ_2,τ_3) be a tri-topological space. Let A ⊆ X. Then tri- $b\hat{g}$ int(A) = A if A is tri- $b\hat{g}$ open set.
Proof: Suppose A is a tri- $b\hat{g}$ open set in X, then A^c is tri- $b\hat{g}$ closed set in X. (i.e) tri- $b\hat{g}$ cl (A^c) ⊆ A^c. By the definition, A^c ⊆ tri- $b\hat{g}$ cl(A^c). Therefore tri- $b\hat{g}$ cl(A^c) = A^c ⇒ (tri- $b\hat{g}$ int(A))^c = A^c ⇒ tri- $b\hat{g}$ int(A) = A.

Remark 4.14 The tri- $b\hat{g}$ interior of a set A is not always tri- $b\hat{g}$ open set.

Example 4.15 Let X = {a, b, c}, τ_1 = {X, ϕ}, τ_2 = τ_3 = {X, ϕ, {a}}, tri- $b\hat{g}$ C(X) = {X, ϕ, {b}, {c}, {a, b}, {b, c}, {a, c}}; tri- $b\hat{g}$ O(X) = {X, ϕ, {a}, {b}, {c}, {a, b}, {a, c}}. Here, tri- $b\hat{g}$ int ({b, c}) = {b, c} is not a tri- $b\hat{g}$ open set.

Proposition 4.16
1) Every tri- open set is tri- $b\hat{g}$ open set.
2) Every tri- b open set is tri- $b\hat{g}$ open set.
3) Every tri- semi open set is tri- $b\hat{g}$ open set.
4) Every tri- α open set is tri- $b\hat{g}$ open set.
5) Every tri- g*bω open set is tri- $b\hat{g}$ open set.
6) Every tri- $b\hat{g}$ open set is tri- bτ open set.

Proof: By proposition – 3.17, 3.19, 3.21, 3.23, 3.25, 3.27 we get the results.

5. Conclusions

In this paper, we dealt with tri- $b\hat{g}$ closed sets and tri- $b\hat{g}$ open sets. In future we wish to do our research work in tri- $b\hat{g}$ continuous functions, tri- $b\hat{g}$ separated, tri- $b\hat{g}$ connected sets, tri- $b\hat{g}$ compact and so on.

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