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Inverse Domination Parameters of Jump Graph

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Abstract

Let $G = (V, E)$ be a connected graph. Let D be a minimum dominating set in G. If $V - D$ contains a dominating set D' of G, then D' is called an inverse dominating set with respect to D. Theminimum cardinality of an inverse dominating set of G is called inverse domination number of G. In this article, we determine inverse domination parameters of jump graph of a graph.

Keywords: domination number, inverse domination number, non-split inverse domination number, connected inverse domination number, jump graph.

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1.Introduction

The *undirected graph*$G = (V, E)$ discussed in this paper is simple and connected. The *orde*r and *size* are denoted by n and m respectively. For basic graph theoretical reference, we refer [2]. If uv is an edge of G, then two vertices u and v are said to be adjacent. If $uv \in E(G)$, then u is $v's$ neighbour, and the set of $v's$ neighbours is denoted by $N(v)$. Vertex $v \in V$ has a degree of $deg(v) = |N(v)|$. If $deg(v) = n - 1$, a vertex v is referred to as a universal vertex.

Globally, the domination conception in graphs start off its root in 1850s along the concern of certain chess players. Domination has diverse function consists of the morphological analysis, social network theory, CCTV installation and the most generally argued is the computer network communication. The aforementioned network comprises communication links among a firm number of slots. The trouble is to prime a least number of location where transmitters are installed such that a network as a whole united by a direct communication link to the transmitter site. In alternative words, the issue is to locate the least dominant set in the graph that corresponds to this network.

V.R. Kulli and C. Sigarkanthi [10] were the first to propose the idea of an inverse domination number. A set D of vertices in a graph $G = (V, E)$ is a dominating set if every vertex in $V - D$is adjacent to some vertex in D. The domination number $\gamma(G)$of $G$is the minimum cardinality of a dominating set of$G$. If $V - D$ contains a dominating set D' of G, then D' is called an inverse dominating set with respect to D. The inverse domination number $\gamma'(G)$of $G$is the minimum cardinality of an inverse dominating set of G.

The dominating set S of G is connected dominating set of G if induced sub graph $\langle S \rangle$ is connected. The connected domination number $\gamma_c(G)$ of G is referred to as a minimum cardinality of connected dominating set.

The dominating set S of G is a non-split dominating set of G if induced sub graph $\langle V - S \rangle$ is connected. The non-split domination number $\gamma_{ns}(G)$ of G is referred to as a minimum cardinality of non-split dominating set.[7]

The n-sunlet graph is a graph on $2n$ vertices isobtained by attaching n-pendant edges to the cycle C_n and it is denoted by S_n.

Let P_n be a path graph in n vertices. The comb graph is defined as $P_n \odot K_1$.It has $2n$ vertices and $2n - 1$ edges.

Fan graph F_nn ≥ 2 determined by joining all vertices of a path P_n to a different vertex, called centre. Thus $F_n$has $n + 1$ vertices, such as $u, u_1, u_2, u_3, \ldots. u_n$and $(2n - 1)$ edges, such as $uu_i, 1 \leq i \leq n - 1$. [12]

The line graph $L(G)$ of G has the edges of G as its vertices which are adjacent in $L(G)$ if and only if the corresponding edges are adjacent in G. We call the complement of line graph $L(G)$as the jump graph $J(G)$ of G, found in [11]. The jump graph $J(G)$ of a graph G is the graph defined on$E(G)$ and in which two vertices are adjacent if and only if they are not adjacent in G. Since both $L(G)$ and $J(G)$ are defined on the edge set of a graph G.

2. Main Results

Inverse domination number of jump graph of n–sunlet graph

Theorem 2.1. For the graph $G = J(S_n)$ $(n \geq 4)$, $\gamma'(G)) = 2$.

Proof. For $n \geq 4$, the number of vertices of n-sunlet graph is $2n$. Then it has $2n$ edges. Let $G = J(S_n)$. The number of vertices of jump graph of the n-sunlet graph is $2n$. Let the vertices of the graph is labeled as $\{v_1, v_2, \dots, v_n, u_1, u_2, u_3, \dots. u_n\}$, Since G contains no universal vertices, $\gamma(G) \geq 2$. Let $D = \{u_1, u_3\}$. Then D is a dominating set of G and so $\gamma(G)) = 2$. Let $D' = \{u_2, u_4\}$. Then D' is a inverse dominating set of G so that $\gamma'(G) = 2$. ■

Theorem 2.2. For the graph $G = J(S_n)$ $(n \geq 4)$, $\gamma'_{ns}(G)) = 2$.

Proof. For $n \geq 4$, the number of vertices of n-sunlet graph is $2n$. Then it has $2n$ edges. Let $G = J(S_n)$. The number of vertices of jump graph of the n-sunlet graph is $2n$. Let the vertices of the graph is labeled as $\{v_1, v_2, \dots, v_n, u_1, u_2, u_3, \dots. u_n\}$, Since G contains no universal vertices, $\gamma(G) \geq 2$. Let $D = \{u_1, u_3\}$. Then D is a dominating set of G and so $\gamma(G) = 2$. Let $D' = \{u_2, u_4\}$. Then D' is a non-split inverse dominating set of G so that $\gamma'_{ns}(G) = 2$. ■

Theorem 2.3. For the graph $G = J(S_n)$ $(n \geq 4)$, $\gamma'_c(G) = 2$.

Proof. For $n \geq 4$, the number of vertices of n-sunlet graph is $2n$. Then it has $2n$ edges. Let $G = J(S_n)$. The number of vertices of jump graph of the n-sunlet graph is $2n$. Let the vertices of the graph is labeled as $\{v_1, v_2, \dots, v_n, u_1, u_2, u_3, \dots. u_n\}$, Since G contains no universal vertices, $\gamma(G) \geq 2$. Let $D = \{u_1, u_3\}$. Then D is a dominating set of G and so $\gamma(G)) = 2$. Let $D' = \{u_2, u_4\}$. Then D' is a connected inverse dominating set of G so that $\gamma'_c(G) = 2$. ■

Inverse domination number of jump graph of comb graph

Theorem 2.4. For the graph $G = J(P_n \odot K_1)$ $(n \geq 4)$, $\gamma'(G) = 2$.

Proof. For $n \geq 4$, the number of vertices of combgraph is $2n$. Then it has $2n - 1$ edges. Let $G = J(P_n \odot K_1)$. The number of vertices of jump graph of the comb graph is $2n - 1$. Let the vertices of the graph is labeled as $\{v_1, v_2, \dots, v_{n-1}, u_1, u_2, u_3, \dots. u_n\}$, Since G contains no universal vertices, $\gamma(G) \geq 2$. Let $D = \{u_1, v_1\}$. Then D is a dominating set of G and so $\gamma(G) = 2$. Let $D' = \{u_2, u_4\}$. Then D' is a inverse dominating set of G so that $\gamma'(G) = 2$. ■

Theorem 2.5. For the graph $G = J(P_n \odot K_1)$ $(n \geq 4)$, $\gamma'_{ns}(G) = 2$.

Proof. For $n \geq 4$, the number of vertices of comb graph is $2n$. Then it has $2n - 1$ edges. Let $G = J(P_n \odot K_1)$. The number of vertices of jump graph of the comb graph is $2n - 1$. Let the vertices of the graph is labeled as $\{v_1, v_2, \dots, v_{n-1}, u_1, u_2, \dots, u_n\}$. Since G contains no universal vertices, $\gamma(G) \geq 2$. Let $D = \{u_1, v_1\}$. Then D is a dominating set of G and so $\gamma(G) = 2$. Let $D' = \{u_2, u_4\}$. Then D' is a non-split inverse dominating set of G so that $\gamma'_{ns}(G) = 2$. ■

Theorem 2.6. For the graph $G = J(P_n \odot K_1)$ $(n \geq 4)$, $\gamma'_c(G) = 2$.

Proof. For $n \geq 4$, the number of vertices of comb graph is $2n$. Then it has $2n - 1$ edges. Let $G = J(P_n \odot K_1)$. The number of vertices of jump graph of the comb graph is $2n - 1$. Let the vertices of the graph is labeled as $\{v_1, v_2, \dots, v_{n-1}, u_1, u_2, \dots, u_n\}$, Since G contains no universal vertices, $\gamma(G)) \geq 2$. Let $D = \{u_1, v_1\}$. Then D is a dominating set of G and so $\gamma(G) = 2$. Let $D' = \{u_2, u_4\}$. Then D' is a connected inverse dominating set of G so that $\gamma'_c(G) = 2$. ■

Inverse domination number of jump graph of fan graph

Theorem 2.7. For fan graph $G = J(F_n)$ $(n \geq 5)$, $\gamma'(G) = \begin{cases} 3 & if\ n = 3,4 \\ 2 & if\ n \geq 5 \end{cases}$.

Proof. The number of vertices of fan graph is $n + 1$. Then it has $2n - 1$ edges. Let $G = J(F_n)$. The number of vertices of jump graph of the fan graph is $2n - 1$. Let the vertices of the graph is labeled as $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}\}$, Since G contains no universal vertices, $\gamma(G)) \geq 2$.

Let $n = 3$. It is easily verified that no two element subsets of$J(F_n)$ is not a γ-set of G and so $\gamma(G) \geq 3$.Let $D = \{v_1, v_3, v_4\}$. Then D is a γ-set of G so that $\gamma(G)) = 3$. Let $D' = \{v_2, u_2, u_3\}$. Then D' is a γ'-set of G. Since $\gamma(G) = 3$, we have$\gamma'(G) = 3$.

Let $n = 4$.Let $D = \{v_1, v_4\}$. Then D is a γ-set of G so that $\gamma(G)) = 2$. Let $D' = \{v_2, v_3, u_3\}$. Then D' is a γ'-set of G and so$\gamma'(G) \leq 3$. It is easily observed that no two element subsets of G is not a γ'-set of G. Therefore $\gamma'(G)) = 3$.

Let $n \geq 5$. Let $D = \{v_1, v_4\}$. Then D is a dominatingset of G and so $\gamma(G) = 2$. Let $D' = \{v_2, v_5\}$. Then D' is a inverse dominating set of G and so $\gamma'(G) = 2$. ■

Theorem 2.8. For the graph $G = J(F_n)$ $(n \geq 5)$, $\gamma'_{ns}(G) = \begin{cases} 3 & if\ n = 3 \\ 2 & if\ n \geq 5 \end{cases}$.

Proof. The number of vertices of fan graph is $n + 1$. Then it has $2n - 1$ edges. Let $G = J(F_n)$. The number of vertices of jump graph of the fan graph is $2n - 1$. Let the vertices of the graph is labeled as $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}\}$, Since G contains no universal vertices, $\gamma(G) \geq 2$.

Let $n = 3$. It is easily verified that no two element subsets of $J(F_n)$ is not a γ-set of G and so $\gamma(G) \geq 3$. Let $D = \{v_2, u_2, u_3\}$. Then D is a γ-set of G so that $\gamma(G) = 3$. Let $D' = \{v_1, v_3, u_4\}$. Then D' is a non-split inverse dominating set of G so that $\gamma'_{ns}(G) = 3$.

Let $n = 4$. Let $D = \{v_1, v_4\}$. Then D is a γ-set of G so that $\gamma(G) = 2$. Let $D' = \{v_2, v_3, u_3\}$. Then D' is a not a non-split inverse dominating set of G.

Let $n \geq 5$. Let $D = \{v_1, v_4\}$. Then D is a dominatingset of G and so $\gamma(G) = 2$. Let $D' = \{v_2, v_5\}$. Then D' is a non-split inverse dominating set of G and so $\gamma'_{ns}(G) = 2$. ■

Theorem 2.9. For the graph $G = J(F_n)$ $(n \geq 5)$, $\gamma'_c(G) = \begin{cases} 3 & if\ n = 3 \\ 2 & if\ n \geq 5 \end{cases}$.

Proof. The number of vertices of fan graph is $n + 1$. Then it has $2n - 1$ edges. Let $G =$

$J(F_n)$. The number of vertices of jump graph of the fan graph is $2n-1$. Let the vertices of the graph is labeled as $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}\}$, Since G contains no universal vertices, $\gamma(G) \geq 2$.
Let $n = 3$. It is easily verified that no two element subsets of $J(F_n)$ is not a γ-set of G and so $\gamma(G) \geq 3$. Let $D = \{v_2, u_2, u_3\}$. Then D is a γ-set of G so that $\gamma(G) = 3$. Let $D' = \{v_1, v_3, u_4\}$. Then D' is a connected inverse dominating set of G so that $\gamma_c'(G) = 3$.
Let $n = 4$. Let $D = \{v_1, v_4\}$. Then D is a γ-set of G so that $\gamma(G) = 2$. Let $D' = \{v_2, v_3, u_3\}$. Then D' is a not a connected inverse dominating set of G.
Let $n \geq 5$. Let $D = \{v_1, v_4\}$. Then D is a dominatingset of G and so $\gamma(G) = 2$. Let $D' = \{v_2, v_5\}$. Then D' is a connected inverse dominating set of Gso that$\gamma_c'(G) = 2$. ∎

Inverse domination number of jump graph of $C_n \odot \overline{K}_2$

Theorem 2.10. For the graph $G = J(C_n \odot \overline{K}_2)$ $(n \geq 4)$, $\gamma'(G) = \begin{cases} 3 & if\ n = 3 \\ 2 & if\ n \geq 4 \end{cases}$.

Proof. The number of vertices of $C_n \odot \overline{K}_2$is $3n$. Then it has $3n$ edges. Let $G = J(C_n \odot \overline{K}_2)$. The number of vertices of jump graph of the $C_n \odot \overline{K}_2$ is $3n$. Let the vertices of the graph is labeled as $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$. Since G contains no universal vertices, $\gamma(G) \geq 2$.
Let $n = 3$. It is easily verified that no two element subsets of $J(C_n \odot \overline{K}_2)$ is not a γ-set of G and so $\gamma(G) \geq 3$. Let $D = \{v_2, w_2, w_3\}$. Then D is a γ-set of G so that $\gamma(G) = 3$. Let $D' = \{u_1, u_2, u_3\}$. Then D' is a γ'-set of G. Since $\gamma(G) = 3$, we have $\gamma'(G) = 3$.
Let $n \geq 4$. Let $D = \{u_2, u_4\}$. Then D is a dominatingset of G and so $\gamma(G) = 2$. Let $D' = \{w_2, w_4\}$. Then D' is a inverse dominating set of G so that$\gamma'(G) = 2$. ∎

Theorem 2.11. For the graph $G = J(C_n \odot \overline{K}_2)$ $(n \geq 4)$, $\gamma_{ns}'(G) = \begin{cases} 3 & if\ n = 3 \\ 2 & if\ n \geq 4 \end{cases}$.

Proof. The number of vertices of $C_n \odot \overline{K}_2$ is $3n$. Then it has $3n$ edges. Let $G = J(C_n \odot \overline{K}_2)$. The number of vertices of jump graph of the $C_n \odot \overline{K}_2$ is $3n$. Let the vertices of the graph is labeled as $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$. Since G contains no universal vertices, $\gamma(G) \geq 2$.
Let $n = 3$. It is easily verified that no two element subsets of $J(C_n \odot \overline{K}_2)$ is not a γ-set of G and so $\gamma(G) \geq 3$. Let $D = \{v_2, w_2, w_3\}$. Then D is a γ-set of G so that $\gamma(G) = 3$. Let $D' = \{u_1, u_2, u_3\}$. Then D' is a non-split inverse dominating set of G so that $\gamma_{ns}'(G) = 3$.
Let $n \geq 4$. Let $D = \{u_2, u_4\}$. Then$D$ is a dominating set of G so that $\gamma(G) = 2$. LetD'= $\{w_2, u_4\}$. Then D' is a non-split inverse dominating set of G so that $\gamma_{ns}'(G) = 2$.

Theorem 2.12. For the graph $G = J(C_n \odot \overline{K}_2)$ $(n \geq 4)$, $\gamma_c'(G) = \begin{cases} 3 & if\ n = 3 \\ 2 & if\ n \geq 4 \end{cases}$.

Proof. The number of vertices of $C_n \odot \overline{K}_2$ is $3n$. Then it has $3n$ edges. Let $G =$

$J(C_n \odot \overline{K}_2)$. The number of vertices of jump graph of the $C_n \odot \overline{K}_2$ is $3n$. Let the vertices of the graph is labeled as $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$. Since G contains no universal vertices, $\gamma_c'(G) \geq 2$.
Let $n = 3$. It is easily verified that no two element subsets of $J(C_n \odot \overline{K}_2)$ is not a γ-set of G and so $\gamma(G) \geq 3$. Let $D = \{v_2, w_2, w_3\}$. Then D is a γ-set of G so that $\gamma(G) = 3$. Let $D' = \{u_1, u_2, u_3\}$. Then D' is a non-split inverse dominating set of G so that $\gamma_c'(G) = 2$.
Let $n \geq 4$. Let $D = \{u_2, u_4\}$. Then D is a dominating set of G so that $\gamma(G) = 2$. Let $D' = \{w_2, w_4\}$. Then D' is a connected inverse dominating set of G so that $\gamma_c'(G) = 2$. ■

Inverse domination number of jump graph of $P_n \odot \overline{K}_2$

Theorem 2.13. For the graph $G = J(P_n \odot \overline{K}_2)$ $(n \geq 3)$, $\gamma'(G) = 2$.

Proof. For$n \geq 3$,the number of vertices of $P_n \odot \overline{K}_2$is $3n$. Then it has $3n - 1$ edges. Let $G = J(P_n \odot \overline{K}_2)$. The number of vertices of jump graph of the $P_n \odot \overline{K}_2$ is $3n - 1$. Let the vertices of the graph is labeled as $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots v_{n-1}, w_1, w_2, \dots, w_n\}$. Since G contains no universal vertices, $\gamma(G) \geq 2$. Let $D = \{u_1, u_3\}$. Then D is a γ-set of G so that $\gamma(G) = 2$. Let $D' = \{w_1, w_3\}$. Then D' is a inverse dominating set of G so that$\gamma'(G) = 2$. ■

Theorem 2.14. For the graph $G = J(P_n \odot \overline{K}_2)$ $(n \geq 3)$, $\gamma_{ns}'(G) = 2$.

Proof. For$n \geq 3$,the number of vertices of $P_n \odot \overline{K}_2$is $3n$. Then it has $3n - 1$ edges. Let $G = J(P_n \odot \overline{K}_2)$. The number of vertices of jump graph of the $P_n \odot \overline{K}_2$ is $3n - 1$. Let the vertices of the graph is labeled as $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}, w_1, w_2, \dots, w_n\}$. Since G contains no universal vertices, $\gamma(G) \geq 2$. Let $D = \{u_1, u_3\}$. Then D is a dominating set of $G$and so $\gamma(G) = 2$. Let $D' = \{w_1, w_3\}$. Then D' is a non-split inverse dominating set of G so that $\gamma_{ns}'(G) = 2$. ■

Theorem 2.15. For the graph $G = J(P_n \odot \overline{K}_2)$ $(n \geq 3)$, $\gamma_c'(G) = 2$.

Proof. For$n \geq 3$,the number of vertices of $P_n \odot \overline{K}_2$is $3n$. Then it has $3n - 1$ edges. Let $G = J(P_n \odot \overline{K}_2)$. The number of vertices of jump graph of the $P_n \odot \overline{K}_2$ is $3n - 1$. Let the vertices of the graph is labeled as$\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}, w_1, w_2, \dots, , w_n\}$. Since G contains no universal vertices, $\gamma(G) \geq 2$. Let $D = \{u_1, u_3\}$. Then D is a dominating set of G and so $\gamma(G) = 2$. Let $D' = \{w_1, w_3\}$. Then D' is a connected inverse dominating set of G so that $\gamma_c'(G) = 2$. ■

3. Conclusions

In this article, we determined some inverse domination parameter for jump graph of some special graphs. We will determine some more inverse domination parameters for jump graph of some special graph in future work.

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