AUTHOR:
Dr Rina Durandt[1]
Dr Sheldon Herbst[1]
Mr Majane Seloane[1]

AFFILIATION:
[1]University of Johannesburg, South Africa

DOI: http://dx.doi.org/10.18820/2519593X/pie.v40.i1.9

e-ISSN 2519-593X
Perspectives in Education
2022 40(1): 143-163

PUBLISHED:
04 March 2022

RECEIVED:
16 August 2021

ACCEPTED:
09 December 2021

Published by the UFS
http://journals.ufs.ac.za/index.php/pie

Teaching and learning first-year engineering mathematics at a distance: A critical view over two consecutive years

Abstract

This article reports on the empirical results and practical aspects concerning the teaching and learning of first-year engineering mathematics at a distance. The investigation over two consecutive years (2020 and 2021) is meant to yield prospects and contribute to the development of suitable pedagogies for online mathematics teaching and learning for engineering students in South Africa in the future. In 2020, lecturers were faced with a "sudden" shift from face-to-face to online teaching and the focus was to save the academic year and leave no student behind; in 2021 the situation was unchanged. Lecturers had to consider key aspects (such as the module structure, teaching theory and practice, and perspectives on "what can work") in the transition and continuation from face-to-face to fully online and developing a suitable teaching and learning approach. Approximately 1000 first-year engineering students at the University of Johannesburg, South Africa, were exposed to the newly developed online teaching and learning approach in both years. The approach includes the following key elements: an evaluation of prior knowledge by a diagnostic test; an investigation of students' attitudes towards mathematics by means of the SATM questionnaire; weekly virtual lecture and tutorial sessions; weekly homework tasks; additional online resources; discussion forums between all role players (students, tutors and lecturers); online tutor support; the use of e-textbooks and online assessments via the university learning management system and an external platform (such as WeBWorK). Both years followed a similar approach, although slight changes were implemented in 2021. Descriptive statistics from key elements were used to monitor the students' involvement and progress in both years. Results show the teaching and learning approach is effective but has room for improvement. Particularly, the results encourage addressing the needs of the students and lecturers when implementing pedagogical aspects in learning mathematics at a distance.

Keywords: *COVID-19; distance education; engineering mathematics; first-year students; learning management system; teaching and learning online.*

1. Introduction

In March 2020, lecturers and students globally and in South Africa experienced a "sudden" shift from teaching and learning in a traditional (face-to-face) approach, to teaching and learning at a distance (online), as restrictions caused by the COVID-19 pandemic affected the tertiary education sector. Locally and globally, universities were expected to shift unprecedentedly in all their sectors (see Parekh, 2021). One sector is in the classroom, and particularly the teaching and learning of engineering mathematics. “Business as usual” could not be continued and the lecturers and students had to depend largely on the university’s learning management system (LMS) and other educational technologies. During this time, at the University of Johannesburg (UJ), the focus was to save the academic year and leave no student behind (see QA report on the transition to remote teaching and learning, 2020; Parekh, 2021). With this focus in mind, the lecturers (including the authors in this article) responsible for a large engineering mathematics module made innovative changes despite disruptions.

In the first semester of 2021 the situation remained unchanged and teaching and learning at a distance became the “new” normal. One difference between the 2020 and 2021 academic years, particularly in the first semester for first-year students, is the initial experiences at university. In 2020, first-year students could experience the learning culture at university physically on campus, which is very different from the secondary educational environment (compare Rach & Heinze, 2017); meet lecturers and fellow students in person and experience the situational context (e.g., university systems and support structures). Whereas in 2021, all these experiences were at a distance from the beginning. Another difference is the teaching and learning approach students were exposed to in the pre-tertiary year; the 2020 first-year cohort was exposed to a traditional face-to-face teaching and learning approach while the 2021 first-year cohort was exposed to a blended learning, or an online, approach. In constructing a classroom for engineering mathematics at a distance, lecturers reflected on these differences and expected the 2021 first-year cohort to be more prepared for this “new” approach.

It is well documented that first-year students at university, locally and internationally, are often under-prepared for tertiary education, particularly in the science and engineering fields, and underperform academically. Du Plessis and Gerber (2012) conducted a study on the academic achievement of two cohorts of first-year students at a public university in South Africa and concluded that a combination of aspects is related to students’ under-preparedness in the academic domain. These aspects include English reading or writing ability, mathematical ability and effective study habits. Will these aspects also be relevant in 2020 and in 2021 in a first-year engineering mathematics online classroom? Will some aspects become more prominent? Others, such as Leong *et al*. (2021) view student preparedness as an essential component of transitioning to university; they describe this idea by using a combination of aspects such as academic aptitude, prior knowledge, self-efficacy, self-confidence and a complex assortment of study and life skills. Additionally, the formal education of engineering students requires the development of mathematical competency, in particular, with a focus on higher cognitive skills such as arguing or problem solving. According to the latest TIMSS[1] findings (Reddy *et al.*, 2020), South African school learners at the Grade 9 level are not well prepared and lack basic knowledge of mathematics and science. It seems reasonable to expect gaps in prior knowledge in mathematics at first-year level; thus, it is necessary to determine these gaps to have a more effective approach in developing further mathematical competency and higher-order skills.

1 TIMSS stands for the Trends in International Mathematics and Science Study. This is a comparative study in educational achievement in mathematics and science and involves approximately 50 countries and thousands of students in each participating country.

The online classroom has different aspects related to the cognitive and affective domain and different role players, such as the university structure, systems and processes, the module structure, lecturers, students and tutors. In 2020 and 2021, the authors aimed to develop and implement a new pedagogy for teaching a large first-year engineering mathematics module at a distance. The initiative was guided by the following research questions:

i. How can a first-year engineering mathematics classroom be structured for teaching at a distance?

ii. What elements are fundamental in this “new” classroom environment?

iii. How can students’ involvement and progress be monitored?

In the present, practice-oriented article, we attempt to answer the second and third research questions (see section 4). In the methodology section (sub-section 3.2) we introduce the online classroom design and its contents (e.g., weekly virtual lecture and tutorial sessions; discussion forums between all role players [students, tutors, and lecturers] and online tutor support) to give the reader a view of the “new” teaching approach and we further discuss key elements of this approach (see sub-section 3.3). Then, we report on the empirical results from key elements (see section 4); students’ engagement in online activities and the online assessments via an external platform, WeBWorK.

2. Conceptual framework

The authors followed a pragmatic approach (see Creswell, 2013) in developing a “new” pedagogy for teaching and learning engineering mathematics at a distance. The idea was also to align the workable pedagogy with the University of Johannesburg’s primary perspective, “*learning to be*” (originally from Bruner, 1959). For example, to expose first-year engineering mathematics students to learning *about* the facts, concepts and procedures of the knowledge domain (particularly engineering mathematics), and learning *to be* the practices of the knowledge domain (in this context the new online approach and some application to the real-world context of engineers). Two underlying theoretical perspectives are relevant: (i) the teaching and learning of mathematics at tertiary level and (ii) the key characteristics of distance education.

2.1 Teaching and learning mathematics at tertiary level

Learning opportunities are largely dependent on what happens in the classroom and the theory describing by the *criteria for quality teaching* (compare Blum, 2015) seems important for mathematics learning in general, but also for learning in authentic situations. These criteria refer to the necessary conditions (applicable to the secondary and tertiary environment) that must be fulfilled if teaching ought to have visible effects on students’ knowledge, skills and abilities. The five *criteria for quality teaching* (Blum, 2015) that directly influence our online classroom practice are: i) *Effective classroom management*, which refers to aspects largely independent of a specific subject (e.g., structuring a lesson, or using time effectively, incorporating technology); (ii) *Student orientation*, which considers the unique and specific circumstance of students (e.g., level of prior knowledge, or strategic support to a student or a group); (iii) *Cognitive activation of students*, which refers to the mental stimulation of students (e.g., the types of questions asked, or the balance between adaptive teacher interventions and students working on their own); (iv) *Meta-cognitive activation of students*, which refers to advancing learning and working strategies (e.g., by the use of strategic aids or steps or technology sources) and (v) *Demanding orchestration of topics*, which refers to often creating opportunities for students to develop practice, and apply the desired competencies as well as linking between mathematical topics and a variety of subject areas, and the real world (e.g., by solving real-life open-ended tasks). The authors of this article considered the criteria for quality teaching in designing an online classroom for learning engineering mathematics at a distance.

Learning mathematics includes not only cognitive aims but also non-cognitive aims and these relate to the *affective domain* that influences students and teachers. The theory of the affective domain (Chamberlin, 2019) plays a role in the development of critical and creative minds and more so in dealing with authentic activities or unfamiliar situations (see, for example, Jacobs & Durandt, 2016). Moreover, attitudinal aspects seem important for students in the transition between school and university (Leong *et al.*, 2021). The attitudinal domain is multidimensional and relates to beliefs, attitudes and emotions. Attitudes are generally regarded as less cognitive than beliefs, but more cognitive than emotions, and may involve positive or negative feelings. Attitudes are more generally manners of acting, feeling or thinking. For an overview of the literature, see Hannula (2002). The individual dimensions of this domain that particularly influenced our teaching pedagogy were the students' view on: (i) the estimated difficulty of mathematical activities, (ii) the subjective value (including usefulness and relevance in personal/professional life), (iii) the level of personal interest in studying mathematics, (iv) the effort (related to motivation) that s/he is willing to invest in learning mathematics and (v) his/her level of cognitive competence of intellectual skills and abilities.

2.2 Characteristics of distance education

Engelbrecht, Llinares and Borba (2020) reflect on different perspectives, developed in the last decade, in the domains of (i) principles of design of new settings; (ii) social interactions and construction of knowledge and (iii) tools and resources. Their work provided evidence of the advances in theoretical frameworks and support in the generation of new meanings for old constructs such as "tool", "resources" or "learning setting", which were used by the lecturers in this study as a guideline for making changes to the new online classroom. The work from Engelbrecht *et al.* (2020) regarding the transformation of the mathematics classroom with the growing use of the internet in educational contexts, and the work from Quinn and Aaräo (2020) focusing on blended learning environments in first-year engineering mathematics, largely informed the design of the "new" teaching and learning approach reported on in this article. This "new" approach was also informed by the initiatives from the Centre for Academic Technologies at the University of Johannesburg. For the last few years, academics were prepared for a modern context in teaching, learning and assessment; COVID-19 pushed all role players towards implementation. This modern context, which the internet has mostly transformed, includes aspects of socio-economic adaptability, familiarity with social media, happenings in the Twittersphere, availability of Massive Open Online Courses (MOOCs), the use of learning management systems (e.g., Blackboard), instant knowledge application and gratification from Google and YouTube, immediate feedback from computer processes to the user and more (see Louw, 2021). For many students the shift to online teaching and learning could have been easy, provided they had the technology and infrastructure available; however, for lecturers, this might have been very challenging.

In building a learning environment for distance education three main strands should be addressed (see Borba *et al.*, 2016; Engelbrecht *et al.*, 2020: 827):

i. *Principles of design:* How mathematics educators enact the principles of design in MOOCs and blended approaches to designing professional development opportunities and mathematics teaching contexts. A variety of conceptual frameworks can influence the design and implementation of MOOCs (e.g., Hollebrands & Lee, 2020; Taranto & Arzarello, 2020). In this intervention the authors were influenced by the criteria for quality teaching (see sub-section 2.1).

ii. *Social interaction and construction of knowledge:* How technologies in online contexts support social interaction among participants as a medium to support mathematical knowledge construction and teaching competencies. Vygotsky developed the idea of the social constructivist learning theory and is widely supported in the literature (e.g., Schmidt, 2013). In this intervention social interaction was purposefully addressed (see sub-section 3.3) although the overarching view from the researchers in this intervention was pragmatic.

iii. *Tools and resources:* Different meanings associated with the idea of online resources and how their use is conceptualised in different mathematics teaching contexts, given the emergence of new online mathematics resources and ways of teaching (e.g., Oechsler & Borba, 2020). In this intervention, the lecturers used tools available on the UJ LMS (Blackboard) and other platforms (see sub-section 3.3).

The initial idea with the introduction of blended learning environments was to enrich and improve efficiency in traditional face-to-face teaching by making minor changes to pedagogy. This was usually done by adding resources and supplementary materials (compare Graham, 2006). Also, the change between environments is challenging, specifically if the intention is to create a rich and effective domain. COVID-19 forced lecturers at universities to make a sudden change from a traditional environment to a fully online environment; this was specifically challenging for lecturers of large class groups (see the QA report on the transition to remote teaching and learning, 2020). Quinn and Aaräo (2020) emphasise the teaching and learning approach for first-year engineering mathematics students should encourage self-regulation of learning. The latter idea includes an investigation of the characteristics of the student, his/her perceptions of the learning context and approaches to learning. Quinn and Aaräo (2020) experimented with online quizzes on two different first-year engineering mathematics classes in an Australian context. They investigated changing attitudes of students, refreshing assumed knowledge and teaching foundational concepts, archiving online lecture options in addition to face-to-face lectures, adopting board tutorials supported by a problem-solving approach for more complex engineering modelling problems and scaffolding with online interactive problems. Furthermore, shared websites for online and face-to-face cohorts allowed the amount of blending to be determined by the individual student. The study by Quinn and Aaräo (2020) was conducted before the COVID-19 pandemic and the focus at the time was to provide more flexible study options and more active learning opportunities.

3. Study context and design

The authors followed a pragmatic approach (Creswell, 2013), namely considering the criteria for quality teaching and characteristics of distance education, to establish “what works” in developing and implementing a new pedagogy for teaching a large first-year engineering mathematics module at a distance. The pragmatic approach aligns with the quantitative nature of the data that were collected from key elements in the learning approach, in 2020 and 2021, from the LMS and WeBWorK platforms. A limitation of the study is the differences in the sample size from the different platforms. We address this aspect later (see section 4) and, as a result, direct comparisons through standardised statistical tests were not possible.

3.1 Characteristics of participants and module structure

The teaching and learning approach was developed for two cohorts of first-year engineering mathematics students from a large public university in South Africa. In both years, 2020 and 2021, the representatives were enrolled for the same engineering mathematics module that is offered by the Faculty of Science. Approximately 1000 students (998 students in 2020 and 988 students in 2021) per year are enrolled in the module. In both years, the students speak mostly African languages (e.g., isiZulu or Northern Sotho) as their home language, and the language of instruction in the engineering mathematics module is English.

The group is divided into five sub-groups according to qualification: (i) Mechanical and Industrial Engineering, (ii) Construction and Civil Engineering, (iii) Extraction and Physical Metallurgy and Chemical Engineering, (iv) Electrical Engineering and (v) Mining Engineering and Mineral Surveying; consequently, the student numbers per sub-group are not equal. In this article the data are not presented per sub-group. In a traditional face-to-face environment, each sub-group had a lecturer (following his/her own teaching approach) and two tutors (managed by the sub-group lecturer); however, with the change to online teaching the entire cohort shared one electronic learning environment, with all lecturers (5) and tutors (10) sharing the same classroom. All lecturers, in both years, in the first-year engineering mathematics module are qualified and experienced in teaching at tertiary level. All tutors are selected through a departmental system and are competent in the module content. Traditionally lecturers have large autonomy in most teaching activities, including learning material, in-class arrangements and continuous assessments. Formal assessments and the examination are aligned between all sub-groups and follow standard departmental moderation procedures. Traditionally all assessments are completed pen-on-paper and sit down, and online/electronic assessments have not been used in this module prior to 2020. The lecturers have very little experience in online assessments. Before COVID-19, the module had one electronic learning environment via Blackboard, but this space was only used in a limited way by some lecturers and mainly for communication purposes. Although the university has a strong and reliable electronic learning environment and the idea of a blended learning approach was encouraged during the last few years, lecturers used this approach by choice, infrequently and in different ways.

Each of the sub-groups has a unique timetable according to the university's structure, which only overlaps sometimes during an academic week. Each sub-group has six periods per week (either four periods for lectures and two periods for tutorials, or five periods for lectures and one tutorial period, all depending on the lecturer's individual approach and preference). In both years the modules had one coordinator, who is also a lecturer and the first author of this article.

The content of the module includes precalculus and calculus components and is structured over 13 academic weeks. The precalculus components include *functions*, *conic sections*, *complex numbers*, *The Binomial Theorem* and *Cramer's Rule*. The calculus components include *limits*, *differentiation rules*, *basic integration* and some *applications* related to differentiation (such as curve sketching) and integration (such as area problems). Due to low pass rates in previous years and the prerequisite role of this first-year engineering mathematics module for other engineering modules, it was highlighted as "high-risk". The Academic Development Centre of the university supported the lecturers and students regularly and monitored the students' progress. With the sudden change to online teaching, not all lecturers or students had reliable devices and infrastructure (e.g., connectivity and data) to work from home and to work during convenient times of the day (see Section 4 for some empirical data).

3.2 Design of the "new" teaching and learning strategy

The "new" teaching and learning approach was designed for and implemented on the university's LMS (Blackboard) in the first semesters of 2020 and 2021. Although all lecturers provided input, the module coordinator was responsible for designing the new online classroom. The first idea was to change the traditional face-to-face approach, presented as five sub-groups in an individual approach with an inconsistent "look & feel", to a fully online approach presented as "one" with a combined approach and a consistent "look & feel". Table 1 shows the different elements included in the teaching and learning approach, as well as a short description of how these elements were implemented and used in both years.

Table 1: Elements included in the "new" teaching and learning approach in 2020 and 2021

Elements	Instruments and/or tools	2020/2021 Implementation
Prior knowledge	Diagnostic test	Week 1 (2020 sit down, 2021 online)
Attitudes towards mathematics	SATM	Week 1 (online in 2020 & 2021; in 2020 only some sub-groups participated and in 2021 all sub-groups participated)
Communication	Announcements	Weekly (in both years; the layout of the information was improved in 2021)
Administrative Information	Module schedule, learning guide, policy documents, lecturer/tutor information	Available from week 1 (in both years; in 2021 more information was added)
Tutor support	Discussion forum	Weekly and aligned with content (in both years; in 2021 the support was more structured with a lecturer monitoring the forum regularly and tutors working according to a predetermined schedule)
Content	Step-by-step instructions, lecture slides, homework, virtual lectures/tutorials, online resources, e-textbook	Arranged by week (in both years; in 2021 the structure was improved, and archived options were available from the beginning)
Assessments	Written tasks (submitted via Blackboard) and electronic tasks (submitted via WeBWorK)	Continuous assessment (in 2020 only halfway through the semester, in 2021 from the beginning); several opportunities (almost weekly in both years; in 2021 the structure was improved)

In 2020, students were exposed to the new online approach from weeks seven to 13 (since the COVID-19 restrictions changed the tertiary education sector) and the 2021 cohort were exposed to the new approach from the first academic week up to the end of the first semester. One of the ideas was to use a diagnostic test, which was already developed before COVID-19, to determine students' prior knowledge in certain content areas.

3.3 Some key elements of the 'new' teaching and learning approach

a) *Diagnostic test*

The *test* was designed by Durandt and Blum in 2019 based on guidelines from Stewart (2016) and included the content areas *algebra*, *analytical geometry*, *functions, trigonometry,* *elementary calculus,* and *modelling*. The test consisted of 25 tasks (with 32 items altogether, and a maximum of 38 marks) of which the format and level of difficulty ought to be mostly familiar to high school students in South Africa. The only unfamiliar task was the second of two modelling (real-life) tasks in the final section of the test. The idea behind the modelling question was to expose first-year engineering students to a real-life problem-solving task. Table 2 provides an overview of the key aspects of the test.

Table 2: Features of the diagnostic test with example items used in 2020 and 2021

Knowledge component	Number of items/marks	Example items
Algebra	9/12	Factor the expression $x^3 - 3x^2 - 4x + 12$.
Analytical geometry	6/7	Find an equation for the line that passes through the point $(2, -5)$ and is parallel to the line $2x - 4y = 3$.
Functions	8/8	Find the domain of the function h with $h(x) = \sqrt{x^2 - 1}$.
Trigonometry	3/3	Find all value(s) of x such that $sin2x = sinx$ and $0 \leq x \leq 360^{\circ}$.
Calculus	2/2	Calculate $\lim_{x \to -1} \frac{x^2+4x+13}{x+3}$.
Modelling	4/6	Calculate how much air, approximately, is in the hot-air balloon on the photo (the photo shows a balloon and a man on top of it).

In both years the test was administered in the first week of the academic semester. In 2020, it was administered during an official 45-minute lecture period (sit down, pen-on-paper environment) and participants were not informed of the test ahead of time. In 2021, it was administered on the online platform (via Blackboard) and participants knew about the test beforehand. In both years scientific calculators were only allowed in the final test section (on modelling), although this restriction could not be controlled in 2021. Additionally, in 2021, students had more time to plan for technical difficulties. They were expected to download the test paper, complete the test by writing their solutions on paper, scan and save their work, and finally upload the document onto the Blackboard environment. Numerous problems were reported relating to where to find or submit the paper on the online platform; scanning and saving their work as a PDF file; how to upload documents; uploading a wrong file; computer illiteracy; how to work from a phone; a slow PC; load shedding; network problems and more. To accommodate the numerous problems reported, the test was extended for a further 24-hour period to allow students to master the online environment. Furthermore, they had three attempts and only the last attempt was marked. The differences in the implementation conditions are a limitation and the recorded data cannot be compared.

b) *Survey of Attitudes Towards Mathematics (SATM)*

In 2020 and 2021 we measured the participants' *attitudes* towards mathematics at the beginning of the semester using an internationally well-established instrument, the Survey of Attitudes Towards Statistics (SATS-36, Schau *et al.,* 1995; Schau, 2003), adapted for mathematics. Analogous to the original instrument, six dimensions are differentiated: Affect (6 items, e.g., "I am scared of mathematics"), Cognitive competence (6 items, e.g., "I can learn mathematics"), Value (9 items, e.g., "Mathematics should be a required part of my professional training"), Difficulty (7 items, e.g., "Mathematics is highly technical"), Interest (4 items, e.g., “I am interested in using mathematics”), and Effort (4 items, e.g., “I plan to, or I did, attend every mathematics class session”). The instrument was administered on the online platform (via Blackboard).

c) *Electronic assessments (via WeBWorK)*

Developed at the University of Rochester, WeBWorK is an online-based system for the delivery of homework. It introduces an element of blended learning by providing students with a first-hand look at how computer programs read typed commands. While this is not the focus of the program it teaches the importance of syntax. As a teaching tool, WeBWorK offers an extensive library of questions organised per topic, automated manipulation of task settings (e.g., number of attempts, duration of task availability) and most importantly and conveniently, automated marking. The last point has played a crucial role given the large number of students in the first-year engineering mathematics module. Not only could markers save time, but students received immediate feedback and could learn from their mistakes. The study of Hauk and Segalla (2005) revealed that the use of WeBWorK was at least as useful as pen and paper. However, in the context of South Africa, the issue of internet access and other technical difficulties remain a greatly debated topic (see Mathipa & Mukhari, 2014). See Figure 1 for an example task from a calculus section showing the question, student answer and correct answer.

8. (1 point) Compute the derivative of the given function in two different ways.

$$h(x) = \frac{x^3 - 9x^2}{2x^2}$$

a) Use the Quotient Rule, $\left[\frac{f}{g}\right]' = \frac{g \cdot f' - f \cdot g'}{g^2}$. (Fill in each blank, then simplify.)

$h'(x) = [(\_\_) \cdot (\_\_) - (\_\_) \cdot (\_\_)]/(\_\_)$

$=$ ________.

b) Use algebra first to simplify h, then differentiate without the Quotient Rule.

$h'(x) =$ ________.

Answer(s) submitted:

- `2x^2`
- `3x^2 -18x`
- `x^3 -9x^2`
- `4x`
- `(2x^2)^2`
- `1/2`
- `1/2`

(correct)

Correct Answers:

- `2 x^2`
- `3x^2 + -18 x`
- `x^3 + -9 x^2`
- `4 x`
- `4 x^4`
- `1/2`
- `1/2`

Figure 1. Example item from a WeBWork task in 2021 showing the question, student answer, correct answer and syntax

3.4 Ethical considerations

Standard ethical practices were implemented according to Creswell (2013) and the procedures at the university. The initiative reported on in this article forms part of a larger study that seeks the ongoing development of first-year mathematics students' mathematical and modelling competencies. Participation in the diagnostic test and SATM instrument was voluntary, and participants were informed of their respective purposes. The authors of this article were also lecturers in the first-year engineering mathematics modules in 2020 and 2021. This is a limitation and was purposefully addressed by balancing the roles. Due to technical reasons data collected from some teaching elements could not be reported on, nor compared, which is another limitation of this study.

4. Discussion and results

Most of the empirical data reported in this section of this article were collected from the 2021 cohort. Some data from the 2020 cohort have already been published (see Durandt, Blum & Lindl, 2021a). This section of the paper provides an answer for the second and third research questions. All data processing and analyses were conducted using the statistical software R (R Core Team, 2020), Excel or the statistical analysis tool in Blackboard.

4.1 Engineering students' prior knowledge and attitudes towards mathematics

Some elements fundamental in the "new" online classroom is to determine students' prior-knowledge and attitudes towards mathematics. The results from the diagnostic test in 2020 are reported elsewhere (see Durandt *et al.,* 2021a), but it is worth mentioning here that, on average, the participants achieved only about one-third of all possible marks in the overall test. Previously, researchers reported one reason for this result could have been a too strict time frame. The 2021 results show that students performed better, on average, achieving almost half of all possible marks (including all test items). A direct comparison between results over the two years is not possible due to the different implementation conditions (see sub-section 3.3). Table 3 below shows the descriptive statistics and the mark distribution per interval from the 2021 cohort. It is interesting to note the large number of students (approximately 65%) in the intervals less than 50% and the small number of students (approximately 2%) in the intervals above 70%. One reason might be that students did not take the diagnostic test seriously.

Table 3: The descriptive statistics and mark distribution of the 2021 cohort in the diagnostic test

Descriptive statistics		**Mark distribution (Interval/number)**	
Count (N)	738	90–100	0
Minimum Value	0.00	80–89	1
Maximum Value	31.00	70–79	14
Range	31.00	60–69	68
Average	16.03	50–59	169
Median	16.00	40–49	155
Standard Deviation	5.48	30–39	189
Variance	29.89	20–29	98
		10–19	28
		1–9	16

Table 4 shows the descriptive statistics of the SATM collected from the 2021 cohort. A 7-point rating scale was used (1 = strongly disagree, …., 7 = strongly agree). Variations of this survey (SATS and SATMM) have been used in the South African context (compare Durandt, Blum & Lindl, 2021b; van Appel & Durandt, 2018). As can be seen in Figure 2 below, positive attitudes exist for first-year students in 2021 in terms of affect, cognitive competence and value, while strong positive attitudes exist in interest and effort. More negative attitudes are shown for difficulty. It is interesting to note the number of outliers shown in all six dimensions.

Table 4: The descriptive statistics of the Survey of Attitudes Towards Mathematics (SATM) results from the 2021 cohort

	N	mean	Sd	median	Trimmed	min	max	range	skew	kurtosis
aff	525	5.65	1.09	5.83	5.75	1.5	7	5.5	-0.81	0.28
cog	522	5.72	0.89	5.83	5.78	2.83	7	4.17	-0.6	-0.21
val	523	5.97	0.74	6.11	6.04	2.89	7	4.11	-0.89	0.85
diff	523	3.19	0.74	3.14	3.18	1.29	6	4.71	0.21	0.19
int	525	6.55	0.74	7	6.7	1.5	7	5.5	-2.56	9.4
eff	526	6.51	0.75	7	6.66	2.5	7	4.5	-1.85	3.56

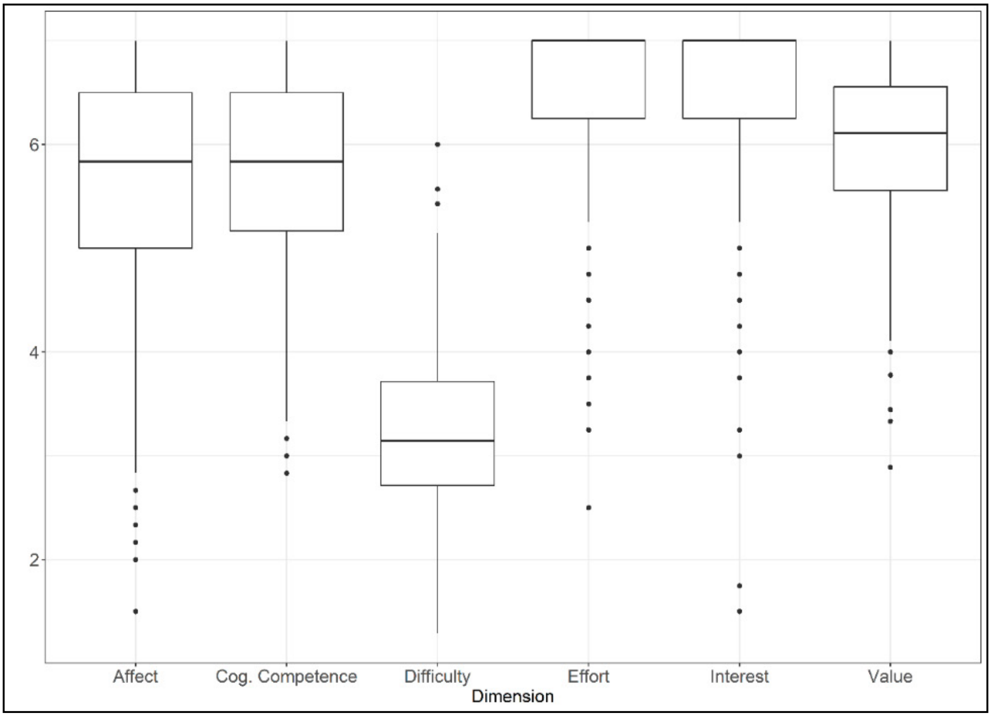

Figure 2: Attitudes towards mathematical modelling with six dimensions (7-point rating scale: 1 = strongly disagree, …, 7 = strongly agree)

Both aspects, namely prior knowledge and attitude, are indicators for academic success (Quinn & Aaräo, 2020) and it is advisable for educators to collect information on these elements early in the academic semester.

4.2 Engineering students' engagement in online activities

The student engagement in online activities, in 2021, were monitored and are reported here. Due to technical reasons the data from the 2020 cohort could not be analysed which is a limitation. Figure 3a shows the percentage of hits comparing continuous and formal assessments (diagnostic test, semester test 1 and 2, sick tests and deferred examination), support elements (tutor support and discussion forum) and standard university documents (module evaluation and policies).

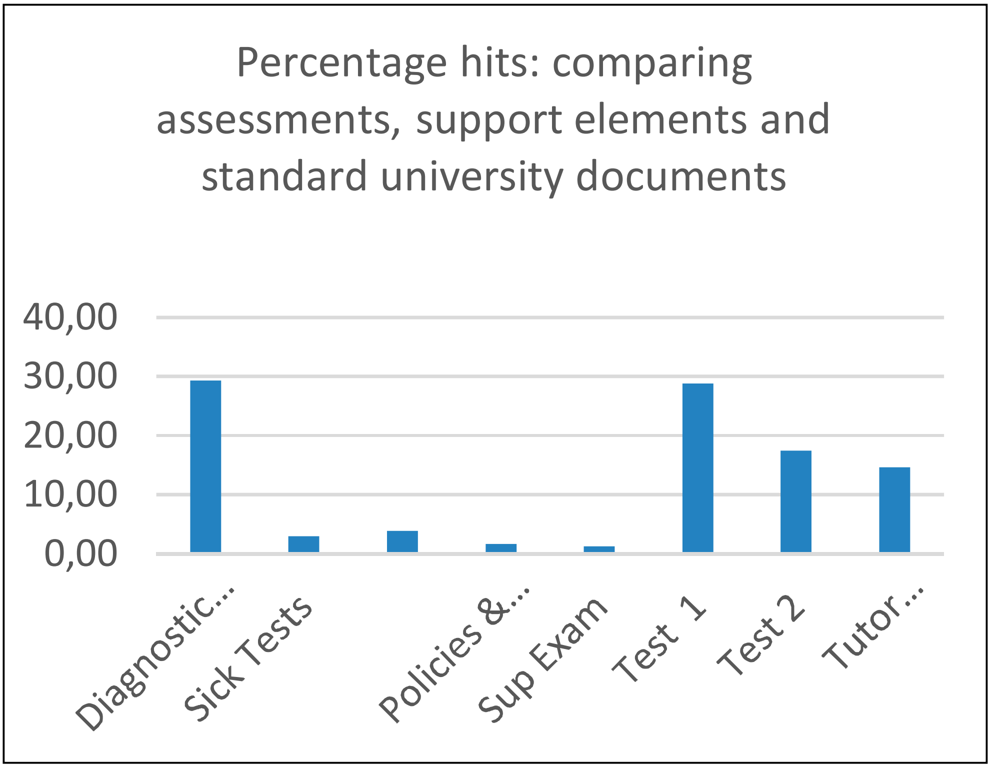

Figure 3a: Comparing percentage hits

According to expectations, much more hits are reported for assessments than for standard university documents. The assessments shown here were written on paper and submitted online on the LMS (Blackboard). For each assessment students were allowed three submissions to allow for technical difficulties. It seems that students became more comfortable with the environment throughout the semester (comparing test 2 hits with the diagnostic and test 1 hits). The support element accounts for approximately 14% of hits.

Figure 3b shows the number of students attending the virtual classroom per week divided into three categories: less than 30 minutes, between 30 and 60 minutes and longer than 60 minutes.

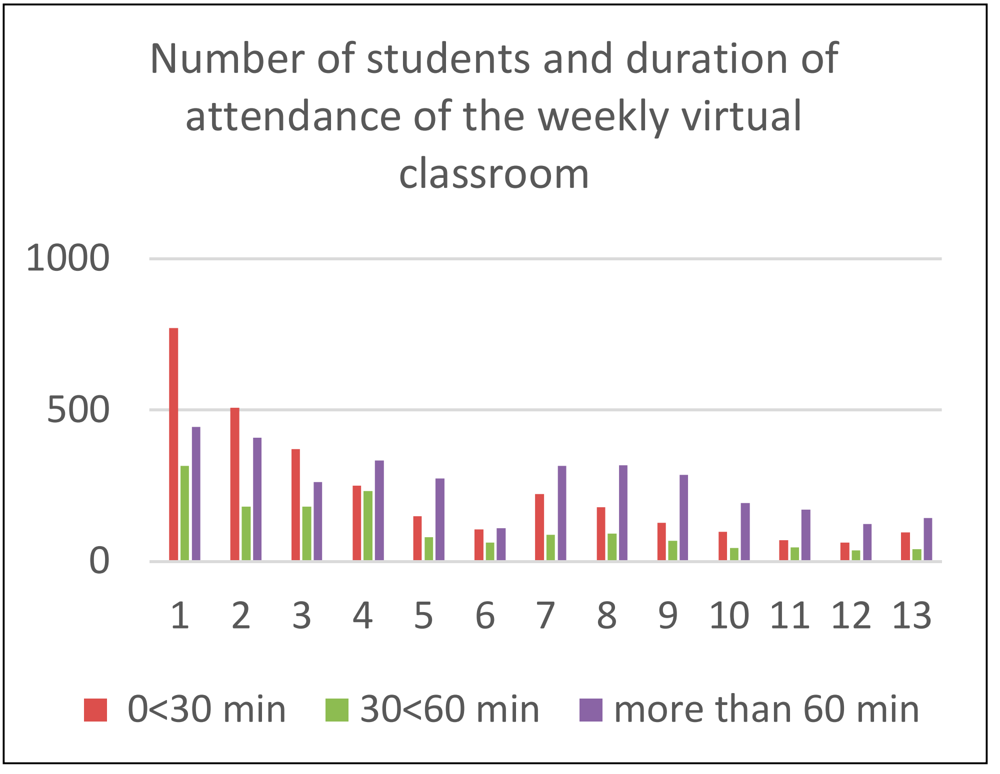

Figure 3b: Attendance and duration of the virtual classroom

Three live sessions (90 minutes each) were available to students each week and according to their timetables they could attend one or more sessions. It is noticeable that some students attended more than one session per week especially during the first two weeks. We report 1529 and 1095 attendees for the first and second week, respectively. These numbers exceed the official number of students registered on the LMS. More students attended less than 30 minutes during the first three weeks. This pattern was reversed from week four, with many students attending more than 60 minutes. One reason might be that students attended longer as the content became more challenging. Overall, student attendance decreased over 13 weeks whilst the duration and percentage of those students who attended more than 60 minutes increased. The average attendance duration increased from approximately 24 minutes in the first three weeks of the semester to approximately 39 minutes during the last three weeks of the semester. On average approximately 33% of students attended for more than 60 minutes in the first three weeks of the semester (week 1 – 29%; week 2 – 37%; week 3 – 32%) compared to an average attendance of approximately 56% during the last three weeks of the semester (week 11 – 60%; week 12 – 56%; week 13 – 51%). One reason for this pattern might be that a large portion of students became more comfortable studying on their own and relying mainly on online material and archived lectures. However, there was a surge around scheduled semester test weeks (weeks 4 and 9). For these two weeks in particular, students' typical attendance was approximately 31 minutes with an average attendance of more than 60 minutes of 50%.

Figure 3c shows the user activity per week in content areas on the LMS (Blackboard). The data also includes uploads per week.

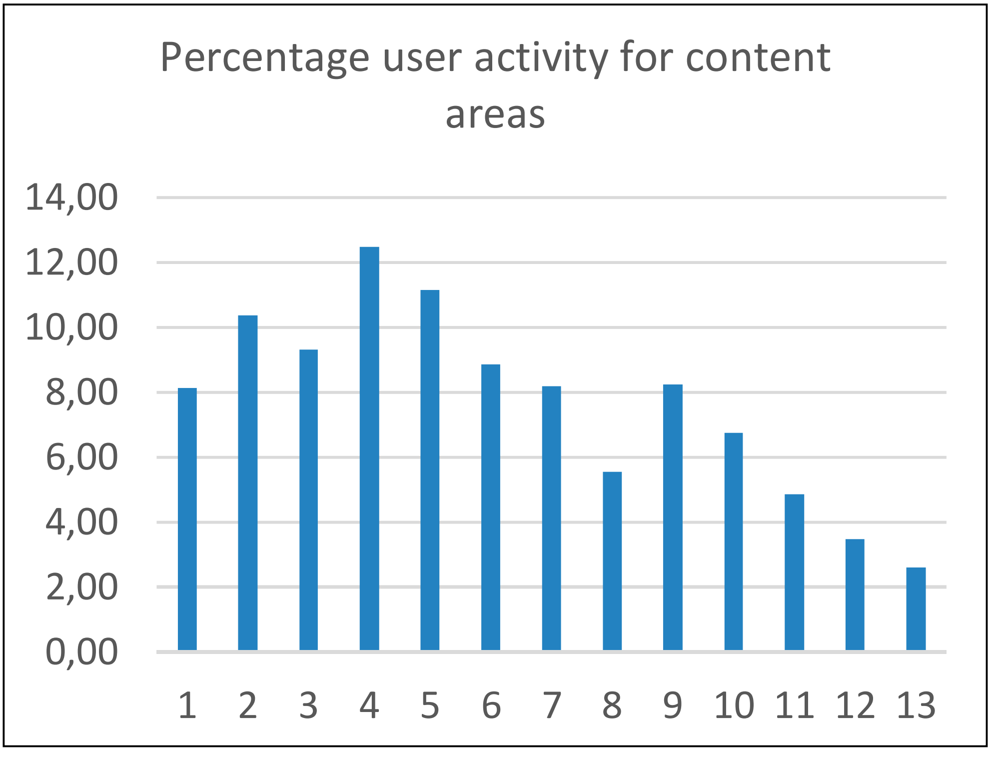

Figure 3c: User activity per content area per week

An increase in activity can be seen in weeks four and nine, which corresponds with the scheduled semester tests. The average of these two weeks is approximately 10% compared to the overall average of approximately 8%. The overall range is calculated at approximately 10%, with a median of approximately 8%. Overall, from week four the user activity decreased. This is slightly unexpected but can indicate that the students became more familiar with the environment. They could find online material with greater ease later in the semester.

Figure 3d shows the number of messages posted per week on the tutor support and discussion forum platform.

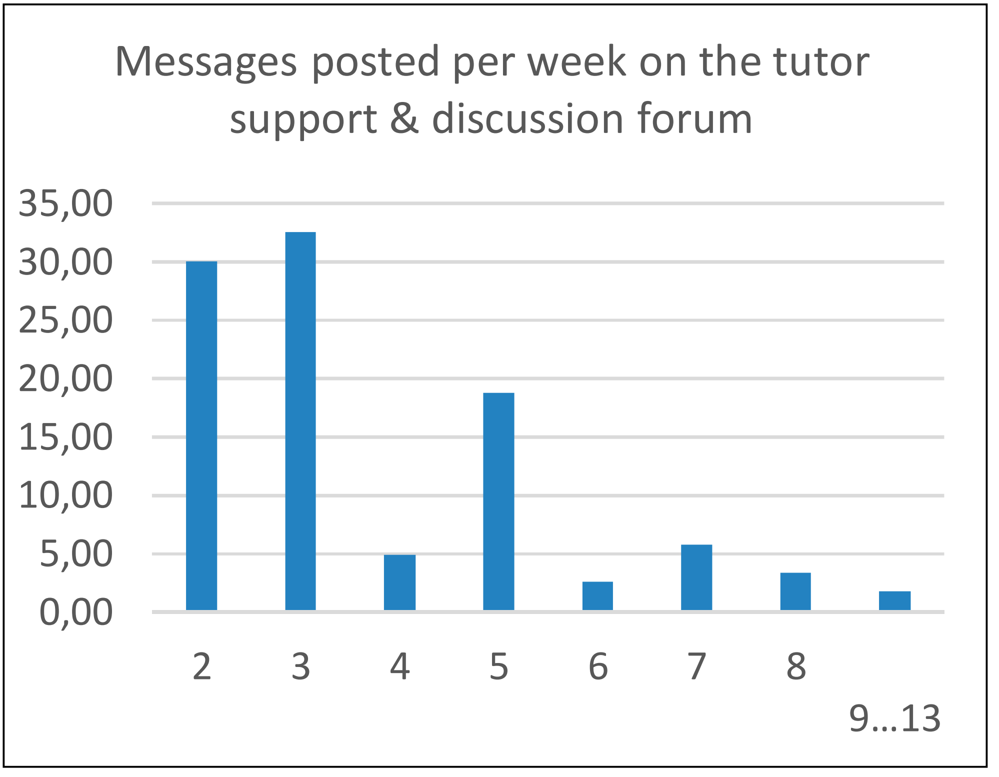

Figure 3d: Messages on the tutor support and discussion forum

Overall, a low number of messages were posted, which was rather disappointing. Surprisingly, unlike shown in figures 3b and 3c, not much activity is recorded in weeks four and nine (week 4 – approximately 5% messages compared to an average of approximately 8% per week). One reason might be that students are unfamiliar with asking questions and discussing mathematical content on a public forum. The user activity also decreased towards the end of the semester, which corresponds with data reported in Figures 3a, 3b and 3c.

4.3 Monitoring engineering students' progress on WeBWorK

In this section, we report on data generated from the WeBWorK platform over two years (2020 & 2021), in order to investigate its usefulness as a tool for online teaching and learning. WeBWorK makes two forms of statistical information available. First, information provided directly by the platform that is purely related to the performance of active students and lacks the more traditional statistics such as participation and pass rates. Second, information by the marksheet that WeBWorK allows for export as a .csv file. The latter of these data provided the basis for some useful statistical information which is displayed in Table 5 below, while the former is presented in scatter plots (see Figures 4a and 4b).

Data extracted over the two years provide some insight into how students perceive the use of WeBWorK and its value as a tool for learning at a distance. The main idea was to utilise the tool for continuous assessments, although in both years the tool was also used for a formal assessment near the end of the semester. We note three differences over two years: (i) in

2021, continuous assessment tasks were given from the beginning of the semester, whereas the migration to online teaching and learning forced the class of 2020 into it. (ii) In 2021, the tasks took on a weekly structure, whereas a more sectional approach with variable task sizes was followed in 2020, and lastly, (iii) in 2021, the tasks carried a larger contribution of marks (therefore the need to fix the task size and settings), whereas in 2020, the number of tasks, and questions per task, varied.

The WeBWorK platform is entirely disconnected from the university's learning management system meaning that students are added to WeBWorK manually. This has implications on the data because students who later deregister are not necessarily manually removed from the platform's list of students. To this end, data presented in Table 5 required some refinement in which students who did not participate in any task were removed from the list. In both cases, this reduced the number of students on the class list from 1017 to 876 for the 2020 class, and from 1417 to 958 for the 2021 class.

Table 5: Statistical data for WeBWorK activities over the two-year period; 2020 and 2021

Descriptor	Year 2020	Year 2021
Number of active students	876	958
Total number of continuous assessment (CA) tasks	8	7
Minimum number of questions per CA task	10	15
Maximum number of questions per CA task	15	15
Average percentage across CA tasks	70.22%	80.93%
Average pass rate across CA tasks	78.06%	92.67%
Formal test total	45	40
Formal test number of questions	45	18
Formal test average	80.05%	80.98%
Formal test pass rate	89.38%	89.68%
Formal test highest pass rate	88.93%	93.84%
Formal test lowest pass rate	35.50%	66.84%
Highest scoring section	Introduction to complex numbers	Functions
Worst scoring section	Polar form of complex numbers	Prior knowledge components focusing on algebra

From Table 5 the following observations are noticeable:

- The number of active students on the WeBWork platform in 2021 was slightly larger than the number of active students in 2020, this is likely due to the acclimatisation to lockdown conditions.
- The class average and pass rate were significantly higher for the 2021 cohort which was likely due to the class starting the WeBWorK tasks from the beginning of the semester.

- The performance on the WeBWorK formal test was almost identical across both years with 2021, again, taking the lead by a very small margin. This is despite the test structure being dramatically different across the two years: the 2020 formal test was given over three days and had 45 individual items (with a mark each) and the 2021 formal test was more conventional with only 18 items carrying a total of 40 marks. This test was only open for a day.
- The 2021 cohort saw higher, best and worst averages for the continuous assessment tasks (with the lowest averages corresponding to the first task). One reason might be the slow start to the 2021 academic year with many students joining the platform well after the first task was closed. This contrasts with the 2020 group that experienced its lowest average toward the end of the first term, with the sudden shift from face-to-face to online teaching.

Figures 4a and 4b show a relationship between the percentage of active students with correct answers and the number of attempts. We notice a similar and obvious trend emerging from both years that the average number of attempts decrease with the percentage of active students with correct answers.

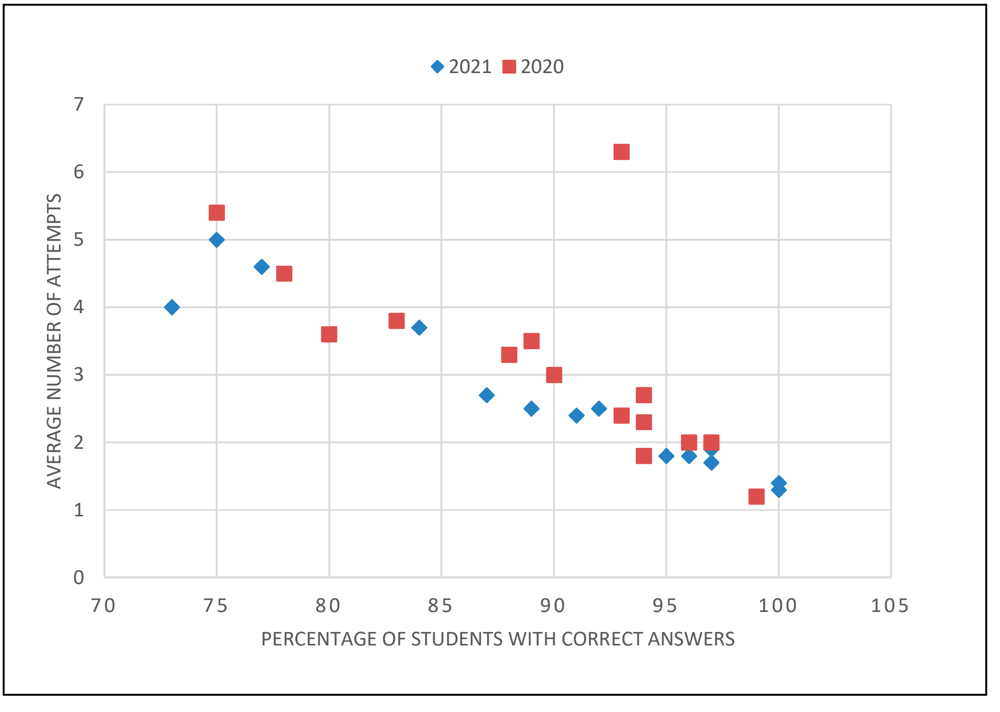

Figure 4a: A plot of the percentage of students with correct answers versus the average number of attempts per question for continuous assessment tasks with an unlimited number of attempts offered (2020 & 2021)

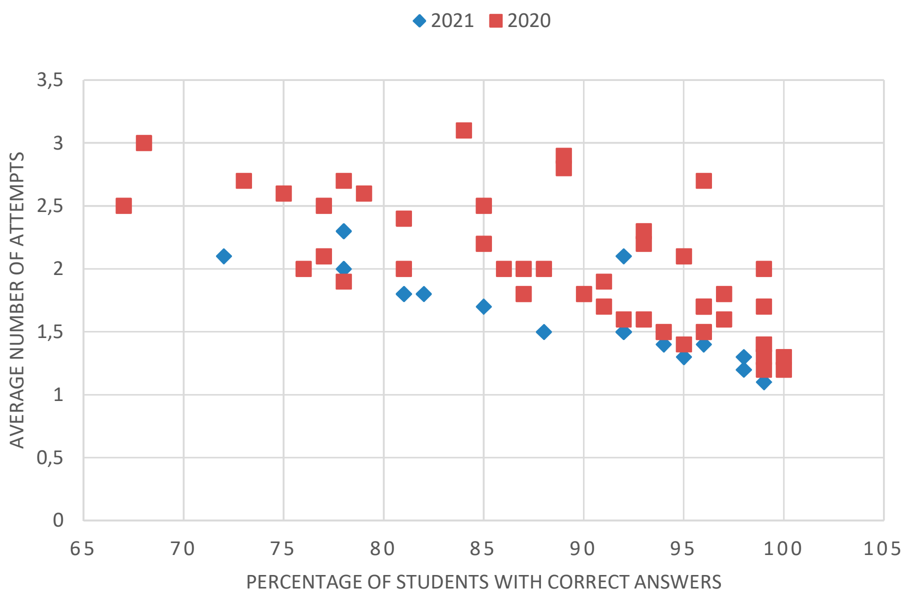

Figure 4b: A plot of the percentage of students with correct answers versus the average number of attempts for the formal test on WeBWorK (2020 & 2021)

In Figure 4a, students had an unlimited number of attempts available for the continuous assessment tasks, while for the formal tests (see Figure 4b) the maximum number of attempts was limited to five. A greater spreading of the 2020 data is noticed in the latter figure owing to the large number of questions in the formal test. It is interesting to note that, from the idea that the slope of a linear regression line through the data is indicative of the perceived difficulty of the task.

5. Conclusion

This article reported on empirical results collected from the online teaching and learning environment of a large group of first-year engineering mathematics students when learning at a distance, after the "sudden" shift from face-to-face to fully online teaching early in 2020. The investigation took place over two consecutive years (2020 and 2021). The main aim was to identify fundamental elements in this "new" classroom environment, and to monitor students' involvement and progress over the period, and on different platforms (Blackboard and WeBWorK).

One of the fundamental elements is to determine students' level of prior knowledge that seems low, as expected, and another element is to determine students' attitudes. In this study students showed an interest in studying mathematics and a willingness to try, although they view mathematics as a difficult subject. Quinn and Aaräo (2020) reported that first-year students do not have the correct attitude to be successful when dealing with first-year engineering mathematics and that the online environment (through quizzes) can be used to catch up on assumed knowledge for students transitioning to university.

Furthermore, results indicate that students became more familiar with online learning as the semester progressed. This result supports the growth in mathematics students' organisational maturity towards the end of the academic semester as reported in the study by Yani, Harding and Engelbrecht (2019). Learning conversations can be automated through the discussion forum on Blackboard, between students, tutors and lecturers, although it seems that students need to develop more confidence in using this tool. Students accessed the environment more often in the time of formal assessments.

In a survey of over 2000 students, Roth, Ivanchenko and Record (2008) found that the use of WebWorK resulted in a reduction in calculation error. The major complaint from students, as mentioned in this work, was related to syntactic difficulties. However, in our study, the main complaint came from login difficulties. Nevertheless, regular continuous assessment tasks on WeBWorK appear to be valuable. In this study, improvement in student performance is demonstrated in the overall high marks.

More in-depth analysis of student participation and performance related to all teaching and learning elements could add further insight. We argue teaching over a distance requires a well-organised environment with a variety of tools and assessment opportunities to keep students interested and connected.

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