*Proceedings of Engineering and Technology Innovation, vol. 10, 2018, pp. 13 - 18*

# Robust Control of Delayed Fin Stabilizer Stochastic Systems of a Ship

Cheung-Chieh Ku*, Chin-Yuan Hsu

Department of Marine Engineering, National Taiwan Ocean University, Keelung, Taiwan, ROC

Received 02 March 2018; received in revised form 24 June 2018; accepted 29 July 2018

## Abstract

In this paper, the robust control problem of delayed fin stabilizer stochastic system of a ship with uncertainty is discussed and investigated. To describe the system, Linear Parameter Varying (LPV) modelling approach and multiplicative noise term are used to establish the corresponding polynomial model. For simulating the general operating environment, the delay effect is considered as time-varying case. Moreover, the gain-scheduled control scheme is employed to discuss the delay-dependent stabilization problem and to design the corresponding controller. Moreover, a novel Lyapunov-Kravoskii function is proposed by using parameter-dependent matrix and integral Lyapunov function to reduce the conservatism of the derived stability conditions. In order to apply the convex optimization algorithm, the derived conditions are converted into Linear Matrix Inequality (LMI) form. By solving the conditions, some feasible solutions can be obtained to establish the controller to guarantee robust stability of the delayed fin stabilizer stochastic system of a ship in the mean square.

**Keywords:** LPV system, stochastic behavior, LMI, fin stabilizer system of a ship

## 1. Introduction

Generally, the comfort of passengers is deeply affected by sea waves according to violent rolling. Therefore, an important issue of reducing the roll motion of the ship is usually discussed for building a liner or a passenger ship. Besides, a large amplitude rolling motion may cause damage to the cargoes and vessels during the shipping process. For the above reasons, an effectiveness of fin stabilizer was investigated in [1-7] to reduce the rolling motions. Generally, a pair of fins consists a ship fin stabilizer which locates approximately amidships on the bilge of the hull. Those fins can be used to control changes of ship roll angle and its rate. Through changing the angle [5-6], the hydrodynamic forces are induced on the fins to produce a moment that can reduce the wave induced roll motion. Furthermore, the state feedback control method [3-4] is usually used to deal with the fin stabilizer system. However, it is difficult for reaching control target of the fin stabilizer system because the uncertainties appear in calculating the fin lift from fin angle. For the reason, LPV system is applied to describe the dynamics of the fin stabilizer system in this paper and to discuss the stabilization problem.

LPV system [8-13] has been built by several linear systems and a specific weighting function to represent uncertain systems or nonlinear systems. According to the structure of LPV system, a general description for uncertain systems can be proposed to represent complex uncertainties. In [12-13], a Gain-Schedule (GS) control scheme has been applied to deal with the stabilization problem of LPV system. Because the structure of GS controller is similar to LPV system, the robustness of the described system can be increased. It means that the GS scheme is vary suitable control scheme for systems described by numerous sub-systems. Therefore, many robust stability criteria [10-13] have been proposed for LPV systems via applying the GS design scheme. Furthermore, many practical robust control applications [9, 13] have been achieved via LPV system and

* Corresponding author. E-mail address: ccku@mail.ntou.edu.tw

Tel.: +886-2-24622192; Fax: +886-2-24620724

GS control scheme. In this paper, the LPV system is applied to represent the uncertain fin stabilizer system and the GS control scheme is used to discuss the stabilization problem of the system.

In addition to uncertainty, effect of time delay always exists in dynamic process such as propagation/transportation of material, information or energy. The similar effect of time delay appears in the fin stabilizer system due to signal transportation. Generally, delay-dependent criterion [11, 14-16] is discussed for general effect of time delay due to conservatism caused by constant delay. Besides, some relaxed technologies [15-16] for delay-dependent criteria are developed to reduce conservatism of stability conditions. Moreover, Lyapunov-Krasovskii function is often applied to derive sufficient conditions to analyze the stability of delayed system. Thus, time delay effecting on state is also concerned with the considered control problem of the uncertain fin stabilizer system.

Moreover, stochastic behaviour [17-20] caused by high sea states often exists in operation during the shipping process. However, stochastic behaviour cannot be measured and predicted in the practical operation. Referring to [17], stochastic differential equation provides a powerful tool to describe the stochastic behaviours. In the stochastic differential equation, stochastic behaviour is modelled as multiplicative noise term which is consisted by system states and white noise. Hence, the stochastic difference equation has been applied in many control fields to extend design methods from deterministic systems to stochastic systems. Based on the stochastic differential equation, many stability criteria have been developed to deal with stability issue of stochastic systems. From [21-22], one can find that the LPV system with multiplicative noise term can be successfully employed to represent uncertain stochastic systems. Therefore, a stability criterion of delayed uncertain fin stabilizer stochastic system of a ship is discussed in this paper.

To discuss the considered criterion, a delayed LPV stochastic system is built to describe practical fin stabilizer system of a ship with time delay, time-varying parameters and stochastic behaviors. Based on the modelling approach and stochastic differential equation, the linear systems with multiplicative noise term in LPV stochastic system can be modelled by setting varying range and number of time-varying parameters. In order to analyze stability of the system, some relaxed sufficient conditions are derived via Lyapunov-Krasovskii function and Jensen inequality [14]. In order to apply convex optimization algorithm, those stability conditions are converted into Linear Matrix Inequality (LMI) form [23]. Through solving the conditions, the feasible solutions can be obtained to build GS controller such that the considered fin stabilizer system of a ship is robust stable. Based on the simulation result, the fin stabilizer system of a ship can be stabilized with the added delay effect and stochastic behavior.

## 2. System Description and Problem Formulation

Referring to [4], the dynamics of fin stabilizer system of a ship can be represented as Fig. 1. Based on Fig. 1, the dynamic equation of fin stabilizer system can be described as follows:

$$\left(I_x+\Delta I_x\right)\ddot{\psi}+B_1\dot{\psi}+B_2\left|\dot{\psi}\right|\dot{\psi}+C_1\psi+C_3\psi^3+C_5\psi^5=M_d-M_s \tag{1}$$

where $\psi$ denotes the ship's rolling angle, $I_x$ and $\Delta I_x$ denote the mass inertia moment and affixing mass inertia moment relative to the vertical axes of ship, $M_d$ denotes the disturbance moment of sea waves, $M_s$ denotes the stable moment of lift feedback fins. Besides, $C_1$, $C_3$, $C_5$, $B_1$, $B_2$ are constants and $C_1=Dh$, where $D$ denotes the ship's tonnage and $h$ denotes the height of steady center of ship's rolling. In this paper, the parameters of a certain ship are assumed as $D=1457.26t$, $h=1.15m$, $I_x+\Delta I_x=3.4383\times10^6$, $C_3=2.097\times10^6$, $C_5=4.814\times10^6$, $B_1=0.636\times10^6$, $B_2=0.79\times10^6$ .Let $M_t=M_d+M_s$, the nonlinear model of this ship's rolling can be described as follows:

$$\ddot{\psi}+0.185\dot{\psi}+0.23|\dot{\psi}|\dot{\psi}+0.4874\psi+0.61\psi^3+1.4\psi^5=2.9084\times10^{-7}M_t \tag{2}$$

To consider Eq. (2) operated around $0^0$, the nonlinear fin stabilizer system can be rewritten as a linear system. Moreover, the delay effect, uncertainties and stochastic behavior are added to describe the natural unstable source. Furthermore, the following delayed LPV stochastic systems is provided to represent (2) with the considered effects.

$$dx(t)=\sum_{i=1}^{2}\alpha_i(t)\Big(\big(\mathbf{A}_i x(t)+\mathbf{A}_{ti}x(t-d(t))+\mathbf{B}_i u(t)\big)dt+\mathbf{E}_i x(t)d\beta(t)\Big) \tag{3}$$

where $\mathbf{A_1}=\begin{bmatrix}0 & 1\\ -0.458156 & -0.185\end{bmatrix}$, $\mathbf{A_2}=\begin{bmatrix}0 & 1\\ -0.516644 & -0.185\end{bmatrix}$, $\mathbf{A_{t1}}=\begin{bmatrix}0 & 1\\ -0.0195 & 0\end{bmatrix}$, $\mathbf{A_{t2}}=\begin{bmatrix}0 & 0\\ -0.0195 & 0\end{bmatrix}$, $\mathbf{B_1}=\begin{bmatrix}0\\ 2.9084\times10^{-7}\end{bmatrix}$, $\mathbf{B_2}=\begin{bmatrix}0\\ 2.9084\times10^{-7}\end{bmatrix}$, $\mathbf{E_1}=\begin{bmatrix}0 & 0\\ 0 & 0.0698\end{bmatrix}$, $\mathbf{E_2}=\begin{bmatrix}0 & 0\\ 0 & 0.0698\end{bmatrix}$, $a_1(t)=|\sin(t)|$ and $a_2(t)=1-|\sin(t)|\cdot$ $0\le d(t)\le d_M$, and $\dot{d}(t)\le\varepsilon<1$. $d\beta(t)$ is scalar continuous type Brownian motion satisfying the independent increment property [18].

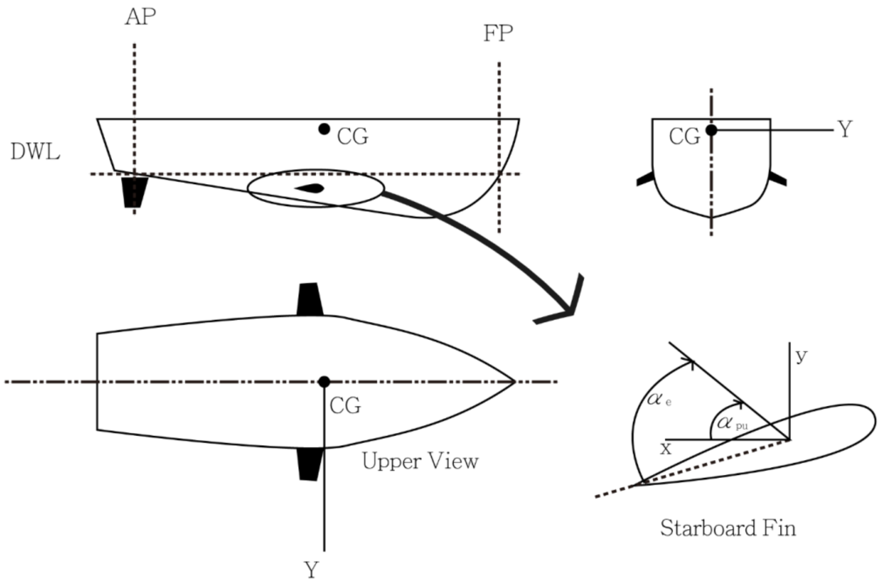


Fig. 1 Dynamic of fin stabilizer system

To deal with stabilization problem of Eq. (3), the following gain-scheduled controller is designed via state-feedback control concept.

$$u(t)=\sum_{j=1}^{2}\alpha_j(t)\big(-\mathbf{F}_j x(t)\big) \tag{4}$$

Substituting Eq. (4) into Eq. (3), the following closed-loop system can be inferred:

$$\begin{aligned}dx(t)&=\sum_{i=1}^{2}\sum_{j=1}^{2}\alpha_i(t)\alpha_j(t)\Big(\big((\mathbf{A}_i-\mathbf{B}_i\mathbf{F}_j)x(t)+\mathbf{A}_{ti}x(t-d(t))\big)dt+\mathbf{E}_i x(t)d\beta(t)\Big)\\ &=\sum_{i=1}^{2}\sum_{j=1}^{2}\alpha_i(t)\alpha_j(t)\Big(\big((\mathbf{G}_{ij})x(t)+\mathbf{A}_{ti}x(t-d(t))\big)dt+\mathbf{E}_i x(t)d\beta(t)\Big)\end{aligned} \tag{5}$$

where $\mathbf{G}_{ij}=\mathbf{A}_i-\mathbf{B}_i\mathbf{F}_j$.

### *2.1. Lemma 1*

For any constant matrix $\mathbf{M}>0$, scalars $r_1$ and $r_2$ satisfying $r_2>r_1$, a vector function $x(t)$: $[0,\bar{\tau}]\to\Re^{n_x}$ such that the integrals concerned are well defined, the following inequality holds.

$$\left(\int_{r_1}^{r_2} x(s)ds\right)^{\mathrm{T}} \mathbf{M}\left(\int_{r_1}^{r_2} x(s)ds\right) \le (r_2 - r_1)\int_{r_1}^{r_2} x(s)\mathbf{M}x(s)ds$$

In the following section, a stability criterion is proposed to deal with the stability issue of the close-loop system (5). Based on the proposed stability criterion, the robust stability of the closed-loop system (5) can be guaranteed.

## 3. Delay-Dependent Stability Criterion

In this section, the stability criterion subject to time-varying delay performances for closed-loop system Eq. (5) is developed. Some sufficient conditions are derived via applying Lyapunov-Krasovskii function and Itô's formula in the following theorem.

### 3.1. Theorem 1

For given scalars $d_M$ and $\varepsilon$, the closed-loop system (5) is asymptotically stable if there exists $\mathbf{P}_i = \mathbf{P}_i^{\mathrm{T}} > 0$, $\mathbf{Q} > 0$, and $\mathbf{R} > 0$ such that the following conditions are satisfied.

$$\begin{bmatrix} \mathbf{P}_i\mathbf{G}_{ij} + \mathbf{G}_{ij}^{\mathrm{T}}\mathbf{P}_i + \mathbf{E}_i^{\mathrm{T}}\mathbf{P}_i\mathbf{E}_i + \mathbf{Q} + d_M\mathbf{R} & \mathbf{P}_i\mathbf{A}_{ti} & 0 \\ * & -(1-\varepsilon)\mathbf{Q} & 0 \\ * & * & -\frac{(1-\varepsilon)}{d_M}\mathbf{R} \end{bmatrix} < 0 \;\; for \;\; i,j = 1,2 \tag{6}$$

In order to limitation of pages, the proof of this theorem is omitted. Although the condition in Theorem 1 belongs to bilinear matrix inequality, the similar conversion and algorithm in [21] can be used to convert Eq.(6) into LMI problem. In the following section, the numerical simulation is proposed.

## 4. Simulation Results

To apply the proposed design method, $d_M = 10$ and $\varepsilon = 0.7$ are set. Moreover, the following feasible solutions can be obtained.

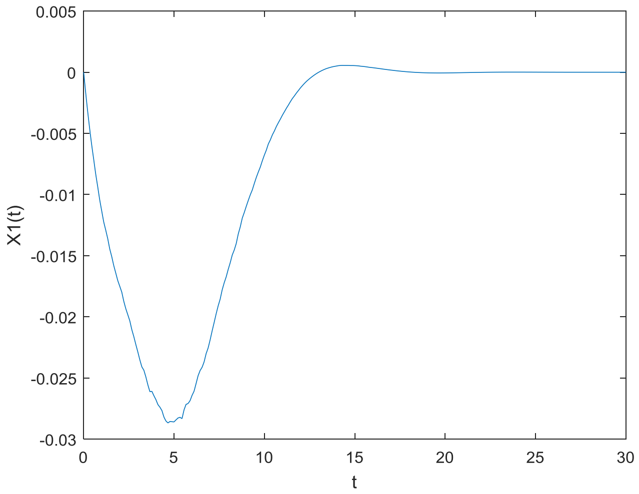


Fig. 2 Responses of $\chi_1(t)$

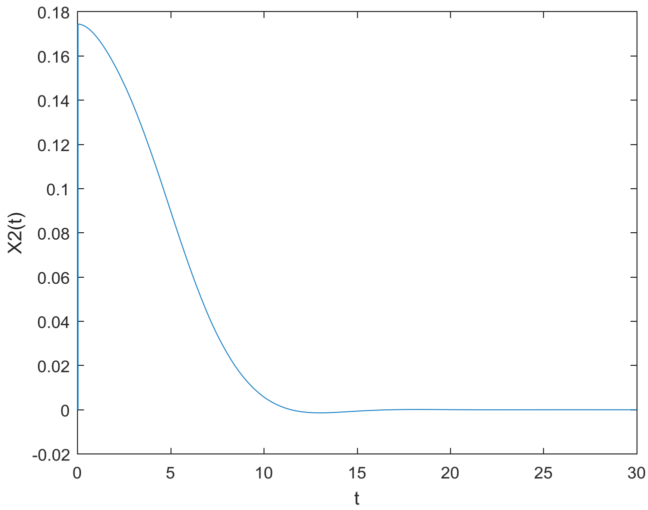


Fig. 3 Responses of $\chi_2(t)$

$\mathbf{P_1} = \begin{bmatrix} 0.9271 & 0.8592 \\ 0.8592 & 6.1864 \end{bmatrix} \times 10^7$, $\mathbf{P_2} = \begin{bmatrix} 0.3741 & 0.8592 \\ 0.8592 & 6.6161 \end{bmatrix} \times 10^7$, $\mathbf{F}_1 = |-1.2315 \quad 3.1461| \times 10^6$, $\mathbf{F}_2 = |-1.2315 \quad 3.1461| \times 10^6$, $\mathbf{Q} = \begin{bmatrix} 0.2910 & 0.1170 \\ 0.1170 & 7.5274 \end{bmatrix} \times 10^6$, $\mathbf{R} = \begin{bmatrix} 0.0151 & 0.1212 \\ 0.1212 & 7.6452 \end{bmatrix} \times 10^6$, and $\alpha = -0.1287$ with the above feasible solutions, the GS controller (4) can be designed. Based on the designed GS controller, the responses of (3) are stated in Fig. 2-3 with initial condition $x(0) = \begin{bmatrix} \frac{\pi}{18} & 0 \end{bmatrix}^T$. From Figs. (2)-(3), the state of Eq. (3) are converged to zero. According to simulation result, the proposed design method can be used to stabilize the uncertain fin stabilizer stochastic system with time delay.

## 5. Conclusions

In this paper, a GS controller design method was proposed to deal with stabilization problem of uncertain fin stabilizer stochastic system with time delay. Based on the modelling approaches, a LPV system with multiplicative noise was provided to describe the considered system. For the system, some conditions were derived via Lyapunov-Krasovskii function and Jensen inequality. By solving the derived conditions, the feasible solutions can be obtained to build GS controller such that the closed-loop system is robust stability. Finally, a simulation result has been provided to demonstrate the effectiveness and usefulness of the proposed design method.

## Acknowledgement

This work was supported by the Ministry of Science and Technology, under Contract MOST 106-2221-E-019-006.

## References

[1] H. Jin, Y. Gu, B. Wang, and X. Hou, "Variable parameters PID controller design of fin stabilizer," Ship Engineering, vol. 4, pp. 55-60, 1994. (In Chinese)

[2] H. Jin, Y. Luo, and Z. Wang, "Design of sliding mode variable structure controller for electric servo system of fin stabilizer," Proc. of the Sixth World Congress on Intelligent Control and Automation, vol. 1, 2006, pp. 2086-2090.

[3] Y. H. Liang, H. Z. Jin, and L. H. Liang, "Fuzzy-PID controlled lift feedback fin stabilizer," Journal of Marine Science and Application, vol. 7, no. 2, pp. 127-134, June 2008.

[4] T. Perez and G. C. Goodwin, "Constrained predictive control of ship fin stabilizers to prevent dynamic stall," Control Engineering Practice, vol. 16, no. 4, pp. 482-494, April 2008.

[5] C. Y. Tzeng and C. Y. Wu, "On the design and analysis of ship stabilizing fin controller," Journal of Marine Science and Technology, vol. 8, no. 2, pp. 117-124, January 2000.

[6] N. Wang, X. Meng, and Q. Xu, "Fuzzy control system design and stability analysis for ship lift feedback fin stabilizer," Proc. of 7th World Congress on Intelligent Control and Automation, IEEE Press, August 2008, pp. 1223-1228.

[7] Z. H. Xiu and G. Ren, "Fuzzy controller design and stability analysis for ship's lift-feedback-fin stabilizer," Proc. of the 2003 IEEE International Conf. Intelligent Transportation Systems, IEEE Press, December 2003, pp. 1692-1697.

[8] F. Blanchini, D. Casagrande, S. Miani, and U. Viaro, "Stable LPV realization of parametric transfer functions and its application to gain-scheduling control design," IEEE Transactions on Automatic Control, vol. 55, no. 10, pp. 2271-2281, October 2010.

[9] L. Liu, X. Wei, and X. Liu, "LPV control for the air path system of diesel engines," IEEE International Conf. Control and Automation, November 2007, pp. 873-878.

[10] J. Daafouz, J. Bernussou, and J. C. Geromel, "On inexact LPV control design of continuous-time polytopic systems," IEEE Transactions on Automatic Control, vol. 53, no. 7, pp. 1674-1678, August 2008.

[11] F. Wu and K. M. Grigoriadis, "LPV systems with parameter-varying time delays: analysis and control," Automatica, vol. 37, no. 2, pp. 221-229, February 2001.

[12] M. G. Yoon, V. A. Ugrinovskii, and M. Pszczel, "Gain-Scheduling of minimax optimal state-feedback controllers for uncertain LPV systems," IEEE Transactions on Automatic Control, vol. 52, no. 2, pp. 311-317, February 2007.

[13] T. Yue, L. Wang, and J. Ai, "Gain self-scheduled $H_\infty$control for morphing aircraft in the wing transition process based on an LPV model," Chinese Journal of Aeronautics, vol. 26, no. 4, pp. 909-917, August 2013.

[14] X. L. Zhu and G. H. Yang, "Jensen integral inequality approach to stability analysis of continuous-time systems with time-varying delay," IET Control Theory and Applications, vol. 2, pp. 524-534, 2008.

[15] B. Chen, X. Liu, and S. Tong, "New delay-dependent stabilization conditions of T-S fuzzy systems with constant delay," Fuzzy Sets and Systems, vol. 158, no. 20, pp. 2209-2224, October 2007.

[16] W. J. Chang, C. C. Ku, and P. H. Huang, "Robust fuzzy control for uncertain stochastic time-delay Takagi-Sugeno fuzzy models for achieving passivity," Fuzzy Sets and Systems, vol. 161, no. 15, pp. 2012-2032, August 2010.

[17] L. E. Ghaoui, "State-Feedback control of systems with multiplicative noise via linear matrix inequalities," Systems & Control Letters, vol. 24, no. 3, pp. 223-228, February 1995.

[18] E. Gershon, U. Shaked, and I. Yaesh, $H_\infty$ control and estimation of state-multiplicative linear systems, London: Springer, 2005.

[19] Y. Y. Cao, J. Lam, and L. Hu, "Delay-dependent stochastic stability and $H_\infty$ analysis for time-delay systems with Markovian jumping parameters," Journal of the Franklin Institute, vol. 340, no. 6-7, pp. 423-434, September-November 2003.

[20] C. C. Ku, "Robust mixed $H_2/Passivity$ performance controller design for uncertain drum-boiler system," Journal of Marine Science and Technology, vol. 23, no. 5, pp. 1003-1010, October 2016.

[21] C. C. Ku and G. W. Chen, "Gain-Scheduled controller design for discrete-time linear parameter varying systems with multiplicative noises," International Journal of Control, Automation and Systems, vol. 13, no. 6, pp. 1382-1390, December 2015.

[22] C. C. Ku and C. I Wu, "Gain-scheduled $H_\infty$ control for linear parameter varying stochastic systems," ASME, Journal of Dynamic Systems, Measurement and Control, vol. 137, no. 11, pp. 111012-1-111012-12, August 2015.

[23] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, Philadelphia: SIAM, PA, 1994.

[24] W. J. Chang, Y. C. Chang, and C. C. Ku, "Passive fuzzy control via fuzzy integral Lyapunov function for nonlinear ship drum-boiler systems," ASME, Journal of Dynamic Systems, Measurement and Control, vol. 137, no. 4, pp. 041008-1-041008-10, April 2015.