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Empirical Models for Estimation of Rain Rate in the Fifteen ITU Rain Zones

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Abstract

Rain rate data are essential for the computation of rain attenuation that can be experienced by wireless signal passing through a given area. The International Telecommunication Union (ITU) divided the world into fifteen rain zones and for each rain zone ITU published rain rate data for just seven different link percentage availability, namely: 99%, 99.7%, 99.9%, 99.97%, 99.99%, 99.997% and 99.999%. In this paper, two empirical models are developed for estimating the rain rate for any given link percentage availability in all the fifteen ITU rain zones. The goodness of fit of the models are indicated in terms of coefficient of determination (otherwise called r^2), root mean square error and prediction accuracy. In all, 90.3777% is the lowest prediction accuracy recorded for Model 1 for rain zone C and 91.6306% is the lowest prediction accuracy recorded for Model 2 for rain zone B. The best prediction accuracy recorded for Model 1 is 98.2456% for rain zone Q and best prediction accuracy recorded for Model 2 is 95.3553% for rain zone H. The models are useful for the estimation of rain rate and hence rain attenuation for any given link percentage availability in all the fifteen ITU rain zones.

Keywords: Rain rate, rain attenuation, percentage availability, rain zone, empirical models.

1 Introduction

When propagating through rain, radio waves suffer from power loss (attenuation) due to rain [1,2,3]. According to research findings, rain attenuation is the dominant propagation impairment at frequencies above about 10 GHz [4,5,6,7,8,9]. Furthermore, rain attenuation depends on the rainfall rate and the raindrop size distribution. Consequently, rain attenuation is computed with knowledge of rain rate for a given region [1,10,5,11,12].

However, the amount of rainfall an area receives depends on the geographic location and the climate of the area [13,14,15]. As such, in the International Telecommunication Union (ITU) recommendations, ITU-R P. 837-1 and ITU-R P838, the climatic map for the whole world is divided into 15 climatic zones on rain rate distribution or rain intensities at 1% to 0.001% probabilities [16,17,18,19,20,21]. However, for each of the fifteen rain zones, ITU published rain rate data for just seven different link percentage availability, namely: 99%, 99.7%, 99.9%, 99.97%, 99.99%, 99.997% and 99.999%.

In this paper, empirical models are developed and validated for the estimation of the rain rate for each of the fifteen rain zones and for any given percentage of time the rain rate is exceeded. Such models will make it possible for researchers and wireless network designers to effectively study the variation of rain rate and rain attenuation with percentage of time exceeded in any of the rain zones. The models will also enable researchers and wireless network designers to effectively study the relationship between rain rate, percentage of time exceeded and wireless communication link outages or link availability. The two empirical models are developed based on the actual rain rate data published by ITU for the fifteen rain zones.

2. Methodology

In this paper, the available ITU rain rate data for the 15 ITU rain zones are analysed in order to select the appropriate empirical model for estimating the rain rate for any given percentage of time exceeded. Specifically, three different preliminary trend line curves are fitted on the plot of rain rate versus the percentage of time exceeded (Fig. 1) for rain zone F. The error analysis on the trend line equations shows that power trend line model has the highest Coefficients of Determination (r^2) of above 0.98 followed by logarithmic trend line model with $0.92 < r^2 < 0.98$ and lastly the exponential trend line model with $0.73 < r^2 < 0.92$.

Table 1. ITU Rain Rate and Percentage of Time Exceeded for Rain Zone E (Source: [17, 18])

Percentage of Time Exceeded (%)	Rain Rate (mm/h)
1	0.6
0.3	2.4
0.1	6
0.03	12
0.01	22
0.003	41
0.001	70

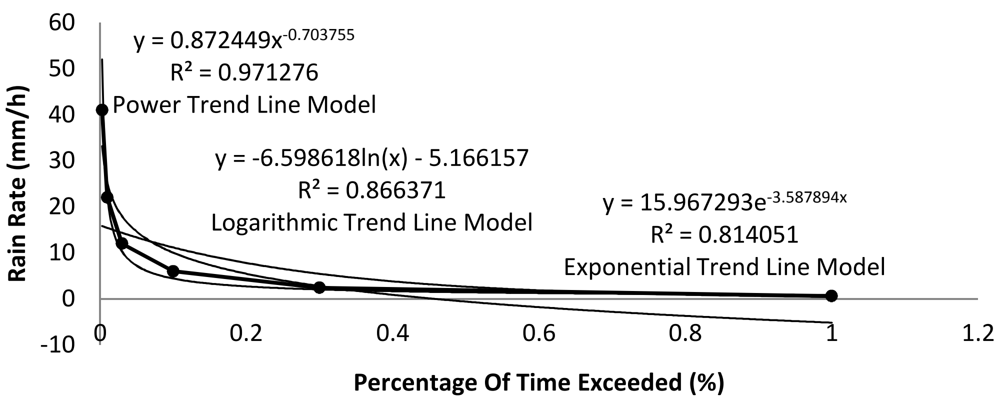

Figure 1. The Graph Of ITU Rain Rate and Percentage of Time Exceeded for Rain Zone E.

Consequently, in order to predict rain rate for the ITU rain zones, two empirical models are used, namely:

$$\text{MODEL 1:} \quad Y = aX^2 e^{cx} \tag{1}$$

$$\text{MODEL 2:} \quad Y = \frac{1}{aX^b + c} \tag{2}$$

Model 1 is a product of power and exponential components, whereas Model 2 is the inverse of the sum of power and constant components. In the two models Y is the predicted rain rate in a particular rain zone, X is the percentage of time the rain rate is exceeded (otherwise called percentage of time exceeded), a, b and c are empirical constants determined for each model in each rain zone. (Note, x = 100% – link percentage availability. So, link percentage availability of 99% amounts to X = 1%). Statistical error analysis parameters such as Coefficients of Determination (r^2), Root mean square error (RMSE) and Prediction Accuracy (PA) for each of the models for each rain zone are determined to quantify the goodness of fit of the model for predicting the rain rate in the given rain zone. After selecting the two empirical models, Matlab program is used to find the values of the empirical constants a, b and c for each of the model. Afterwards, the nonlinear optimisation option in Microsoft Excel Solver is used to determine a constant (Kopt) that will be added to the model in order to minimise the prediction error. Essentially, the effective models used for estimating the rain rate for each of the rain zones are:

$$\text{MODEL 1:} \quad Y = aX^2 e^{cx} + \text{Kopt} \tag{3}$$

$$\text{MODEL 2:} \quad Y = \frac{1}{aX^b + c} + \text{Kopt} \tag{4}$$

3. Goodness of Fit Measures

The Coefficients of Determination (r^2):

If n is the number of data items x and y, the Correlation Coefficient (r) is given as:

$$r = \left(\frac{\left(n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)\right)}{\left[n\left(\sum x^2\right) - \left(\sum x^2\right)\right]\left[n\left(\sum y^2\right) - \left(\sum y^2\right)\right]} \right) \tag{5}$$

Also, the Coefficients of Determination (r^2) is given as:

$$r = \left(\frac{\left(n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)\right)}{\left[n\left(\sum x^2\right) - \left(\sum x^2\right)\right]\left[n\left(\sum y^2\right) - \left(\sum y^2\right)\right]} \right)^2 \tag{6}$$

Prediction Accuracy:

The prediction accuracy (PA in %) is calculated as follows:

$$PA = \left\{ 1 - \frac{1}{n} \left(\sum_{i=1}^{i=n} \left| \frac{\left| Y_{(actaul)(i)} - \left| Y_{(predicted)(i)} \right| \right|}{\left| Y_{(actaul)(i)} \right|} \right| \right) \right\} * 100\% \tag{7}$$

Root Mean Square Error (RMSE):

The Root Mean Square Error (RMSE) is calculated as follows:

$$RMSE = \sqrt[2]{\frac{1}{n} \left(\sum_{i=1}^{i=n} \left(Y_{(actaul)(i)} - Y_{(predicted)(i)} \right)^2 \right)} \tag{8}$$

where $Y_{(actaul)(i)}$ is the actual rain rate given by ITU and $Y_{(predicted)(i)}$ is model predicted rain rate.

4. Results and Discussions

Table 2a and Table 2b show the values for the empirical constants a, b, and c, the model optimization constant (Kopt), and the goodness of fit parameters, Coefficients of Determination (r^2), Root mean square error (RMSE) and prediction accuracy (PA) of the two models for each rain zone. For rain zone A,C,D,E,G,J,P and Q Model 1 has better prediction accuracy and hence it is used for predicting the rain rate for the rain zone A,C,D,E,G,J,P and Q (shown in Table 2a). On the other hand, Model 2 has better prediction accuracy for rain zones B,F,H,K,L,M and N and hence, Model 2 is used for predicting the rain rate for the rain zones B,F,H,K,L,M and N (shown in Table 2b).

Table 2a. MODEL 1: - Values for the empirical constants a, b, and c, the model optimization constant (Kopt), and the goodness of fit parameters, namely: Coefficients of Determination (r^2), Root mean square error (RMSE) and prediction accuracy (PA) of the two models for each ITU rain zone

	a	b	c	Kopt	RMSE	r^2	Prediction Accuracy (%)
A	1.261	-0.4148	-3.887	0.093228	0.1673	0.9996	93.0066
C	2.387	-0.4148	-3.887	0.749839	0.7819	0.9983	90.3777
D	3.976	-0.3416	-0.7553	0.161511	0.2257	0.9999	97.3823
E	2.309	-0.4945	-1.877	0.234747	0.3251	1.0000	96.8494
G	6.264	-0.3392	-0.8824	0.318893	0.4509	0.9998	97.0029
J	13.71	-0.2027	-0.6551	0.762792	1.0774	0.9977	95.9201
P	49.09	-0.2379	-2.091	3.741203	4.9247	0.9980	92.2942
Q	53.91	-0.1668	-0.8625	1.118311	1.5616	0.9995	98.2456

Table 2b. MODEL 2: - Values for the empirical constants a, b, and c, the model optimization constant (Kopt) the goodness of fit parameters, namely: Coefficients of Determination (r^2), Root mean square error (RMSE), and prediction accuracy (PA) of the two models for each ITU rain zone

	a	b	C	Kopt	RMSE	r^2	Prediction Accuracy (%)
B	1.392	0.6512	0.01577	-0.25442	0.3640	0.9995	91.6306
F	0.6567	0.6953	0.007375	0.169725	0.8671	0.9990	93.9921
H	0.4108	0.601	0.005593	-0.48374	0.6301	0.9997	95.3553
K	0.397	0.6739	0.006244	-0.76367	0.9533	0.9996	93.0746
L	0.3157	0.7038	0.004226	-0.52586	0.6459	0.9999	93.7753
M	0.1787	0.6364	0.006129	-0.74685	0.9745	0.9997	95.1533
N	0.1083	0.6198	0.004093	-2.72616	3.4914	0.9982	92.4932

From Eq.3 and Table 2a, the empirical model for the rain rates in rain zones A,C , D, E, G, J, P and Q can be expressed as:

$$Y_A = 1.261X^{-0.4148} e^{-3.887X} + 0.093228 \quad (9)$$
$$Y_C = 2.387X^{-0.4148} e^{-3.887X} + 0.749839 \quad (10)$$
$$Y_D = 3.976X^{-0.3416} e^{-0.7553X} + 0.161511 \quad (11)$$
$$Y_E = 2.309X^{-0.4945} e^{-1.877X} + 0.234747 \quad (12)$$
$$Y_G = 6.264X^{-0.3392} e^{-0.8824X} + 0.318893 \quad (13)$$
$$Y_J = 13.71X^{-0.2027} e^{-0.6551X} + 0.762792 \quad (14)$$
$$Y_P = 49.09X^{-0.2379} e^{-2.091X} + 3.741203 \quad (15)$$
$$Y_Q = 53.91X^{-0.1668} e^{-0.8625X} + 1.118311 \quad (16)$$

Similarly, from Eq4 and Table 2b, the empirical model for the rain rates in rain zones B, F, H, K, L,M and N can be expressed as:

$$Y_B = \frac{1}{1.392X^{0.6512} + 0.01577} - 0.25442 \quad (17)$$
$$Y_F = \frac{1}{0.6567X^{0.6953} + 0.007375} + 0.169725 \quad (18)$$
$$Y_H = \frac{1}{0.4108X^{0.601} + 0.005593} - 0.48374 \quad (19)$$
$$Y_K = \frac{1}{0.397X^{0.6739} + 0.006244} - 0.76367 \quad (20)$$
$$Y_L = \frac{1}{0.3157X^{0.7038} + 0.004226} - 0.52586 \quad (21)$$
$$Y_M = \frac{1}{0.1787X^{0.6364} + 0.006129} - 0.74685 \quad (22)$$
$$Y_N = \frac{1}{0.1083X^{0.6198} + 0.004093} - 2.72616 \quad (23)$$

Fig. 2 and Table 3 show the graph of the actual and Model 1 predicted rain rate versus percentage of time exceeded for rain zone E in which Model 1 has prediction accuracy of 96.8494% (as shown in Table 2a).

Similarly, Fig. 3 and Table 4 show the graph of the actual and Model 2 predicted rain rate versus percentage of time exceeded for rain zone F in which Model 2 has prediction accuracy of 93.9921% (as shown in Table 2b).

Table 3. Actual and Model 1 Predicted Rain Rate for Rain Zone E.

Percentage of Time Exceeded (%)	Actual Rain Rate (mm/h) for Rain Zone E	Model 1 Predicted Rain Rate (mm/h) for Rain Zone E
1	0.6	0.588136
0.3	2.4	2.619461
0.1	8	6.210696
0.03	12	12.59514
0.01	22	22.32864
0.003	41	40.83624
0.001	70	70.39785

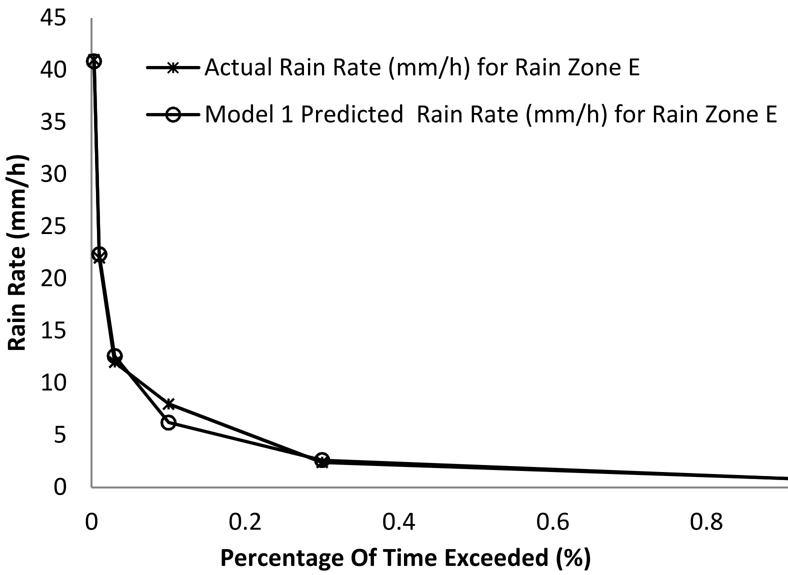

Figure 2. The Actual and Model 2 Predicted Rain Rate for Rain Zone E

Table 4. The Actual and Model 2 Predicted Rain Rate for Rain Zone F.

Percentage of Time Exceeded (%)	Actual Rain Rate (mm/h) for Rain Zone F	Model 1 Predicted Rain Rate (mm/h) for Rain Zone F
1	1.7	1.505854
0.3	4.5	3.428223
0.1	8	7.151562
0.03	15	15.45074
0.01	28	29.33349
0.003	54	52.79358
0.001	78	78.34848

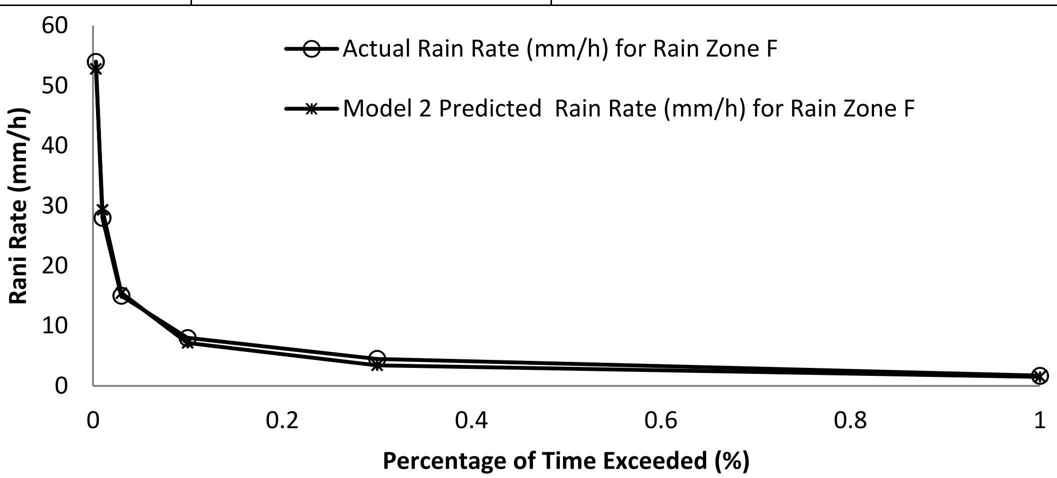

Fig. 3 The Actual and Model 2 Predicted Rain Rate For Rain Zone F

5 Conclusion

In this paper, two different empirical models referred to as Model 1 and Model 2 are developed and evaluated for their suitability for predicting the rain rate for the fifteen ITU rain zones across the world. The goodness of fit of the models are indicated in terms of coefficient of determination (otherwise called r^2), root mean square error and prediction accuracy. For rain zone A,C,D,E,G,J,P and Q Model 1 has better prediction accuracy and hence it is used for predicting the rain rate for the rain zone A,C,D,E,G,J,P and Q. On the other hand, Model 2 has better prediction accuracy for rain zones B,F,H,K,L,M and N and hence, Model 2 is used for predicting the rain rate for the rain zones B,F,H,K,L,M and N.

The models are useful for the estimation of rain rate and hence rain attenuation for any given link percentage availability in all the fifteen ITU rain zones.

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