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A Note on Large Cycles in Graphs Around Conjectures of Bondy and Jung

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Abstract

New sufficient conditions are derived for generalized cycles (including Hamilton and dominating cycles as special cases) in an arbitrary k-connected ($k = 1, 2, ...$) graph, which prove the truth of Bondy's (1980) famous conjecture for some variants significantly improving the result expected by the given hypothesis. Similarly, new lower bounds for the circumference (the length of a longest cycle) are established for the reverse hypothesis proposed by Jung (2001) combined inspiring new improved versions of the original conjectures of Bondy and Jung.

Keywords: Hamilton cycle, Dominating cycle, Longest cycle, Large cycle.
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1. Introduction

We consider only finite undirected graphs without loops or multiple edges. The set of vertices of a graph G is denoted by $V(G)$; the set of edges by $E(G)$. For a subset S of $V(G)$, we denote by $G-S$ the maximum subgraph of G with the vertex set $V(G)-S$. For a subgraph H of G, we use $G-H$, short for $G-V(H)$. A good reference for any undefined terms is [3].

Let α and δ be the independence number and the minimum degree of a graph G, respectively. We define σ_k by the minimum degree sum of any k independent vertices if $\alpha \geq k$; if $\alpha < k$, we set $\sigma_k = +\infty$. In particular, we have $\sigma_1 = \delta$.

A simple cycle (or just a cycle) Q of order t (the number of vertices) is a sequence $v_1v_2...v_tv_1$ of distinct vertices $v_1, ..., v_t$ with $v_iv_{i+1} \in E(G)$ for each $i \in \{1, ..., t\}$, where $v_{t+1} = v_1$. When $t = 1$, the cycle v_1 coincides with the vertex v_1. So, by this standard definition, all vertices and edges in a graph can be considered as cycles of orders 1 and 2, respectively. Such an extension of the cycle definition allows to avoid unnecessary repetition "let G be a graph of order $n \geq 3$" in a large number of results. Further, a simple path (or just a path) of order t is a sequence $v_1v_2...v_t$ of distinct vertices $v_1, ..., v_t$ with $v_iv_{i+1} \in E(G)$ for each $i \in \{1, ..., t-1\}$.

A graph G is Hamiltonian if G contains a Hamilton cycle, i.e., a cycle of order $|V(G)|$.

Now let Q be an arbitrary cycle in G. We say that Q is a dominating cycle in G if $V(G-Q)$ is an independent set of vertices.

The first type of generalized cycles, including Hamilton and dominating cycles as special cases, was introduced by Bondy [4]. For a positive integer λ, Q is said to be a D_λ-cycle if $|H| \leq \lambda - 1$ for every component H of $G-Q$. Alternatively, Q is a D_λ-cycle of G if and only if every connected subgraph of order λ of G has at least one vertex with Q in common. Thus, a D_λ-cycle dominates all connected subgraphs of order λ. By this definition, Q is a Hamilton cycle if and only if Q is a D_1-cycle. Analogously, Q is a dominating cycle if and only if Q is a D_2-cycle.

We now present another two types of more interesting generalized cycles that form the main topic of this paper. For a positive integer λ, the cycle Q is called a PD_λ-cycle (PD - Path Dominating) if each path of order at least λ in G has at least one vertex with Q in common. Similarly, we call the cycle Q a CD_λ-cycle (CD - Cycle Dominating; introduced in [13]) if each cycle of order at least λ has at least one vertex with Q in common. In fact, a PD_λ-cycle dominates all paths of order λ in G; and a CD_λ-cycle dominates all cycles of order λ in G. In terms of PD_λ and CD_λ-cycles, Q is a Hamilton cycle if and only if either Q is a PD_1-cycle or a CD_1-cycle. Further, Q is a dominating cycle if and only if either Q is a PD_2-cycle or a CD_2-cycle.

Throughout the paper, we consider a graph G on n vertices with minimum degree δ and connectivity κ. Further, let C be a longest cycle in G with $c = |C|$, and let $\overline{p}$ and $\overline{c}$ denote the orders of a longest path and a longest cycle in $G-C$, respectively. In particular, C is a Hamilton cycle if and only if $\overline{p} \leq 0$ or $\overline{c} \leq 0$. Similarly, C is a dominating cycle if and only if $\overline{p} \leq 1$ or $\overline{c} \leq 1$.

In 1980, Bondy [4] conjectured a common generalization of some well-known degree-sum conditions for PD_λ-cycles (called $(\sigma, \overline{p})$-version) including Hamilton cycles (PD_1-cycles) and dominating cycles (PD_2-cycles) as special cases.

Conjecture 1. *(Bondy [4],1980): $(\sigma, \overline{p})$-version*
Let C be a longest cycle in a λ-connected $(1 \leq \lambda \leq \delta)$ graph G of order n. If $\sigma_{\lambda+1} \geq n + \lambda(\lambda-1)$, then $\overline{p} \leq \lambda - 1$.

Parts of Conjecture 1 were proved for $\lambda = 1, 2, 3$.

$$
\begin{array}{lllll}
(a) & \kappa \geq 1, \ \ \sigma_2 \geq n & \Longrightarrow & \overline{p} \leq 0 & (Ore[15], 1960), \\
(b) & \kappa \geq 2, \ \ \sigma_3 \geq n+2 & \Longrightarrow & \overline{p} \leq 1 & (Bondy[4], 1980), \\
(c) & \kappa \geq 3, \ \ \sigma_4 \geq n+6 & \Longrightarrow & \overline{p} \leq 2 & (Zou[17], 1987).
\end{array}
$$

For the general case, Conjecture 1 is still open.

The long cycles analogue (the so called reverse version) of Bondy's conjecture (Conjecture 1) can be formulated as follows.

Conjecture 2. *(reverse, σ, $\overline{p}$)-version*
Let C be a longest cycle in a λ-connected $(1 \leq \lambda \leq \delta)$ graph G. If $\overline{p} \geq \lambda - 1$, then $c \geq \sigma_\lambda - \lambda(\lambda-2)$.

Parts of Conjecture 2 were proved for $\lambda = 1, 2, 3, 4$.

$$
(d) \ \ \kappa \geq 1, \ \overline{p} \geq 0 \ \ \Longrightarrow \ \ c \geq \sigma_1 + 1 \quad (Dirac[6], 1952),
$$

(e) $\kappa \geq 2,\ \overline{p} \geq 1 \implies c \geq \sigma_2$ $(Bondy[2], 1971; Bermond[1], 1976; Linial[11], 1976)$,
(f) $\kappa \geq 3,\ \overline{p} \geq 2 \implies c \geq \sigma_3 - 3$ $(Fraisse, Jung[8], 1989)$,
(g) $\kappa \geq 4,\ \overline{p} \geq 3 \implies c \geq \sigma_4 - 8$ $(Chiba, Tsugaki, Yamashita[5], 2014)$.

Note that the initial motivations of Conjecture 1 and Conjecture 2 come from their minimal degree versions - the most popular and much studied versions, which also remain unsolved.

Conjecture 3. *(Bondy [4],1980): $(\delta, \overline{p})$-version*
Let C be a longest cycle in a λ-connected $(1 \leq \lambda \leq \delta)$ graph G of order n. If $\delta \geq \frac{n+2}{\lambda+1} + \lambda - 2$, then $\overline{p} \leq \lambda - 1$.

Conjecture 4. *(Jung [10], 2001): (reverse, δ, $\overline{p}$)-version*
Let C be a longest cycle in a λ-connected $(1 \leq \lambda \leq \delta)$ graph G. If $\overline{p} \geq \lambda - 1$, then $c \geq \lambda(\delta - \lambda + 2)$.

Parts of Conjecture 3 were proved for $\lambda = 1, 2, 3$.

(h) $\kappa \geq 1,\ \delta \geq \frac{n}{2} \implies \overline{p} \leq 0$ $(Dirac[6], 1952)$,
(i) $\kappa \geq 2,\ \delta \geq \frac{n+2}{3} \implies \overline{p} \leq 1$ $(Nash-Williams[12], 1971)$,
(j) $\kappa \geq 3,\ \delta \geq \frac{n+6}{4} \implies \overline{p} \leq 2$ $(Fan[7], 1987)$.

Parts of Conjecture 4 were proved for $\lambda = 1, 2, 3, 4$.

(k) $\kappa \geq 1,\ \overline{p} \geq 0 \implies c \geq \delta + 1$ $(Dirac[6], 1952)$,
(l) $\kappa \geq 2,\ \overline{p} \geq 1 \implies c \geq 2\delta$ $(Dirac[6], 1952)$,
(m) $\kappa \geq 3,\ \overline{p} \geq 2 \implies c \geq 3\delta - 3$ $(Voss, Zuluaga[16], 1977)$,
(n) $\kappa \geq 4,\ \overline{p} \geq 3 \implies c \geq 4\delta - 8$ $(Jung[9], 1990)$.

Note that CD_λ-cycles are more suitable for research than PD_λ-cycles since cycles in $G - C$ are more symmetrical than paths in view of the connections between $G - C$ and CD_λ-cycles. This is the main reason why some minimum degree versions of Conjectures 1 and 2 have been solved just for CD_λ-cycles.

According to the above arguments, it is natural to consider the exact analogues of Bondy's generalized conjecture (Conjecture 1) and its reverse version (Conjecture 2) for CD_λ-cycles, which we call $(\sigma, \overline{c})$ and $(reverse, \sigma, \overline{c})$-versions, respectively.

Conjecture 5. *$(\sigma, \overline{c})$-version*
Let C be a longest cycle in a λ-connected $(1 \leq \lambda \leq \delta)$ graph G of order n. If $\sigma_{\lambda+1} \geq n + \lambda(\lambda - 1)$, then $\overline{c} \leq \lambda - 1$.

Conjecture 6. *(reverse, σ, $\overline{c}$)-version*
Let C be a longest cycle in a λ-connected $(1 \leq \lambda \leq \delta)$ graph. If $\overline{c} \geq \lambda - 1$, then $c \geq \sigma_\lambda - \lambda(\lambda - 2)$.

In 2009, the author proved [14] the validity of minimum degree versions of Conjectures 5 and 6.

Theorem 1. *([14], 2009): $(\delta, \overline{c})$-version*
Let C be a longest cycle in a λ-connected $(1 \leq \lambda \leq \delta$ graph G of order n. If $\delta \geq \frac{n+2}{\lambda+1} + \lambda - 2$, then $\overline{c} \leq \lambda - 1$.

Theorem 2. *([14], 2009): (reverse, δ, $\overline{c}$)-version*
Let C be a longest cycle in a λ-connected $(1 \leq \lambda \leq \delta)$ graph. If $\overline{c} \geq \lambda - 1$, then $c \geq \lambda(\delta - \lambda + 2)$.

Actually, in [14], a significantly stronger result than Theorem 1 was proved showing that the conclusion $\overline{c} \leq \lambda - 1$ in Theorem 1 can be strengthened to $\overline{c} \leq \min\{\lambda - 1, \delta - \lambda\}$, called $\overline{c}$-improvement.

Theorem 3. *([14], 2009): $(\delta, \overline{c})$-version, $\overline{c}$-improvement*
Let C be a longest cycle in a λ-connected $(1 \leq \lambda \leq \delta)$ graph G of order n. If $\delta \geq \frac{n+2}{\lambda+1} + \lambda - 2$, then $\overline{c} \leq \min\{\lambda - 1, \delta - \lambda\}$.

Analogously, the condition $\overline{c} \geq \lambda - 1$ in Theorem 2 was weakened [14] to $\overline{c} \geq \min\{\lambda - 1, \delta - \lambda + 1\}$.

Theorem 4. *([14], 2009): (reverse, δ, $\overline{c}$)-version, $\overline{c}$-improvement*
Let C be a longest cycle in a λ-connected $(1 \leq \lambda \leq \delta)$ graph G. If $\overline{c} \geq \min\{\lambda - 1, \delta - \lambda + 1\}$, then $c \geq \lambda(\delta - \lambda + 2)$.

In this paper, we present new analogous further improvements of Theorems 1, 2, 3, 4 inspiring new conjectures in forms of improvements of the initial generalized conjectures of Bondy and Jung.

2. Results

First, we prove that the connectivity condition $\kappa \geq \lambda$ in Theorem 1 can be weakened to $\kappa \geq \min\{\lambda, \delta - \lambda + 1\}$.

Theorem 5. *$(\delta, \overline{c})$-version, κ-improvement*
Let C be a longest cycle in a graph G of order n and λ a positive integer with $1 \leq \lambda \leq \delta$. If $\kappa \geq \min\{\lambda, \delta - \lambda + 1\}$ and $\delta \geq \frac{n+2}{\lambda+1} + \lambda - 2$, then $\overline{c} \leq \lambda - 1$.

Analogously, we prove that the connectivity condition $\kappa \geq \lambda$ in Theorem 2 can be weakened to $\kappa \geq \min\{\lambda, \delta - \lambda + 2\}$.

Theorem 6. *(reverse, δ, $\overline{c}$)-version, κ-improvement*
Let C be a longest cycle in a graph G and λ a positive integer with $1 \leq \lambda \leq \delta$. If $\kappa \geq \min\{\lambda, \delta - \lambda + 2\}$ and $\overline{c} \geq \lambda - 1$, then $c \geq \lambda(\delta - \lambda + 2)$.

Next, we prove that the conclusion $\overline{c} \leq \lambda - 1$ in Theorem 5 can be strengthened to $\overline{c} \leq \min\{\lambda - 1, \delta - \lambda\}$.

Theorem 7. *$(\delta, \overline{c})$-version, $(\overline{c}, \kappa)$-improvement*
Let C be a longest cycle in a graph G of order n and λ a positive integer with $1 \leq \lambda \leq \delta$. If $\kappa \geq \min\{\lambda, \delta - \lambda + 1\}$ and $\delta \geq \frac{n+2}{\lambda+1} + \lambda - 2$, then $\overline{c} \leq \min\{\lambda - 1, \delta - \lambda\}$.

Finally, we prove that the condition $\overline{c} \geq \lambda - 1$ in Theorem 6 can be weakened to $\overline{c} \geq \min\{\lambda - 1, \delta - \lambda + 1\}$.

Theorem 8. *(reverse, δ, $\overline{c}$)-version, $(\overline{c}, \kappa)$-improvement*
Let C be a longest cycle in a graph G and λ a positive integer with $1 \leq \lambda \leq \delta$. If $\kappa \geq \min\{\lambda, \delta - \lambda + 2\}$ and $\overline{c} \geq \min\{\lambda - 1, \delta - \lambda + 1\}$, then $c \geq \lambda(\delta - \lambda + 2)$.

3. Generalized Improvements of Conjectures of Bondy and Jung

Motivated by Theorems 5, 6, 7, 8 (minimum degree versions) with Conjectures 1 and 2, in this section we propose their exact analogs in terms of degree sums as generalized improvements of Bondy and Jung Conjectures.

Conjecture 7. *$(\sigma,\overline{c})$-version, $(\overline{c},\kappa)$-improvement*
Let C be a longest cycle in a graph G of order n and λ a positive integer. If $\kappa \geq \min\{\lambda, \delta-\lambda+1\}$ and $\sigma_{\lambda+1} \geq n+\lambda(\lambda-1)$, then $\overline{c} \leq \min\{\lambda-1, \delta-\lambda\}$.

Conjecture 8. *(reverse, σ, $\overline{c}$)-version, $(\overline{c},\kappa)$-improvement*
Let C be a longest cycle in a graph G and λ a positive integer. If $\kappa \geq \min\{\lambda, \delta-\lambda+2\}$ and $\overline{c} \geq \min\{\lambda-1, \delta-\lambda+1\}$, then $c \geq \sigma_\lambda - \lambda(\lambda-2)$.

Conjecture 9. *$(\sigma,\overline{p})$-version, $(\overline{p},\kappa)$-improvement*
Let C be a longest cycle in a graph G of order n and λ a positive integer. If $\kappa \geq \min\{\lambda, \delta-\lambda+1\}$ and $\sigma_{\lambda+1} \geq n+\lambda(\lambda-1)$, then $\overline{p} \leq \min\{\lambda-1, \delta-\lambda\}$.

Conjecture 10. *(reverse, σ, $\overline{p}$)-version, $(\overline{p},\kappa)$-improvement*
Let C be a longest cycle in a graph G and λ a positive integer. If $\kappa \geq \min\{\lambda, \delta-\lambda+2\}$ and $\overline{p} \geq \min\{\lambda-1, \delta-\lambda+1\}$, then $c \geq \sigma_\lambda - \lambda(\lambda-2)$.

4. Proofs

Proof of Theorem 7. We shall prove that $\overline{c} \leq \min\{\lambda-1, \delta-\lambda\}$ under the conditions

$$\kappa \geq \min\{\lambda, \delta-\lambda+1\}, \quad \delta \geq \frac{n+2}{\lambda+1} + \lambda - 2$$

for each $1 \leq \lambda \leq \delta$. If $\min\{\lambda, \delta-\lambda+1\} = \lambda$, that is $\lambda \leq \lfloor \frac{\delta+1}{2} \rfloor$, then we shall prove that $\overline{c} \leq \lambda - 1$ under the conditions

$$\kappa \geq \lambda, \quad \delta \geq \frac{n+2}{\lambda+1} + \lambda - 2.$$

But the latter follows from Theorem 1 for all $\lambda = 1, 2, ..., \lfloor \frac{\delta+1}{2} \rfloor$ immediately.

Now let $\min\{\lambda, \delta-\lambda+1\} = \delta-\lambda+1$, that is $\lambda \geq \lfloor \frac{\delta+2}{2} \rfloor$. To conclude the proof, it remains to show that

$$\kappa \geq \delta-\lambda+1, \ \delta \geq \frac{n+2}{\lambda+1} + \lambda - 2 \ \Rightarrow \ \overline{c} \leq \delta - \lambda \quad \left(\lambda = \delta, \delta-1, ..., \left\lfloor \frac{\delta+2}{2} \right\rfloor\right). \tag{1}$$

Put $\delta-\lambda+1 = \mu$. Acording to this notation, (1) is equivalent to

$$\kappa \geq \mu, \ \delta \geq \frac{n+2}{\delta-\mu+2} + \delta - \mu - 1 \ \Rightarrow \ \overline{c} \leq \mu - 1 \quad \left(\mu = 1, 2, ..., \left\lfloor \frac{\delta+1}{2} \right\rfloor\right). \tag{2}$$

In (2), the inequality

$$\delta \geq \frac{n+2}{\delta-\mu+2} + \delta - \mu - 1$$

is equivalent to

$$\delta \geq \frac{n+2}{\mu+1} + \mu - 2,$$

implying that (2) is equivalent to

$$\kappa \geq \mu, \ \delta \geq \frac{n+2}{\mu+1} + \mu - 2 \ \Rightarrow \ \overline{c} \leq \mu - 1 \quad \left(\mu = 1, 2, ..., \left\lfloor \frac{\delta+1}{2} \right\rfloor\right). \tag{3}$$

Observing that (3) follows from Theorem 1 immediately, we obtain

$$(1) \equiv (2) \equiv (3) \ \Leftarrow \ \text{"Theorem 1"}.$$

Theorem 7 is proved. ■

Proof of Theorem 5. Let G be a graph with

$$\kappa \geq \min\{\lambda, \delta - \lambda + 1\}, \quad \delta \geq \frac{n+2}{\lambda+1} + \lambda - 2$$

for each $1 \leq \lambda \leq \delta$. We shall prove that $\overline{c} \leq \lambda - 1$. Observing that $\min\{\lambda - 1, \delta - \lambda\} \leq \lambda - 1$, we can weaken the conclusion $\overline{c} \leq \min\{\lambda - 1, \delta - \lambda\}$ in Theorem 7 to $\overline{c} \leq \lambda - 1$ and the result follows immediatly. ■

Proof of Theorem 8. Let G be a graph with

$$\kappa \geq \min\{\lambda, \delta - \lambda + 2\}, \quad \overline{c} \geq \min\{\lambda - 1, \delta - \lambda + 1\}$$

for each $1 \leq \lambda \leq \delta$. We shall prove that $c \geq \lambda(\delta - \lambda + 2)$. If $\lambda = 1$, then the result follows from the fact that each graph has a cycle of length at least $\delta + 1$ [6]. Let $\lambda \geq 2$. Further, if $\min\{\lambda, \delta - \lambda + 2\} = \lambda$, then we are done by Theorem 2. Now let $\min\{\lambda, \delta - \lambda + 2\} = \delta - \lambda + 2$, that is $\lambda \geq \lfloor \frac{\delta+3}{2} \rfloor$. Then it remains to prove that

$$\kappa \geq \delta - \lambda + 2, \ \ \overline{c} \geq \delta - \lambda + 1 \ \Rightarrow \ c \geq \lambda(\delta - \lambda + 2) \quad \left(\lambda = \delta, \delta - 1, ..., \left\lfloor \frac{\delta+3}{2} \right\rfloor\right). \tag{4}$$

Put $\delta - \lambda + 2 = \mu$. By this notation, the statement (4) is equivalent to

$$\kappa \geq \mu \ \ \overline{c} \geq \mu - 1 \ \Rightarrow \ c \geq \mu(\delta - \mu + 2) \quad \left(\mu = 2, 3, ..., \left\lfloor \frac{\delta+2}{2} \right\rfloor\right), \tag{5}$$

which follows from Theorem 2 immediately. So, $(4) \equiv (5) \Leftarrow$ "Theorem 2". Theorem 8 is proved. ■

Proof of Theorem 6. Let G be a graph with

$$\kappa \geq \min\{\lambda, \delta - \lambda + 2\}, \quad \overline{c} \geq \lambda - 1$$

for each $1 \leq \lambda \leq \delta$. We shall prove that $c \geq \lambda(\delta - \lambda + 2)$. Observing that $\min\{\lambda - 1, \delta - \lambda + 1\} \leq \lambda - 1$, we can strengthen the condition $\overline{c} \geq \min\{\lambda - 1, \delta - \lambda + 1\}$ in Theorem 8 to $\overline{c} \geq \lambda - 1$ and the result follows immediately. Theorem 6 is proved. ■.

5. Conclusion

In 2009 [14], a minimum degree sufficient condition for large cycles in graphs is established showing that the famous conjecture of Bondy principally is improvable. In the same paper, a lower bound for the length of a longest cycle (the circumference) is derived showing that the conjecture of Jung (reverse version of Bondys conjecture) principally is improvable as well. In this note, two new analogous sufficient conditions for large cycles and two new lower bounds for the circumference are derived inspiring four new improved versions of Bondys and Jungs conjectures.

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Ակնարկ գրաֆներում մեծ ցիկլերի մասին Բոնդիի և Յունգի վարկածների շուրջ

Ժորա Գ. Նիկողոսյան

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Ամփոփում

Ստացվել են նոր բավարար պայմաններ գրաֆի ընդհանրացված ցիկլերի համար (ընդգրկելով Համիլթոնյան և դոմինանտ ցիկլերը որպես մասնավոր դեպքեր) կամայական k-կապակցված $(k = 1, 2, ...)$ գրաֆում, որոնք ապացուցում են Բոնդիի (1980) հայտնի վարկածի ճշմարտացիությունը որոշ տարբերակների դեպքում, ինչի շնորհիվ զգալիորեն լավացվում է տվյալ վարկածով ակնկալվող արդյունքը: Համանմանորեն, ամենաերկար ցիկլի երկարության համար ստացվել են նոր ստորին գնահատականներ հակադարձ վարկածի համար, որն առաջ է քաշել Յունգը 2001-ին: Ստացված արդյունքները բավարար հիմքեր են տալիս առաջ քաշելու նոր լավացված տարբերակներ Բոնդիի և Յունգի նախնական վարկածների փոխարեն:

Բանալի բառեր՝ Համիլթոնի ցիկլ, դոմինանտ ցիկլ, ամենաերկար ցիկլ, մեծ ցիկլ:

Заметка о больших циклах в графах вокруг гипотез Бонди и Юнга

Жора Г. Никогосян

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Аннотация

Получены новые достаточные условия для обобщенных циклов (включая гамильтоновые и доминантные циклы как частные случаи) в произвольном k-

связном графе ($k = 1, 2, ...$), доказывающие справедливость известной гипотезы Бонди (1980) для некоторых вариантов, значительно улучшив ожидаемый по данной гипотезе результат. Аналогично, получены новые нижние оценки для длины длиннейшего цикла графа для обратной гипотезы, предложенной Юнгом (2001). Полученные результаты в сочетании дают основания выдвижения новых улучшенных вариантов для исходных гипотез Бонди и Юнга.

Ключевые слова: Гамильтонов цикл, доминантный цикл, длиннейший цикл, большой цикл.