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On Locally-Balanced 2-Partitions of Complete Multipartite Graphs

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Abstract

A 2-partition of a graph G is a function $f : V(G) \to \{\mathbf{White}, \mathbf{Black}\}$. A 2-partition f of a graph G is locally-balanced with an open neighborhood if for every $v \in V(G)$,

$$||\{u \in N_G(v) \colon f(u) = \mathbf{White}\}| - |\{u \in N_G(v) \colon f(u) = \mathbf{Black}\}|| \leq 1,$$

where $N_G(v) = \{u \in V(G) \colon uv \in E(G)\}$. A 2-partition f' of a graph G is locally-balanced with a closed neighborhood if for every $v \in V(G)$,

$$||\{u \in N_G[v] \colon f'(u) = \mathbf{White}\}| - |\{u \in N_G[v] \colon f'(u) = \mathbf{Black}\}|| \leq 1,$$

where $N_G[v] = N_G(v) \cup \{v\}$. In this paper we give necessary and sufficient conditions for the existence of locally-balanced 2-partitions of complete multipartite graphs.

Keywords: 2-partition, Locally-balanced 2-partition, Equitable coloring, Complete multipartite graph.

1. Introduction

All graphs considered in this work are finite, undirected, and have no loops or multiple edges. Let $V(G)$ and $E(G)$ denote the sets of vertices and edges of a graph G, respectively. The set of neighbors of a vertex v in G is denoted by $N_G(v)$. Let $N_G[v] = N_G(v) \cup \{v\}$. A graph G is called a complete n-partite ($n \geq 2$) graph if its vertices can be partitioned into n nonempty independent sets $X_1, \dots, X_n$ such that each vertex in X_i is adjacent to all the other vertices in X_j for $1 \leq i < j \leq n$. Let $K_{r_1,r_2,\dots,r_n}$ denote a complete n-partite graph with independent sets $X_1, X_2, \dots, X_n$ of sizes $r_1, r_2, \dots, r_n$. The terms and concepts that we do not define can be found in [8, 15].

A 2-*partition* of a graph G is a function $f : V(G) \to \{\mathbf{White}, \mathbf{Black}\}$. A 2-partition f of a graph G is *locally-balanced with an open neighborhood* if for every $v \in V(G)$,

$$||\{u \in N_G(v) \colon f(u) = \mathbf{White}\}| - |\{u \in N_G(v) \colon f(u) = \mathbf{Black}\}|| \leq 1.$$

A 2-partition f' of a graph G is *locally-balanced with a closed neighborhood* if for every $v \in V(G)$,

$$||\{u \in N_G[v] \colon f'(u) = \mathbf{White}\}| - |\{u \in N_G[v] \colon f'(u) = \mathbf{Black}\}|| \leq 1.$$

The concept of locally-balanced 2-partition of graphs was introduced by Balikyan and Kamalian [12] in 2005, and it can be considered as a special case of equitable colorings of hypergraphs [1]. In [1], Berge obtained some sufficient conditions for the existence of equitable colorings of hypergraphs. In [7, 9, 10, 14], the authors considered the problems of the existence and construction of proper vertex-coloring of a graph for which the number of vertices in any two color classes differ by at most one. In [11], 2-vertex-colorings of graphs were considered for which each vertex is adjacent to the same number of vertices of every color. In particular, in [11], it was proved that the problem of the existence of such a coloring is NP-complete even for the $(2p, 2q)$-biregular $(p, q \geq 2)$ bipartite graphs. In [12], Balikyan and Kamalian proved that the problem of existence of locally-balanced 2-partition with an open neighborhood of bipartite graphs with maximum degree 3 is NP-complete. Later, they also proved [13] the similar result for locally-balanced 2-partitions with a closed neighborhood. In [2, 3, 4], the necessary and sufficient conditions for the existence of locally-balanced 2-partitions of trees were obtained. In [6], the authors obtained some necessary conditions for the existence of locally-balanced 2-partitions of Eulerian graphs. In particular, they proved some results on the existence of locally-balanced 2-partitions of rook's graphs and cycle powers.

In the present paper we give necessary and sufficient conditions for the existence of locally-balanced 2-partitions of complete multipartite graphs. A preliminary version of this paper was presented at the 6th Polish Combinatorial Conference, Bedlewo, Poland, 2016 [5].

2. Main Results

Before we formulate and prove our results, we introduce some terminology and notation. For any 2-partition φ of a graph G, we define $\overline{\varphi}$ as follows: for any $v \in V(G)$, let

$$\overline{\varphi}(v) = \begin{cases} \textbf{Black}, & \text{if } \varphi(v) = \textbf{White}, \\ \textbf{White}, & \text{if } \varphi(v) = \textbf{Black}. \end{cases}$$

If φ is a 2-partition of a graph G and $v \in V(G)$, then define $\#(v)$ and $\#[v]$ as follows:

$$\#(v) = |\{u \in N_G(v)\colon \varphi(u) = \textbf{White}\}| - |\{u \in N_G(v)\colon \varphi(u) = \textbf{Black}\}|,$$
$$\#[v] = |\{u \in N_G[v]\colon \varphi(u) = \textbf{White}\}| - |\{u \in N_G[v]\colon \varphi(u) = \textbf{Black}\}|.$$

Clearly, φ is a locally-balanced 2-partition with an open neighborhood (with a closed neighborhood) if for every $v \in V(G)$, $|\#(v)| \leq 1$ $(|\#[v]| \leq 1)$.

If G is a complete n-partite graph and X is a part of G, then X is called an odd (even) part if $|X|$ is odd ($|X|$ is even). If G is a complete n-partite graph, then by m_1, m_2, and $m_{\geq 3}$ we denote the number of parts of G with one vertex, two vertices and at least three vertices, respectively. If φ is a 2-partition of a complete n-partite graph G and X is a part of G, then by W_X (B_X) we denote the number of **White** (**Black**) vertices of the part X. If φ is a 2-partition of a complete n-partite graph G, then by W (B) we denote the number of **White** (**Black**) vertices in G.

We begin with the following simple lemma.

Lemma 1. *If φ is a locally-balanced 2-partition of G, then $\overline{\varphi}$ is also a locally-balanced 2-partition of G.*

Lemma 2. *If $K_{r_1,r_2\ldots,r_n}$ has a locally-balanced 2-partition with an open neighborhood, then for any part X, $|W_X - B_X| \leq 1$.*

Proof. Let φ be a a locally-balanced 2-partition with an open neighborhood of $K_{r_1,r_2\dots,r_n}$. Suppose, to the contrary, that there exists a part X' such that either $W_{X'} - B_{X'} \geq 2$ or $B_{X'} - W_{X'} \geq 2$. By Lemma 1., we can assume that there exists a part X' such that

$$W_{X'} - B_{X'} \geq 2 \tag{1}$$

It is easy to see that for any $v \in X$,

$$\#(v) = W - B - W_X + B_X.$$

From this and taking into account that for any $v \in V(K_{r_1,r_2\dots,r_n}), -1 \leq \#(v) \leq 1$, we obtain that for any part X,

$$-1 \leq W - B - W_X + B_X \leq 1. \tag{2}$$

By (1) and (2), we have

$$W - B \geq 1. \tag{3}$$

For any part Y $(Y \neq X')$, by (2), we have

$$-1 \leq W - B - W_Y + B_Y \leq 1.$$

From this and (3), we obtain that for any part Y $(Y \neq X')$,

$$W_Y - B_Y \geq 0. \tag{4}$$

Let us consider any $v \in X$ $(X \neq X')$. Clearly,

$$\#(v) = \sum_{Y, Y \neq X} (W_Y - B_Y) = \sum_{Y, Y \neq X, X'} (W_Y - B_Y) + W_{X'} - B_{X'}.$$

By (1) and (4), we get $\#(v) \geq 2$, which is a contradiction. ■

Theorem 1. *The graph $K_{r_1,r_2\dots,r_n}$ has a locally-balanced 2-partition with an open neighborhood if and only if the number of odd parts is even or one.*

Proof. Assume that $K_{r_1,r_2\dots,r_n}$ has k even parts and s odd parts. Suppose that $K_{r_1,r_2\dots,r_n}$ has a locally-balanced 2-partition with an open neighborhood, but contains odd number s $(s \geq 3)$ odd parts. Lemma 2 implies that for any part X of $K_{r_1,r_2\dots,r_n}$, $|W_X - B_X| \leq 1$. We decompose all the vertices of the graph into three groups as follows:

1. all even parts X_i (here, we have $W_{X_i} - B_{X_i} = 0$, by Lemma 2);

2. all odd parts X'_i with $W_{X'_i} > B_{X'_i}$ (here, we have $W_{X'_i} - B_{X'_i} = 1$, by Lemma 2);

3. all odd parts X''_i with $W_{X''_i} < B_{X''_i}$ (here, we have $W_{X''_i} - B_{X''_i} = -1$, by Lemma 2.).

Let $X_1, X_2, \dots, X_k$ be parts of the first group; $X'_1, X'_2, \dots, X'_m$ be parts of the second group; $X''_1, X''_2, \dots, X''_t$ be parts of the third group. Without loss of generality, we may assume that $m > t$. Consider two cases.

Case 1: $t = 0$.
Clearly, $m \geq 3$. Let us consider a vertex $v \in X_1'$. By 1 and 2, we have

$$\#(v) = \sum_{Y, Y \neq X_1'} (W_Y - B_Y) = \sum_{i=1}^{k} (W_{X_i} - B_{X_i}) + \sum_{i=2}^{m} (W_{X_i'} - B_{X_i'}) = m - 1 \geq 2,$$

which is a contradiction.

Case 2: $t > 0$.
Let us consider a vertex $v \in X_1''$. By 1, 2 and 3, we have

$$\#(v) = \sum_{Y, Y \neq X_1''} (W_Y - B_Y) = \sum_{i=1}^{k} (W_{X_i} - B_{X_i}) + \sum_{i=1}^{m} (W_{X_i'} - B_{X_i'}) +$$

$$\sum_{i=2}^{t} (W_{X_i''} - B_{X_i''}) = m - (t - 1) = (m - t) + 1 \geq 2,$$

which is a contradiction.

Now we show that if the number of odd parts is even or one, then $K_{r_1, r_2 \ldots, r_n}$ has a locally-balanced 2-partition with an open neighborhood.

First we color even parts uniformly.

Next we consider two cases.

Case A) The number of odd parts is $2l$. First l odd parts we color as follows: if X is such a part with $2p + 1$ ($p \geq 0$) vertices, then any p vertices get the color **Black** and the rest vertices get the color **White**. Next l odd parts we color similarly by taking the color **White** instead of **Black**, and vice versa.

Case B) X is the only odd part with $2p + 1$ ($p \geq 0$) vertices.
In this case we color any p vertices of X with the color **Black** and the rest vertices with the color **White**.

It is easy to see that the above-mentioned 2-partition is a locally-balanced 2-partition of $K_{r_1, r_2 \ldots, r_n}$ with an open neighborhood. ■

Theorem 2. *If $m_1 = 0$, then the graph $K_{r_1, r_2 \ldots, r_n}$ has a locally-balanced 2-partition with a closed neighborhood if and only if it has no odd part.*

Proof. Let φ be a locally-balanced 2-partition with a closed neighborhood of $K_{r_1, r_2 \ldots, r_n}$. Suppose, to the contrary, that there exists an odd part X. Since $m_1 = 0$, we have $|X| \geq 3$. Without loss of generality, we may assume that $W_X > B_X$. We consider two cases.

Case 1: $B_X = 0$.
Consider a vertex $v \in X$. Clearly, $\varphi(v) =$ **White**. This implies that

$$\#[v] = W - B - W_X + B_X + 1.$$

Since φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1, r_2 \ldots, r_n}$, we have

$$-1 \leq W - B - W_X + B_X + 1 \leq 1.$$

This implies that

$$W - B \geq W_X - 2 > 0.$$

Case 2: $B_X > 0$.
Consider a vertex $v \in X$ with $\varphi(v) = \textbf{Black}$. This implies that

$$\#[v] = W - B - W_X + B_X - 1.$$

Since φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\ldots,r_n}$, we have

$$-1 \leq W - B - W_X + B_X - 1 \leq 1.$$

This implies that $W - B \geq W_X - B_X > 0$.

In any case we obtain

$$W - B > 0. \tag{5}$$

First we consider the case when for any part Y, $W_Y \geq B_Y$. Let us consider a vertex $v \in X'$ ($X' \neq X$) with $\varphi(v) = \textbf{White}$. This implies that

$$\#[v] = \sum_{Y, Y \neq X'} (W_Y - B_Y) + 1 =$$

$$W_X - B_X + \sum_{Y, Y \neq X, X'} (W_Y - B_Y) + 1 \geq 2,$$

which is a contradiction.

So, we may assume that there exists a part X' such that $W_{X'} < B_{X'}$.

If $W_{X'} = 0$, then we consider a vertex $v \in X'$. Clearly, $\varphi(v) = \textbf{Black}$. From this and taking into account that φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\ldots,r_n}$, we have

$$-1 \leq W - B - W_{X'} + B_{X'} - 1 \leq 1.$$

Hence, $W - B \leq 2 - B_{X'} \leq 0$, which contradicts (5).

If $W_{X'} > 0$, then we consider a vertex $v \in X'$ with $\varphi(v) = \textbf{White}$. From this and taking into account that φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\ldots,r_n}$, we have

$$-1 \leq W - B - W_{X'} + B_{X'} + 1 \leq 1.$$

Hence, $W - B \leq W_{X'} - B_{X'} < 0$, which contradicts (5).

Let $K_{r_1,r_2\ldots,r_n}$ be a complete n-partite graph without an odd part. In this case we color each part uniformly. ■

Lemma 3. *If $m_1 > 0$ and $K_{r_1,r_2\ldots,r_n}$ has a locally-balanced 2-partition with a closed neighborhood, then $|W - B| \leq 1$.*

Proof. Let φ be a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2...,r_n}$. Consider a vertex v of the part X with $|X| = 1$. Since φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2...,r_n}$, we have $\#[v] = W - B$, and hence

$$-1 \leq W - B \leq 1.$$

■

Remark 1. *Clearly, Lemma 3. implies that if $|V(K_{r_1,r_2...,r_n})|$ is even, then $W = B$, and if $|V(K_{r_1,r_2...,r_n})|$ is odd, then $|W - B| = \pm 1$ (in fact, by Lemma 1., we may assume that $W - B = 1$).*

Theorem 3. *If $m_1 > 0$ and $|V(K_{r_1,r_2...,r_n})|$ is even, then $K_{r_1,r_2...,r_n}$ has a locally-balanced 2-partition with a closed neighborhood if and only if it has no odd part X with at least three vertices.*

Proof. Let φ be a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2...,r_n}$. Suppose that there exists an odd part X with at least three vertices. We may assume that $W_X > B_X$. We consider two cases.

Case 1: $B_X = 0$.

Consider a vertex $v \in X$. Clearly, $\varphi(v) =$ **White**. This implies that

$$\#[v] = W - B - W_X + B_X + 1.$$

By Remark 1, we get $\#[v] = 1 - W_X \leq -2$, which is a contradiction.

Case 2: $B_X > 0$.

Consider a vertex $v \in X$ with $\varphi(v) =$ **Black**. This implies that

$$\#[v] = W - B - W_X + B_X - 1.$$

By Remark 1 and taking into account that $W_X > B_X$, we obtain

$$\#[v] = B_X - W_X - 1 \leq 2,$$

which is a contradiction.

Now let $K_{r_1,r_2...,r_n}$ be a complete n-partite graph with $m_1 > 0$ odd parts. Clearly, m_1 is even.

Each even part we color uniformly. Then we color $\frac{m_1}{2}$ odd parts with the color **Black** and the other $\frac{m_1}{2}$ odd parts with the color **White**.

It is easy to see that this 2-partition is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2...,r_n}$. ■

Theorem 4. *If the graph $K_{r_1,r_2...,r_n}$ has an odd number of vertices, $m_1 > 0$ and there exists a part X such that $|X| = 2 + 2k$ ($k \in \mathbf{N}$), then $K_{r_1,r_2...,r_n}$ has no locally-balanced 2-partition with a closed neighborhood.*

Proof. Suppose that φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2...,r_n}$. By Remark 1 and Lemma 1, we get

$$W - B = 1.$$

Let us consider four cases.

Case 1: $B_X = 0$.
Let us consider a vertex $v \in X$. Since φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\dots,r_n}$, we have

$$\#[v] = W - B - W_X + B_X + 1 = 2 - W_X \leq -2,$$

which is a contradiction.

Case 2: $W_X = 0$.
Let us consider a vertex $v \in X$. Since φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\dots,r_n}$, we have

$$\#[v] = W - B - W_X + B_X - 1 = B_X \geq 4,$$

which is a contradiction.

Case 3: $W_X > B_X > 0$.
Let us consider a vertex $v \in X$ with $\varphi(v) = \mathbf{Black}$. Since φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\dots,r_n}$, we have

$$\#[v] = W - B - W_X + B_X - 1 = B_X - W_X \leq -2,$$

which is a contradiction.

Case 4: $0 < W_X \leq B_X$.
Let us consider a vertex $v \in X$ with $\varphi(v) = \mathbf{White}$. Since φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\dots,r_n}$, we have

$$\#[v] = W - B - W_X + B_X + 1 = 2 + B_X - W_X \geq 2,$$

which is a contradiction. ■

Theorem 5. *If the graph $K_{r_1,r_2\dots,r_n}$ has an odd number of vertices, $m_1 > 0$ and each part X of the graph has either two vertices or an odd number of vertices, then $K_{r_1,r_2\dots,r_n}$ has a locally-balanced 2-partition with a closed neighborhood if and only if $m_1 \geq 2m_2 + m_{\geq 3} - 1$.*

Proof. Suppose that φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\dots,r_n}$. By Remark 1 and Lemma 1, we have

$$W - B = 1. \tag{6}$$

Let us consider a part X with only two vertices u and v.

If $\varphi(v) = \mathbf{White}$ and $\varphi(u) = \mathbf{Black}$, then since φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\dots,r_n}$, we have

$$\#[v] = W - B - W_X + B_X + 1 = 2,$$

a contradiction.

Similarly, if $\varphi(v) = \mathbf{Black}$ and $\varphi(u) = \mathbf{White}$, then we consider the vertex u.

If $\varphi(v) = \varphi(u) = \mathbf{Black}$, then since φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\dots,r_n}$, we have

$$\#[v] = W - B - W_X + B_X - 1 = 2,$$

a contradiction.

This implies that $\varphi(v) = \varphi(u) =$ **White**.

Let X be an odd part with at least three vertices.

If $W_X = 0$, then by considering a vertex $v \in X$ and taking into account that φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\ldots,r_n}$, we obtain

$$\#[v] = W - B - W_X + B_X - 1 = B_X \geq 3,$$

a contradiction.

If $0 < W_X < B_X$, then by considering a vertex $v \in X$ with $\varphi(v) =$ **White** and taking into account that φ is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\ldots,r_n}$, we obtain

$$\#[v] = W - B - W_X + B_X + 1 = 2 + B_X - W_X \geq 3,$$

a contradiction. This shows that

$$W_X > B_X. \tag{7}$$

Let us consider two cases.

Case 1: $|X| = 3$.

By (7), we have that there are two possible cases: all vertices of X are **White** or two of them are **White** and the last one is **Black**.

If $W_X = 3$ and $B_X = 0$, then for any $v \in X$, we have

$$\#[v] = W - B - W_X + B_X + 1 = -1.$$

This implies that for any $v \in X$, $-1 \leq \#[v] \leq 1$.

If $W_X = 2$ and $B_X = 1$, then for any $v \in X$, we have either $\#[v] = W - B - W_X + B_X + 1 = 1$ (if $\varphi(v) =$ **White**) or $\#[v] = W - B - W_X + B_X - 1 = -1$ (if $\varphi(v) =$ **Black**). This implies that for any $v \in X$, $-1 \leq \#[v] \leq 1$.

Case 2: $|X| \geq 5$.

By (7), we have $W_X - B_X \geq 1$.

Let us show that $W_X - B_X = 1$.

Suppose, to the contrary, that $W_X - B_X \geq 3$.

If $B_X = 0$, then for any $v \in X$, we have

$$\#[v] = W - B - W_X + B_X + 1 \leq -3,$$

which is a contradiction.

If $W_X > B_X > 0$. For any $v \in X$ with $\varphi(v) =$ **Black**, we have

$$\#[v] = W - B - W_X + B_X - 1 \leq -3,$$

which is a contradiction.

This shows that $W_X - B_X = 1$.

So, we obtain that if X is a part with at least two vertices, then we have the following three possible cases:

a) If X is an odd part with three vertices, then

$$\text{either} \quad W_X - B_X = 1 \quad \text{or} \quad W_X - B_X = 3; \tag{8}$$

b) If X is an odd part with at least five vertices, then

$$W_X - B_X = 1; \tag{9}$$

c) If X has two vertices, then

$$W_X - B_X = 2. \tag{10}$$

By (6), (8),(9),(10), we get

$$m_1 = \sum_{X,|X|=1} (B_X + W_X) \geq \sum_{X,|X|=1} (B_X - W_X) = B - W + \sum_{X,|X|=2} (W_X - B_X) +$$

$$\sum_{X,W_X-B_X=1} (W_X - B_X) + \sum_{X,W_X-B_X=3} (W_X - B_X) = B - W + 2|\{X\colon X \text{ is a part with } |X| = 2\}| +$$

$$|\{X\colon X \text{ is a part with } W_X - B_X = 1\}| + 3|\{X\colon X \text{ is a part with } W_X - B_X = 3\}| \geq$$

$$B - W + 2|\{X\colon X \text{ is a part with } |X| = 2\}| + |\{X\colon X \text{ is a part with } W_X - B_X = 1\}| +$$

$$|\{X\colon X \text{ is a part with } W_X - B_X = 3\}| = B - W + 2|\{X\colon X \text{ is a part with } |X| = 2\}| +$$

$$|\{X\colon X \text{ is an odd part with at least three vertices}\}| = -1 + 2m_2 + m_{\geq 3}.$$

Now let $m_1 \geq 2m_2 + m_{\geq 3} - 1$. We construct a 2-partition of $K_{r_1,r_2\dots,r_n}$ as follows:

1) Each vertex $v \in X$ ($|X| = 2$), we color with the color **White**;

2) For any odd part X with $|X| = 2p + 1$ ($p \in \mathbf{N}$) vertices, we color $p + 1$ vertices of X with the color **White**, and the rest p vertices we color with the color **Black**;

3) We take any $(2m_2 + m_{\geq 3} - 1)$ parts with only one vertex of the graph and we color each vertex in these parts with the color **Black**. Since $|V(K_{r_1,r_2\dots,r_n})|$ is odd and $m_1 \geq 2m_2 + m_{\geq 3} - 1$, we have that $m_1 - (2m_2 + m_{\geq 3} - 1)$ is even and non-negative. The vertices of the remaining $m_1 - (2m_2 + m_{\geq 3} - 1)$ parts we color uniformly.

It is not difficult to see that this 2-partition is a locally-balanced 2-partition with a closed neighborhood of $K_{r_1,r_2\dots,r_n}$. ■

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Լրիվ բազմակողմանի գրաֆների լոկալ-հավասարակշռված 2-տրոհումների մասին

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Ամփոփում

$f : V(G) \to \{\mathbf{White}, \mathbf{Black}\}$ ֆունկցիան կոչվում է G գրաֆի 2-տրոհում: G գրաֆի f 2-տրոհումը կոչվում է լոկալ-հավասարակշռված 2-տրոհում բաց շրջակայքով, եթե կամայական $v \in V(G)$ գագաթի համար տեղի ունի

$$||\{u \in N_G(v) \colon f(u) = \mathbf{White}\}| - |\{u \in N_G(v) \colon f(u) = \mathbf{Black}\}|| \le 1,$$

որտեղ $N_G(v) = \{u \in V(G) \colon uv \in E(G)\}$: G գրաֆի f' 2-տրոհումը կոչվում է լոկալ-հավասարակշռված տրոհում փակ շրջակայքով, եթե կամայական $v \in V(G)$ գագաթի համար տեղի ունի

$$||\{u \in N_G[v] \colon f'(u) = \mathbf{White}\}| - |\{u \in N_G[v] \colon f'(u) = \mathbf{Black}\}|| \le 1,$$

որտեղ $N_G[v] = N_G(v) \cup \{v\}$: Այս աշխատանքում տրվում են լրիվ բազմակողմանի գրաֆների լոկալ-հավասարակշռված տրոհումների գոյության անհրաժեշտ և բավարար պայմաններ:

О локально-сбалансированных 2-разбиениях полных многодольных графов

А. Гарибян и П. Петросян

Аннотация

2-разбиением графа G называется функция $f : V(G) \to \{\mathbf{White}, \mathbf{Black}\}$. 2-разбиение f графа G называется локально-сбалансированным с открытой окрестностью, если для любой вершины $v \in V(G)$

$$||\{u \in N_G(v) \colon f(u) = \mathbf{White}\}| - |\{u \in N_G(v) \colon f(u) = \mathbf{Black}\}|| \le 1,$$

где $N_G(v) = \{u \in V(G) \colon uv \in E(G)\}$. 2-разбиение f' графа G называется локально-сбалансированным с закрытой окрестностью, если для любой вершины $v \in V(G)$

$$||\{u \in N_G[v] \colon f'(u) = \mathbf{White}\}| - |\{u \in N_G[v] \colon f'(u) = \mathbf{Black}\}|| \le 1,$$

где $N_G[v] = N_G(v) \cup \{v\}$. В настоящей работе даются необходимые и достаточные условия существования локально-сбалансированных 2-разбиений полных многодольных графов.