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On Church-Rosser Property of Notion of $\beta\delta$-Reduction for Canonical Notion of δ-Reduction

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Abstract

In this paper the canonical notions of δ-reduction for typed λ-terms are considered. Typed λ-terms use variables of any order and constants of order ≤ 1, where constants of order 1 are strongly computable, monotonic functions with indeterminate values of arguments. The canonical notions of δ-reduction are the notions of δ-reduction that are used in the implementations of functional programming languages. It is shown that for the main canonical notion of δ-reduction the notion of $\beta\delta$-reduction has the Church-Rosser property. It is also shown that there exists a canonical notion of δ-reduction such that the notion of $\beta\delta$-reduction does not have Church-Rosser property.

Keywords: Canonical notion of δ-reduction, Church-Rosser property, $\beta\delta$-reduction.

1. Typed λ-Terms, Canonical Notion of δ-Reduction

The definitions of this section can be found in [1]–[4]. Let M be a partially ordered set, which has a least element $\perp$, which corresponds to the indeterminate value, and each element of M is comparable only with $\perp$ and itself. Let us define the set of types (denoted by Types).

1. $M \in Types$,
2. If β,α_1,...,$\alpha_k \in$ Types ($k > 0$), then the set of all monotonic mappings from $\alpha_1 \times ... \times \alpha_k$ into β (denoted by $[\alpha_1 \times ... \times \alpha_k \to \beta]$) belongs to Types.

Let $\alpha \in Types$, then the order of type α (denoted by $ord(\alpha)$) will be a natural number, which is defined in the following way: if $\alpha = M$ then $ord(\alpha) = 0$, if $\alpha = [\alpha_1 \times ... \times \alpha_k \to \beta]$, where $\beta, \alpha_1, ..., \alpha_k \in Types, k > 0$, then $ord(\alpha) = 1 + max(ord(\alpha_1), ..., ord(\alpha_k), ord(\beta))$. If x is a variable of type α and constant $c \in \alpha$, then $ord(x) = ord(c) = ord(\alpha)$.

Let $\alpha \in$ Types and V_α be a countable set of variables of type α, then $V = \bigcup_{\alpha \in Types} V_\alpha$ is the set of all variables. The set of all terms, denoted by $\Lambda = \bigcup_{\alpha \in Types} \Lambda_\alpha$, where Λ_α is the set of terms of type α, is defined the in following way:

1. If $c \in \alpha, \alpha \in$ Types, then $c \in \Lambda_\alpha$,
2. If $x \in V_\alpha$, $\alpha \in$ Types, then $x \in \Lambda_\alpha$,

3. If $\tau \in \Lambda_{[\alpha_1 \times ... \times \alpha_k \to \beta]}$, $t_i \in \Lambda_{\alpha_i}$, where $\beta, \alpha_i \in Types, i = 1, ..., k, k \geq 1$, then $\tau(t_1, ..., t_k) \in \Lambda_\beta$ (the operation of application),

4. If $\tau \in \Lambda_\beta, x_i \in V_{\alpha_i}$ where $\beta, \alpha_i \in Types,\ i \neq j \Rightarrow x_i \neq x_j, i, j = 1, ..., k, k \geq 1$, then $\lambda x_1 ... x_k[\tau] \in \Lambda_{[\alpha_1 \times ... \times \alpha_k \to \beta]}$ (the operation of abstraction).

The notion of free and bound occurrences of variables as well as free and bound variable are introduced in the conventional way. The set of all free variables in the term t is denoted by $FV(t)$. Terms t_1 and t_2 are said to be congruent (which is denoted by $t_1 \equiv t_2$) if one term can be obtained from the other by renaming bound variables.

Let $t \in \Lambda_\alpha, \alpha \in Types$ and $FV(t) \subset \{y_1, ..., y_n\}, \overline{y}_0 = (y_1^0, ..., y_n^0)$, where $y_i \in V_{\beta_i}, y_i^0 \in \beta_i, \beta_i \in Types, i = 1, ..., n, n \geq 0$. The value of the term t for the values of the variables $y_1, ...y_n$ equal to $\overline{y}_0 = (y_1^0, ..., y_n^0)$, is denoted by $Val_{\overline{y}_0}(t)$ and is defined in the conventional way, see [2].

Let terms $t_1, t_2 \in \Lambda_\alpha, \alpha \in Types$, $FV(t_1) \cup FV(t_2) = \{y_1, ..., y_n\}, y_i \in V_{\beta_i}, \beta_i \in Types, i = 1, ..., n, n \geq 0$, then terms t_1 and t_2 are called equivalent (denoted by $t_1 \sim t_2$) if for any $\overline{y}_0 = (y_1^0, ..., y_n^0)$, where $y_i^0 \in V_{\beta_i}, i = 1, ..., n$ we have the following: $Val_{\overline{y}_0}(t_1) = Val_{\overline{y}_0}(t_2)$. A term $t \in \Lambda_\alpha, \alpha \in Types$, is called a constant term with value $a \in \alpha$ if $t \sim a$.

Further, we assume that M is a recursive set, and the considered terms use variables of any order and constants of order ≤ 1, where constants of order 1 are strongly computable, monotonic functions with indeterminate values of arguments. A function $f : M^k \to M, k \geq 1$, with indeterminate values of arguments, is said to be strongly computable if there exists an algorithm, which stops with value $f(m_1, ..., m_k) \in M$ for all $m_1, ..., m_k \in M$, see [1].

To show mutually different variables of interest $x_1, ..., x_k, k \geq 1$, of a term t, the notation $t[x_1, ..., x_k]$ is used. The notation $t[t_1, ..., t_k]$ denotes the term obtained by the simultaneous substitution of the terms $t_1, ..., t_k$ for all free occurrences of the variables $x_1, ..., x_k$ respectively, where $x_i \in V_{\alpha_i}, i \neq j \Rightarrow x_i \not\equiv x_j,\ t_i \in \Lambda_{\alpha_i}, \alpha_i \in Types, i, j = 1, .., k, k \geq 1$. A substitution is said to be admissible if all free variables of the term being substituted remain free after the substitution. We will consider only admissible substitutions.

A term of the form $\lambda x_1 ... x_k[\tau[x_1, ..., x_k]](t_1, ..., t_k)$, where $x_i \in V_\alpha, i \neq j \Rightarrow x_i \not\equiv x_j, \tau \in \Lambda, t_i \in \Lambda_{\alpha_i}, \alpha_i \in Types, i, j = 1, ..., k, k \geq 1$, is called a β-redex, its convolution is the term $\tau[t_1, ..., t_k]$. The set of all pairs (τ_0, τ_1), where τ_0 is a β-redex and τ_1 is its convolution, is called a notion of β-reduction and is denoted by β. A one-step β-reduction ($\to_\beta$) and β-reduction ($\to\to_\beta$) are defined in the conventional way. A term containing no β-redexes is called a β-normal form. The set of all β-normal forms is denoted by β-NF.

δ-redex has a form $f(t_1, ..., t_k)$, where $f \in [M^k \to M]$, $t_i \in \Lambda_M, i = 1, ..., k, k \geq 1$, its convolution is either $m \in M$ and in this case $f(t_1, ..., t_k) \sim m$ or a subterm t_i and in this case $f(t_1, ..., t_k) \sim t_i, i = 1, ..., k$. A fixed set of term pairs (τ_0, τ_1), where τ_0 is a δ-redex and τ_1 is its convolution, is called a notion of δ-reduction and is denoted by δ. A one-step δ-reduction ($\to_\delta$) and δ-reduction ($\to\to_\delta$) are defined in the conventional way.

A one-step $\beta\delta$-reduction ($\to$) and $\beta\delta$-reduction ($\to\to$) defined in the conventional way. A term containing no $\beta\delta$-redexes is called a normal form. The set of all normal forms is denoted by NF.

A notion of δ-reduction is called a single-valued notion of δ-reduction, if δ is a single-valued relation, i.e., if $(\tau_0, \tau_1) \in \delta$ and $(\tau_0, \tau_2) \in \delta$, then $\tau_1 \equiv \tau_2$, where $\tau_0, \tau_1, \tau_2 \in \Lambda_M$. A notion of δ-reduction is called an effective notion of δ-reduction if there exists an algorithm, which for any term $f(t_1, ..., t_k)$, where $f \in [M^k \to M]$, $t_i \in \Lambda_M, i = 1, ..., k, k \geq 1$, gives its convolution if $f(t_1, ..., t_k)$ is a δ-redex and stops with a negative answer otherwise.

Definition 1: *[2] An effective, single-valued notion of δ-reduction is called a canonical notion of δ-reduction if:*

1. $t \in \beta\text{-}NF,\ t \sim m,\ m \in M \setminus \{\bot\} \Rightarrow t \twoheadrightarrow_\delta m,$
2. $t \in \beta\text{-}NF,\ FV(t) = \emptyset,\ t \sim \bot \Rightarrow t \twoheadrightarrow_\delta \bot.$

Theorem 1: *[2] For every recursive set of strong computable, monotonic functions with indeterminate values of arguments there exists a canonical notion of δ-reduction.*

2. Main Canonical Notion of δ-Reduction, Church-Rosser Property

Let C be a recursive set of strongly computable, monotonic functions with indeterminate values of arguments. Let us fix the following notion of δ-reduction, which contains only the following pairs, where for every $f \in C$, $f : M^k \to M$, $k \geq 1$ we have:

1. if $f(m_1, ..., m_k) = m$, where $m, m_1, ..., m_k \in M, m \neq \bot$, then $(f(\mu_1, ..., \mu_k), m) \in \delta$, where $\mu_i = m_i$ if $m_i \neq \bot$, and $\mu_i \equiv t_i, t_i \in \Lambda_M$ if $m_i = \bot, i = 1, ..., k, k \geq 1$.
2. if $f(m_1, ..., m_k) = \bot$, where $m_1, ..., m_k \in M$, then $(f(m_1, ..., m_k), \bot) \in \delta$.

From the proof of Theorem 1 it follows that the δ is a canonical notion of δ-reduction. The δ is called a main canonical notion of δ-reduction.

Definition 2: *The term $t \in \Lambda$ is said to be strongly normalizable, if the length of each $\beta\delta$-reduction chain from the term t is finite.*

Theorem 2: *[3] Every term is strongly normalizable.*

Theorem 3: *[3] For every term $t \in \Lambda$, if $t \twoheadrightarrow_\beta t', t \twoheadrightarrow_\beta t''$ and $t', t'' \in \beta$-NF then $t' \equiv t''$.*

Definition 3: *Let $t \in \Lambda_\alpha, \alpha \in Types$ and $t \equiv t_1 \to ... \to t_n, n \geq 1$, where $t_i \in \Lambda_\alpha, i = 1, ..., n$, then the sequence $t_1, ..., t_n$ is called the inference of the term t_n from the term t and n is called the length of that inference.*

Definition 4: *The inference tree of the term t is an oriented tree with the root t and if a term τ is some node of the tree and $\tau_1, ..., \tau_k, k \geq 0$ are all $\beta\delta$-redexes of τ, then $\tau_{\tau'_1}, ..., \tau_{\tau'_k}$ are all descendants of the node τ, where τ'_i is the convolution of $\tau_i, i = 1, ..., k$.*

It is easy to see that each node in the inference tree of the term t has a finite number of descendants and if τ is a leaf of that tree, then $\tau \in NF$. The height of an inference tree of the term t is the length of the longest path from the root t to a leaf. The set of all terms, the height of the inference tree of which is equial to $n - 1$, is denoted by $\Lambda^{(n)}$, $n \geq 1$.

Definition 5: *The notion of $\beta\delta$-reduction has the Church-Rosser property (CR-property) if for every term $t \in \Lambda_\alpha, \alpha \in Types$, if $t \twoheadrightarrow t_1$ and $t \twoheadrightarrow t_2$, $t_1, t_2 \in \Lambda_\alpha$, then there exists a term $t' \in \Lambda_\alpha$ such that $t_1 \twoheadrightarrow t'$ and $t_2 \twoheadrightarrow t'$.*

Theorem 4: *For the main canonical notion of δ-reduction the notion of $\beta\delta$-reduction has the Church-Rosser property.*

Proof. Let $t \in \Lambda^{(1)}$, then $t \in NF$ and $t \equiv t_1 \equiv t_2 \equiv t'$. Now, let us suppose that CR-property holds for every term $\tau \in \Lambda^{(k)}, k \leq n-1, n \geq 2$ and show that it holds for every term $t \in \Lambda^{(n)}$. If $t \equiv t_1$, then $t_1 \twoheadrightarrow t_2$ and $t' \equiv t_2$. If $t \equiv t_2$, then $t_2 \twoheadrightarrow t_1$ and $t' \equiv t_1$. If $t_1 \not\equiv t$ and $t_2 \not\equiv t$, then there exist terms $t'_1, t'_2 \in \Lambda$, such that $t \to t'_1 \twoheadrightarrow t_1$ and $t \to t'_2 \twoheadrightarrow t_2$. Therefore, there exist $\beta\delta$-redexes $\tau_1, \tau_2 \in \Lambda$ such that $t \equiv t_{\tau_1} \equiv t_{\tau_2}$, $t'_1 \equiv t_{\tau'_1}$ and $t'_2 \equiv t_{\tau'_2}$, where terms τ'_1, τ'_2 are the convolutions of τ_1 and τ_2, accordingly. Let us show that there exists a term t' such that $t'_1 \twoheadrightarrow t'$ and $t'_2 \twoheadrightarrow t'$.

If τ_1 is not a subterm of τ_2 and τ_2 is not a subterm of τ_1, then $t_{\tau_1,\tau_2} \to t_{\tau'_1,\tau_2} \to t_{\tau'_1,\tau'_2}$ and $t_{\tau_1,\tau_2} \to t_{\tau_1,\tau'_2} \to t_{\tau'_1,\tau'_2}$. Therefore $t' \equiv t_{\tau'_1,\tau'_2}$.

If τ_2 is a subterm of τ_1 or τ_1 is a subterm of τ_2, then the following cases are possible:

1) τ_1 and τ_2 are both δ-redexes. Without loss of generality we suppose that τ_2 is a subterm of τ_1 ($\tau_1 \equiv \tau_{1_{\tau_2}}$). Let $\tau_{1_{\tau_2}} \equiv f(\mu_1, ..., \mu_{j_{\tau_2}}, ..., \mu_k), f \in [M^k \to M], t_i \in \Lambda_M, i = 1, ..., k, 1 \leq j \leq k$. Since τ_2 is a δ-redex, then $\mu_{j_{\tau_2}} \notin M$ and since τ_1 is a δ-redex, then from Definition 2 it follows that there exists $m \in M$, $m \neq \bot$ such that $(\tau_1, m) \in \delta$. Therefore $\tau_1 \to_\delta m$. Since $\mu_{j_{\tau_2}} \notin M$ and $(\tau_1, m) \in \delta$, where $m \neq \bot$, then from Definition 2 it follows that $f(\mu_1, ..., \mu, ..., \mu_k) \in \delta$ for every $\mu \in \Lambda_M$. Therefore $(f(\mu_1, ..., \mu_{j_{\tau'_2}}, ..., t_k), m) \in \delta$ and $\tau_{1_{\tau'_2}} \to_\delta m$. Therefore, $t_{\tau_{1_{\tau_2}}} \to_\delta t_{\tau'_1} \equiv t_m$ and $t_{\tau_{1_{\tau_2}}} \to_\delta t_{\tau_{1_{\tau'_2}}} \to_\delta t_m$, where $m \in M \backslash \{\bot\}$ and $t' \equiv t_m$.

2) τ_1 and τ_2 are both β-redexes. From Theorem 2 it follows that there exist terms $t'_1, t'_2 \in \beta - NF$ such that $t \to_\beta t_1 \twoheadrightarrow_\beta t'_1$ and $t \to_\beta t_2 \twoheadrightarrow_\beta t'_2$. Therefore from Theorem 3 it follows that $t'_1 \equiv t'_2 \equiv t'$.

3) τ_1 is δ-redex and τ_2 is β-redex or τ_1 is β-redex and τ_2 is δ-redex. Without lose of generality we suppose that τ_1 is δ-redex and τ_2 is β-redex. Let $\tau_2 \equiv \lambda x_1, ..., x_n[\tau[x_1, ... , x_n]](\mu_1, ..., \mu_n)$, $\tau \in \Lambda, x_i \in V_{\alpha_i}, \alpha_i \in Types$, $i = 1, ..., n$. The following cases are possible:

3.1) $\tau_1 \equiv \tau_{1_{\tau_2}}$. It can be shown that $\tau_1 \to m$ and $\tau_{1_{\tau'_2}} \to m$ as shown in case 2. Therefore $t_{\tau_{1_{\tau_2}}} \to_\delta t_{\tau'_1} \equiv t_m$ and $t_{\tau_{1_{\tau_2}}} \to_\beta t_{\tau_{1_{\tau'_2}}} \to_\delta t_m$, where $m \in M \backslash \{\bot\}$. Therefore $t' \equiv t_m$.

3.2) $\tau_2 \equiv \lambda x_1 ... x_n[\tau_{\tau_1}[x_1, ..., x_n]](\mu_1, ..., \mu_n)$, then it is easy to see, that if $\tau_1[x_1, ..., x_k] \to_\delta \tau'_1$ then $\tau_1[\mu_1, ..., \mu_k] \to_\delta \tau'_1$ and we have:

$\theta \equiv \lambda x_1 ... x_n[\tau_{\tau_1}[x_1, ..., x_n]](\mu_1, ..., \mu_n) \to_\delta \lambda x_1 ... x_n[\tau_{\tau'_1}[x_1, ..., x_n]](\mu_1, ..., \mu_n) \equiv \theta_1 \to_\beta \tau_{\tau'_1}[\mu_1, ..., \mu_n]$;

$\theta \equiv \lambda x_1 ... x_n[\tau_{\tau_1}[x_1, ..., x_n]](\mu_1, ..., \mu_n) \to_\beta \tau_{\tau_1}[\mu_1, ..., \mu_n] \equiv \theta_2 \to_\delta \tau_{\tau'_1}[\mu_1, ..., \mu_n]$;

Therefore $t' \equiv t_{\tau_{\tau'_1}[\mu_1, ..., \mu_n]}$ since $t \to_\delta t_{\theta_1} \to_\beta t_{\tau_{\tau'_1}[\mu_1, ..., \mu_n]}$ and $t \to_\beta t_{\theta_2} \to_\delta t_{\tau_{\tau'_1}[\mu_1, ..., \mu_n]}$.

3.3) $\tau_2 \equiv \lambda x_1 ... x_n[\tau[x_1, ..., x_n]](\mu_1, ..., \mu_{i_{\tau_i}}, ..., \mu_n)$. Without loss of generality we suppose that $i = 1$.

$\theta \equiv \lambda x_1 ... x_n[\tau[x_1, ..., x_n]](\mu_{1_{\tau_1}}, ..., \mu_n) \to_\delta \lambda x_1 ... x_n[\tau[x_1, ..., x_n]](\mu_{1_{\tau'_1}}, ..., \mu_n)$

$\equiv \theta_1 \to_\beta \tau[\mu_{1_{\tau'_1}}, ..., \mu_n] \equiv \theta'$;

$\theta \to_\beta \tau[\mu_{1_{\tau_1}}, ..., \mu_n] \equiv \theta_2 \twoheadrightarrow_\delta \theta'$;

Therefore $t \to_\delta t_{\theta_1} \equiv t_1 \to_\beta t_{\theta'}$, $t \to_\beta t_{\theta_2} \equiv t_2 \twoheadrightarrow_\delta t_{\theta'}$ and $t' \equiv t_{\theta'}$.

Since $t'_1 \twoheadrightarrow t_1$, $t'_1 \twoheadrightarrow t'$ and $t'_1 \in \Lambda^{(k_1)}, 1 \leq k_1 \leq n-1$, then from the induction hypothesis it follows that there exists a term t''_1 such that $t_1 \twoheadrightarrow t''_1$ and $t' \twoheadrightarrow t''_1$. Since $t'_2 \twoheadrightarrow t_2$, $t'_2 \twoheadrightarrow t'$ and $t'_2 \in \Lambda^{(k_2)}, 1 \leq k_2 \leq n-1$, then from the induction hypothesis it follows that there exists a term t''_2 such that $t_2 \twoheadrightarrow t''_2$ and $t' \twoheadrightarrow t''_2$. Since $t' \twoheadrightarrow t''_1$, $t' \twoheadrightarrow t''_2$ and $t' \in \Lambda^{(k_3)}, 1 \leq k_3 \leq n-1$, then from the induction hypothesis it follows that there exists a term t'' such that $t''_1 \twoheadrightarrow t''$ and $t''_2 \twoheadrightarrow t''$. Therefore $t_1 \twoheadrightarrow t''$ and $t_2 \twoheadrightarrow t''$. ■

Theorem 5: *There exists a canonical notion of δ-reduction such that $\beta\delta$-reduction does not have Church-Rosser property.*

Proof. Let us fix $M = N \cup \{\perp\}$, where $N = \{0, 1, 2, ...\}$ and $C = \{min, dec\}$ where $dec \in [M \to M], min \in [M^2 \to M]$ and for every $m, m_1, m_2 \in M$ we have:

$$min(m_1, m_2) = \begin{cases} m_1, & if\ m_1, m_2 \in N\ and\ m_1 < m_2 \\ m_2, & if\ m_1, m_2 \in N\ and\ m_1 \geq m_2 \\ \perp, & otherwise \end{cases}$$

$$dec(m) = \begin{cases} 0, & if\ m = 0 \\ m - 1, & if\ m \in N\ and\ m \neq 0 \\ \perp, & otherwise \end{cases}$$

It is easy to see that min and dec are strongly computable, naturally extended functions with indeterminate values of arguments (a function is said to be naturally extended, if its value is $\perp$ whenever the value of at least one of the arguments is $\perp$). Let us consider the main canonical notion of δ-reduction δ for the set C:

δ is: $(min(n_1, n_2), n_1) \in \delta$, where $n_1, n_2 \in N$ and $n_1 < n_2$

$(min(n_1, n_2), n_2) \in \delta$, where $n_1, n_2 \in N$ and $n_1 \geq n_2$

$(min(n, \perp), \perp) \in \delta$, where $n \in N$

$(min(\perp, n), \perp) \in \delta$, where $n \in N$

$(min(\perp, \perp), \perp) \in \delta$

$(dec(0), 0) \in \delta$

$(dec(n_1), n_2) \in \delta$, where $n_1, n_2 \in N$ and $n_1 > 0, n_2 = n_1 - 1$

$(dec(\perp), \perp) \in \delta$

Let us consider the notion of δ-reduction δ'.

δ' is: $(min(n_1, n_2), n_1) \in \delta'$, where $n_1, n_2 \in N$ and $n_1 < n_2$

$(min(n_1, n_2), n_2) \in \delta'$, where $n_1, n_2 \in N$ and $n_1 \geq n_2$

$(min(t, \perp), \perp) \in \delta'$, where $t \in \Lambda$

$(min(\perp, t), \perp) \in \delta'$, where $t \in \Lambda$

$(dec(0), 0) \in \delta'$

$(dec(n_1), n_2) \in \delta'$, where $n_1, n_2 \in N$ and $n_1 > 0, n_2 = n_1 - 1$

$(dec(\perp), \perp) \in \delta'$

$(min(dec(x), x), dec(x)) \in \delta'$, where $x \in V$

It is easy to see that δ' is an effective, single-valued notion of δ-reduction. Therefore, to show that δ' is a canonical notion of δ-reduction it suffices to show that $\delta \subset \delta'$, which is obvious.

Let us show that for the δ' the notion of $\beta\delta$-reduction does not have Church-Rosser property. For the term $t \equiv \lambda x[min(dec(x), x)](dec(y))$ we have:

$$\lambda x[min(dec(x), x)](dec(y)) \to_\beta min(dec(dec(y)), dec(y)) \in NF;$$

$$\lambda x[min(dec(x), x)](dec(y)) \to_{\delta'} \lambda x[dec(x)](dec(y)) \to_\beta dec(dec(y)) \in NF;$$

Let $t_1 \equiv min(dec(dec(y)), dec(y))$ and $t_2 \equiv dec(dec(y))$. Since $t_1, t_2 \in NF$ and $t_1 \not\equiv t_2$, then there does not exist a term t' such that $t_1 \twoheadrightarrow t'$ and $t_2 \twoheadrightarrow t'$. Therefore, for δ' the notion of $\beta\delta$-reduction does not have Church-Rosser property. ■

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Կանոնիկ δ-ռեդուկցիայի գաղափարի դեպքում $\beta\delta$-ռեդուկցիայի գաղափարի Չորչ-Ռոսսերի հատկության մասին

Դ. Գրիգորյան

Ամփոփում

Աշխատանքում դիտարկվում է կանոնիկ δ-ռեդուկցիայի գաղափարը տիպիզացված λ-թերմերի համար: Տիպիզացված λ-թերմերը օգտագործում են ցանկացած կարգի փոփոխականներ և հաստատուններ, որոնց կարգը ≤ 1, որտեղ 1-ին կարգի հաստատունները հանդիսանում են ուժեղ հաշվարկելի, արգումենտների անորոշ արժեքներով ֆունկցիաներ: Կանոնիկ δ-ռեդուկցիայի գաղափարը այն δ-ռեդուկցիայի գաղափարն է, որն օգտագործվում է ֆունկցիոնալ ծրագրավորման լեզուների իրականացման մեջ: Ապացուցված է, որ հիմնական կանոնիկ δ-ռեդուկցիայի գաղափարի դեպքում $\beta\delta$-ռեդուկցիայի գաղափարը օժտված է Չորչ-Ռոսսերի հատկությամբ: Ապացուցված է նաև, որ գոյություն ունի կանոնիկ δ-ռեդուկցիայի գաղափար, որի դեպքում $\beta\delta$-ռեդուկցիայի գաղափարը օժտված չէ Չորչ-Ռոսսերի հատկությամբ:

О свойстве Чёрча-Россера понятия $\beta\delta$-редукции в случае каноническом понятии δ-редукции

Д. Григорян

Аннотация

В данной работе рассматривается основное каноническое понятие δ-редукции для типизированных λ-термов. Типизированные λ -термы используют переменные любых порядков и константы, порядок которых ≤ 1, причем константы порядка 1 являются сильно вычислимыми, монотонными функциями с неопределеными значениями аргументов. Каноническое понятие δ-редукции используется при реализации функциональных языков программирования. Доказана, что в случае основного канонического понятия δ-редукции понятие $\beta\delta$-редукции имеет свойство Чёрча-Россера. Доказана, что существует каноническое понятие δ-редукции, в случае которого понятие $\beta\delta$-редукции не имеет свойство Чёрча-Россера.