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On the Comparative Complexity of Primitive Recursive Arithmetical and String Functions[1]

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Abstract

Formal languages LA and LW are introduced as in [1] for the representation of primitive recursive arithmetical and string functions. Shannon functions SH_{AW} and SH_{WA} describing the relations between the complexities of functions representations in these languages are defined as in [1]. A new proof of the upper bounds for SH_{AW} is presented; it is based on a new method giving in some cases new possibilities for applications in comparison with the methods considered in [1].

Keywords: string function, arithmetical function, term, alphabetic enumeration, Shannon function, primitive recursive function.

Investigations described in this paper may be considered as the continuation of those presented in [1]. Let us recall definitions of some notions given in [1]. We suppose that an alphabet $A = \{a_1, a_2, ..., a_p\}$, where $p > 1$, is fixed. The set of all strings in this alphabet (including the empty string Λ) is denoted by A^*; the set of all k-tuples $(Q_1, Q_2, ..., Q_k)$, where $Q_i \in A^*$ for $1 \le i \le k$, will be denoted by $(A^*)^k$. The set of all non-negative integers {0, 1, 2, ... } will be denoted by *N;* the set of all k-tuples $(x_1, x_2, ..., x_k)$, where $x_i \in N$ for $1 \le i \le k$, will be denoted by N^k. *k-dimensional string function* in A is defined ([1], [2]) as a mapping of $(A^*)^k$ into A^*; *k-dimensional arithmetical function* is defined as a mapping of $(N)^k$ into *N. Primitive recursive string functions* in A as well as *primitive recursive arithmetical functions* are defined in a usual way as in [1] and [2]. *The alphabetic enumeration* of the set A^* is defined as in [1] and [2]; let us recall that this enumeration defines a one-to-one correspondence between the sets A^* and *N.* The non-negative integer, corresponding to a string Q in the alphabetic enumeration is denoted by $\pi(Q)$. The string in A^* corresponding to the number n in this enumeration is denoted by $\alpha_p(n)$ or αn. The length of a string Q is denoted by $|Q|$. All these notations are used in [1].

The alphabetic enumeration of strings gives also a one-to-one correspondence between n-dimensional string functions in A, and n-dimensional arithmetical functions.

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Namely, we say ([1], [2]) that an n-dimensional arithmetical function *f represents* an n-dimensional string function *F,* if

$$F(\alpha x_1,\ \alpha x_2,\ \ldots,\ \alpha x_n) = \alpha f(x_1,\ x_2,\ \ldots,\ x_n)$$

for all $x_1, x_2, \ldots, x_n$ in *N*. In this case we say also that *F* and *f correspond* to one another.

The mentioned correspondence gives also a one-to-one correspondence between primitive recursive string functions in *A* and primitive recursive arithmetical functions ([1], [2]).

In [1] the formal languages *LA* and *LW* are introduced for the representation of primitive recursive arithmetical functions and primitive recursive string functions. The formal expressions in these languages are said to be *terms*; by $t \in LA$ and $r \in LW$ we denote the statements "*t* is a term in *LA*", "*r* is a term in *LW*". In the definition of *LA* the symbols *S* and *R* are used for the operators of superposition and primitive recursion of arithmetical functions; in the definition of *LW* the symbols ***S*** and ***R*** are used for the operators of superposition and alphabetic primitive recursion of string functions ([1], [2]). Special notations for some modifications of the mentioned operators (*Sbl, Sbr, Sel, Ser, Sb, Se* in *LA;* ***Sbl, Sbr, Sel, Ser, Sb, Se*** in *LW*) are also included in *LA* and *LW* ([1]). We shall consider below special cases of the implementation of the modifications ***Sb*** and ***Se*** of the operator ***S*** (see [1]); these cases are described in the following points (1), (2), (3). Let us note that all the terms considered in (1), (2), (3) are terms in the language *LW.*

(1) If $\tilde{f}$ and $\tilde{g}$ are terms expressing correspondingly a $v-$dimensional function *f* (where $v \geq 2$) and a one-dimensional function *g*, then the term $\boldsymbol{Se}(\tilde{f}, \tilde{g})$ expresses the $v-$dimensional function *h* such that

$$h(Q_1, Q_2, \ldots, Q_v) = f(Q_1, Q_2, \ldots, Q_{v-1},\ g(Q_v))$$

for all values of the variables $Q_1,\ Q_2,\ \ldots, Q_v$.

(2) If $\tilde{f}$ and $\tilde{g}$ are terms expressing correspondingly a 2-dimensional function *f* and a *k*-dimensional function *g* (where $k \geq 1$), then the term $\boldsymbol{Sb}(\tilde{f}, \tilde{g})$ expresses the $(k+1)$-dimensional function *h* such that

$$h(Q_1, Q_2, \ldots, Q_{k+1}) = f(g(Q_1, Q_2, \ldots, Q_k),\ Q_{k+1})$$

for all values of the variables $Q_1,\ Q_2,\ \ldots, Q_{k+1}$.

(3) If $\tilde{f},\ \tilde{g}_1,\ \tilde{g}_2$ are terms expressing correspondingly a $v-$dimensional function *f* (where $v \geq 2$) and one-dimensional functions g_1 and g_2, then the term $\boldsymbol{Sb}(\tilde{f}, \tilde{g}_1, \tilde{g}_2)$ expresses the $(v-1)$ dimensional function *h* such that

$$h(Q_1, Q_2, \ldots, Q_{v-1}) = f(g_1(Q_1),\ g_2(Q_1),\ Q_2, \ldots,\ Q_{v-1})$$

for all values of the variables $Q_1,\ Q_2,\ \ldots, Q_{v-1}$.

As it will be seen below, it is convenient to represent the list of variables for the function *h* in the following form: $R,\ Q_3,\ Q_4, \ldots, Q_v$. Using this list, we can write the expression for *h* as follows:

$$h(R,\ Q_3,\ Q_4, \ldots, Q_v) = f(g_1(R),\ g_2(R),\ Q_3,\ Q_4, \ldots,\ Q_v).$$

In [1] Shannon functions $SH_{AW}(n)$ and $SH_{WA}(n)$ are introduced; these functions describe the relations between the lengths of terms expressing arithmetical functions (in *LA*) and string functions (in *LW*) when the considered functions correspond to one another. Namely, if $t \in LW$, then by $LA(t)$ we denote the set of all terms in *LA* expressing the arithmetical function corresponding to the string function expressed by *t.* Similarly, if $r \in LA$, then by $LW(r)$ we denote the set of all terms in *LW* expressing the string function corresponding to the arithmetical function expressed by *r.* Now we can give (see [1]) the definitions of $SH_{AW}(n)$ and $SH_{WA}(n)$ as follows:

$$SH_{WA}(n) = \max_{(t \in LW)\&(|t| \le n)} \left(\min_{r \in LA(t)} |r| \right);$$

$$SH_{AW}(n) = \max_{(r \in LA)\&(|r| \le n)} \left(\min_{t \in LW(r)} |t| \right).$$

In [1] the following statement is established (see the main theorem in [1]): there are upper and lower bounss for $SH_{AW}(n)$ and $SH_{WA}(n)$ such that each of them has the form $cn+d$, where c and d are some constants.

We shall consider the function $SH_{AW}(n)$. There are some defects in the proof of the upper bouns for this function in [1]; their removal requires essential changes in the proof. Below we give another proof of the mentioned bouns based on a method which is different from those used in [1]. Namely, we shall give a new proof of the following theorem.

Theorem. *There are constants c and d such that for any non-negative integer n*

$$SH_{AW}(n) \le cn+d.$$

We shall use three Lemmas in the proof given in [1] (similar statements are proved also in [2]). By $\nu(n)$ we denote the function such that $\nu(0)=\Lambda$, $\nu(n)=\overbrace{a_1a_1...a_1}^{n\ times}$ for any positive integer *n*.

Lemma 1. *There are constants c' and d' such that for any term $t \in LA$ expressing a function $\tau(x_1,x_2,...,x_m)$, a term $\Phi \in LW$ expressing some function $\varphi(Q_1,Q_2,...,Q_m)$ can be constructed such that the following conditions are satisfied:*

1. $\varphi(\nu(x_1),\nu(x_2),...,\nu(x_m)) = \nu(\tau(x_1,x_2,...,x_m))$, *for any* $x_1,x_2,...,x_m$ *in N.*
2. $|\Phi| \le c'|t| + d'$.

Lemma 2. *There is a primitive recursive string function G such that $G(\nu(m)) = \alpha m$ for any $m \in N$.*

Lemma 3. *The one-dimensional string function $\gamma(Q) = \nu(\pi(Q))$ is primitive recursive.*

Proof of Theorem. Let *t* be any term in *LA* expressing some function $\tau(x_1,\ x_2,\ ...\ ,\ x_m)$. As it is proved in [1], the following inequality holds: $m \le |t|$.

The string function corresponding to τ let us denote by $\psi(Q_1,\ Q_2,\ ...\ ,\ Q_m)$. We shall construct a term Ω in *LW* having the length mentioned in Theorem and expressing the function ψ.

Using Lemma 1 we construct a term Φ in LA such that $|\Phi| \le c'|t| + d'$, where c' and d' are constants (fixed in Lemma 1), and Φ expresses a function φ satisfying the condition

$$\varphi(\nu(x_1),\nu(x_2),...,\nu(x_m)) = \nu(\tau(x_1,x_2,...,x_m))$$

for any $x_1,x_2,...,x_m$ in *N*.

Using Lemmas 1 and 2 we obtain the following equalities

$$\psi(Q_1,Q_2...Q_m) = \alpha\tau(\pi(Q_1),\pi(Q_2)...\pi(Q_m)) =$$
$$= G\big(\nu\big(\tau(\pi(Q_1),\pi(Q_2)...\pi(Q_m))\big)\big) =$$
$$= G\big(\varphi\big(\nu\big(\pi(Q_1)\big),\nu\big(\pi(Q_2)\big)...\nu\big(\pi(Q_m)\big)\big)\big) =$$
$$= G\big(\varphi\big(\gamma(Q_1),\gamma(Q_2)...\gamma(Q_m)\big)\big),$$

By $\tilde{G}$ and $\tilde{\gamma}$ we denote the terms in *LW* expressing the functions *G* and γ.

Let us consider the well-known primitive recursive arithmetical functions *c, l, r,* defining a one-to-one correspondence between N^2 and *N*. Such functions we define by the following equalities:

$$c(x,y)=\frac{(x+y)(x+y+1)}{2}+x,$$
$$c(l(z),r(z))=z,$$
$$l(c(x,y))=x,\ r(c(x,y))=y.$$

We consider also the following functions (where $n\geq 2,\ 2\leq k\leq n$):

$$c^n(x_1,x_2,...,x_n)=\underbrace{c(...c(c(}_{(n-1)\text{ times}}x_1,x_2),x_3),...,x_n);$$
$$c_{n1}(z)=\overbrace{l(l(...l(}^{(n-1)\text{ times}}z)...));$$
$$c_{nk}(z)=r(\overbrace{l(l(...l(}^{(n-k)\text{ times}}z)...))).$$

Obviously, for any $x_1,x_2,...,x_n,z$ in N and for $1\leq k\leq n$, the following equalities hold:

$$c^n(c_{n1}(z),c_{n2}(z),...,c_{nn}(z))=z;$$
$$c_{nk}(c^n(x_1,x_2,...,x_n))=x_k.$$

Using Lemma 1 we construct string functions $\sigma,\ \lambda,\ \rho$, such that for any *x, y, z* in *N*

$$\sigma(\nu(x),\nu(y))=\nu(c(x,y));$$
$$\lambda(\nu(z))=\nu(l(z));$$
$$\rho(\nu(z))=\nu(r(z)).$$

Let us note a peculiarity of these functions.

If some strings $Q,\ Q_1,\ Q_2$ in A do not contain other letters except a_1. then the following equalities hold: $\sigma(\lambda(Q),\rho(Q))=Q,\ \lambda(\sigma(Q_1,Q_2))=Q_1,\ \rho(\sigma(Q_1,Q_2))=Q_2$. However, in general such equalities are not valid.

Let us consider also the following string functions (where $n\geq 2,\ 2\leq k\leq n$)

$$\sigma^n(Q_1,Q_2,...,Q_n)=\sigma(...\sigma(\sigma(Q_1,Q_2),Q_3),...,Q_n);$$
$$\lambda_{n1}(Q)=\overbrace{\lambda(\lambda(...\lambda(}^{(n-1)\text{ times}}Q)...));$$
$$\lambda_{nk}(Q)=\rho(\overbrace{\lambda(\lambda(...\lambda(}^{(n-k)\text{ times}}Q)...))).$$

The terms in LW expressing the functions $\sigma,\ \lambda,\ \rho,\ \sigma^n,\ \lambda_{n1},\ \lambda_{nk}$ (where $n\geq 2,\ 2\leq k\leq n$) we denote, correspondingly, by $\tilde{\sigma},\ \tilde{\lambda},\ \tilde{\rho},\ \tilde{\sigma}^n,\ \tilde{\lambda}_{n1},\ \tilde{\lambda}_{nk}$.

If some strings $Q_1,\ Q_2,\ ...\ Q_n,\ Q$ in A do not contain other letters except a_1. then the following equalities hold (where $n\geq 2,\ 2\leq k\leq n$):

$$\sigma^n(\lambda_{n1}(Q),\lambda_{n2}(Q),...,\lambda_{nn}(Q))=Q,$$
$$\lambda_{n1}(\sigma^n(Q_1,Q_2,...,Q_n))=Q_1;$$
$$\lambda_{nk}(\sigma^n(Q_1,Q_2,...,Q_n))=Q_k.$$

In general such equalities are not valid.

Now in the case, when $m\geq 2$, let us construct the term $\mathbb{C}^m$ as follows:

$$\mathbb{C}^m=\overbrace{\boldsymbol{Se}(\boldsymbol{Sb}(\tilde{\sigma},...,\boldsymbol{Se}(\boldsymbol{Sb}(}^{(m-1)\text{ times}}\tilde{\sigma},\tilde{\gamma}),\overbrace{\tilde{\gamma})),...,\tilde{\gamma})}^{(m-2)\text{ times}}),\tilde{\gamma}).$$

Here the group of symbols $\boldsymbol{Se}(\boldsymbol{Sb}(\tilde{\sigma},$ is repeated (m-1) times; after this the group $\tilde{\gamma})$ is repeated once; after this the group $,\tilde{\gamma}))$ is repeated (m-2) times; finally, the group $,\tilde{\gamma})$ is repeated once. It is easily seen that the length of the term $\mathbb{C}^m$ does not exceed $c_{10}m + d_{10}$, where c_{10} and d_{10} are some constants. Let us consider some subterms of the term $\mathbb{C}^m$ as well as functions expressed by them. It is easily seen that the following statements are valid.

The term $\boldsymbol{Sb}(\tilde{\sigma},\tilde{\gamma})$ expresses the function $\sigma\left(\gamma(Q_1), Q_2\right)$.

The term $\mathbb{C}^2 = \boldsymbol{Se}(\boldsymbol{Sb}(\tilde{\sigma},\tilde{\gamma}),\tilde{\gamma})$ expresses the function $\sigma\left(\gamma(Q_1),\gamma(Q_2)\right)$, that is, the function $\sigma^2\left(\gamma(Q_1),\gamma(Q_2)\right)$.

The term $\boldsymbol{Sb}(\tilde{\sigma},\boldsymbol{Se}(\boldsymbol{Sb}(\tilde{\sigma},\tilde{\gamma}),\tilde{\gamma}))$ expresses the function $\sigma\left(\sigma\left(\gamma(Q_1),\gamma(Q_2)\right),Q_3\right)$.

The term $\mathbb{C}^3 = \boldsymbol{Se}(\boldsymbol{Sb}(\tilde{\sigma},\boldsymbol{Se}(\boldsymbol{Sb}(\tilde{\sigma},\tilde{\gamma}),\tilde{\gamma})),\tilde{\gamma})$ expresses the function $\sigma\left(\sigma\left(\gamma(Q_1),\gamma(Q_2)\right),\gamma(Q_3)\right)$, that is, the function $\sigma^3\left(\gamma(Q_1),\gamma(Q_2),\gamma(Q_3)\right)$.

Using similar considerations, we conclude that the term $\mathbb{C}^m$ expresses the function

$$\sigma(...\sigma(\sigma(\gamma(Q_1),\gamma(Q_2)),\gamma(Q_3)),...,\gamma(Q_m)),$$

that is, the function

$$\sigma^m(\gamma(Q_1),\gamma(Q_2),\gamma(Q_3),...,\gamma(Q_m)).$$

Further, let us construct the term $\Im^m$ (where $m \geq 1$) as follows:

$$\Im^m = \overbrace{\boldsymbol{Sb}(\boldsymbol{...Sb}(\boldsymbol{Sb}(}^{(m-1)\text{ times}}\Phi,\overbrace{\tilde{\lambda},\tilde{\rho}),\tilde{\lambda},\tilde{\rho})...,\tilde{\lambda},\tilde{\rho})}^{(m-1)\text{ times}}.$$

It is easily seen that the length of the term $\Im^m$ does not exceed $|\Phi| + c_{11}m + d_{11}$, where c_{11} and d_{11} are some constants. Using the inequalities $|\Phi| \leq c'|t| + d'$ and $m \leq |t|$ we conclude that the length $|\Im^m|$ does not exceed $c_{12}|t| + d_{12}$, where c_{12} and d_{12} are some constants. Let us consider some subterms of the term $\Im^m$, as well as functions expressed by them. It is easily seen that the following statements are valid.

As it is said above, the term Φ expresses the function φ depending on m variables. The function φ we denote also by φ_0. The term $\Im^1$ is defined as the term which is equal to Φ.

The term $\Im^2 = \boldsymbol{Sb}(\Phi,\tilde{\lambda},\tilde{\rho})$ expresses some function φ_1 depending on $(m-1)$ variables; the list of variables for this function we denote by $R_1, Q_3,...,Q_m$. Using such notations we can represent the equality describing the function $\varphi_1(R_1, Q_3,...,Q_m)$ as follows:

$$\varphi_1(R_1,Q_3,...,Q_m) = \varphi(\lambda(R_1),\rho(R_1),Q_3,...,Q_m),$$

that is

$$\varphi_1(R_1,Q_3,...,Q_m) = \varphi(\lambda_{21}(R_1),\lambda_{22}(R_1),Q_3,...,Q_m).$$

The term $\Im^3 = \boldsymbol{Sb}(\boldsymbol{Sb}(\Phi,\tilde{\lambda},\tilde{\rho}),\tilde{\lambda},\tilde{\rho})$ expresses the function $\varphi_2(R_2,Q_4,Q_5,...,Q_m)$ depending on $(m-2)$ variables; the equality describing this function can be represented as follows:

$$\varphi_2(R_2,Q_4,Q_5,...,Q_m) = \varphi(\lambda(\lambda(R_2)),\rho(\lambda(R_2)),\rho(R_2),Q_4,Q_5,...,Q_m),$$

that is

$$\varphi_2(R_2,Q_4,Q_5,...,Q_m) = \varphi(\lambda_{31}(R_2),\lambda_{32}(R_2),\lambda_{33}(R_2),Q_4,Q_5,...,Q_m).$$

The term $\Im^4 = \boldsymbol{Sb}(\boldsymbol{Sb}(\boldsymbol{Sb}(\Phi, \tilde{\lambda}, \tilde{\rho}), \tilde{\lambda}, \tilde{\rho}), \tilde{\lambda}, \tilde{\rho})$ expresses the function $\varphi_3(R_3, Q_5, Q_6, ..., Q_m)$ depending on $(m-3)$ variables; the equality describing this function can be represented as follows:

$$\varphi_3(R_3, Q_5, Q_6, ..., Q_m) = \varphi(\lambda(\lambda(\lambda(R_3))), \rho(\lambda(\lambda(R_3))), \rho(\lambda(R_3)), \rho(R_3), Q_5, Q_6, ..., Q_m),$$

that is

$$\varphi_3(R_3, Q_5, Q_6, ..., Q_m) = \varphi(\lambda_{41}(R_2), \lambda_{42}(R_2), \lambda_{43}(R_2), \lambda_{44}(R_2), Q_5, Q_6, ..., Q_m).$$

Using similar considerations, we conclude that the term $\Im^m$ expresses the function $\varphi_{(m-1)}$ depending on one variable (we shall denote this variable by $R_{(m-1)}$). The equality describing this function can be represented as follows:

$$\varphi_{(m-1)}(R_{(m-1)}) = \varphi(\underbrace{\lambda(\lambda(...\lambda(}_{(m-1)} R_{(m-1)} \underbrace{)...))}_{(m-1)}, \rho(\underbrace{\lambda(\lambda(...\lambda(}_{(m-2)} R_{(m-1)} \underbrace{)...))}_{(m-2)}), ..., \rho(R_{(m-1)})),$$

that is

$$\varphi_{(m-1)}(R_{(m-1)}) = \varphi(\lambda_{m1}(R_{(m-1)}), \lambda_{m2}(R_{(m-1)}), ..., \lambda_{mm}(R_{(m-1)})).$$

Now let us construct the term

$$\boldsymbol{S}(\Im^m, \mathbb{C}^m).$$

This term expresses the function

$$\varphi(\lambda_{m1}(\sigma^m(\gamma(Q_1), \gamma(Q_2), ..., \gamma(Q_m))), \lambda_{m2}(\sigma^m(\gamma(Q_1), \gamma(Q_2), ..., \gamma(Q_m))), ...$$
$$..., \lambda_{mm}(\sigma^m(\gamma(Q_1), \gamma(Q_2), ..., \gamma(Q_m)))).$$

But the strings $\gamma(Q_1), \gamma(Q_2), ..., \gamma(Q_m)$ do not contain other letters except a_1. So, we can conclude that the function expressed by $\boldsymbol{S}(\Im^m, \mathbb{C}^m)$, is equal to

$$\varphi(\gamma(Q_1), \gamma(Q_2), ..., \gamma(Q_m)).$$

Hence the term

$$\Omega = \boldsymbol{S}(\tilde{G}, \boldsymbol{S}(\Im^m, \mathbb{C}^m))$$

expresses the function

$$G(\varphi(\gamma(Q_1), \gamma(Q_2), ..., \gamma(Q_m))),$$

that is, the function

$$\psi(Q_1, Q_2, ..., Q_m).$$

Clearly,

$$|\Omega| \le c_{13}|t| + d_{13},$$

where c_{13} and d_{13} are some constants. So, the statement of Theorem is proved for $m \ge 2$. The cases $m = 1$ and $m = 0$ are considered in a similar way. This completes the proof of Theorem.

Note. Applying usual methods of the recursive functions theory, we can obtain essentially more simple and more natural expressions for the term Ω than those considered above, for example

$$\Omega = \boldsymbol{S}(\tilde{G}, \boldsymbol{S}(\Phi, \boldsymbol{S}(\tilde{\gamma}, \tilde{I}_1^m), \boldsymbol{S}(\tilde{\gamma}, \tilde{I}_2^m), ..., \boldsymbol{S}(\tilde{\gamma}, \tilde{I}_m^m))),$$

where any term $\tilde{I}_k^m$ for $1 \le k \le m$ expresses the function

$$I_k^m(Q_1, Q_2, ..., Q_m) = Q_k.$$

However, such expressions do not give the required bounds of $|\Omega|$. For this aim special methods should be used. One of such methods is implemented above.

References

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Պարզագույն անդրադարձ (ռեկուրսիվ) թվաբանական և բառային ֆունկցիաների համեմատական բարդության մասին

Ի. Զասլավսկի և Մ. Խաչատրյան

Ամփոփում

Դիտարկվում են [1]-ում սահմանված պարզագույն անդրադարձ (ռեկուրսիվ) թվաբանական և բառային ֆունկցիաների ներկայացման *LA* և *LW* ձևային լեզուները: Շենոնի SH_{AW} և SH_{WA} ֆունկցիաները, որոնք բնութագրում են թվաբանական և բառային ֆունկցիաների ներկայացումների բարդությունների միջև եղած կապերը նշված լեզուներում, սահմանվում են, ինչպես [1]-ում: Մի նոր մեթոդով տրվում է SH_{AW} ֆունկցիայի վերին գնահատականի ապացույցը: Այդ մեթոդը որոշ դեպքերում ապահովում է կիրառությունների ավելի լայն հնարավորություններ, քան՝ [1]-ում դիտարկվող մեթոդները:

О сравнительной сложности примитивно рекурсивных арифметических и словарных функций

И. Д Заславский и М. Хачатрян

Аннотация

Рассматриваются формальные языки *LA* и *LW,* введенные в [1] для представления примитивно рекурсивных арифметических и словарных функций. Функции Шеннона SH_{AW} и SH_{WA}, выражающие соотношения между сложностями представления арифметических и словарных функций в этих языках, определяются так же, как в [1]. Дается новое доказательство верхней оценки для SH_{AW}, основанное на методе, дающем в ряде случаев новые возможности для приложений по сравнению с методами, рассматриваемыми в [1].