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Statistical Simmulation of Definite Integral Based on Uppersum and Lowersum Random Partitions using Geogebra

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Abstract

This paper aims to demonstrate the ability of Geogebra application in presenting statistical model simulations. Statistical simulation on Geogebra is based on the value of the random partition of the upper sums and lower sums on definite integral concepts. Random partitioning of the upper sums and lower sums in determining a definite integral value is used to determine the statistical distribution of the resulting random variables. Because the partition value data is obtained randomly, statistical tests can be carried out to determine the type of distribution on the random data obtained. Based on the Kolmogorov Smirnov test, the data for the subinterval partition random variables at the upper sums and lower sums using Goegebra follow a specific statistical distribution. It was found that the statistical simulations of random partitions of the upper sums and lower sums are based on definite integral concepts following the Burr 4, Log-Pearson 3, and Pearson 6 distribution parameters. The results of this statistical test simulation show that apart from being an application for visualizing mathematical concepts geometrically, Geogebra can also analyze mathematical concepts statistically.

Keywords: uppersum; lowersum; integral; Geogebra

INTRODUCTION

The need to visualize mathematics concepts by forming the images (either manually with pencil and paper or with technology) or using such images has effectively discovered the understanding of mathematics (Caligaris, 2014). To achieve deep understanding, the students have to learn how can represent their ideas. The teacher's creativity is needed to choose and use the appropriate learning media to bridge the student's mathematical ability. Milovanović (2011) showed that using multimedia in math classes about definite integral is highly interesting for students and showed better theoretical, practical, and visual knowledge because it can give visualization possibilities, animations, and illustrations.

Along with the development of technology, many choices of mathematics learning media can be used. The software that can be obtained from the internet freely is Geogebra. Geogebra is a dynamic mathematical software that can be used for all the education levels from the primary to the university (Hohenwarter, 2007).

GeoGebra is already used by more than 100 million students around the world. GeoGebra can minimize the difficulties of students who get Calculus subjects, especially students majoring in Natural Sciences and Engineering. Arini (2019) stated that the advantages of using GeoGebra are helping convey the

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Calculus concept material to be more interesting. The material, especially concepts of functions, limits, derivatives, and integrals, provides a more realistic image, especially for more complex calculus material, and provides a faster and more accurate solution.

Caligaris (2014) stated that understanding the definite integral definitions and theorems in mathematics takes symbolic representation or graphics to describe. With Geogebra, an interactive application can illustrate the mathematics materials that require visualization, especially on Calculus material. Furthermore, Serhan (2015) stated that most students only knew the procedure and calculation steps to solve the calculus problem, especially the indefinite integral problem. In fact, to understand the concept of definite integral, many things that students can explore and get a piece of additional knowledge, both its relationship with calculus itself or its relationship with the other science, for example, statistics.

Nur'aini (2017) used Geogebra to draw and calculate the geometries mathematically as a catalyst to make mathematics learning to be more realistic. The implementation of Geogebra can be used to visualize and determine: the application of the Pythagoras Theorem, the angle of the clock, and geometric transformation. Sari (2016) did the research about Geogebra-assisted learning media (module) that was developed received an assessment for the interesting category and was worthy of being used as a learning media for derivative.

Integrating educational technology in the teaching and learning of definite integral creates a conceptually rich learning environment. Kado & Dem (2020) found that GeoGebra software can enhance and significantly improve students' conceptual understanding of definite integral. The computer-assisted instruction method using GeoGebra was found to contribute to teaching the definite integral topic positively. Because all the study about using GeoGebra to visualize the materials of mathematics gives the preliminary study about visualizing the definite integral concept, this study gives the exploration in using Geogebra to construct the definite integral concept.

Two things that become the basic idea to define and construct the definite integral are the case to calculate the traveled distance of a moving object with the velocity and the case to calculate the area. Calculating the traveled distance of an object that moves at certain time intervals will provide how to calculate the area under the curve. Caligaris (2015) used GeoGebra to illustrate the integral function depends on the upper limit o integration and constructed the value changes to improve the presentation of content taught, allowing dynamic visualization. The paper concludes that incorporating the Geogebra Applets is a much more effective teaching methodology than traditional one to facilitate the learning of the fundamental concepts of Calculus.

Defining the definite integral concept is based on the partition of the interval as the lower sum and upper sum with the same and random subinterval length. Calculation of the area under the curve becomes the basic idea for understanding the definite integral materials. The results of the exploration and material design of the definite integral are expected to form new knowledge about concepts and

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application of the definite integral concept material by visualizing it on Geogebra (Sur, 2020).

GeoGebra can design the same and random subinterval to present the definite integral dynamically. The length of the same or random subinterval of the integral concept was the basic idea in this research. Since GeoGebra can create the partitions randomly with an unlimited number of partitions, the authors were interested in knowing whether the random partitions in Geogebra statistically have a particular distribution.

This study illustrates and presents definite integral concepts with analytical and statistical approaches by using GeoGebra. Exploration of the concepts is related to the comparation of the lower sum and upper sum of the integran function based on the subinterval length. Exploration is presented to construct and discover new things in understanding the definite integral concepts, particularly about the statistical model of an upper sum and lower sum partitions of the definite integral concept.

RESEARCH METHOD

This research appropriates the definite integral concept materials to make the simulation, illustration, and then analyze the comparison between upper sum and lower sum geometrically by using Geogebra 5.0 version. After the simulation, illustration, and analysis, we perform the statistical test of upper sum and lower sum random partition associated with the definite integral concepts to find out the statistical distribution of its generated random variable. To obtain the numerical statistic of random partitions, we used with Kolmogorov Smirnov test. Based on the test results, we will determine if the random partition data follows the specified distribution or does not follow the specified distribution. This anaysis uses the following hypothesis:

H_0: Data follows the specified distribution
H_1: Data do not follow the specified ditribution

If data follows the specified distribution, then the distribution of random variable data in every subinterval partition can be determined using EasyFit version 5.5.

RESULTS AND DISCUSSION

Elementary Concepts About Uppersum and Lowersum by Using Geogebra

In case to define the definite integral concepts, let the definition of Uppersum and Lowersum of a function. Given P is a finite ordered points between a dan b, $P = \{x_0, x_1, x_2, \dots, x_{n-1}, x_n\}$, with $a = x_0 < x_1 < x_2, \dots, < x_{n-1} < x_n = b$. The set P is said a partition of interval $[a, b]$, that devides $[a, b]$ into n subinterval,with i^{th} subinterval is $[x_{-1}, x_i]$. The length of i^{th} from P is $\Delta x_i = x_i - x_{i-1}$, for $1 \leq i \leq n$. (Salas, 2007)

Based on theorem Adams (2010) about minimum and maksimum value of a function, we know if $f: [a, b] \to \mathbb{R}$ is continu on $[a, b]$, then in every subinterval

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$[x_{i-1}, x_i]$, there exist $l_i, u_i \in [x_{i-1}, x_i]$, such that f minimum at $f(l_i)$ and maximum at $f(u_i)$

Defining the definite integral based on the partision of the interval as the lower and the upper limit with the same and random subinteval length, can be explored by using geogebra. Consider the region A is an area bounded by the graph of a continuous function $y = f(x)$,the x-axis and between vertikal line $x = a$, and $x = b$. Region A can be estimated by dividing region A into n subregion $A_1, A_2, \dots, A_n$. By the worksheet of Geogebra, we can draw the following ilustration

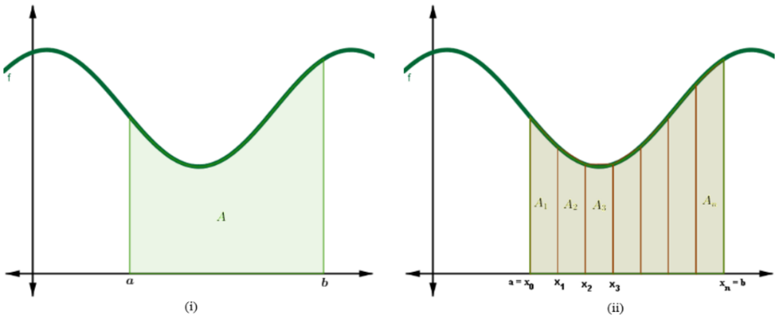

Fig. 1. Ilustration of dividing region A into n subregions

Interval [a,b] is divided into a finite number of subintervals $[a = x_0, x_1]$; $[x_1, x_2]$; ... $[x_{n-1}, x_n = b]$. Because f continu on interval [a, b], then f continu on every subintervals $[x_{i-1}, x_i]$, with $i = 1,2, \dots, n$. It means f maksimum and minimum at every points on that subinterval. Because of that, there exist the numbers l_i and u_i at $[x_{i-1}, x_i]$, such that $f(l_i) \leq f(x) \leq f(u_i)$ with $x_{i-1} \leq x \leq x_i$ (Bartle, 2000).

Let's denote $M_i = f(u_i)$ the maximum value of f on $[x_{i-1}, x_i]$, and $m_i = f(l_i)$ the minimum valueof f on$[x_{i-1}, x_i]$. If we take any i^{th}subitervals on $[a, b]$, then we can consider the rectangles r_i and R_i, with $r_i \subseteq A_i \subseteq R_i$. It means area of $r_i \leq$ area of $A_i \leq$ area of R_i. Since the area of regctamgle is the length times the width, then $m_i \Delta x_i \leq$ area of $A_i \leq M_i \Delta x_i$, with $\Delta x_i = (x_i - x_{i-1})$. It holds for $i = 1,2, \dots, n$. The sum of the minimum value approaches $m_1 \Delta x_1 + m_2 \Delta x_2 + \cdots + m_n \Delta x_n \leq$ area of A, and the sum of the maximum value approaches area of $A \leq M_1 \Delta x_1 + M_2 \Delta x_2 + \cdots + M_n \Delta x_n$ (Stewart, 2010).

The sum of minimum values on every subintervals is defined as Lowersum, and the sum of maximum values one every subintervals is defined as Uppersum. By using Geogebra, we can prove that the more particies we make, the more we can reach one and only one number between the uppersum and the lowersum value. This such number will be an area of A, which is defined as definite integral.

In case to determine the uppersum and lowersumm of a function, the subinterval length of partition P can be divided not only into n subinterval with the same length $\Delta x_i = \frac{b-a}{n}, \forall i = 1,2, \dots, n$, but also with the difference or random subinterval length(Stewart, 2008). By using geogebra, we can make a simmulation

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that is compare the integral function based on the uppersum and lowersum between the same subinterval length with random subinterval length.

Simmulation of Integrable Function Based on Lowersum and Uppersum Partition

The bounded function of definite interval, can be defined as integrable function if the supremum of its lowersum is equal to infimum of its uppersum. By using Geogebra, we can make the simmulation that is presenting for all ε>0, the bounded function f:[a,b]→R is integrable on [a,b] if the difference of its uppersum and lowersum less than ε.

Integrable Function Based on Uppersum dan Lowersum Partitions with The Same Subinterval Length

We created a simulation for a bounded function $f: [1,2] \rightarrow \mathbb{R}$, with $f(x) = \frac{1}{x}; \forall x \in [1,2]$. The simulation by Geogebra is divided into 2 cases, i.e the same subintervals, and random subintervals. We compare the minimum partition obtained of uppersum and lowersumt, with $\varepsilon = \frac{1}{100}, \frac{1}{200}, \dots, \frac{1}{1000}$. Suppose $\Pi[1,2]$ is the set of all partitions P at $[1,2]$ with $\Pi[1,2] = \{P_1, P_2, \dots, P_{10}\}$. $U(f,P)$ and $L(f,P)$ are uppersum and lowersum of $f = \frac{1}{x}, x \in [1,2]$ on partition P. Then, for all the same subinterval length with $\Delta x = \frac{1}{n}$ we obtain :

a. Given $\varepsilon = \frac{1}{100} = 0{,}001 > 0$, then the difference of uppersum and lowersum is $U(f,P_1) - L(f,P_1) = 0,6981 - 0{,}6883 = 0,0098 < \varepsilon$, with the number of minimum partition is n = 51. Partition $P_1\ on\ [1,2\ ]$when $n = 51$ is written by $P_1 = \{1, 1.02, 1.03, \dots, 1.98, 1.99, 2\}$.
b. Given $\varepsilon = \frac{1}{200} = 0{,}005 > 0$, then the difference of uppersum and lowersum is $U(f,P_2) - L(f,P_2) = 0,6956 - 0{,}6907 = 0,0049 < \varepsilon$, with the number of minimum partition is n = 102. Partition $P_1\ on\ [1,2]$when $n = 51$ is written by $P_1 = \{1, 1.02, 1.03, \dots, 1.98, 1.99, 2\}$.
c. Given $\varepsilon = \frac{1}{300} = 0{,}0033 > 0$, then the difference of uppersum and lowersum is $U(f,P_3) - L(f,P_3) = 0{,}6948 - 0,6915 = 0,0032 < \varepsilon$, with the number of minimum partition is n = 154. Partition $P_3\ on\ [1,2]$when $n = 154$ is written by $P_3 = \{1, 1.01, 1.01, ..1.98, 1.99, 1.99, 2\}$.
d. The same process is continued till for partition P_{10}, with $\varepsilon = \frac{1}{1000}$
e. Given $\varepsilon = \frac{1}{1000} = 0{,}001 > 0$, then the difference of uppersum and lowersum is $U(f,P_{10}) - L(f,P_{10}) = 0{,}6936 - 0,6927 = 0,0009 < \varepsilon$, with the number of minimum partition is n =527. Partition P_{10} on $[1,2]$when $n = 527$ is written by $P_{10} = \{1,1,1,1,1, 1.01, 1.01, \dots, 1.99,2,2,2,2\}$.

Simulation for the integrable function based on any value $\varepsilon > 0\ on\ f(x) = 1/x\ ; x \in [1,2]$, with the same subinterval length is presented by Figure 2 (S, 2018)

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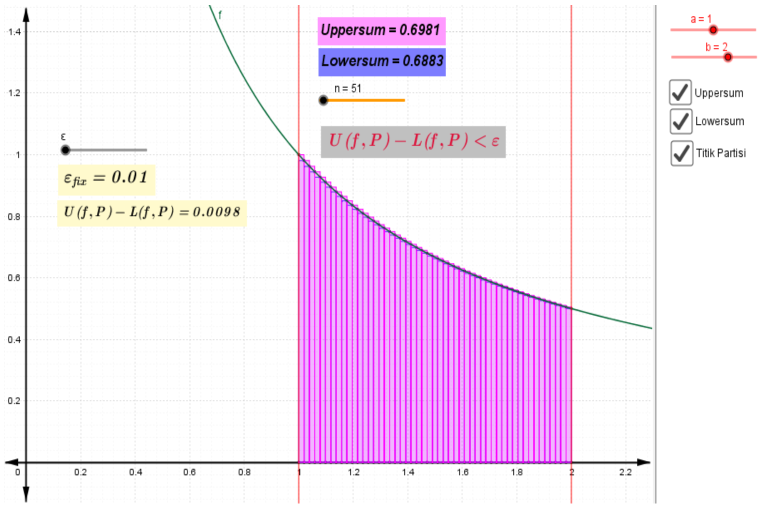

Fig. 2. Simulation of integrable function for $\varepsilon = \frac{1}{100}$ with the same length subintervals

Integrable Function Based on Uppersum dan Lowersum Partitions with The Random Subinterval Length

With the same way, we can simulate the integrable function based on the lowersum and uppersums, for the random subinterval length. With the same function, suppose $\Pi[1,2]$ is the set of all partitions P at $[1,2]$ with $\Pi[1,2] = \{P_1, P_2, \dots, P_{10}\}$. $U(f,P)$ and $L(f,P)$ are uppersum and lowersum of $f = \frac{1}{x}, x \in [1,2]$ on partition P. Then, for all the random subinterval length Δx, we obtain:

a. Given $\varepsilon = \frac{1}{100} = 0{,}01 > 0$, then the difference of uppersum and lowersum is $U(f,P_1) - L(f,P_1) = 0,6980 - 0{,}6884 = 0,0096 < \varepsilon$, with the number of minimum partition is n = 94.
b. Partition P_1 *on* $[1,2]$when $n = 94$ is written by $P_1 = \{1, 1.01, 1.01, 1.05, \dots, 1.94, 1.98, 1.99, 2\}$
c. Given $\varepsilon = \frac{1}{200} = 0{,}005 > 0$, then the difference of uppersum and lowersum is $U(f,P_2) - L(f,P_2) = 0,6955 - 0{,}6908 = 0,0048 < \varepsilon$, with the number of minimum partition is n = 206.
d. Partition P_1 *on* $[1,2]$when $n = 51$ is written by $P_1 = \{1, 1.01, 1.03, \dots, 1.96, 1.98, 2, 2\}$
e. Given $\varepsilon = \frac{1}{300} = 0{,}0033 > 0$, then the difference of uppersum and lowersum is $U(f,P_3) - L(f,P_3) = 0{,}6947 - 0,6915 = 0,0032 < \varepsilon$, with the number of minimum partition is n = 306.
f. Partition P_3 *on* $[1,2]$when $n = 306$ is written by $P_3 = \{1,1,1,1\ 1.01, \dots, 1.98, 1.98, 1.98, 2\}$
g. the same process is continued till for partition P_{10}, with $\varepsilon = \frac{1}{1000}$
h. Given $\varepsilon = \frac{1}{1000} = 0{,}001 > 0$, then the difference of uppersum and lowersum is $U(f,P_{10}) - L(f,P_{10}) = 0{,}6936 - 0,6927 = 0,0009 < \varepsilon$, with the number of minimum partition is n =1050.

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Partition P_{10} on $[1,2]$when $n = 1050$ is written by $P_{10} = \{1,1,1,1.01, \dots 1.99,2,2,2,2,2\}$.

Table 1. The comparison of the partition between the same and random subintervals

$\Pi[1,2]$	$\varepsilon > 0$	$U(f,P) - L(f,P) < \varepsilon$	Subinterval Length	
			Same	Random
P_1	1/100	0,0098	n=51	n=94
P_2	1/200	0,0049	n=102	n=206
P_3	1/300	0,0032	n=154	n=306
P_4	1/400	0,0024	n=205	n=435
P_5	1/500	0,0019	n=257	n=571
P_6	1/600	0,0017	n=304	n=635
P_7	1/700	0,0013	n=371	n=735
P_8	1/800	0,0012	n=415	n=872
P_9	1/900	0,0011	n=477	n=982
P_{10}	1/1000	0,0009	n=527	n=1050

When we compared the partitions between random subintervals and the same subintervals length, it can be said that there are more partitions needed for the length of the random subinterval, to produce the uppersum and lowersum convergen to a certain value, than the same subinterval length. The comparison of the partition between random and the same subintervals for the integrable function $f(x) = \frac{1}{x}, \forall x \in [a,b]$ is provided by Table 1.

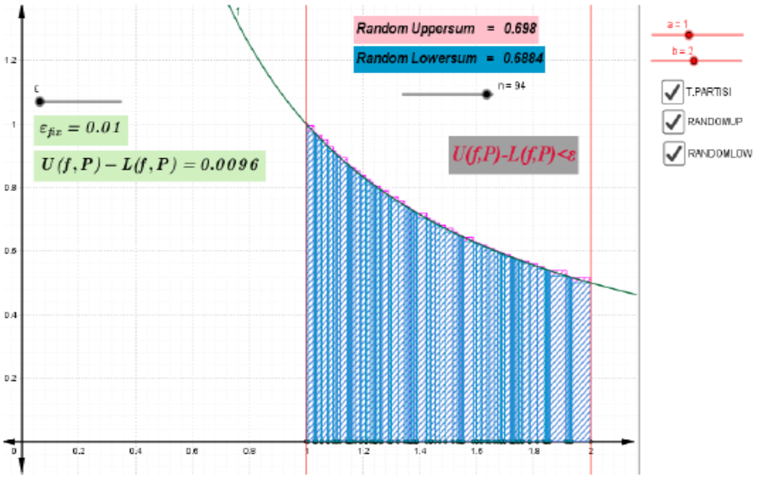

Fig. 3. Simulation of integrable function for $\varepsilon = \frac{1}{100}$ with the random length subintervals

From the explanation about the same and random subintervals, it can be seen that the smaller ε value is taken, the larger n partition it takes for the lowersum and uppersum to converge to a certain value. In other words, the difference between U(f, P) and L(f, P) gets smaller and closer to ε, if n partition point gets bigger. That is how the theorem about the bounded function $f: [a,b] \to \mathbb{R}$ is integrabel on$[a,b]$ iff for all $\varepsilon > 0$, there exist a partition $P \in \Pi[a,b]$ such that $U(f,P) - L(f,P) < \varepsilon$.(Bartle, 2000)

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Statistical Test about Integral Value based on Random Subinterval Length

Relating with the calculation of lower sum and upper sum based on random subinterval, we can present the statistical model to know the type of distribution on the poligons as result of a random subinterval partition. Because the length of subinterval gives the random value, we can make the number of partition with the difference value of partitions. As an example, on interval [1, 3], with number of partition is n = 10, we can note the m sets of all partitions P written by $\Pi[1,3] = \{P_1, P_2, P_3, \dots, P_m\}$ with:

$$\begin{gathered}P_1 = \{1, 1.23, 1.3, 1{,}35, 1.68, 2.01, 2.64, 2.71, 2.77, 2.97, 3\}\\ P_2 = \{1, 1.11, 1.3, 1.55, 1.77, 1.97, 2.04, 2.43, 2.76, 2.84, 3\}\\ P_3 = \{1, 1.33, 1.48, 1.83, 2, 2.22, 2.4, 2.6, 2.8, 3\}\\ P_4 = \{1, 1.02, 1.18, 1.72, 1.74, 1.5, 2.05, 2.33, 2.71, 2.83, 3\}\\ P_5 = \{1, 1.02, 1.27, 1.72, 1.85, 1.86, 2.05, 2.19, 2.54, 2.95, 3\}\\ \vdots\\ P_m = \{1 = x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} = 3\}\end{gathered}$$

Based on that example, with random subinterval on$[a, b]$, we can make $m\ sets$ parition P with n partition points. It can be written as:

$$\begin{gathered}P_1 = \{a = x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, \dots, x_{1n} = b\}\\ P_2 = \{a = x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, \dots, x_{2n} = b\}\\ P_3 = \{a = x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, \dots, x_{3n} = b\}\\ P_4 = \{a = x_{40}, x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, \dots, x_{4n} = b\}\\ P_5 = \{a = x_{50}, x_{51}, x_{52}, x_{53}, x_{54}, x_{55}, \dots, x_{5n} = b\}\\ \vdots\\ P_m = \{a = x_{m0}, x_{m1}, x_{m2}, x_{m3}, x_{m4}, x_{m5}, \dots, x_{mn} = b\}\end{gathered}$$

with x_{mn} is n^{th} partition point at partition m , with $n = 0{,}1{,}2, \dots$, dan $m = 1{,}2, \dots$. Then, we can obtain the area of the poligon as a lowersum of the function for every random subinterval. It can be presented as $L_{m\times n}$ matrix as bellow

$$\begin{bmatrix} l_{11} & l_{12} & l_{13} & l_{14} & \cdots & l_{1n}\\ l_{21} & l_{22} & l_{23} & l_{24} & \cdots & l_{2n}\\ l_{31} & l_{32} & l_{33} & l_{34} & \cdots & l_{3n}\\ l_{41} & l_{42} & l_{43} & l_{44} & \cdots & l_{4n}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ l_{m1} & l_{m2} & l_{m3} & l_{m4} & \cdots & l_{mn}\end{bmatrix}$$

l_{ij} is a lowersum at j^{th} subinterval on i^{th} partition, with j = 1, 2,..., n, and i = 1, 2,...,m.

We can take a real valued function as an example. Let A⊆ ℝ, and $g: A \to \mathbb{R}$, with $g(x) = 2x\sqrt{1 + x^2}, \forall x \in A$. By using geogebra, we form the random variable of L as a lower sum of f on interval [1,3], with the number of patition is $n = 50$. If we made 5 types of random partition of subinterva length, then we presented the matrix as follow:

$$L_{m\times n} = \begin{bmatrix} 0{,}00043 & 0{,}00044 & 0{,}00025 & 0{,}00029 & \cdots & 0{,}16519\\ 0{,}00110 & 0{,}00118\ 0 & 0{,}00125 & 0{,}00033 & \cdots & 0{,}13901\\ 0{,}30661 & 0{,}37606 & 0{,}41889 & 0{,}00061 & \cdots & 0{,}2318\\ 0{,}00052 & 0{,}00053 & 0{,}00066 & 0{,}00066 & \cdots & 0{,}00027\\ 0{,}00001 & 0{,}00029 & 0{,}00031 & 0{,}00031 & \cdots & 0{,}00241\end{bmatrix} \quad (1)$$

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Based on the statitical analysis, with Kolmogorov Smirnov test by software *EasiFit*, we obtained the numerical statistic as follow at Table 2

Table 2. The comparison of the Statistical Test of Random Subinterval with Sign 0,05

Statisctical Test	P_1	P_2	P_3	P_4	P_5
D	0.02638	0,02356	0,02328	0,02653	0,2113
P-Value	0,86794	0,938	0,94335	0,8636	0,97549
Range	0,81023	0,41953	0,41881	0,35934	0,91562
Mean	0,3001	0,03784	0,03819	0,03914	0,0399
Variansi	0,00394	0,00212	0,00223	0,00243	0,00373
StdDeviasi	0,06276	0,04607	0,04717	0,0493	0,06111
Std. Error	0,00281	0,00206	0,00211	0,0022	0,00273

Based on the results, we made the hypotesis tes on data of $L_{m\times n}$ as follows:

H_0: Data follows the specifeid distribution
H_1: Data do not follow the specified ditribution

Statistical tes value of D is represented by:
Statistical test result of P_1: $D = 0{,}02638$
Statistical test result of P_2: $D = 0{,}02356$
Statistical test result of P_3: $D = 0{,}02328$
Statistical test result of P_4: $D = 0{,}02653$
Statistical test result of P_5: $D = 0{,}02113$
Signifince Level : $\alpha = 0{,}05$
Critical Value : 0,0607
Critical Region : Reject H_0 if $D > 0{,}0607$
Because all partitions gave statstical test $D < 0{,}0607$, we can make a conclussion tht H_0 is accepted. In other words, the random variable data of subinterval partitions on lowersum by using goegebra follwed the certain distribution. The distribution of all partition presented as:
The first partition has a distribution *Burr 4 parameter*, with probability density function (pdf) is:

$$f(x) = \frac{\alpha k \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}} \qquad (2)$$

with $k = 1{,}9582$, $\alpha = 1{,}1482$, $\beta = 0{,}03125$, and $\gamma = 0{,}0000824$ for $\gamma \le x \le +\infty$
The second partition, has distribution $Burr\ 4\ parameter$, same as with the first partition ditribution. Its parameters are $k = 6{,}266$, $\alpha = 0{,}97426$, $\beta = 0{,}20374$, and $\gamma = 0{,}00008$ $\gamma \le x \le +\infty$ The third partition has a distribution $Log - Person$ 3, with probability dnsity function (pdf) is:

$$f(x) = \frac{1}{x|\beta|\Gamma(\alpha)}\left(\frac{\ln(x)-\gamma}{\beta}\right)^{\alpha-1} \exp\left(-\frac{ln(x)-\gamma}{\beta}\right) \qquad (3)$$

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with $\alpha = 9{,}574$, $\beta = -0{,}41003$, and $\gamma = 0{,}01728$ for $0 < x \leq e^{\gamma}$; $\beta < 0$ and $e^{\gamma} \leq x < +\propto$; $\beta > 0$
The fourth partition has ditribution $Pearson$ 6 ,with probability density function (pdf) is:

$$f(x) = \frac{\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta B(\alpha_1, \alpha_2)(1 + \left(\frac{x-\gamma}{\beta}\right)^{\alpha_1+\alpha_2}} \qquad (4)$$

with $\alpha_1 = 0{,}9371$, $\alpha_2 = 3{,}9549$, $\beta = 0{,}12504$ and $\gamma = 0$ for $\gamma \leq x < +\infty$
The fifth partition has distribution $Log - Pearson$ 3 which is same with the distribution of third partition. Its paramaters are $\alpha = 6{,}3222$, $\beta = -0{,}56464$, and $\gamma = -0{,}4218$ for $0 < x \leq e^{\gamma}$; $\beta < 0$ and $e^{\gamma} \leq x < +\infty$; $\beta > 0$.

With the same process, we can determine the disitbution of uppersum based on random subinterval length of integral value of a real function. We can make $m\ sets$ parition P with n partition points
We can obtain the area of the poligon as an uppersum of the function for every random subinterval. It can be presented as $U_{m\times n}$ matrix as bellow

$$U_{m\times n=}\begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & u_{24} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & u_{34} & \cdots & u_{3n} \\ u_{41} & u_{42} & u_{43} & u_{44} & \cdots & u_{4n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_{m1} & u_{m2} & u_{m3} & u_{m4} & \cdots & u_{mn} \end{bmatrix} \qquad (5)$$

u_{ij} is an uppersum at j^{th} subinterval on i^{th} partition, with j = 1, 2,..., n, and i = 1, 2,...,m.

We can take a real valued function as an example. Let A⊆ ℝ, and $g: A \rightarrow \mathbb{R}$, with $g(x) = 2x\sqrt{1+x^2}, \forall x \in A$. By using geogebra, we form the random variable of U as an uppersum of f on interval [1,3], with the number of partition is $n = 50$.

CONCLUSION

Based on the simulation, we know that Geogebra can not only visualize the basics of definite integral concepts. Geogebra, with its combined ability to visualize the concept of the upper and lower sum of a function defined at certain intervals, can be used to explore the concept of definite integrals. One of the explorations is to do partitions of the same length and partitions of different lengths. Partitions of the same length give the number of partitions less than random partitions to determine the value of the integral approximation. Because of its ability to generate random partitions, Geogebra can also be used to explore statistical models of these random partitions. Based on the Kolmogorov Smirnov Normality test, we know that the upper sum and lower sum values of the functions that can be integrated are obtained with certain statistical distributions. Random partitions of upper and lower sum following a statistical distribution are Burr 4, Log-Pearson 3, and Pearson 6.

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