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Mathematical Construction of Definite Integral Concepts by Using GeoGebra

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Abstract

The need to visualize concepts and support the materials in Mathematics learning by forming the pictures or by using existing draw, becoming what is needed by the students in improving their mathematical development and understanding. GeoGebra can combine dynamic visualization of geometry and the results of mathematical calculations simultaneously. The author examines the definite integral concept material of course, to then be explored with GeoGebra so that it is easily understood by students. The author studies the procedures for using GeoGebra, studies and constructs the concept of definite integral concepts, into a simulation and visualization in the form of GeoGebra files, which are ready to be used in the learning of definite integral concepts..

Keywords: GeoGebra; definite integral; mathematical construction.

INTRODUCTION

The use of technology in learning mathematics is not a new thing to use. At present, there are many Mathematics software specifically designed to facilitate the learning of Mathematics, both paid and which can be obtained free from the internet. Computer Algebra Systems (such as Derive, Mathematica, Maple or MuPAD), Dynamic Geometry Software (such as Geometer's Sketchpad or Cabri Geometry), Matlab, Autograph, and GeoGebra are some examples of software that are widely used in Mathematics learning.

There is a need to visualize Math concepts and materials by imagining, shaping the images (manually or with the help of technology) or by using existing images, which is the reason why in Mathematics, we need media that can help the learning process. Furthermore, to gain a deeper understanding, visualization must be represented. Students must learn how mathematical ideas are represented by symbols, numbers, and graphics, as a form of understanding of mathematical concepts (Caligaris, 2014).

For the reasons mentioned above, it is very necessary for a teacher's creativity in choosing and using appropriate learning media to bridge the mathematical abilities of their students. Along with the development of technology, there are many choices of media for learning mathematics that can be used for this. One of the software that can be obtained free from the internet is GeoGebra. GeoGebra is a dynamic mathematical software that can be used at various levels of education, from elementary school level to university level.

The GeoGebra application combines geometry, algebra, spreadsheets, graphs, statistics, and calculus, in one package that is easy to use. This makes GeoGebra have quite a several users spread in almost every country throughout the world. Nowadays, GeoGebra has become a provider of software leading mathematical that supports the

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advancement of science, technology, engineering, and mathematics education, as well as providing innovation in teaching and learning processes throughout the world. In this study, the application of GeoGebra 5, which is a new edition of GeoGebra, is used because it is easily obtained, and its application can reach various domains of Mathematics.

Besides being able to visualize mathematical concepts that are analytic geometrically, GeoGebra is also able to perform mathematical and statistical calculations and represent them. To facilitate users, GeoGebra facilitates users to have an account and community on the GeoGebra website, so it is very easy to share and access Mathematics material that has been created by users in various countries. The user just has to choose the type of material and the desired material and can modify it according to what is needed. All of which are available completely and free of charge on the official GeoGebra website(Caligaris, 2014).

Calculus is one of the mathematical materials that requires an understanding of concepts for students who study it, given the application and use of calculus that is very much needed in various branches of science (Serhan, 2015). However, the concept of Integral is certainly also difficult to understand and requires visualization to be able to imagine it, so it is appropriate if the writer chooses the topic of Integral and its application, to design simulations and learning media

The aim of this research is to construct the Concept of Definite Integral, as well as design visualization and simulation of its material using GeoGebra. The number of errors in interpreting the meaning of Integral is certainly good among high school students or college students, making the writer feel the need to visualize the concept of the Integral of course. Of course by exploring and then designing simulation material in the form of dynamic learning media with GeoGebra. This article will present the results of the conceptual integration of Integral concepts with GeoGebra and visualization of concepts that are dynamic, interesting, easy to understand and provide a complete picture for understanding Integral concepts.

RESEARCH METHOD

The method of this article is literature studies. Data and documents obtained from various sources are compiled, analyzed, then the material produced is constructed and then visualized and simulated the material using GeoGebra.

RESULT AND DISCUSSION

The Overview of GeoGebra

GeoGebra is a computer application program that combines geometry, algebra, tables, graphs, statistics, and calculus in an easy-to-use package so that it can be utilized in learning mathematics at various levels of education. GeoGebra is a dynamic program that can visualize or demonstrate and construct mathematical concepts, into various forms, types, styles for all levels of mathematical material. Besides being able to be obtained for free from the internet, GeoGebra is also easy to use and learn. There are many examples of material and tutorials that can be accessed on the official GeoGebra page. The files that are processed in GeoGebra can be saved in ".ggb" format, and or in the form of dynamic

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web pages. GeoGebra can also produce ".png" or animated ".gif" files with quality illustrations.

GeoGebra has received various educational software awards in Europe and the USA. Developed since 2001 by Markus Hohenwarter, a mathematician and Professor of Mathematics in Austria, who examines the special use of technology in mathematics education. GeoGebra continues to develop until now. Its inventors and designers continue to improve and add to the shortcomings of this GeoGebra program. The latest development is, GeoGebra 5., which can display images in 3-dimensional shapes. Aside from being a computer application, GeoGebra also has a special version for tablet or android users. It can even be used online without having to install the application first on a PC / Laptop.[3]

With the development of applications and features on GeoGebra, its users are also expanding in various countries, ranging from students, students, teachers, and lecturers. Each user can create their account on the GeoGebra page and can join various GeoGebra user communities around the world. Users and owners of GeoGebra accounts can easily share and obtain a variety of materials or mathematical material in the form of GeoGebra, which can be downloaded and developed according to their learning needs. GeoGebra is formed on the Cartesian coordinate system and can accept geometrical commands (drawing lines through two points, forming curved fields, algebraic commands (drawing curves with given equations). Some of the advantages of GeoGebra compared to other mathematical software are (1) Can produce paintings - Geometry paintings quickly and thoroughly, even complex ones (2) The ability to represent algebra, two-dimensional geometry, and 3-dimensional geometry at once so that changes in given equations or changes in images in Geometry views will also change the shape of equations in Algebra display and position on the 3D display. (3) The existence of animation facilities and manipulation movements can provide visual experience in understanding the concept of geometry (4) Make it easy to investigate or show the properties that apply to a geometry object, (5) Can be used to make into learning material acts as a web page, and can be directly distributed to the official GeoGebra web site, www.GeoGebra.org.

With these advantages, a teacher can guide their students to know the relationship between an algebraic form and geometry. Teachers can first design demonstrations for understanding students' concepts, and students can directly work from the materials needed and can see visually the meaning of the concepts being taught, thus helping them to be able to imagine the analytical meaning of a material being taught.

When you open the GeoGebra application, Graph and Algebra displays and GeoGebra's standard menus will appear. Worksheets for displaying applications that have been created on GeoGebra are called applets. On the right side, there is an arrow which if clicked will bring up a box in the GeoGebra Math Application dialog. The dialog box provides display options or screen forms to be displayed, according to user needs. Where each view has its Toolbar which contains a selection of tools and commands that allow

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users to create dynamic constructions in representing mathematical objects. There are 6 views on GeoGebra, which can be selected based on user needs, namely:

1) Graphing Calculator
It is an Algebraic and Graphical display, as a standard display on GeoGebra. The left side, which is the Algebra display which is a place to display the algebraic object of the input object or equation, while the graph display will display the object geometrically
2) Computer Algebra System (CAS)
It is a computer algebraic system display that allows users to do mathematical calculations for symbolic calculations. The CAS display consists of lines that each have an input.
3) Geometry
Is a graphic display that only displays the geometric shapes of objects/equations that are inputted
4) 3D Grapher
Almost the same as Graphing Calculator, it's just that the graph shown is a 3-dimensional graph, with the left side is an Algebraic view.
5) Spreadsheet
It is a display form of a number processor table consisting of rows and columns. In this view, matrices, tables and other mathematical objects can be made that contain columns and rows. Not only numbers, but all types of mathematical objects can also be inputted into spreadsheet cells, such as point coordinates, functions, and mathematical calculation commands supported by GeoGebra. If possible, GeoGebra will immediately display a graphical representation of the object entered in the spreadsheet cell.
6) Probability
Is a display of statistical forms, which displays statistical distribution forms and statistical test calculations.
(White, 2016).

The following image is the home page of GeoGebra worksheet.

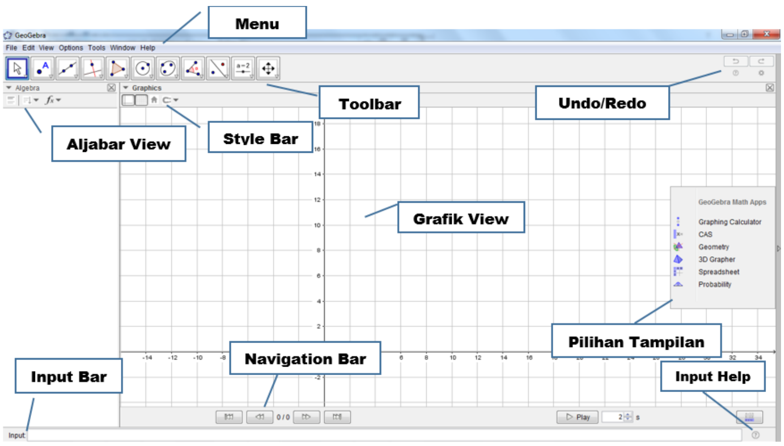

Figure 1 Home Page of GeoGebra Worksheet

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Visualization of Definite Integral Concepts with GeoGebra

After knowing and exploring some basic material that is important to know in the understanding integral, of course, the following is given the construction of the presentation of the material of the concept of Integral Naturally by visualizing it on GeoGebra. Two things that become the basic idea of defining and constructing Integral of course are the Mileage and Space Area under the curve as follows :

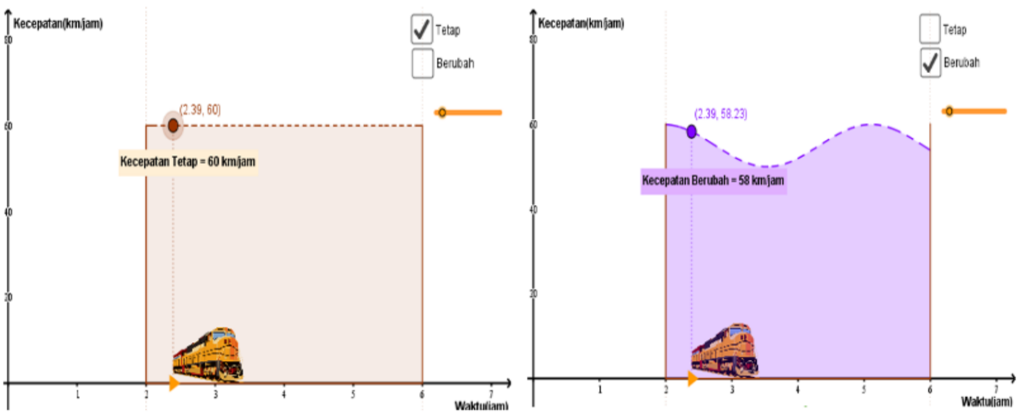
a. The Distance of Moving Objects

In addition to the area under the curve, the integral idea is of course also based on the distance and speed of an object. Departing from real problems, the integral idea is certainly based on observing the distance of an object that moves (moves) within a certain time interval. If an object moves with a fixed speed at a certain time interval, then the total distance traveled can be calculated using a formula that has been known, namely

$$\text{Distance} = \text{speed} \times \text{time}$$

For example, a train travels at a constant speed of 60km/ hour. Then the total distance traveled by train between 02:00 and 06:00 is 60 x 4 = 240 km. Calculation of the train's mileage, if visualized in the form of a speed function graph, can be seen in Figure 2 (i). Because distance can be obtained by multiplying speed and time interval, then the total distance traveled is nothing but the area of the rectangle formed from the velocity function at that time interval. With the length of the interval as the length of the rectangle and the velocity value as the width of the rectangle.

Calculation of the total distance in the interval of time taken by multiplying the speed and time used is if the object's speed remains (constant) 60 km/hour. But in reality, objects that move like the train, it is not possible to move at a constant speed within a certain time interval. There will be times when objects move slowly or quickly. If the velocity of the object changes within a certain time interval, then the visualization in the case of the train could be described as a function of speed in Figure 2(ii)



(i) (ii)

Figure 2 Visualization of the Distance of Moving Objects with GeoGebra

Figure 2 (ii) shows that the distance of the train is the same as the product of time inteval with the speed of the train. Therefore simply multiplying the length and width of the rectangle formed. However, in Figure 2 (ii) the shaded part is no longer rectangular. So to calculate it also requires a different approach. The distance traveled

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can be calculated with the approach that best approaches the actual mileage. The approach taken is to partition the area into small rectangles, then add up the total area of the resulting rectangles.

The time interval [2,6] can be partitioned into a number of very small sub-intervals, so that each of sub-intervals will show a constant velocity, because of very small time intervals. Geometrically, the subinterval division will produce small rectangles, along the area formed under the given speed function curve.

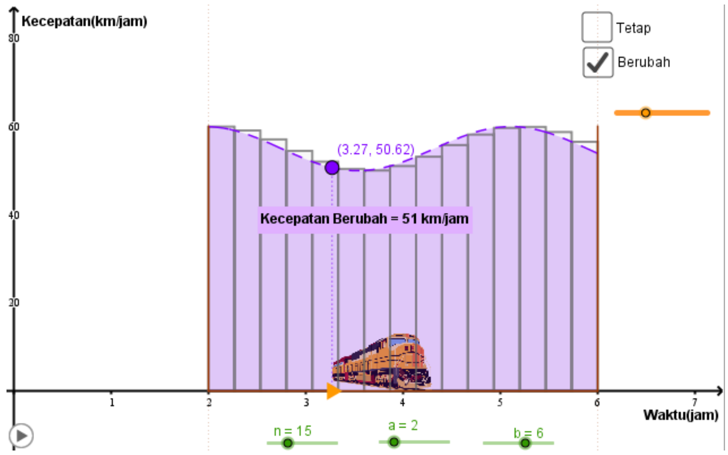

Figure 3 Visualization of Interval Time Partition of Moving Object

For the case of a railroad in Figure 3, the form of time interval division becomes area of the rectangles from this partition, then added up, so the results will approach the actual mileage. In this case, the area under the curve.

If the train problem above is abstracted, then let's say an object moves continuously in the time interval [a, b], with a non-constant speed. Time lapse [a; b] can be partitioned into smaller time intervals, namely:

$[t_0, t_1], [t_1, t_2], \ldots [t_{\{n-1\}}, t_n]$ with $a = t_0 < t_1 < \ldots < t_n = b$

for each subintervals $[t_{i-1}, t_i]$, the object reaches M_i maximum speed and reaches m_i minimum speed. So, if the object moves constantly at a minimum speed, the object will travel a distance of $m_i \Delta t_i$ unit of distance. Conversely, if the object is constantly moving at maximum speed, then the object will travel a distance $M_i \Delta t_i$ unit of distance. Meanwhile, the actual distance, denoted s_i, *the* distance must be located between the maximum distance and the minimum distance, to obtain:

$$m_i \Delta t_i \le s_i \le M_i \Delta t_i$$

The total distance traveled along the time interval [a,b] called s, is the sum of the distance traveled at each sub interval $[t_{\{i-1\}}, t_i]$, then:

$$s = s_1 + s_2 + \cdots + s_n$$

$$m_1 \Delta t_1 \le s_1 \le M_1 \Delta t_1$$

$$m_2 \Delta t_2 \le s_2 \le M_2 \Delta t_2$$

$$\vdots$$

$$m_n \Delta t_n \le s_n \le M_n \Delta t_n$$

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which in turn, add up the obtained inequality:

$$m_1\Delta t_1 + m_2\Delta t_2 + \cdots + m_n\Delta t_n \leq s \leq M_1\,\Delta t_1 + M_2\,\Delta t_2 + \cdots + M_n\,\Delta t_n \qquad (1)$$

The sum of $m_1\Delta t_1 + m_2\Delta t_2 + \ldots + m_n\Delta t_{n\,is}$ called the sum down for the velocity function and $M_1\,\Delta t_1 + M_2\,\Delta t_2 + \ldots + M_n\,\Delta t_n$ called the sum of the velocity functions. The inequality above shows that s must be greater than or equal to each bottom sum, and smaller or equal to each top sum. As with the area, there is exactly one number that satisfies the inequality, and that number is the total distance traveled.

b. Area Under The Curve

Area under the curve becomes the basic idea of the formation of an understanding of Definite integral. Taking into account the following figure, suppose that A is the area under the curve $y = f\,(x)$ and above the axis $x -$ which is between the vertical lines $x = a$ and $x = b$, then the area can be estimated by dividing region A into n subregions A_1, $A_2, \ldots,$ A_n as in the following figure, presented on the GeoGebra worksheet as:

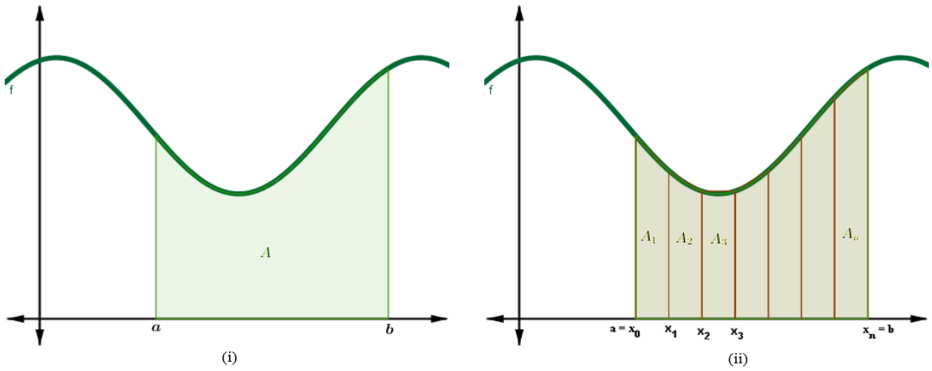

Figure 4 Estimation of Area Under the Curve $y = f(x)$

Because f *is* continuous at intervals $[a, b]$, then f *is* continuous at each subinterval $[x_{\{i-1\}}, x_i]$ on P. This means, f reaches the maximum and minimum at the points that exist in the subinterval. Therefore, there are numbers l_i and u_i in $[x_{\{i-1\}}, x_i]$ such that holds:

$$f(l_i) \leq f\,(x) \leq f(u_i) \; with \; x_{\{i-1\}} \leq x \leq x_i$$

Denoted M_i, in this case as $f(u_i)$ is the maximum value f in the subinterval $[x_{\{i-1\}}, x_i]$, and $m_i$in this case as $f(l_i)$ is the minimum value f in the subinterval $[x_{\{i-1\}}, x_i]$. If taking any subinterval to-i from the interval [a,b] the function f of the following figure, it can be formed rectangle-rectangle r_i and R_i as shown below:

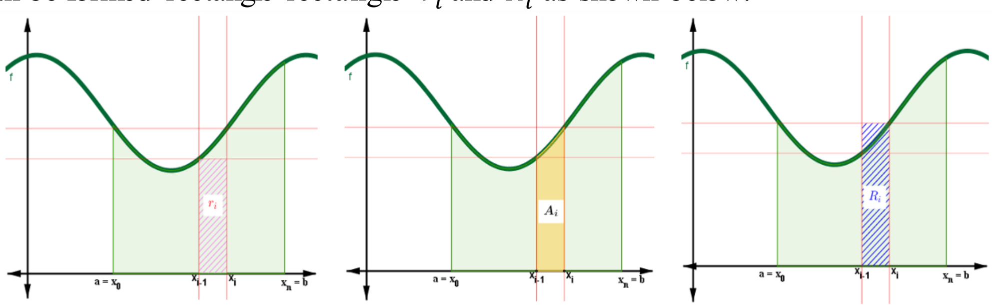

Figure 5 Rectangle Area of Maksimum and Minimun Values of a Function

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According to Figure 5, rectangular area r_i and R_i are formed on subintervals $[x_i, x_{\{i-1\}}]$, which is then compared to the area A_i at the same subinterval. It is clearly known if $r_i \subseteq A_i \subseteq R_i$, so that it is obtained:
Area of $r_i \leq$ Area of area $A_i \leq$ Area of area R_i
Because the area of a quadrilateral is a product of its length and width, we obtain

$$m\_i\left(x_i - x_{\{i-1\}}\right) \leq \; Area\ area\ A_i \; \leq \; M_i\left(x_i - x_{\{i-1\}}\right)$$

With $\triangle x_i = x_i - x_{\{i-1\}}$, we get:

$$m_i \triangle x_i \; Area\ of\ A_i \; \leq \; M_i \triangle x_i$$

This inequality applies to every $i = 1.2, \ldots, n$. So, from the sum of the minimum values obtained

$$m_1 \triangle x_1 + m_2 \triangle x_2 + \ldots + m_n \triangle x_n \; uasThe\ area\ of\ A_i$$

and from the sum of the maximum values obtained

$$Area\ of\ A_i \; \leq \; M_1 \triangle x_1 + M_2 \triangle x_2 + \ldots + M_n \triangle x_n$$

The sum of the minimum values is referred to as the Lowersum and the sum of the maximum values is called the Uppersum. Analytically, it can be proven that if f *is* continuous at intervals $[a, b]$, then there is one number that satisfies the equation. Using GeoGebra, it can be proven that with more partitions being formed, i.e. with $n \rightarrow \infty$, we will get exactly the same value from the uppersum and the lowersum. This same value is exactly the area of A_i. Visualization of the lowersum ad uppersum can be seen in the following figure:

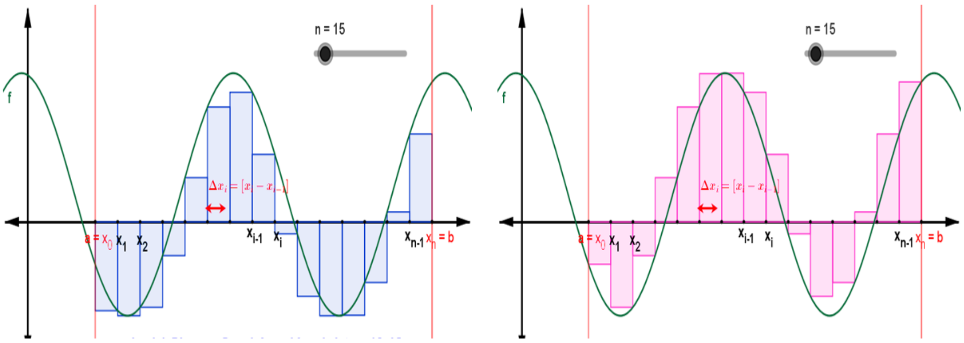

Figure 6 Visualization of Uppersum and Lowersum of a Function

c. Definite Integral of Continuous Function

Based on the area of the curve and the area of the curve and the distance calculation, then the problem is then abstracted into a continuous function.

It is assumed that a function $f\ (x)$ is defined and continuous at a finite closed interval [a, b]. Furthermore, given P is a finite set of points arranged sequentially between a and b, i.e

$P \ = \{x_0, x_1, x_2, x_3, \ldots, x_{\{n-1\}}, x_n\}$

with $a \ = x_0 \ < \ x_1 < x_2 < x_3 < \ldots < x_{\{n-1\}} < x_n = \ b$

The set P *is* called the partition of the interval [a, b] which divides [a, b] into n subintervals, where the i-th subinterval is $[x_{\{i-1\}}, x_i]$. This subinterval is called the subinterval of the P partition, which can then be formed into rectangular arrangements, with the length and width being the length of the subinterval on the axis X and its functional value on the axis Y as shown in the following figure:

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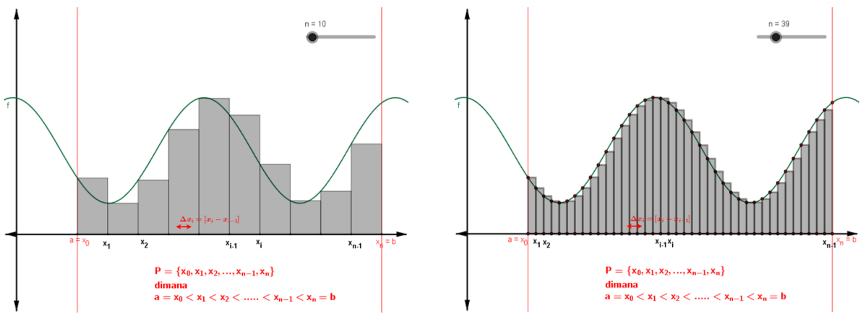

Figure 7 Visualization of *P* Partition on Interval $[a, b]$

The length of the i-th subinterval of P is $\Delta x_i = x_i - x_{\{i-1\}}$, for 1≤i≤ n. Furthermore, for each subinterval $[x_i, x_{\{i-1\}}]$, the function f reaches the maximum value at M_i and the minimum value m_i.

Definition

(Adams, 2010) Lowersum $L\ (f, P)$ and Uppersum $U\ (f, P)$ on partition P to the function f, defined as:

$$L(f,P) = m_1 \Delta\, x_1\ +\ m_2 \Delta x_2 + \ldots + m_n \Delta x_n = \sum_{i=1}^{n} m_i \Delta x_i$$

$$U(f,P) = M_1 \Delta\, x_1\ +\ M_2 \Delta x_2 + \ldots + M_n \Delta x_n = \sum_{i=1}^{n} M_i \Delta x_i$$

By Using Geogebra, we can calculate the lowersum and upersum of a function. We give the steps to calculate the lowersum and uppersum of a function by the following example

Will be calculated uppersum and lowersum value for the function f (x) $= \frac{1}{x}$ in the interval P $= [1,2]$, which is divided into four subintervals with the same distance to know the value of its uppersum and lowersum. First defined the function f at the input Bar

>> f (x) = 1 / x

Next, draw the boundary line interval, namely:

>> x = 1

>> x = 2

If we want to find the partition in the interval [1,3], then register the partition points with the sequence command. The sequence command is useful for displaying values and visualizing those values on the cartesian axis. If partition points are displayed on the X-axis, the following command is used:

>> Sequence [<Expression>, <Variable>, <Start Value>, <End Value>, <Increment>] become

>> Sequence [(a + (i (b - a)) / n, 0), i, 0, n]

In this case, $a + \frac{i(b-a)}{n}$ is the n^{th} partition point formula, at intervals $[a, b]$ with the same point distance. Then in the Algebra window, all partition points will be displayed in the form of coordinates on the X-axis in the Graphics window. And its values in the Algebra window.

Furthermore, if we don't want to know the interval first, then the lowersum and uppersum values can be directly calculated, namely:

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>> LowerSum [<Function>, <Start x-Value>, <End x-Value>, <Number of Rectangles>]
>> UpperSum [<Function>, <Start x-Value>, <End x-Value>, <Number of Rectangles>]
becomes
>> LowerSum [f, 1,2,4] for the lowersum and
>> LowerSum [f , 1,2,4] for uppersum
Its calculation results can be seen in the window Algebra, and its figure on the Cartesian axes can be seen in the window Graphics
The results can be visualized as follows:

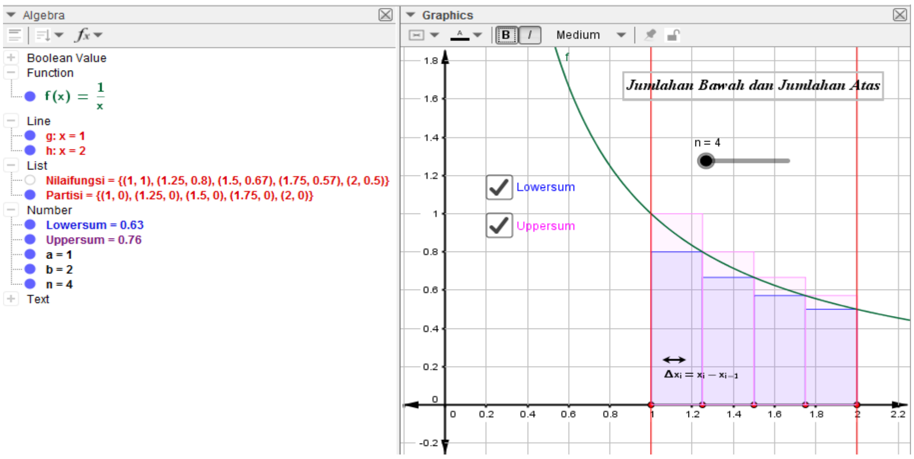

Figure 8 Calculation of Uppersum and Lowersum by using Geoebra

According to the calculation of GeoGebra, then obtained points in the interval [1,2] can be seen on the Algebra display in the List partition, namely:
$x_0 = 1, x_1 = 1.25, x_2 = 1.25, x_3 = 1.25, x_4 = 1.25,$
Obtained Lower Sums (Lowersum) = 0.63 and Upper Sums (Uppersum) = 0.76. Meanwhile, using the definition of the lower and upper Sum,
$L(f,P) = m_1 \Delta x_1 + m_2 \Delta x_2 + m_3 \Delta x_3 + m_4 \Delta x_4$ and
$U(f,P) = M\Delta x_1 + M_2 \Delta x_2 + M_3 \Delta x_3 + M_4 \Delta x_4,$
But the function value is calculated first at each point. To make it easier to know the value of its function, then you can add the following command GeoGebra:
>> Sequence [<Expression>, <Variable>, <Start Value>, <End Value>, <Increment>] become
>> Sequence [(a + (i (b - a)) / n, f (a + (i (b - a))), i, 0, n]
Then the points and function values given for each part of the point will be displayed, in Algebra's view, in the List Value Function, namely:
$f(x_0) = 1, f(x_1) = 0.8, f(x_2) = 0.67, f(x_3) = 0.57, f(x_4) = 0.5$
With the value of the function, you will be able to calculate the Lower Addition and Top Addition based on the definition. The results obtained will be the same as the Lowersum and Uppersum results in the Figure above display.
Based on the above example, either by calculation Lowersum and Uppersum on GeoGebra or by using the definition, the difference between $L(f,P) =$ and $U(f0.63, P)$ =0.76, still fairly large. However, when the partitions of points added in this case n gets bigger, the difference between $L(f,P)$ and $U(f,P)$ will be smaller.
If more points are added to the interval, the bottom sum will increase, and vice versa the top sum decreases. Besides, the more the partition points, the difference in value between $L(f,P)$ and $U(f,P)$ on the partition P of the function f *is* getting smaller.

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So the values of $L\ (f, P)$ and $U\ (f, P)$ will approach the area under the curve at interval $[a, b]$ that is known. Visualization can be seen in the figure below:

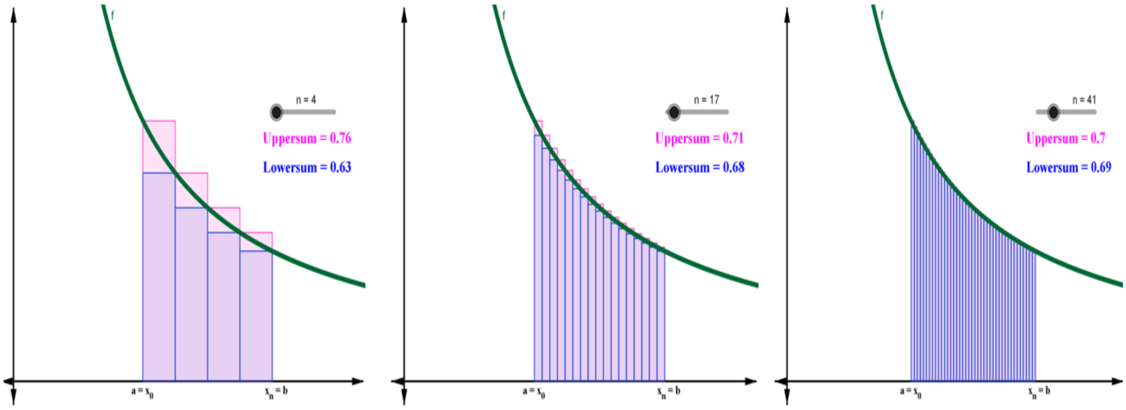

Figure 8 Calculation of Uppersum and Lowersum with different number of n

Therefore, it can be proven that if f is continuous on $[a$,b], then there is exactly one number I that satisfies the inequality $L(f, P) \leq I \leq U(f, P)$, for each partition P in $[a, b]$

Analytically, inequality can be proven as:

Lemma: *If P and Q are partitions on* $[a, b]$, *then* $L\ (f, P)\ \leq U\ (f, Q)$[6]

Proof: Take any $P \cup Q$ partitions on $[a, b]$ containing points the partition points at P and Q, so that they apply: $L\ (f, P)\ \leq L\ (f, P \cup Q)\ \leq U\ (f, P \cup Q)\ \leq U\ (f, Q)$

It is clear that $L\ (f, P)\ \leq U\ (f, Q)$

From the lemma, it can be said that $L\ (f, P)\ is$ limited to the top and $U\ (f, P)\ is$ limited to the bottom. Since $L\ (f, P)$ limited to the above, there are a number L so that if the result of the sum taken any number under $L\ (f, P)$ will always be less than or equal to a number. L the In this case $L\ is$ called infimum from the sum below $L\ (f, P)$, where for any partition P in [a, b], L satisfies the inequality

$$L\ (f, P)\ \leq L \leq U\ (f, P)$$

In the same way, because $U\ (f, P)\ is$ limited to the bottom, there is a U number so that if any number is added to the sum of U$(f, P)\ it$ will always be greater than or equal to the number U. In this case, $U\ is$ called as a supremum from the sum below $U\ (f, P)$, where for any partition P in [a, b], U satisfies the inequality

$$L\ (f, P)\ \leq U \leq U\ (f, P)$$

Furthermore, it would be proved that if f is continuous on $[a$,b], then there is exactly one number I that satisfies the inequality

$L(f, P) \leq I \leq U(f,)$ P, for each partition P in $[a, b]$

Theorem:

If f is continuous on $[a$*,b], then for each partition P in* $[a, b]$ *there is exactly one number I that satisfies the inequality* $L(f, P) \leq I \leq U(f,)$

Proof:

to prove the inequality, first demonstrated the existence of $the\ first$ number.

Based on (2) and (3), then for each partition P applies

$$L\ (f, P) \leq\ L \leq U \leq U\ (f, P)$$

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So there must be a number I that satisfies, namely L and U itself.
Furthermore, to prove the singularity of the number I, it will be proven that $L = U$
Taken any $\varepsilon > 0$
Because f is a continuous function, then f continuous is uniform at $[a, b]$ $which$ means, there is$\delta > 0$, so $\forall x, y \in [a, b]$, with $|xy| < \delta$, then $|f(x) - f(y)| < \varepsilon$
Taken the partition $P = \{x_0, x_1, \ldots, x_n\}$ with a maximum of $\Delta x_i < \delta$
So, for this partition P applies

$$U(f,P) - L(f,P) = \sum_{i=1}^{n} M_i \Delta x_i - \sum_{i=1}^{n} m_i \Delta x_i$$

$$= \sum_{i=1}^{n} f(u_i)\Delta x_i - \sum_{i=1}^{n} f(l_i)\Delta x_i$$

$$= \sum_{i=1}^{n} (f(u_i) - f(l_i))\ \Delta x_i$$

$$< \sum_{i=1}^{n} \frac{\varepsilon}{b-a}\ \Delta x_i = \frac{\varepsilon}{b-a} \sum_{i=1}^{n}\ \Delta x_i = \frac{\varepsilon}{b-a}\ (b-a) = \varepsilon$$

Retrieved $U(f,P) - L(f,P) < \varepsilon$
And since $L\,(f,P) \leq L \leq U \leq U\,(f$,P), the obtained $0 \leq UL \leq U(f,P) - L(f,P) < \varepsilon$
In other words, $0 \leq UL < \varepsilon$
it applies to every $\varepsilon > 0$, the obtained $UL = 0 \Leftrightarrow U = L$
Retrieved U =L, which means the number $first$ that satisfies the inequality $L(f,P) \leq I \leq U(f,P)$ is single. From the evidence of the existence and one number, $first$ of the evidence that there are a number I, which meets $L(f,P) \leq I \leq U(f,P)$
In GeoGebra, we can show numbers I that meet these inequalities, by moving the slider n. The greater the value of n, the uppersum value will be smaller and the lowersum value will be greater, so that at an certain, the Uppersum and Lowersum value will be closer to the same value. The closer the same value is, the closer the area under the curve is. This area of the same value will then become an integral value of the function f at intervals $[a, b]$. From the numbers that satisfy the inequality, then it is defined Integral.

Definitions of Definite Integral:
Known f is continuous at intervals $[a, b]$.single number I that satisfies the inequality $L(f,P) \leq I \leq U(f,)$ P, for each partition P in $[a, b]$ called Fefinite integral (integral) function f from a to b and is denoted as

$$\int_a^b x\ f(x)dx$$

(Adams, 2010)
If it exists with a single I that satisfies the inequality, then the continuous function f is called the function **integrated** in $[a, b]$. The symbol $\int$ is an **integral symbol** that represents the letter S from the word sum or sum, which indicates that the Integral is a representation of the sum as has been explained in the visualization and construction of definitions of the integral certainly above. Numbers a and b are **integral limits** (a lower limit and b upper limit of the integral). The function f integrated is called the

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integrant, x is **the variable** integrated, dx as a derivative of x which replaces the position Δx in the lower and upper sums. If the integrand is a function that has more than one variable, the symbol is dx used as a reference to the variable to be integrated. For the function of one variable $\int_a^b f(x)dx$, the variable x can be replaced by another variable, as well as with dx, even though some also write integral symbols without using dx.
The examples of definite intergal writing of a single variable function are:
$\int_a^b f(x)dx$, $\int_a^b f(t)dt$,$\int_a^b f(t)dt$, $\int_a^b f(z)dz$,atau$\int_a^b f$saja.

Visualization of Definite Integral Properties

Acording to definition of Definite integral, then we get the properties of Definite integral which are generally used in calculating the Integral of a function. These integral properties are based on the sum of the top and bottom of a function. Because the steps are quite long and use a lot of calculations (especially for partitions and high boundaries), then originally, the calculation of the integral value is certainly based on the following properties that are clearly by the results of the sum calculation above and below. The following will be presented with a visualization of the definite integral properties of course, by using GeoGebra. The visualization presented combines the calculation of the integral value of course based on the sum of the top and bottom sum of the functions. In GeoGebra, the calculation of integral values can certainly be done by inputting the values in the following command:
>> Integral[<Function>, <Start x-Value>, <End x-Value>]

***Property 1: If any constant k**,then* $\int_a^b k\ dx = k(b-a)$ [7]
With GeoGebra, the ilustration as the following figure:

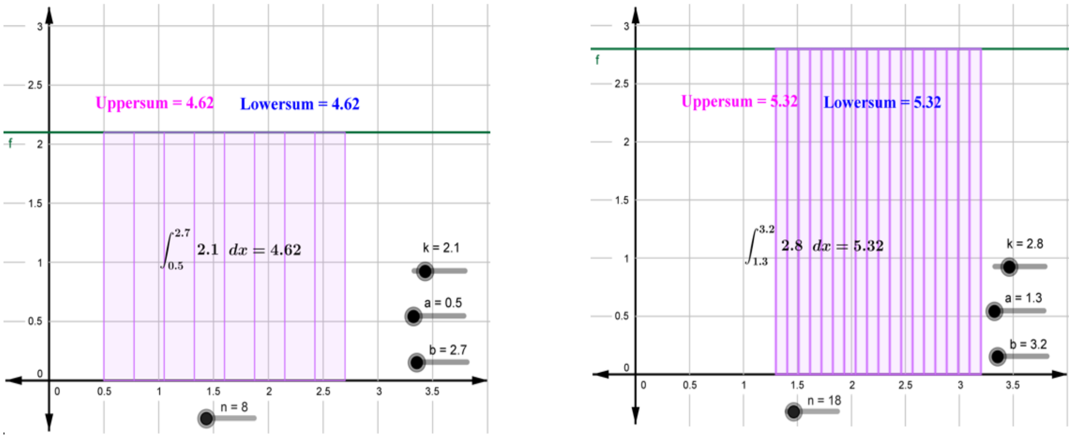

Figure 9 Visualization of Definte Integral Property 1

Property 2 : *Let f be integrable function,the definite integral over an interval of zero length is zero*

$$\int_a^a f(x)\,dx = 0$$

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With GeoGebra, the ilustration is

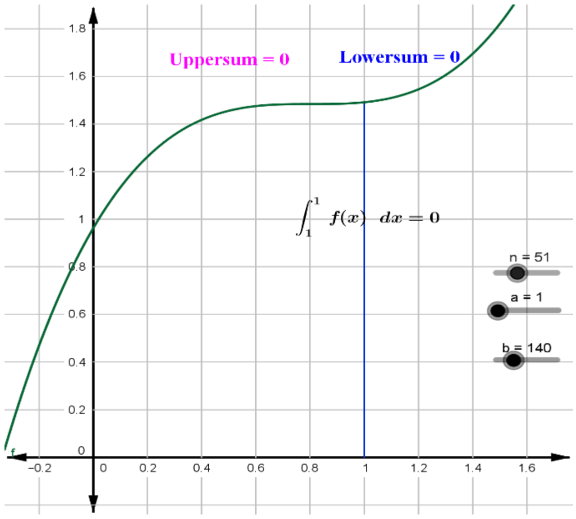

Figure 9 Visualization of Definte Integral Property 2

Property 3 *Reversing the limits of integration changes the sign of integral*

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

With GeoGebra, the ilustration is

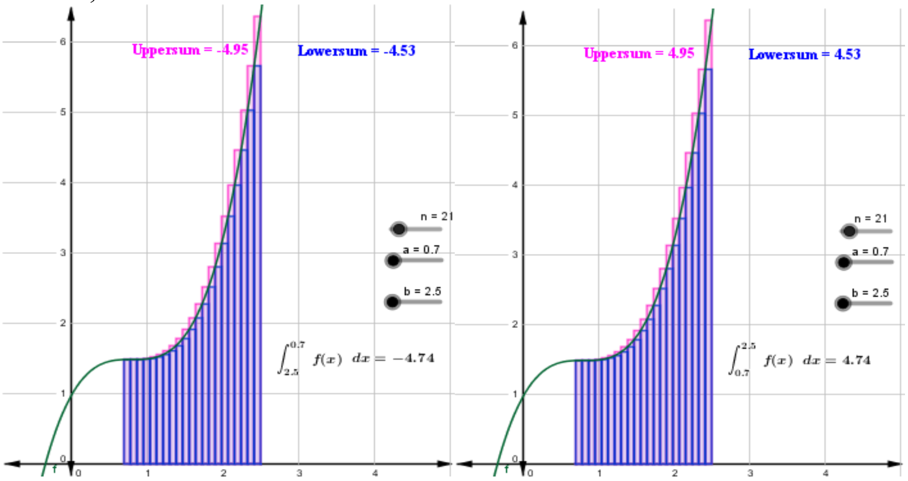

Figure 10 Visualization of Definte Integral Characteristic 2.3

Property 2.1 *Let f and g integrable on $[a, b]$ and k is any constant, then kf and $f + g$ integrable on $[a, b]$:*

2.4.1 $\int_b^a kf(x)\ dx = k\int_a^b f(x)\ dx$

2.4.2 $\int_b^a [f(x) + g(x)]\ dx = \int_b^a f(x)\ dx + \int_b^a g(x)\ dx$

with GeoGebra, the ilustration is

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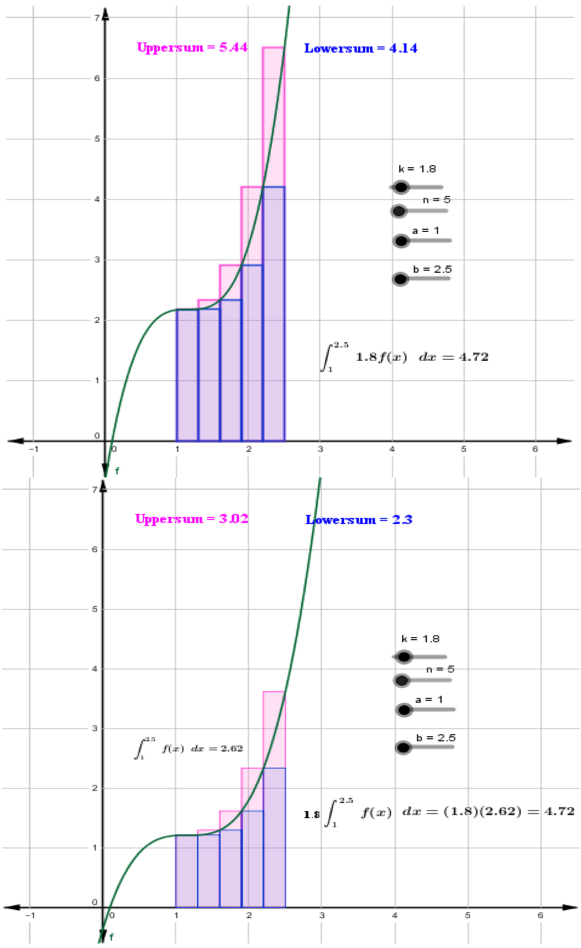

Figure 11 Visualization of Definte Integral Characteristic 2.4.1

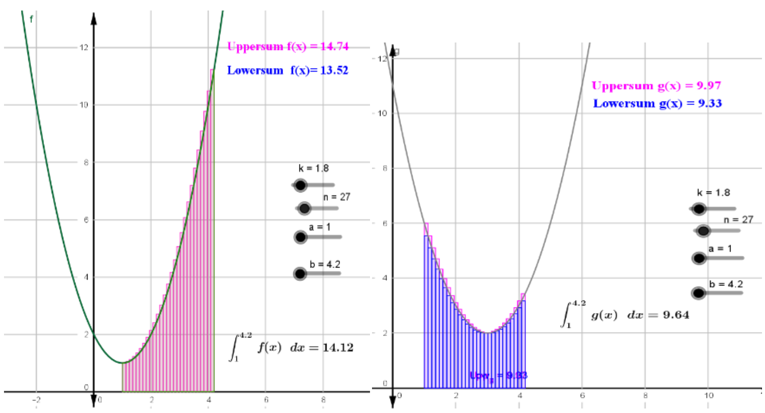

Figure 12 Visualization of Definte Integral Characteristic 2.4.2 (x) and $g(x)$

The picture shows the results of the integral of the function f (x) and g (x), then the following is given a visualization of the integral results of the function f (x) + g (x)

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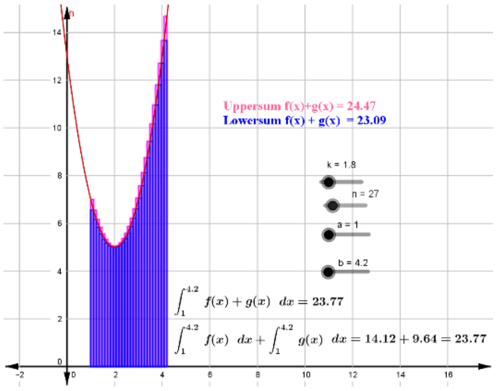

Figure 13 Visualization of Definite Integral Characteristic 2.4.2 f$(x) + g(x)$

The properties that have been visualized with GeoGebra above are some of the traits that are often used in solving problems in determining Integral values of course. It is also from these properties that theorems relating to Integral will emerge. For evidence in analysis, it may be usual to teach students, but not necessarily students understand the analytical meaning.

By making the visualization on GeoGebra, it is expected that students' understanding of the meaning and things underlying the Integral is certainly. In addition to a clear picture, GeoGebra is also able to display dynamic visualization, thus allowing users to see a variety of changes in values that occur, as well as animations that make students more understanding and interested in understanding Mathematics. Students not only memorize the Integral formula given and then use it, but students know how to construct the material and what lies behind the appearance of the formulas and properties used.

CONCLUSION

The concept of Integral can certainly be abstracted from the problem of the area under the curve and the calculation of the distance of an object. Understanding the concept of definite integral using visualization of mathematical software such as GeoGebra will be able to interpret the meaning of the concept of mathematical material specifically definite integral. With the visualization of GeoGebra, it will be obtained that the integral can certainly be constructed from the lowersum and uppersum sum values of a continuous function f at intervals [a, b]. Where the integral certainly is a single value I that satisfies the inequality L (f,P) ≤I≤U (f,P), where L (f,P) is the lowersum and U (f,P) is uppersum. From the integral value, then comes the properties and theorems in Calculus which are then used as formulas in solving integral problems

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