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On the Local Edge Antimagic Coloring of Corona Product of Path and Cycle

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ABSTRACT

Let $G(V, E)$ be a nontrivial and connected graph of vertex set V and edge set E. A bijection $f: V(G) \to \{1,2,3, \dots, |V(G)|\}$ is called a local edge antimagic labeling if for any two adjacent edges e_1 and $e_2, w(e_1) \neq w(e_2)$, where $e = uv \in E(G), w(e) = f(u) + f(v)$. Thus, the local edge antimagic labeling induces a proper edge coloring of G if each edge e assigned the color $w(e)$. The color of any edge $e = uv$ is assigned by $w(e)$ which is defined by the sum of both vertices labels $f(u)$ and $f(v)$. The local edge antimagic chromatic number, denoted by $\gamma_{lae}(G)$ is the minimum number of colors taken over all colorings induced by local edge antimagic labeling of G. In our paper, we present the local edge antimagic coloring of corona product of path and cycle, namely path corona cycle, cycle corona path, path corona path, and cycle corona cycle.

Keywords: Local antimagic; edge coloring; corona product; path; cycle.

INTRODUCTION

The local antimagic vertex coloring of a graph G introduced by Arumugam *et. al* [1]. Furthermore, Agustin, *et. al* [2] defined local edge antimagic coloring of the graph. A bijection $f: V(G) \to \{1,2,3, \dots, |V(G)|\}$, is called a local edge antimagic labeling if every two adjacent edges e_1 and e_2, $w(e_1) \neq w(e_2)$, where $e = uv \in E(G)$, and $w(e) = f(u) + f(v)$. Thus, the local edge antimagic labeling induces a proper edge coloring of G if any edge e is assigned the color $w(e)$. The color of each edge $e = uv$ are assigned by $w(e)$ which is defined by the sum of label both and vertices $f(u)$ and $f(v)$. The local edge antimagic chromatic number, denoted by $\gamma_{lae}(G)$, is the minimum number of colors taken over all colorings induced by local edge antimagic labeling of graph G. Agustin, *et. al* [2] establish the local edge antimagic chromatic number of path graph and cycle graph. Dettlaff, *et. al* in [3], a corona graph of G and H, denoted by $G \odot H$, is obtained by joining each vertex of H_i to the vertex u_j of G.

Ramya in [4], discussed an acyclic coloring of a corona graph and Yero in [5] studied coloring, location, and domination of a corona graph. Kristiana, et.al in [6] found the lower bound of the r-dynamic chromatic number of a corona product by wheel graphs. Many papers presented a corona product topics for example in [7], [2], and [8]. However, the local edge antimagic coloring of corona product still has nothing to discuss. In our paper, we investigate the local edge antimagic coloring of corona product of path and cycle, namely

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path corona cycle, cycle corona path, path corona path, and cycle corona cycle. The results of local edge antimagic labeling are as follows.

Conjecture 1.1. Every connected graph other than K_2 is local antimagic.
Observation 1.1. [6]For any graph $G, \gamma_{lae}(G) \geq \gamma(G) - 1$.
Observation 1.2. [8]For any graph G, $\chi_{la}(G) \geq \chi(G)$, where $\chi(G)$ is a vertex chromatic number of G.
Observation 1.3. [6] For any graph G, $\gamma_{lae}(G) \geq \gamma(G)$, where $\gamma(G)$ is an edge chromatic number of G.
Theorem 1.1. [6] If $\Delta(G)$ is maximum degrees of G, then we have $\gamma_{lae}(G) \geq \Delta(G)$.

Proposition 1.1. [6] Let G be a connected graph, we have

a) If $G \cong P_n$, then $\gamma_{lae}(G) = 2$.
b) If $G \cong C_n$, then $\gamma_{lae}(G) = 3$.
c) If $G \cong L_n$, then $\gamma_{lae}(G) = 3$.
d) If $G \cong K_n$, then $\gamma_{lae}(G) = 2n - 3$.
e) If $G \cong W_n$, then $\gamma_{lae}(G) = n + 2$.
f) If $G \cong S_n$, then $\gamma_{lae}(G) = n$.
g) If $G \cong F_n$, then $\gamma_{lae}(G) = 2n + 1$.
h) If $G \odot mK_1$, then $\gamma_{lae}(G \odot mK_1) = \gamma_{lae}(G) + m$.

RESULTS AND DISCUSSION

In our paper, we consider the local edge antimagic chromatic number of a corona product of path and cycle, including path corona cycle, cycle corona path, path corona path, cycle corona cycle. Furthermore, we determine the exact values of local edge antimagic chromatic number of corona product in the following theorems.

Theorem 2.1. The local edge antimagic chromatic number of $P_n \odot P_m$ for n odd and $n, m \geq 3$ is $\gamma_{lae}(P_n \odot P_m) = 2(n + 1) + m$.

Proof. The graph $P_n \odot P_m$ is a connected graph with vertex set $V(P_n \odot P_m) = \{x_i: 1 \leq i \leq n\} \cup \{x_j{}^i: 1 \leq j \leq m;\ 1 \leq i \leq n\}$ and edge set $E(P_n \odot P_m) = \{x_i x_{i+1}: 1 \leq i \leq n - 1\} \cup \{x_j^i x_{j+1}^i: 1 \leq j \leq m - 1;\ 1 \leq i \leq n\} \cup \{x_i x_j^i: 1 \leq j \leq m;\ 1 \leq i \leq n\}$. The cardinality of the vertex set is $|V(P_n \odot P_m)| = n + mn$ and the cardinality of the edge set is $|E(P_n \odot P_m)| = 2mn - 1$. We define a bijection $f: V(P_n \odot P_m) \to \{1,2,3, \ldots, |V(P_n \odot P_m)|\}$ for the graph $P_n \odot P_m$ to be local edge antimagic labeling as follows.

$$f(x_i) = \begin{cases} \dfrac{i+1}{2}, & \text{if } i \text{ is odd} \\ n - \left(\dfrac{i-2}{2}\right), & \text{if } i \text{ is even} \end{cases}$$

$$f(x_j^i) = \begin{cases} n+1+\left(\frac{i-2}{2}\right)+\left(\frac{j-2}{2}\right)n, & \text{if } i \text{ and } j \text{ are even} \\ 2n+n\left\lceil\frac{m}{2}\right\rceil+1+\left(\frac{i-2}{2}\right)-n\left(\frac{j-1}{2}\right), & \text{if } i \text{ is even and } j \text{ is odd} \\ 2n-\left(\frac{i-1}{2}\right)+\left(\frac{j-2}{2}\right)n, & \text{if } i \text{ is odd and } j \text{ is even} \\ mn+n-\left(\frac{i-1}{2}\right)-n\left(\frac{j-1}{2}\right), & \text{if } i \text{ and } j \text{ are odd} \end{cases}$$

It is clear that f is a local antimagic labeling of $P_n \odot P_m$ and the edge weights are as follows:

$$w(x_i x_{i+1}) = \begin{cases} n+1, & \text{if } i \text{ is odd} \\ n+2, & \text{if } i \text{ is even} \end{cases}$$

$$w(x_j^i x_{j+1}^i) = \begin{cases} mn+3n-(i-1), & \text{if } i \text{ and j are odd} \\ mn+2n-(i-1), & \text{if } i \text{ is odd and } j \text{ is even} \\ mn+n+i, & \text{if } i \text{ is even and } j \text{ is odd} \\ mn+i, & \text{if } i \text{ and } j \text{ are even} \end{cases}$$

$$w(x_i x_j^i) = \begin{cases} mn+1+n\left(\frac{j-3}{2}\right), & \text{if } j \text{ is odd} \\ 2n+1+n\left(\frac{j-2}{2}\right), & \text{if } j \text{ is even} \end{cases}$$

Hence, we get that the upper bound of the local edge antimagic chromatic number of $P_n \odot P_m$ is $\gamma_{lae}(P_n \odot P_m) \le 2(n+1)+m$. Furthermore, we prove that lower bound of the local edge antimagic chromatic number of $P_n \odot P_m$ is $\gamma_{lae}(P_n \odot P_m) \ge 2(n+1)+m$. By contradiction, we assume that $\gamma_{lae}(P_n \odot P_m) < 2(n+1)+m$. Without lost of generality, we assume that $w(x_i x_{i+1}) \ne w(x_j^i x_{j+1}^i) \ne w(x_i x_j^i)$. Based on Proposition 1, $\gamma_{lae}(P_n) = 2$ and $\gamma_{lae}(P_m) = 2$ then we get $|\{w(e); e \in E(P_n)\}| = 2$, $|\{w(x_j^i x_{j+1}^i)\}| = m$ and $|\{w(e); e \in E((P_m)_i), 1 \le i \le n-1\}| = 2(n-1)$, $|\{w(e); e \in E((P_m)_n)\}| = 1$ such that

$$|w(e); e \in E(P_n \odot P_m)| = |\{w(e); e \in E(P_n)\}|$$
$$+|\{w(x_j^i x_{j+1}^i)\}| + |\{w(e); e \in E((P_m)_i), 1 \le i \le n-1\}| + |\{w(e); e \in E((P_m)_n)\}|$$
$$|w(e); e \in E(P_n \odot P_m)| = 2+m+2(n-1)+1$$
$$|w(e); e \in E(P_n \odot P_m)| = m+2n+1$$

If $|\{w(e); e \in E((P_m)_n)\}| = 1$, then we obtain at least two edges which have same edge weight, it is a contradiction. Thus, we receive that the lower bound of the local edge antimagic chromatic number of $P_n \odot P_m$ is $\gamma_{lae}(P_n \odot P_m) \ge m+2n+2 = 2(n+1)+m$. It concludes that the local antimagic edge chromatic number of $P_n \odot P_m$ is $\gamma_{lae}(P_n \odot P_m) = 2(n+1)+m$. ■

Theorem 2.2 The local edge antimagic chromatic number of $P_n \odot C_m$ for n, m odd and $n, m \ge 4$ is $\gamma_{lae}(P_n \odot C_m) = 2+3n+m$.

Proof. The graph $P_n \odot C_m$ is a connected graph with vertex set $V(P_n \odot C_m) = \{x_i : 1 \le i \le n\} \cup \{x_j{}^i : 1 \le j \le m;\ 1 \le i \le n\}$ and edge set $E(P_n \odot C_m) = \{x_i x_{i+1} : 1 \le i \le n-1\} \cup \{x_j^i x_{j+1}^i : 1 \le j \le m-1;\ 1 \le i \le n\} \cup \{x_m^i x_1^i, 1 \le i \le n\} \cup \{x_i x_j^i : 1 \le j \le m;\ 1 \le i \le n\}$. The cardinality of the vertex set is $|V(P_n \odot C_m)| = n+mn$ and the cardinality of the edge set is $|E(P_n \odot C_m)| = 2mn+n-1$. We define a function bijection $f: V(P_n \odot C_m) \to \{1,2,3,\dots,|V(P_n \odot C_m)|\}$ for the graph $P_n \odot C_m$ to be local edge antimagic labeling as follows.

$$f(x_i) = \begin{cases} \dfrac{i+1}{2}, & \text{if } i \text{ is odd} \\ n - \dfrac{i-2}{2}, & \text{if } i \text{ is even} \end{cases}$$

$$f(x_j^i) = \begin{cases} mn + 1 + \left(\dfrac{i-2}{2}\right) - \left(\dfrac{j-2}{2}\right)n, & \text{if } i \text{ and } j \text{ are even} \\ n + 1 + \left(\dfrac{i-2}{2}\right) + \left(\dfrac{j-1}{2}\right)n, & \text{if } i \text{ is even and } j \text{ is odd} \\ mn + n - \left(\dfrac{i-1}{2}\right) - \left(\dfrac{j-2}{2}\right)n, & \text{if } i \text{ is odd and } j \text{ is even} \\ 2n - \left(\dfrac{i-1}{2}\right) + n\left(\dfrac{j-1}{2}\right), & \text{if } i \text{ and } j \text{ are odd} \end{cases}$$

$$f(x_m^i) = \begin{cases} n\left\lceil\dfrac{m}{2}\right\rceil + n - \left(\dfrac{i-1}{2}\right), & \text{if } i \text{ is odd} \\ n\left\lceil\dfrac{m}{2}\right\rceil + 1 + \left(\dfrac{i-2}{2}\right), & \text{if } i \text{ is even} \end{cases}$$

It is easy to see that f is a local edge antimagic labeling of $P_n \odot C_m$ and the edge weights are as follows:

$$w(x_i x_{i+1}) = \begin{cases} n+1, & \text{if } i \text{ is odd} \\ n+2, & \text{if } i \text{ is even} \end{cases}$$

$$w(x_j^i x_{j+1}^i) = \begin{cases} mn + 4n - (i-1), & \text{if } i \text{ and } j \text{ are odd} \\ mn + 3n - (i-1), & \text{if } i \text{ is odd and } j \text{ is even} \\ mn + 2(n+1) + (i-2), & \text{if } i \text{ is even and } j \text{ is odd} \\ mn + n + 2 + (i-2), & \text{if } i \text{ and } j \text{ are even} \end{cases}$$

$$w(x_m^i x_1^i) = \begin{cases} n\left\lceil\dfrac{m}{2}\right\rceil + 3n - (i-1), & \text{if } j \text{ is odd} \\ n\left\lceil\dfrac{m}{2}\right\rceil + n + 2 + (i-2), & \text{if } j \text{ is even} \end{cases}$$

$$w(x_i x_j^i) = \begin{cases} 2n + 1 + n\left(\dfrac{j-1}{2}\right), & \text{if } j \text{ is odd} \\ mn + n - n\left(\dfrac{j-2}{2}\right), & \text{if } j \text{ is even} \end{cases}$$

$$w(x_i x_m^i) = \begin{cases} n\left\lceil\dfrac{m}{2}\right\rceil + n + 1 - \left(\dfrac{i-1}{2}\right), & \text{if } j \text{ is odd} \\ n\left\lceil\dfrac{m}{2}\right\rceil + 2 + \left(\dfrac{i-2}{2}\right), & \text{if } j \text{ is even} \end{cases}$$

Hence, we get that the upper bound of the local edge antimagic chromatic number of $P_n \odot C_m$ is $\gamma_{lae}(P_n \odot C_m) \leq 2 + 3n + m$. Furthermore, we prove that the lower bound of the

local edge antimagic chromatic number of $P_n \odot C_m$ is $\gamma_{lae}(P_n \odot C_m) \geq 2 + 3n + m$. By contradiction, we assume that $\gamma_{lae}(P_n \odot C_m) < 2 + 3n + m$. Without of generality, $w(x_i x_{i+1}) \neq w(x_j^i x_{j+1}^i) \neq w(x_1^i x_m^i) \neq w(x_i x_j^i)$. Based on Proposition 1 that $\gamma_{lae}(P_n) = 2$ and $\gamma_{lae}(C_m) = 3$ then we get $|\{w(e); e \in E(P_n)\}| = 2$, $|\{w(x_i x_j^i)\}| = m$ and $|\{w(e); e \in E((C_m)_i), 1 \leq i \leq n-1\}| = 3(n-1)$, $|\{w(e); e \in E((C_m)_n)\}| = 2$ such that
$|w(e); e \in E(P_n \odot C_m)| = |\{w(e); e \in E(P_n)\}| + |\{w(x_i x_j^i)\}| + |\{w(e); e \in E((C_m)_i), 1 \leq i \leq n-1\}| + |\{w(e); e \in E((C_m)_n)\}|$ $|w(e); e \in E(P_n \odot C_m)|$
$= 2 + m + 3(n-1) + 2$ $|w(e); e \in E(P_n \odot C_m)| = m + 3n + 1$
If $|\{w(e); e \in E((C_m)_n)\}| = 2$, then we obtain at least two edges which have the same edge weight, which is a contradiction. Accordingly, the lower bound of the local edge antimagic chromatic number of $P_n \odot C_m$ is $\gamma_{lae}(P_n \odot C_m) \geq m + 3n + 2$. It concludes that the local edge antimagic chromatic number of $P_n \odot C_m$ is $\gamma_{lae}(P_n \odot C_m) = 2 + 3n + m$. ■

Theorem 2.3. The local edge antimagic chromatic number of $C_n \odot C_m$ for n, m even and $n, m \geq 4$ is $\gamma_{lae}(C_n \odot C_m) = 3(n+1) + m$.

Proof. The graph $C_n \odot C_m$ is a connected graph with vertex set $V(C_n \odot C_m) = \{x_i : 1 \leq i \leq n\} \cup \{x_j{}^i : 1 \leq j \leq m;\ 1 \leq i \leq n\}$ and edge set $E(C_n \odot C_m) = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{x_1 x_n\} \cup \{x_j^i x_{j+1}^i : 1 \leq j \leq m-1;\ 1 \leq i \leq n\} \cup \{x_i x_j^i : 1 \leq j \leq m;\ 1 \leq i \leq n\} \cup \{x_m^i x_1^i, 1 \leq i \leq n\}$. The cardinality of the vertex set is $|V(C_n \odot C_m)| = n + mn$ and the cardinality of the edge set is $|E(C_n \odot C_m)| = 2mn + n$. We define a function bijection $f: V(C_n \odot C_m) \to \{1,2,3, \dots, |VC_n \odot C_m)|\}$ for the graph $C_n \odot C_m$ to be local edge antimagic labeling as follows.

$$f(x_i) = \begin{cases} \dfrac{i+1}{2}, & \text{if } i \text{ is odd} \\ n - \dfrac{i-2}{2}, & \text{if } i \text{ is even} \end{cases}$$

$$f(x_j^i) = \begin{cases} mn + 1 + \left(\dfrac{i-2}{2}\right) - \left(\dfrac{j-1}{2}\right)n, & \text{if } i \text{ and } j \text{ are even} \\ n + 1 + \left(\dfrac{i-2}{2}\right) + \left(\dfrac{j-1}{2}\right)n, & \text{if } i \text{ is even and } j \text{ is odd} \\ mn + n - \left(\dfrac{i-1}{2}\right) - \left(\dfrac{j-2}{2}\right)n, & \text{if } i \text{ is odd and } j \text{ is even} \\ 2n - \left(\dfrac{i-1}{2}\right) + n\left(\dfrac{j-1}{2}\right), & \text{if } i \text{ and } j \text{ are odd} \end{cases}$$

$$f(x_m^i) = \begin{cases} n\left\lceil\dfrac{m}{2}\right\rceil + n - \left(\dfrac{i-1}{2}\right), & \textit{if } i \textit{ is odd} \\ n\left\lceil\dfrac{m}{2}\right\rceil + 1 + \left(\dfrac{i-2}{2}\right), & \textit{if } i \textit{ is even} \end{cases}$$

It is easy to see that f is a local edge antimagic labeling of $C_n \odot C_m$ and the edge weights are as follows:

$$w(x_i x_{i+1}) = \begin{cases} n+1, & if\ i\ is\ odd \\ n+2, & if\ i\ is\ even \end{cases}$$

$$w(x_1 x_n) = \frac{n+2}{2}$$

$$w\left(x_j^i x_{j+1}^i\right) = \begin{cases} mn + 4n - (i-1), & if\ i\ and\ j\ are\ odd \\ mn + 3n - (i-1), & if\ i\ is\ odd\ and\ j\ is\ even \\ mn + 2(n+1) + (i-2), & if\ i\ is\ even\ and\ j\ is\ odd \\ mn + n + 2 + (i-2), & if\ i\ and\ j\ are\ even \end{cases}$$

$$w\left(x_m^i x_1^i\right) = \begin{cases} n\left\lceil\frac{m}{2}\right\rceil + 3n - (i-1), if\ j\ is\ odd \\ n\left\lceil\frac{m}{2}\right\rceil + n + 2 + (i-2), if\ j\ is\ even \end{cases}$$

$$w\left(x_i x_j^i\right) = \begin{cases} 2n + 1 + n\left(\frac{j-1}{2}\right), if\ j\ is\ odd \\ mn + n - n\left(\frac{j-2}{2}\right), if\ j\ is\ even \end{cases}$$

$$w\left(x_i x_m^i\right) = \begin{cases} n\left\lceil\frac{m}{2}\right\rceil + n + 1 - \left(\frac{i-1}{2}\right), if\ j\ is\ odd \\ n\left\lceil\frac{m}{2}\right\rceil + 2 + \left(\frac{i-2}{2}\right), if\ j\ is\ even \end{cases}$$

Hence, we get that the upper bound of the local edge antimagic chromatic number of $C_n \odot C_m$ is $\gamma_{lae}(C_n \odot C_m) \leq 3(n+1) + m$. Furthermore, we prove that lower bound of the local edge antimagic chromatic number of $C_n \odot C_m$ is $\gamma_{lae}(C_n \odot C_m) \geq 3(n+1) + m$. By contradiction, we assume that $\gamma_{lae}(C_n \odot C_m) < 3(n+1) + m$. Without lost of generality, we gives that $w(x_i x_{i+1}) \neq w(x_1 x_n) \neq w\left(x_j^i x_{j+1}^i\right) \neq w\left(x_1^i x_m^i\right) \neq w\left(x_i x_j^i\right)$. Based on Proposition 1 that $\gamma_{lae}(C_n) = 3$ and $\gamma_{lae}(C_m) = 3$ then we get $|\{w(e); e \in E(C_n)\}| = 3$, $\left|\left\{w\left(x_i x_j^i\right)\right\}\right| = m$ and $|\{w(e); e \in E((C_m)_i), 1 \leq i \leq n-1\}| = 3(n-1)$, $|\{w(e); e \in E((C_m)_n)\}| = 2$ such that

$$\begin{aligned} &|w(e); e \in E(C_n \odot C_m)| \\ &\quad = |\{w(e); e \in E(C_n)\}| + \left|\left\{w\left(x_i x_j^i\right)\right\}\right| + |\{w(e); e \in E((C_m)_i), 1 \leq i \leq n-1\}| \\ &\quad + |\{w(e); e \in E((C_m)_n)\}| = 3 + m + 3(n-1) + 2 = m + 3n + 2 \end{aligned}$$

If $|\{w(e); e \in E((C_m)_n)\}| = 2$, then we obtain at least two edges which have same edge weight, which is a contradiction. Thus, we receive that the lower bound of the local edge antimagic chromatic number of $C_n \odot C_m$ is $\gamma_{lae}(C_n \odot C_m) \geq m + 3n + 3$. It concludes that the local edge antimagic chromatic number of $C_n \odot C_m$ is $\gamma_{lae}(C_n \odot C_m) = 3(n+1) + m$. ■

Theorem 2.4. The local edge antimagic chromatic number of $C_n \odot P_m$ for n odd and $n, m \geq 3$ is $\gamma_{lae}(C_n \odot P_m) = 3 + 2n + m$.

Proof. The graph $C_n \odot P_m$ is a connected graph with vertex set $V(C_n \odot P_m) = \{x_i: 1 \leq i \leq n\} \cup \{x_j{}^i: 1 \leq j \leq m;\ 1 \leq i \leq n\}$ and edge set $E(C_n \odot P_m) = \{x_i x_{i+1}: 1 \leq i \leq n-1\} \cup \{x_1 x_n\} \cup \{x_j^i x_{j+1}^i: 1 \leq j \leq m-1;\ 1 \leq i \leq n\} \cup \{x_i x_j^i: 1 \leq j \leq m;\ 1 \leq i \leq n\}$. The cardinality of the vertex set is $|V(C_n \odot P_m)| = n + mn$ and the cardinality of the edge set is $|E(C_n \odot P_m)| = 2mn + n - 1$. We define a function bijection $f: V(C_n \odot P_m) \to \{1,2,3,\ldots,|V(C_n \odot P_m)|\}$ for the graph $C_n \odot P_m$ to be local edge antimagic labeling as follows.

$$f(x_i) = \begin{cases} \dfrac{i+1}{2}, & if\ i\ is\ odd \\ n - \dfrac{i-2}{2}, & if\ i\ is\ even \end{cases}$$

$$f(x_j^i) = \begin{cases} n + 1 + \left(\dfrac{i-2}{2}\right) + \left(\dfrac{j-2}{2}\right)n, & if\ i\ and\ j\ are\ even \\ 2n + n\left\lceil\dfrac{m}{2}\right\rceil + 1 + \left(\dfrac{i-2}{2}\right) - n\left(\dfrac{j-1}{2}\right), & if\ i\ is\ even\ and\ j\ is\ odd \\ 2n - \left(\dfrac{i-1}{2}\right) + \left(\dfrac{j-2}{2}\right)n, & if\ i\ is\ odd\ and\ j\ is\ even \\ mn + n - \left(\dfrac{i-1}{2}\right) - n\left(\dfrac{j-1}{2}\right), & if\ i\ and\ j\ are\ odd \end{cases}$$

It is easy to see that f is a local edge antimagic labeling of $P_n \odot C_m$ and the edge weights are as follows:

$$w(x_i x_{i+1}) = \begin{cases} n+1, & if\ i\ is\ odd \\ n+2, & if\ i\ is\ even \end{cases}$$

$$w(x_1 x_n) = \frac{n+2}{2}$$

$$(x_i x_{i+1}) = \begin{cases} n+1, & if\ i\ is\ odd \\ n+2, & if\ i\ is\ even \end{cases}$$

$$w(x_j^i x_{j+1}^i) = \begin{cases} mn + 3n - (i-1), & if\ i\ and\ j\ are\ odd \\ mn + 2n - (i-1), & if\ i\ is\ odd\ and\ j\ is\ even \\ mn + n + i, & if\ i\ is\ even\ and\ j\ is\ odd \\ mn + 2 + i, & if\ i\ is\ even\ and\ j\ is\ even \end{cases}$$

$$w(x_i x_j^i) = \begin{cases} mn + 1 + n\left(\dfrac{j-3}{2}\right), & if\ j\ is\ odd \\ 2n + 1 + n\left(\dfrac{j-2}{2}\right), & if\ j\ is\ even \end{cases}$$

Hence, we get that the upper bound of the local edge antimagic chromatic number of $C_n \odot P_m$ is $\gamma_{lae}(C_n \odot P_m) \leq 3 + 2n + m$. furthermore, we prove that the lower bound of the local edge antimagic chromatic number of $C_n \odot P_m$ is $\gamma_{lae}(C_n \odot P_m) \geq 3 + 2n + m$. By contradiction, we assume that $\gamma_{lae}(C_n \odot P_m) < 3 + 2n + m$. Without lost of generality, we gives that $w(x_i x_{i+1}) \neq w\left(x_j^i x_{j+1}^i\right) \neq w\left(x_1^i x_m^i\right) \neq w\left(x_i x_j^i\right)$. Based on Proposition 1 that $\gamma_{lae}(C_n) = 3$ and $\gamma_{lae}(P_m) = 2$ then we get $|\{w(e); e \in E(C_n)\}| = 3$, $|\{w\left(x_i x_j^i\right)\}| = m$ and $|\{w(e); e \in E((P_m)_i), 1 \leq i \leq n-1\}| = 2(n-1)$, $|\{w(e); e \in E((P_m)_n)\}| = 1$ such that

$$|w(e); e \in E(C_n \odot P_m)| = |\{w(e); e \in E(C_n)\}| + \left|\{w\left(x_i x_j^i\right)\}\right| +$$
$$|\{w(e); e \in E((P_m)_i), 1 \leq i \leq n-1\}| + |\{w(e); e \in E((P_m)_n)\}|$$
$$|w(e); e \in E(C_n \odot P_m)| = 3 + m + 2(n-1) + 1$$
$$|w(e); e \in E(C_n \odot P_m)| = m + 2n + 2$$

If $|\{w(e); e \in E((P_m)_n)\}| = 1$, then we obtain at least two edges which have same edge weight, Which is a contradiction. Thus, we receive that the lower bound of the local edge antimagic chromatic number of $C_n \odot P_m$ is $\gamma_{lae}(C_n \odot P_m) \geq m + 2n + 3$. It concludes that the local edge antimagic chromatic number of $C_n \odot P_m$ is $\gamma_{lae}(C_n \odot P_m) = 3 + 2n + m$. ■

CONCLUSIONS

In this paper we have given the result on the local edge antimagic chromatic number of corona product of path and cycle, namely path corona cycle, cycle corona path, path corona path, cycle corona cycle.

Open Problem 1. What is the upper bound of local edge antimagic coloring of corona product of a connected graph?

Open Problem 2. What is the lower bound of local edge antimagic coloring of corona product of a connected graph?

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