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On Rainbow Vertex Antimagic Coloring of Graphs: A New Notion

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ABSTRACT

For a bijective function $g: E(G) \to \{1, 2, 3, \cdots, |E(G)|\}$, the associated weight of a vertex $v \in V(G)$ under g is $w_g(v) = \Sigma_{e\in E(v)} g(e)$, where $E(v)$ is the set of vertices incident to v. The function g is called a vertex-antimagic edge labeling if every vertex has distinct weight. A path P in the edge-labeled graph G is said to be a rainbow path if for any two vertices x and x', all internal vertices in the path $x - x'$ have different weight. If for every two vertices x and y of G, there exists a rainbow $x - y$ path, then g is called a rainbow vertex antimagic labeling of G. When we assign each edge xy with the color of the vertex weight $w_g(v)$, thus we say the graph G admits a rainbow vertex antimagic coloring. The smallest number of colors taken over all rainbow colorings induced by rainbow vertex antimagic labelings of G is called rainbow vertex antimagic connection number of G, denoted by $rvac(G)$. In this paper, we initiate to determine the rainbow vertex antimagic connection number of graphs, namely path (P_n), wheel (W_n), friendship ($\mathcal{F}_n$), and fan (F_n).

Keywords: antimagic labeling; rainbow vertex coloring; rainbow vertex antimagic coloring; rainbow vertex antimagic connection number.

INTRODUCTION

We consider a graph $G(V, E)$ in this paper are simple, connected and un-directed graph, where V and E are respectively a vertex set and edge set of G [1]. The Rainbow coloring problem has been studied by many researchers since many years ago. Many good results has been published in some reputable journal [2]. Thus, it has given many contributions in graph theory research of interest. There are many types of rainbow coloring, namely rainbow (edge) coloring, rainbow vertex coloring, strong rainbow edge/vertex coloring. The minimum number of colors for which an edge (vertex) coloring exists such that the graph G is rainbow connected is called the rainbow connection number, denoted by $rc(G)$ for edge coloring and the rainbow vertex connection number, denoted by $rvc(G)$ for vertex coloring, see [3]–[10] for detail. Krivelevich and Yuster [6] gave the lower bound for $rvc(G)$, namely $rvc(G) \geq diam(G) - 1$, where $diam(G)$ is the diameter of graph G. An easy observation is that if G has an order n, then $rvc(G) \leq n - 2$ and $rvc(G) = 0$ if and only if G is a complete graph. Notice that $rvc(G) \geq diam(G) - 1$ with equality if the diameter of G is 1 or 2.

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Meanwhile, In 2003, Hartsfield and Ringel [11] defined antimagic graphs. A graph G is called antimagic if there exists a bijection $f: E(G) \rightarrow \{1,2,\cdots,q\}$ such that the weights of all vertices are distinct [12] . The vertex weight of a vertex v under f, $w_f(v)$, is the sum of labels of edges incident with v, that is, $w_f(v) = \sum_{uv \in E(G)} f(uv)$. In this case, f is called an antimagic labeling. There many results were found for antimagicness of graph. There are extension types of vertex antimagic labeling, namely total vertex antimagic labeling, super total vertex antimagic labeling, (a,d)-vertex antimagic labeling, super (a,d)-vertex antimagic labeling. For detail, see Galian Dynamic Survey of Graph Labeling [13] .

In this study, we initiate to combine the two notion, namely rainbow coloring and antimagic labeling [14][15]. We name for this combination as rainbow vertex antimagic coloring. For a bijective function $g: E(G) \rightarrow \{1,2,3,\cdots,|E(G)|\}$, the associated weight of a vertex $v \in V(G)$ under g is $w_g(v) = \Sigma_{e \in E(v)} g(e)$, where $E(v)$ is the set of vertices incident to v. The function g is called a vertex-antimagic edge labeling if every vertex has distinct weight. A path P in the edge-labeled graph G is said to be a rainbow path if for any two vertices x and x', all internal vertices in the path $x - x'$ have different weight. If for every two vertices x and y of G, there exists a rainbow $x - y$ path, then g is called a rainbow vertex antimagic labeling of G. When we assign each edge xy with the color of the vertex weight $w_g(v)$, thus we say the graph G admits a rainbow vertex antimagic coloring. The rainbow vertex antimagic connection number of G, denoted by $rvac(G)$, is the smallest number of colors taken over all rainbow colorings induced by rainbow vertex antimagic labelings of G.

To determine the rainbow vertex antimagic connection number of any graph is considered to be hard problem. Even, this study fall into NP-hard problem. In this paper, we initiate to determine the rainbow vertex antimagic connection number of graphs, namely path (P_n), wheel (W_n), friendship $(\mathcal{F}_n)$, and fan (F_n) as well as fix the lower bound $rvac(G)$ of any graph.

METHODS

This research includes deductive analytic methods. The procedures to obtain the rainbow vertex antimagic connection number of are as follows.

1. Define a graph G.
2. Determine the cardinality of graph G by obtaining the order and size of graph G.
3. Determine the lower bound of $rvac(G)$ by using the obtained remark of sharpest lower bound.
4. Determine the upper bound of $rvac(G)$ by constructing the bijective function, compute the vertex weight using $w_g(v) = \Sigma_{e \in E(v)} g(e)$, and show that every two different vertices of G satisfy the rainbow vertex antimagic coloring.
5. If the upper bound attains the lower bound, then we obtain the $rvac(G)$. If the upper bound does not attain the lower bound, then we return to determine the upper bound of $rvac(G)$.
6. Finally we can construct a new theorem and its proof after we obtain the rainbow vertex antimagic connection number of graph G.

RESULTS AND DISCUSSION

In this section we have several theorems on the rainbow vertex antimagic coloring. We determine the minimum color taken to the graph such that it has rainbow vertex antimagic coloring. Since we determine the minimum colors such that G has rainbow vertex antimagic coloring, then the lower bound of rainbow vertex antimagic connection number of graph is at least and equal to rainbow vertex connection number. The lower bound of rainbow vertex antimagic connection number of any graph is mathematically written in the Remark 1.

Remark 1
Let G be a connected graph, $rvac(G) \geq rvc(G)$.

Theorem 1
If P_n be a path graph of order n and $n \geq 3$, then

$$rvac(P_n) = \begin{cases} 3, & n = 3,4 \\ n-2, & n \geq 5 \end{cases}$$

Proof. Let P_n be a path graph with vertex set $V(P_n) = \{v_1, v_2, v_3, \cdots, v_n\}$ and edge set $E(P_n) = \{v_i v_{\{i+1\}} : 1 \leq i \leq n-1\}$. The diameter of P_n is $n-1$. We divide into two cases to prove the rainbow vertex antimagic connection number as follows.

Case 1. For $P_n, n = 3,4$
Path graph $P_n, n = 3$ have two edges. If we give labels on it, it gives three different weights on its edges exactly. It concludes that the rainbow vertex antimagic connection number of P_3 is 3. Furthermore for P_4, we determine the all permutation of edge labeling on P_4. Let e_1, e_2, e_3 are the edges of P_4, thus there are six possibilities of edge labeling on P_4 as follows.
1). If $e_1 = 1, e_2 = 2, e_3 = 3$, then $wt(v_1) = 1, wt(v_2) = 3, wt(v_3) = 5, wt(v_4) = 3$.
2). If $e_1 = 1, e_2 = 3, e_3 = 2$, then $wt(v_1) = 1, wt(v_2) = 4, wt(v_3) = 5, wt(v_4) = 2$.
3). If $e_1 = 2, e_2 = 1, e_3 = 3$, then $wt(v_1) = 2, wt(v_2) = 3, wt(v_3) = 4, wt(v_4) = 3$.
4). If $e_1 = 2, e_2 = 3, e_3 = 1$, then $wt(v_1) = 2, wt(v_2) = 5, wt(v_3) = 4, wt(v_4) = 1$.
5). If $e_1 = 3, e_2 = 1, e_3 = 2$, then $wt(v_1) = 3, wt(v_2) = 4, wt(v_3) = 3, wt(v_4) = 2$.
6). If $e_1 = 3, e_2 = 2, e_3 = 1$, then $wt(v_1) = 3, wt(v_2) = 5, wt(v_3) = 3, wt(v_4) = 1$.
Based on edge labelings and vertex weights above, it is easy to determine the rainbow vertex antimagic connection number of P_4 at least 3. Thus $arvc(P_4) = 3$.

Case 2. For $P_n, n \geq 5$
Based on Remark 1, we have $rvac(P_n) \geq rvc(P_n) = diam(P_n) - 1 = n - 1 - 1 = n - 2$. Furthermore, to show the upper bound we construct the bijective function of edge labels. We have two conditions, namely for $n \equiv 1 (\text{mod } 2)$ and $n \equiv 0 (\text{mod } 2)$. For $n \equiv 1 (\text{mod } 2)$, we have

$$g(v_1 v_2) = 3$$
$$g(v_2 v_3) = 1$$
$$g(v_3 v_4) = 2$$
$$g(v_{n-1} v_n) = 4$$
$$g(v_i v_{i+1}) = i + 1 : 4 \leq i \leq n-2$$

From the edge labels above, we have the vertex weight as follows. For P_5, we have $w(v_1, v_2, v_3, v_4, v_5) = (3,4,3,6,4)$. For $P_n : n \geq 6$, we have

$$w(v_1) = 3$$
$$w(v_2) = 4$$
$$w(v_3) = 3$$
$$w(v_4) = 7$$
$$w(v_i) = 2i + 1: 5 \leq i \leq n - 2$$
$$w(v_{n-1}) = n + 3$$
$$w(v_n) = 4$$

For $n \equiv 0(\text{mod } 2)$, we have

$$g(v_1v_2) = 3$$
$$g(v_2v_3) = 1$$
$$g(v_3v_4) = 2$$
$$g(v_iv_{i+1}) = i: 4 \leq i \leq n - 1$$

From the edge labels above, we have the vertex weights in the following:

$$w(v_1) = 3$$
$$w(v_2) = 4$$
$$w(v_3) = 3$$
$$w(v_4) = 6$$
$$w(v_i) = 2i - 1: 5 \leq i \leq n - 1$$
$$w(v_n) = n - 1$$

From the vertex weight above, it is easy to see that the different weight is $n - 2$. It concludes that the rainbow vertex antimagic connection number of $P_n: n = \{3,4\}$ is 3 and the rainbow vertex antimagic connection number of $P_n: n \geq 5$ is $n - 2$.
Furthermore, we show that every two different vertices of P_n is rainbow vertex antimagic coloring. Suppose that $v \in V(P_n)$, refer to the vertex weight the rainbow vertex path is shown in Table 1.

Table 1. The Rainbow Vertex Path of P_n

Case	v	v	**Rainbow Vertex Coloring**
1	v_1	v_n	$v_1, v_2, v_3, \dots, v_i, \dots, v_{n-1}$

Hence, the vertex coloring of P_n is rainbow vertex antimagic coloring. Thus, we obtain $arvc(P_n)$ is 3 for $n = 3,4$ and $arvc(P_n)$ is $n - 2$ for $n \geq 5$. ■

Theorem 2
If W_n be a wheel graph of order $n + 1$ and $n \geq 3$, then $rvac(W_n) = 2$ if $n \equiv 1(\text{mod } 2)$ and $2 \leq rvac(W_n) \leq 3$ if $n \equiv 0(\text{mod } 2)$.

Proof. Let W_n be a wheel graph with vertex set $V(W_n) = \{A, x_1, x_2, x_3, \cdots, x_n\}$ and edge set $E(W_n) = \{Ax_i: 1 \leq i \leq n\} \cup \{x_ix_{\{i+1\}}: 1 \leq i \leq n - 1\} \cup \{x_{n-1}x_1\}$. The diameter of W_n is 2. Based on Remark 1, we have $rvac(W_n) \geq rvc(W_n) = diam(W_n) - 1 = 2 - 1 = 1$. Since the vertex A has degree of much greater than the others, it must have a different vertex weight than the others. The vertex weight of A is the sum of labels of edges which incident to A. From this condition, such that we have $rvac(W_n) \geq 2$. We divide into two cases to show the upper bound of the rainbow vertex antimagic connection number of W_n as follows.

Case 1. For $W_n, n \equiv 1(\text{mod } 2)$
To show the upper bound of $(W_n): n \equiv 1(\text{mod } 2)$, we construct the bijective function of edge labels.

$$g(x_i x_{i+1}) = \begin{cases} \frac{i+1}{2}, & \text{if } i \equiv 1(\text{mod } 2) \\ \left\lceil \frac{n}{2} \right\rceil + \frac{i}{2}, & \text{if } i \equiv 0(\text{mod } 2) \end{cases}$$
$$g(Ax_i) = 2n + 1 - i$$

From the edge labels above, we have the vertex weights in the following:

$$w(x_i) = 2n + 1 + \left\lceil \frac{n}{2} \right\rceil$$
$$w(A) = \frac{n}{2}(3n + 1)$$

From the vertex weights above, it is easy to see that the different weight is 2.

Case 2. For $W_n, n \equiv 0(\text{mod } 2)$
To show the upper bound of $rvac(W_n): n \equiv 0(\text{mod } 2)$, we construct the bijective function of edge labels.

$$g(x_i x_{i+1}) = \begin{cases} \frac{i+1}{2}, & \text{if } i \equiv 1(\text{mod } 2) \\ \left\lceil \frac{n}{2} \right\rceil + \frac{i}{2}, & \text{if } i \equiv 0(\text{mod } 2) \end{cases}$$
$$g(Ax_i) = 2n + 1 - i$$

From the edge labels above, we have the vertex weights in the following.

$$w(x_1) = 3n + 1$$
$$w(x_i) = 2n + 1 + \left\lceil \frac{n}{2} \right\rceil$$
$$w(A) = \frac{n}{2}(3n + 1)$$

From the vertex weight above, it is easy to see that the different weight is 3.

Furthermore, we show that every two different vertices of W_n is rainbow vertex antimagic coloring. Suppose that $x, y \in V(W_n)$, refer to the vertex weight the rainbow vertex $x - y$ path is shown in Table 2.

Table 2. The Rainbow Vertex of $x - y$ Path of W_n

Case	x	y	**Rainbow Vertex Coloring** $x - y$
1	x_i	A	x_i, A
2	x_i	x_i	x_i, A, x_i

Hence, the vertex coloring of W_n is rainbow vertex antimagic coloring. Thus, we obtain $rvac(W_n) = 2$ if $n \equiv 1(\text{mod } 2)$ and $2 \leq rvac(W_n) \leq 3$ if $n \equiv 0(\text{mod } 2)$. ■

Theorem 3
If $\mathcal{F}_n$ be a friendship graph of order $2n + 1$ and $n \geq 3$, then $rvac(\mathcal{F}_n) = 3$.

Proof. Let $\mathcal{F}_n$ be a friendship graph with vertex set $V(\mathcal{F}_n) = \{A\} \cup \{x_1, x_2, x_3, \dots, x_n\} \cup \{y_1, y_2, y_3, \dots, y_n\}$ and edge set $E(\mathcal{F}_n) = \{Ax_i; 1 \leq i \leq n\} \cup \{Ay_i; 1 \leq i \leq n\} \cup \{x_i y_i; 1 \leq i \leq n\}$. The diameter of $\mathcal{F}_n$ is 2. Based on Remark 1, we have $rvac(\mathcal{F}_n) \geq rvc(\mathcal{F}_n) = diam(\mathcal{F}_n) - 1 = 2 - 1 = 1$. Since the vertex A has degree of much greater than the others, it must have a different vertex weight than the others. The vertex weight of A is the sum of labels of edges which incident to A. In the other hand, the vertex x_i and y_i are adjacent, such that based on the edge labeling it can not receive the same weight. From this condition, such that we have $arvc(\mathcal{F}_n) \geq 3$. Furthermore, to show the upper bound we construct the bijective function of edge labels.

$$\begin{aligned} g(Ax_i) &= i & &: 1 \leq i \leq n \\ g(x_i y_i) &= 2n + 1 - i & &: 1 \leq i \leq n \\ g(Ay_i) &= 2n + i & &: 1 \leq i \leq n \end{aligned}$$

From the edge labels above, we have the vertex weights in the following.

$$\begin{aligned} w(x_i) &= 2n + 1 \\ w(y_i) &= 4n + 1 \\ w(A) &= 3n^2 + n \end{aligned}$$

From the vertex weight above, it is easy to see that the different weight is 3. Furthermore, we show that every two different vertices of $\mathcal{F}_n$is rainbow vertex antimagic coloring. Suppose that $x, y \in V(\mathcal{F}_n)$, refer to the vertex weight the rainbow vertex $x - y$ path is shown in Table 3.

Table 3. The Rainbow Vertex of $x - y$ Path of $\mathcal{F}_n$

Case	$\boldsymbol{x}$	$\boldsymbol{y}$	**Rainbow Vertex Coloring $\boldsymbol{x - y}$**
1	x_i	x_i	x_i, A, x_i
2	x_i	y_i	x_i, A, y_i
3	y_i	y_i	y_i, A, y_i
4	y_i	x_i	y_i, A, x_i

Hence, the vertex coloring of $\mathcal{F}_n$ is rainbow vertex antimagic coloring. Thus, we obtain $rvac(\mathcal{F}_n)$ is 3 . ■

Theorem 4
If F_n be a fan graph n+1 and $n \geq 3$, then $rvac(F_n) = 2$ if $n \equiv 1 (\text{mod } 2)$ and $2 \leq rvac(F_n) \leq 3$ if $n \equiv 0 (\text{mod } 2)$.

Proof. Let F_n be a fan graph with vertex set $V(F_n) = \{A, x_1, x_2, x_3, \cdots, x_n\}$ and edge set $E(F_n) = \{Ax_i: 1 \leq i \leq n\} \cup \{x_i x_{\{i+1\}}: 1 \leq i \leq n-1\}$. The diameter of F_n is 2. Based on Remark 1, we have $rvac(F_n) \geq rvc(F_n) = diam(F_n) - 1 = 2 - 1 = 1$. Since the vertex A has degree of much greater than the others, it must have a different vertex weight than the others. The vertex weight of A is the sum of labels of edges which incident to A. From this condition, such that we have $rvac(F_n) \geq 2$. We divide into two cases to show the upper bound of the antimagic rainbow connection number of F_n as follows.

Case 1. For $F_n, n \equiv 1 (\text{mod } 2)$
To show the upper bound of $rvac(F_n)$: $n \equiv 1 (\text{mod } 2)$, we construct the bijective function of edge labels.

$$g(x_i x_{i+1}) = \begin{cases} \dfrac{i}{2}, & \text{if } i \equiv 0 (\text{mod } 2) \\ \dfrac{n+i}{2}, & \text{if } i \equiv 1 (\text{mod } 2) \end{cases}$$

$$g(Ax_i) = \begin{cases} 2n-1, & \text{if } i = n \\ 2n-i-1, & \text{if } 1 \leq i \leq n-1 \end{cases}$$

From the edge labels above, we have the vertex weights in the following.

$$w(x_i) = \frac{5n-3}{2}$$

$$w(A) = \frac{3n^2-n}{2}$$

From the vertex weights above, it is easy to see that the different weight is 2.

Case 2. For $F_n, n \equiv 0(\text{mod } 2)$

To show the upper bound of $rvac(F_n)$: $n \equiv 0(\text{mod } 2)$, we construct the bijective function of edge labels.

$$g(x_i x_{i+1}) = \begin{cases} \frac{i}{2}, & \text{if } i \equiv 0(\text{mod } 2) \\ \frac{n+i-1}{2}, & \text{if } i \equiv 1(\text{mod } 2) \end{cases}$$

$$g(Ax_i) = \begin{cases} 2n-1, & \text{if } i = n \\ 2n-i-1, & \text{if } 1 \leq i \leq n-1 \end{cases}$$

From the edge labels above, we have the vertex weights in the following.

$$w(x_i) = \begin{cases} 3n-2, & \text{if } i = n \\ \frac{5n}{2}-2, & \text{if } 1 \leq i \leq n-1 \end{cases}$$

$$w(A) = \frac{3n^2-n}{2}$$

From the vertex weight above, it is easy to see that the different weight is 3.

Furthermore, we show that every two different vertices of F_n is rainbow vertex antimagic coloring. Suppose that$x, y \in V(F_n)$, refer to the vertex weight the rainbow vertex $x-y$ path is shown in Table 4.

Table 4. The Rainbow Vertex of $x-y$ Path of F_n

Case	x	y	Rainbow Vertex Coloring $x-y$
1	x_i	A	x_i, A
2	x_i	x_i	x_i, A, x_i

Hence, the vertex coloring of F_n is rainbow vertex antimagic coloring. Thus, we obtain $rvac(F_n) = 2$ if $n \equiv 1(\text{mod } 2)$ and $2 \leq rvac(F_n) \leq 3$ if $n \equiv 0(\text{mod } 2)$. ■

The illustration of antimagic rainbow edge labeling can be seen in Figure 1. Based on the Figure 1, we know that wheel graph W_{17} satisfy the rainbow vertex antimagic coloring and rainbow vertex antimagic connection number of W_{17} is 2.

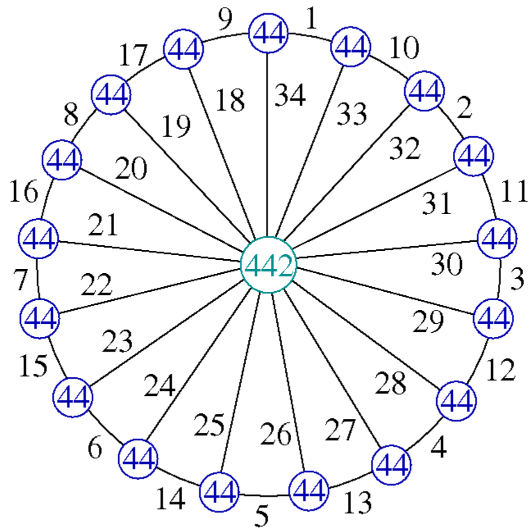

Figure 2. The Illustration Rainbow Vertex Antimagic Coloring of W_{17}

CONCLUSIONS

We have obtained the exact values of rainbow vertex antimagic connection number of some connected graphs, namely path (P_n), wheel (W_n), friendship ($\mathcal{F}_n$), and fan (F_n). However, since obtaining rainbow vertex antimagic connection number of graph is considered to be NP-complete problem, the characterization of the exact value of $arvc(G)$ for any family graph is still widely open. Therefore, we propose the following open problems as follows.

1. Determine the exact value of rainbow vertex antimagic connection number of graphs apart from those families.
2. Determine the exact value of rainbow vertex antimagic connection number of any operation graphs.

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