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MINIMUM DOMINATING SET FOR THE PRISM GRAPH FAMILY

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Abstract. The dominating set of the graph G is a subset D of vertex set V, such that every vertex not in $V - D$ is adjacent to at least one vertex in the vertex subset D. A dominating set D is a minimal dominating set if no proper subset of D is a dominating set. The number of elements in such set is called as domination number of graph and is denoted by $\gamma(G)$. In this work the domination numbers are obtained for family of prism graphs such as prism CL_n, antiprism Q_n and crossed prism R_n by identifying one of their minimum dominating set.

1. Introduction

Domination in graphs is a wide research area in graph theory. The dominating set of the graph G is a subset D of a vertex set V such that every vertex not in $V - D$ is adjacent to at least one vertex in the vertex subset D [6]. A dominating set D is a minimal dominating set if no proper subset of D is a dominating set. The number of elements in such a set is known as the domination number of graph [7]. The basic definition and details about the domination sets and domination numbers of graphs are discussed in [10]. Although the mathematical study of domination in graphs began around 1960, there are some references to domination-related problems about 100 years prior. In 1862, de Jaenisch attempted to determine the minimum number of queens required to cover an $n \times n$ chess board. With reference to [9] in many fields such as school bus routing, computer communication networks, radio stations, the locating radar station problem, modeling biological networks, facility location problems and coding theory, the domination is applied. In 2021, Adel et al. [1] have successfully used the Minimum Dominating Sets (MDSets) method to extract proteins that control Protein-Protein Interaction (PPI) networks, revealing a link between structural analysis and biological functions.

Motivation

Many real-time situations can be modeled as graphs. For instance, each floor in a multi-storey building can be modelled as an $n-$ prism graph [12] by considering the corners of the corridor positions in a floor to be vertices and the side brick walls on both sides as connecting edges. Also, the vertices located in similar at the corner regions of the hallway are connected to each other. This reflects the structure of prism graph. Consider the example of a $4-$prism graph obtained from the following floor plan of a rectangular multi-storey building.

To strengthen the security system of the building, surveillance cameras may be fixed at various positions. In order to minimize the cost of fixing cameras at various parts of the building, we need some leading positions to cover the whole area. Such dominating points can be visualized using graph-theoretical concept of domination. The minimum number of cameras needed to be installed, which is cost-effective, can be achieved through a minimally dominant set. To handle these types of situations,

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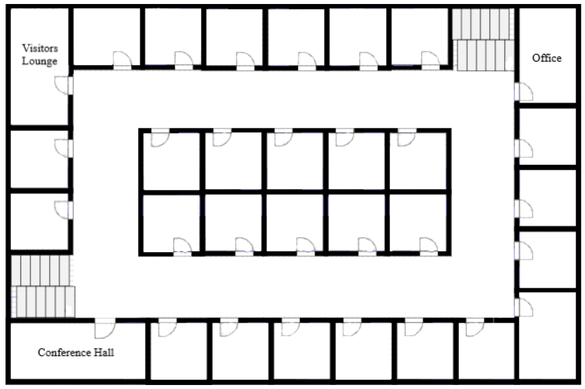


FIGURE 1. Floor plan of a multi-storey building

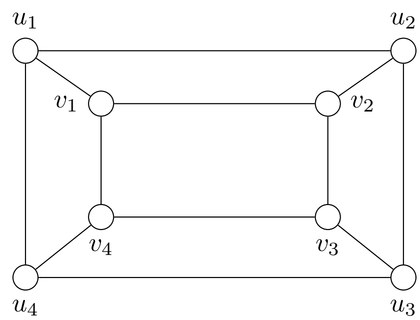


Figure 2: Prism Graph CL_4

domination in graphs can be applied. Consequently, it motivates us to study the minimal domination of the family of prism graphs.

2. Preliminaries

In this section, the literature review and some basic definitions related to our work are given. Here we consider only the graphs that are undirected and cycle-structured. Because of the richness in applications, huge collection of research works has been carried out in this area dealing with domination of graphs. Recently [4] researchers Behzad and et al. studied about an infinite family of regular graphs, the generalized Petersen graphs. The authors [15] in the year 2019 examined the domination of Fibonacci cubes and also they gave the pattern of minimum dominating sets of Fibonacci cubes and in the same year [1] Al-Harere and Breesam investigated the domination number of a new graph called as spinner graph. [3] Alvarado et al. explored about the domination of tree and established the bound for minimal dominating set of forests.

In 2021 [11] Liu, Chanjuan discussed about the upper and lower bounds of domination number for n maximal outer planar graph. [5] Bermudo and others proposed some lower bounds on the domination number of a catacondensed hexagonal system using the number of hexagons and the number of branching hexagons[13]. Sarah et al. found certain bounds for the domination number of Latin square graphs.

A *dominating set* [9] for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one number of D. The domination number $\gamma(G)$ is number of vertices in a smallest dominating set for G.

A *prism graph* [8], denoted by CL_n called also as circular ladder graph, is a graph corresponding to the skeleton of an n-prism graph had $2n$ vertices and $3n$ edges.

An *antiprism graph* [16] is a graph that has one of the antiprism as its skeleton. An n-sided antiprism graph has $2n$ vertices and $4n$ vertices. They are regular, polyhedral. An antiprism graph is a special case of a circulant graph $Ci_{2n}(1,2)$. Let us denote this graph by Q_n.

Let n be a positive even integer of at least 4. An *n-crossed prism graph* [14] is a graph obtained by taking two disjoint cycle graphs on n vertices, namely C_1^n and C_2^n, where $V(C_1^n) = \{x_1x_2,\dots,x_n\}$ and $V(C_2^n) = \{w_1,w_2,\dots,w_n\}$, such that $E(C_1^n) = (x_ix_{i+1}, x_1x_n)$ *where* $i = 1,2,\dots,n-1$ and $E(C_2^n) = (w_iw_{i+1}, w_1w_n)$ such that $i = 1,2,\dots,n-1$ adding some edges w_sx_{s+1} for $s \in \{1,3,\dots,n-1\}$ and w_tx_{t-1} for $t \in \{2,4,\dots,n\}$.

Due to many applications of domination in discrete mathematics, coding theory and networking, etc. It still remains an active research field for many decades. Also it helps in optimal identification of minimum nodes to cover the entire graph. Because of the cyclic structured nature of graphs in this work we have considered this family prism graphs and followed the method of mathematical induction to prove the bounds for domination number. In this paper the domination number for family of prism graphs namely prism graph, antiprism graph and crossed prism graph are obtained.

3. Computation of domination numbers for the prism graph family

Theorem 3.1. *The domination number of the prism graph CL_n, for $n \geq 4$, is*

1. $\gamma(CL_n) = \frac{n}{2}$, *if* $n \equiv 0$ *mod* 4
2. $\gamma(CL_n) = \lceil \frac{n}{2} \rceil$, *if* $n \equiv 1$ *mod* 4
3. $\gamma(CL_n) = \frac{n+2}{2}$, *if* $n \equiv 2$ *mod* 4
4. $\gamma(CL_n) = \lceil \frac{n}{2} \rceil$, *if* $n \equiv 3$ *mod* 4

Proof. Let CL_n be the prism graph with vertex set V and edge set E. The vertex set is given by $V = \{v_j\} \cup \{u_j\}$ where $j = 1,2,\dots,n$. The number of vertices v in CL_n is $|V| = 2n$ and the number of edges in CL_n is $|E| = 3n$. One of the minimum dominating set is identified from the following four cases on the various values of n.

Case 1. When $n \equiv 0$ *mod* 4 the minimum dominating set is given by

$$D_m = \{v_1, v_i\} \cup \{u_3, u_k\} \text{ where } i = 4p+1, k = 4p+3 \text{ and } p = 1,2,\dots,\frac{n-4}{4}.$$

We used mathematical induction method [17], to prove that given above set is one of the minimal dominating sets of prism graph when $n \equiv 0$ *mod* 4. Let $n = 4q$ and $q \geq 1$. When $q = 1$ it gives $n = 4$ and

$$D_1 = \{v_1, u_3\} = \{v_{n-3}, u_{n-1}\}$$

Induction Hypothesis on q: Assume that the result is true for the case $q = l$ and then prove that the result is true for $q = l+1$. By induction hypothesis the given set $D_l = \{v_1, v_i\} \cup \{u_3, u_k\}$ is the minimal dominating set. Therefore

$$p = 1,2,\dots,l-1,\ i = 5,9,\dots,4l-3 \text{ and } k = 7,11,\dots,4l-1.$$

$$\implies D_l = \{v_1, v_5, v_9, \dots, v_{4l-3}\} \cup \{u_3, \dots, u_{4l-1}\} \text{ is true.}$$

To prove: When $q = l+1$ the result is true. Let $q = l+1 \implies n = 4l+4$

$$\implies D_{l+1} = \{v_1, v_5, \dots, v_{4l+1}\} \cup \{u_3, \dots, u_{4l+3}\}.$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_5, v_9, \dots, v_{4l-3}\} \cup \{u_3, \dots, u_{4l-1}\} \cup \{v_1, u_3\} \cup \{v_{4l+1}, u_{4l+3}\}.$$

This proves the result is true for $q = l + 1$.

$$\implies |D_m| = \frac{n}{2}$$

Therefore $\gamma(CL_n) = \frac{n}{2}$.

Case 2. If $n \equiv 1 \mod 4$ then

$$D_m = \{v_1, v_i, v_n\} \cup \{u_3, u_k\} \text{where } i = 4p + 1,\ k = 4p + 3 \text{ and } p = 1, 2, \ldots, \frac{n-5}{4}.$$

Let $n = 4q + 1$ and $q \geq 1$. Assume $q = 1 \implies n = 5$

$$\implies D_1 = \{v_1, v_5, u_3\} = \{v_1, v_n, u_{n-2}\}$$

Induction Hypothesis on q: Assume that the result is true for the case when $q = l$ and then prove that the result is true for $q = l + 1$. By induction hypothesis the given set $D_l = \{v_1, v_i, v_n\} \cup \{u_3, u_k\}$ is the minimal dominating set. We have $p = 1, 2, \ldots, l - 1,\ i = 5, 9, \ldots, 4l - 3$ and $k = 7, 11, \ldots, 4l - 1$

$$\implies D_l = \{v_1, v_5, v_9, \ldots, v_{4l-3}, v_{4l+1}\} \cup \{u_3, \ldots, u_{4l-1}\} \text{ is true.}$$

To prove that when $q = l + 1$ the result is true. Let $q = l + 1 \implies n = 4l + 5$

$$\implies D_{l+1} = \{v_1, v_5, \ldots, v_{4l+1}\} \cup \{u_3, \ldots, u_{4l+3}\}.$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_5, v_9, \ldots, v_{4l-3}\} \cup \{u_3, \ldots, u_{4l-1}\} \cup \{v_1, v_{4l+1}, u_{4l+3}\}$$

This proves that the result is true for $q = l + 1$.

$$\implies |D_m| = \lceil \frac{n}{2} \rceil.$$

Hence $\gamma(CL_n) = \lceil \frac{n}{2} \rceil$.

Case 3. If $n \equiv 2 \mod 4$ then

$$D_m = \{v_1, v_i\} \cup \{u_2, u_k\} \text{ where } i = 4p,\ k = 4p + 2 \text{ and } p = 1, 2, \ldots, \frac{n-2}{4}.$$

Let $n = 4q+2$ and $q \geq 1$. When $q = 1 \implies n = 6$, $D_1 = \{v_1, v_4, u_2, u_6\} = \{v_1, v_{n-2}, u_2, u_n\}$. See Fig.2. Induction Hypothesis on q: Assume that the result is true for the case when $q = l$ and we need to prove that the result is true for $q = l + 1$. By induction hypothesis the given set $D_l = \{v_1, v_i\} \cup \{u_2, u_k\}$ is the minimal dominating set. Therefore $p = 1, 2, \ldots, l,\ i = 4, 8, \ldots, 4l$ and $k = 6, 12, \ldots, 4l + 2$.

$$\implies D_l = \{v_1, v_4, v_8, \ldots, v_{4l}\} \cup \{u_2, \ldots, u_{4l-2}, u_{4l+2}\} \text{ is true.}$$

To prove: The result is true for $q = l + 1$. When $q = l + 1 \implies n = 4l + 6$

$$\implies D_{l+1} = \{v_1, v_4, \ldots, v_{4l+4}\} \cup \{u_2, \ldots, u_{4l+6}\}.$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_4, v_8, \ldots, v_{4l}\} \cup \{u_2, \ldots, u_{4l-2}, u_{4l+2}\} \cup \{v_1, v_{4l+4}, u_{4l+6}\}$$

This proves that the result is true for $q = l + 1$.

$$\implies |D_m| = \frac{n+2}{2}$$

Therefore, it is obtained that

$$\gamma(CL_n) = \frac{n+2}{2}.$$

Case 4. If $n \equiv 3 \mod 4$ then

$$D_m = \{v_1, v_i\} \cup \{u_3, u_k\} \text{ where } i = 4p + 1,\ k = 4p + 3 \text{ and } p = 1, 2, \ldots, \frac{n-3}{4}.$$

Let $n = 4q + 3$ and $q \geq 1$. When $q = 1 \implies n = 7 \implies D_1 = \{v_1, v_5, u_3, u_7\} = \{v_1, v_{n-2}, u_3, u_n\}$. Induction Hypothesis on q: Assume that the result is true for the case when $q = l$ and we need to prove that the result is true for $q = l+1$. By induction hypothesis the given set $D_l = \{v_1, v_i\} \cup \{u_3, u_k\}$ is the minimal dominating set where, $n = 4l+3,\ p = 1, 2, \ldots, l,\ i = 5, 9, \ldots, 4l+1$ and $k = 7, 11, \ldots, 4l+3$

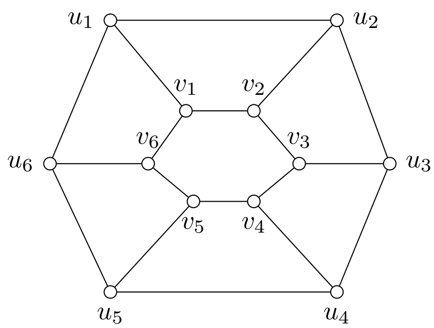
$$\implies D_l = \{v_1, v_5, v_9, \ldots, v_{4l-3}, v_{4l+1}\} \cup \{u_3, \ldots, u_{4l-1}, u_{4l+3}\} \text{ is true.}$$

To prove that the result is true when $q = l+1$. Let $q = l+1 \implies n = 4l+7$

$$\implies D_{l+1} = \{v_1, v_5, \ldots, v_{4l+5}\} \cup \{u_3, \ldots, u_{4l+7}\}.$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_5, v_9, \ldots, v_{4l-3}, v_{4l+1}\} \cup \{u_3, \ldots, u_{4l-1}, u_{4l+3}\} \cup \{v_1, v_{4l+5}, u_3, u_{4l+7}\}.$$

This proves that the result is true for $q = l+1$.
Hence $|D_m| = \lceil \frac{n}{2} \rceil \implies \gamma(CL_n) = \lceil \frac{n}{2} \rceil$. □



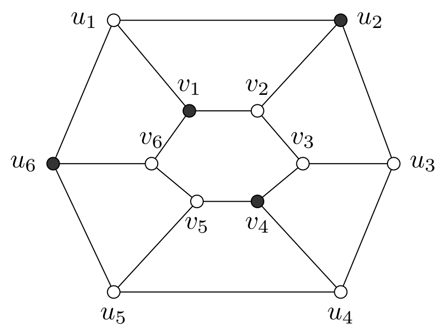


Figure 1: Prism Graph CL_6 Figure 2: Dominating vertices of CL_6

Remark:1 The domination numbers of some prism graphs CL_n of all the cases discussed are summarized in Table 1.

n	V	E	γ	n	V	E	γ	n	V	E	γ
11	22	33	6	16	32	48	8	21	42	63	11
12	24	36	6	17	34	51	9	22	44	66	12
13	26	39	7	18	36	54	10	23	46	69	12
14	28	42	8	19	38	57	10	24	48	72	12
15	30	45	8	20	40	60	10	25	50	75	13

Table:1 Domination numbers (γ) of prism graphs CL_n with vertices V and edges E

Theorem 3.2. *The domination number of the antiprism graph Q_n, where $n \geq 3$, is $\gamma(Q_n) = \lceil \frac{2n}{5} \rceil$.*

Proof. Let Q_n be the antiprism graph with vertex set V and edge set E. The number of vertices in $Q_n, |V| = 2n$ given by $V = \{v_j\} \cup \{u_j\}$ where $j = 1, 2, \ldots, n$ and the number of edges $|E| = 4n$. See Fig.3.

Case 1. If $n \equiv 0 \mod 5$ then

$$D_m = \{v_1, v_i\} \cup \{u_3, u_k\}$$

where $i = 5p+1,\ k = 5p+3$ and $p = 1, 2, 3, \ldots, \frac{n-5}{5}$.

The induction method is taken to prove that the above set is one of the minimal dominating set. Let $n = 5q$ and $q \geq 1$. When $q = 1 \implies n = 5 \implies D_1 = \{v_1, u_3\} = \{v_{n-4}, u_{n-2}\}$. See Fig.4. Induction Hypothesis on q: Assume that the result is true for the case when $q = l$ and then prove that the result is true when $q = l+1$. By induction hypothesis the given set $D_l = \{v_1, v_i\} \cup \{u_3, u_k\}$ is the minimal dominating set. Therefore $p = 1, 2, \ldots, l-1,\ i = 6, 11, \ldots, 5l-4$ and $k = 8, 13, \ldots, 5l-2$.

$$\implies D_l = \{v_1, v_6, v_{11}, \ldots, v_{5l-4}\} \cup \{u_3, \ldots, u_{5l-2}\} \text{ is true.}$$

We need to prove that when $q = l+1$ the result is true. Let $q = l+1 \implies n = 5l+5$

$$\implies D_{l+1} = \{v_1, v_6, \ldots, v_{5l+1}\} \cup \{u_3, \ldots, u_{5l+3}\}.$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_6, \ldots, v_{5l-4}\} \cup \{u_3, \ldots, u_{5l-2}\} \cup \{v_{5l+1}, u_{5l+3}\}$$

This prove that the result is true for $q = l+1$.

$$\implies |D_m| = \lceil \frac{2n}{5} \rceil$$

Hence $\gamma(Q_n) = \lceil \frac{2n}{5} \rceil$.

Case 2. If $n \equiv 1 \mod 5$ then

$$D_m = \{v_1, v_i, v_{n-1}\} \cup \{u_3, u_k\}$$

where $i = 5p+1,\ k = 5p+3$ and $p = 1, 2, 3, \ldots, \frac{n-6}{5}$ is one of the minimal dominating set. By induction method, let $n = 5q+1$ and $q \geq 1$. When $q = 1 \implies n = 6,\ D_1 = \{v_1, v_6, u_3\} = \{v_1, v_n, u_{n-4}\}$. Induction Hypothesis on q: Assume that the result is true for the case $q = l$ and then prove that the result is true for $q = l+1$. By induction hypothesis the given set $D_l = \{v_1, v_i, v_n\} \cup \{u_3, u_k\}$ is the minimal dominating set. Therefore $p = 1, 2, \ldots, l-1,\ i = 6, 11, \ldots, 5l-4$ and $k = 8, 13, \ldots, 5l-2$.

$$\implies D_l = \{v_1, v_6, v_{11}, \ldots, v_{5l-4}, v_{5l+1}\} \cup \{u_3, \ldots, u_{5l-2}\} \text{ is true.}$$

To prove: when $q = l+1$ the result is true. Let $q = l+1 \implies n = 5l+6$

$$\implies D_{l+1} = \{v_1, v_6, \ldots, v_{5l+6}\} \cup \{u_3, \ldots, u_{5l+3}\}$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_6, \ldots, v_{5l-4}, v_{5l+1}\} \cup \{u_3, \ldots, u_{5l-2}\} \cup \{v_{5l+6}, u_{5l+3}\}$$

Hence the result is true when $q = l+1$.

$$\implies |D_m| = \lceil \frac{2n}{5} \rceil$$

Hence $\gamma(Q_n) = \lceil \frac{2n}{5} \rceil$.

Case 3. If $n \equiv 2 \mod 5$ then

$$D_m = \{v_1, v_i, v_{n-1}\} \cup \{u_3, u_k\}$$

where $i = 5p+1,\ k = 5p+3$ and $p = 1, 2, 3, \ldots, \frac{n-7}{5}$.

Prove by induction that the above set is one of the minimal dominating sets. Let $n = 5q+2$ and $q \geq 1$. When $q = 1 \implies n = 7$ and $D_1 = \{v_1, v_6, u_3\} = \{v_1, v_{n-1}, u_{n-4}\}$. Induction Hypothesis on q: Assume that the result is true when $q = l$ and then prove that the result is true when $q = l+1$. By induction hypothesis the given set $D_l = \{v_1, v_i, v_{n-1}\} \cup \{u_3, u_k\}$ is the minimal dominating set. So that $p = 1, 2, \ldots, l-1,\ i = 6, 11, \ldots, 5l-4$ and $k = 8, 13, \ldots, 5l-2$.

$$\implies D_l = \{v_1, v_6, v_{11}, \ldots, v_{5l-4}, v_{5l+1}\} \cup \{u_3, \ldots, u_{5l-2}\} \text{ is true.}$$

To prove that if $q = l+1$ then the result is true. Let $q = l+1 \implies n = 5l+7$.

$$\implies D_{l+1} = \{v_1, v_6, \ldots, v_{5l+6}\} \cup \{u_3, \ldots, u_{5l+3}\}$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_6, v_{11}, \ldots, v_{5l-4}, v_{5l+1}\} \cup \{u_3, \ldots, u_{5l-2}\} \cup \{v_{5l+6}, u_{5l+3}\}$$

Hence result is true for $q = l+1$.

$$\implies |D_m| = \lceil \frac{2n}{5} \rceil$$

Thus $\gamma(Q_n) = \lceil \frac{2n}{5} \rceil$.

Case 4. If $n \equiv 3 \mod 5$ then

$$D_m = \{v_1, v_i\} \cup \{u_3, u_k\}$$

where $i = 5p+1,\ k = 5p+3$ and $p = 1,2,3,\dots,\frac{n-3}{5}$. Prove by induction that the above set is one of the minimal dominating sets. Let $n = 5q+3$ and $q \geq 0$. Assume $q = 0 \implies n = 3$

$$\implies D_1 = \{v_1, u_3\} = \{v_{n-2}, u_n\}$$

Induction Hypothesis on q: Assume that the result is true for the case when $q = l$ and then prove that the result is true for $q = l+1$. By induction hypothesis $D_l = \{v_1, v_i\} \cup \{u_3, u_k\}$ is the minimal dominating set. We have $p = 1,2,\dots,l,\ i = 6,11,\dots,5l-4,5l+1$ and $k = 8,13,\dots,5l-2,5l+3$. $\implies D_l = \{v_1, v_6, v_{11},\dots,v_{5l+1}\} \cup \{u_3,\dots,u_{5l+3}\}$ is true. To prove that when $q = l+1$ the result is true. Let $q = l+1 \implies n = 5l+8$.

$$\implies D_{l+1} = \{v_1, v_6,\dots,v_{5l+6}\} \cup \{u_3,\dots,u_{5l+8}\}$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_6,\dots,v_{5l+1}\} \cup \{u_3,\dots,u_{5l+3}\} \cup \{v_{5l+6}, u_{5l+8}\}$$

This implies the result is true when $q = l+1$.

$$\implies |D_m| = \lceil \frac{2n}{5} \rceil$$

Therefore $\gamma(Q_n) = \lceil \frac{2n}{5} \rceil$.

Case 5. If $n \equiv 4 \mod 5$ then the minimum dominating set

$$D_m = \{v_1, v_i\} \cup \{u_3, u_k\}$$

where $i = 5p+1,\ k = 5p+3$ and $p = 1,2,3,\dots,\frac{n-4}{5}$. Prove by induction that the above set is one of the minimal dominating sets. Let $n = 5q+4$ and $q \geq 0$. Assume the case $q = 0 \implies n = 4$

$$\implies D_1 = \{v_1, u_3\} = \{v_{n-3}, u_{n-1}\}$$

Induction Hypothesis on q: Assume that the result is true for $q = l$ and then prove that the result is true when $q = l+1$. By induction hypothesis the given set $D_l = \{v_1, v_i\} \cup \{u_3, u_k\}$ is the minimal dominating set. Here

$$p = 1,2,\dots,l,\ i = 6,11,\dots,5l-4,5l+1 \text{ and } k = 8,13,\dots,5l-2,5l+3$$

$$\implies D_l = \{v_1, v_6, v_{11},\dots,v_{5l+1}\} \cup \{u_3,\dots,u_{5l+3}\} \text{ is true.}$$

We have to prove that when $q = l+1$ the result is true. Let $q = l+1 \implies n = 5l+9$

$$\implies D_{l+1} = \{v_1, v_6,\dots,v_{5l+6}\} \cup \{u_3,\dots,u_{5l+8}\}.$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_6, v_{11},\dots,v_{5l+1}\} \cup \{u_3,\dots,u_{5l+3}\} \cup \{v_{5l+6}, u_{5l+8}\}$$

This prove that the result is true when $q = l+1$.

$$\implies |D_m| = \lceil \frac{2n}{5} \rceil$$

Thus $\gamma(Q_n) = \lceil \frac{2n}{5} \rceil$. □

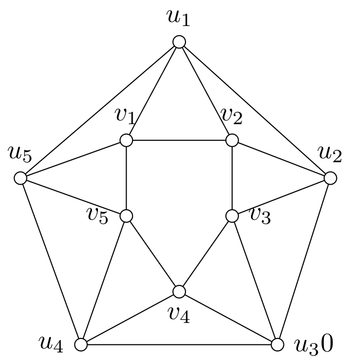


Figure 3: Antiprism Graph Q_5

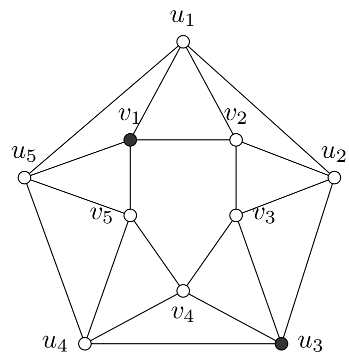


Figure 4: Dominating vertices of Q_5

Remark 2: Table 2 summarises the domination numbers (γ) of few antiprism graphs Q_n with vertices V and edges E.

n	V	E	γ	n	V	E	γ	n	V	E	γ
11	22	44	5	16	32	64	7	21	42	84	9
12	24	48	5	17	34	68	7	22	44	88	9
13	26	52	6	18	36	72	8	23	46	92	10
14	28	56	6	19	38	76	8	24	48	96	10
15	30	60	6	20	40	80	8	25	50	100	10

Table 2: Domination numbers (γ) of crossed prism graph Q_n

Theorem 3.3. *The domination number of the crossed prism graph R_n, where $n \geq 4$ is*

(1) $\gamma(R_n) = \frac{n}{2}$, *if* $n \equiv 0 \mod 4$
(2) $\gamma(R_n) = \frac{n+2}{2}$, *if* $n \equiv 2 \mod 4$.

Proof. Let R_n be the crossed prism graph with vertex set V and edge set E. The number of vertices in R_n is $|V| = 2n$ and the number of edges in R_n, $|E| = 3n$. See Fig.5.

For $n \geq 4$ the form of minimum dominating set is

Case 1. If $n \equiv 0 \mod 4$ then

$$D_m = \{v_1, v_i\} \cup \{u_4, u_k\} \text{ where } i = 4p+1,\ k = 4p+4 \text{ and } p = 1, 2, \ldots, \frac{n-4}{4}.$$

Prove by induction that the above set is one of the minimal dominating sets. Let $n = 4q$ and $q \geq 1$. When $q = 1 \implies n = 4 \implies D_1 = \{v_1, u_4\} = \{v_{n-3}, u_n\}$. Induction Hypothesis on q: Assume that the result is true when $q = l$ and then prove that the result is true for $q = l+1$. By induction hypothesis $D_l = \{v_1, v_i\} \cup \{u_4, u_k\}$ is the minimal dominating set, therefore

$$p = 1, 2, \ldots, l-1,\ i = 5, 9, \ldots, 4l-3 \text{ and } k = 4, 8, \ldots, 4l.$$

$$\implies D_l = \{v_1, v_5, v_9, \ldots, v_{4l-3}\} \cup \{u_4, \ldots, u_{4l-4}\} \text{ is true.}$$

We have to prove that when $q = l+1$ the result is true. Let $q = l+1 \implies n = 4l+4$

$$\implies D_{l+1} = \{v_1, v_5, \ldots, v_{4l+1}\} \cup \{u_3, \ldots, u_{4l+4}\}$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_5, v_9, \ldots, v_{4l-3}\} \cup \{u_4, \ldots, u_{4l}\} \cup \{v_{4l+1}, u_{4l+4}\}$$

This proves that the result is true for $q = l + 1$.

$$\text{Hence } |D_m| = \frac{n}{2} \implies \gamma(R_n) = \frac{n}{2}.$$

Case 2. If $n \equiv 2 \mod 4$ then the one of the minimum dominating set

$$D_m = \{v_1, v_i\} \cup \{u_1, u_k\} \text{ where } i = 4p,\ k = 4p + 1 \text{ and } p = 1, 2, \ldots, \frac{n-2}{4}.$$

The proof is given by mathematical induction method. Let $n = 4q + 2$ and $q \geq 1$. Assume that $q = 1 \implies n = 6$ and $D_1 = \{v_1, v_4, u_1, u_5\} = \{v_1, v_{n-2}, u_1, u_{n-1}\}$. See Fig.6. Induction Hypothesis on q: Assume that the result is true for the case, when $q = l$ and then prove that the result is true for $q = l + 1$. By induction hypothesis $D_l = \{v_1, v_i\} \cup \{u_4, u_k\}$ is the minimal dominating set. Therefore $p = 1, 2, \ldots, l,\ i = 4, 8, \ldots, 4l$ and $k = 5, 9, \ldots, 4l + 1$.

$$\implies D_l = \{v_1, v_4, v_8, \ldots, v_{4l}\} \cup \{u_5, \ldots, u_{4l+1}\} \text{ is true.}$$

To prove: When $q = l + 1$ the result is true. Let $q = l + 1 \implies n = 4l + 6$

$$\implies D_{l+1} = \{v_1, v_5, \ldots, v_{4l+4}\} \cup \{u_3, \ldots, u_{4l+5}\}.$$

$$D_{l+1} = D_l \cup D_1 = \{v_1, v_5, v_9, \ldots, v_{4l}\} \cup \{u_4, \ldots, u_{4l+1}\} \cup \{v_{4l+4}, u_{4l+5}\}$$

This proves that the result is true when $q = l + 1$.

$$\implies |D_m| = \frac{n+2}{2}.$$

Thus $\gamma(R_n) = \dfrac{n+2}{2}$.

□

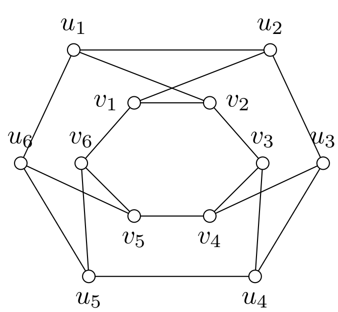


Figure 5: Crossed Prism Graph R_6

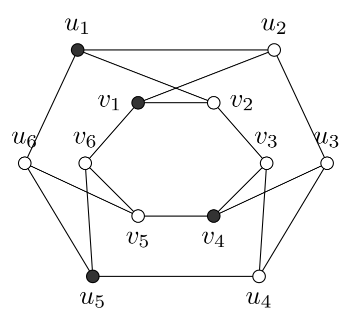


Figure 6: Dominating vertices of R_6

Remark:3 The domination numbers of various crossed prism graphs R_n for some of the cases $n \equiv 0 \mod 4$ and $n \equiv 2 \mod 4$ are summarized in Table 3.

n	V	E	γ	n	V	E	γ	n	V	E	γ
10	20	30	6	20	40	60	10	30	60	90	16
12	24	36	6	22	44	66	12	32	64	96	16
14	28	42	8	24	48	72	12	34	68	102	18
16	32	48	8	26	52	78	14	36	72	108	18
18	36	54	10	28	56	84	14	38	76	114	20

Table 3: Domination number (γ) for crossed prism graph R_n with vertices V and edges E

4. Conclusion

The idea of computing bounds of domination number for graphs remains to be an active area of research for decades. This led to the focus of investigating the domination number of the family of prism graphs. In this work we have generalized the minimum dominating set for each case of considered graphs and proved using mathematical induction method. The upper bound of domination numbers for prism, antiprism and crossed prism graphs are determined. Future work could establish bounds for domination in various structured graphs that can be explored.

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